Research Interests

The dominant motivation of my research has been threefold: (i) to develop general *mathematical methods* for treating complex phenomena occurring in the real world; (ii) to employ the developed mathematical techniques for constructing *physical theories* and formulating principal concepts; (iii) to apply these approaches for describing concrete *nonlinear* and *collective phenomena*. The main results are listed below.

1. **Optimized Perturbation Theory** is advanced for treating problems without small parameters. The optimization is accomplished by introducing *control functions* whose role is to govern the fastest convergence of approximation sequences. The idea of the approach is based on combining the techniques of perturbation theory and optimal control theory. The effectiveness of the method has been demonstrated by numerous applications.

2. **Self-Similar Approximation Theory** is developed for finding accurate approximate solutions of complicated equations. The principal original idea of the approach is to present the passage from one successive approximation to another as the *evolution in approximation space*, with the approximation order playing the role of time. The realization of this idea employs the methods of renormalization-group theory, dynamical-system theory, and optimal control theory. Novel types of approximants are derived: *self-similar factor approximants*, *self-similar root approximants*, *self-similar nested approximants*, *self-similar additive approximants*, and *self-similar exponential approximants*. The theory is demonstrated to be a powerful tool for solving various problems in physics and applied mathematics, being more general and accurate than other approximation techniques.

3. **Method of Self-Similar Prediction** is suggested for analysing and forecasting the behaviour of time series generated by complex systems. The underlying idea of the analysis is the *law of self-similar evolution*. Forecasting presents a spectrum of admissible scenarios weighted with their probabilities. The effectiveness of the method is illustrated for several economic and financial time series containing booms and crashes. A method for predicting the fracture of loaded materials is advanced.

4. **Correlated Iteration Theory** for Green functions of strongly interacting and strongly correlated statistical systems is developed. The theory takes account of particle correlations at the initial step of the procedure, thus having no divergences even for nonintegrable interaction potentials. At the same time it allows for a systematic derivation of any higher-order corrections.

5. **Theory of Heterophase Fluctuations** in statistical systems is elaborated, which takes into account mesoscopic fluctuations of order parameters. The notion of *heterophase turbulence* is introduced. *Theory of melting and crystallization* is developed. Metastable states are described. A model of *turbulent crystal* is suggested. *Anomalous precursor phenomena*, observed around many phase transitions, are ex-

plained by the existence of mesoscopic heterophase fluctuations. The theory is applied to crystals, glasses, magnets, ferroelectrics, superconductors, cold atoms, macromolecules, and other complex systems.

6. **Model of Locally-Disordered Crystals** is proposed, where in a matrix of localized particles there appear nanosize regions of disorder with delocalized atoms. The experimentally observed specific-heat anomaly in solid ^{3}He is explained. The possibility of Bose condensation in solid ^{4}He is demonstrated. Statistical model of a superfluid solid is suggested.

7. **Concept of Stochastic Instability** of quasi-isolated systems is advanced. The basic principle is *nonexistence of isolated systems*, according to which no real system can be completely isolated from its surrounding. It is shown that infinitesimally weak random perturbations can qualitatively change the system properties. In particular, quasi-isolated systems can display *absence of absolute equilibrium*. Stochastic instability of quasi-isolated systems also results in *irreversibility of time arrow*.

8. **Method of Thermodynamic Quasiaverages** is formulated based on the introduction of infinitesimal sources disappearing in the thermodynamic limit. The possibility of obtaining pure as well as mixed statistical states after the sole thermodynamic limit is what distinguishes this technique from the Bogolubov method of quasiaverages, permitting one to obtain only pure states and requiring two limiting procedures.

9. **Operator Order Indices** are introduced, relating the norm and trace of a given operator. The indices allow for a detailed classification of all types of order, off-diagonal as well as diagonal, long-range as well as mid-range. This notion is perfectly defined when order parameters do not exist, e.g. for mid-range order and for finite systems.

10. **Theory of Clustering Matter** is elaborated being based on three pivotal principles: *cluster representation* provides the mathematical foundation for the approach and introduces the notion of cluster probabilities. The concept of *statistical correctness* allows a correct construction of effective Hamiltonians. And the principle of *potential scaling* specifies the interaction potentials for various clusters.

11. **Mixed Quark-Hadron Matter** is described. The appearance of multiquark clusters in nuclear matter is explained. The possibility of the dibaryon Bose condensation is advanced. Provisions for nuclear-matter lasers are estimated. Calculated thermodynamic characteristics are in good agreement with lattice simulations. Deconfinement at finite baryon density, under conditions typical of heavy-ion collisions, is predicted to be a gradual crossover.

12. **Theory of Bose Systems** with global gauge symmetry breaking is elaborated, being completely self-consistent, conserving and gapless in any approximation. This resolves the Hohenberg-Martin dilemma of conserving versus gapless approaches for Bose-condensed systems. *Notion of representative ensembles* is formulated for arbitrary

statistical systems, equilibrium as well as nonequilibrium. The basic idea is to take into account all imposed constraints that are necessary for the unique representation of the considered statistical system. *Theory of disordered superfluids* is worked out for Bose-condensed systems in random potentials. The theory is applicable to strongly interacting particles and arbitrarily strong disorder, being in good agreement with Monte Carlo simulations.

13. **Theorem on Fluctuation of Observables** is proved according to which the variance of a global observable, being a sum of several terms, is thermodynamically normal if and only if all partial variances of its terms are normal, and it is thermodynamically anomalous if and only if at least one of the partial variances is anomalous. The theorem rules out the existence of anomalous fluctuations in stable equilibrium systems. Particle fluctuations in equilibrium Bose-condensed systems are shown to be thermodynamically normal.

14. **Finite Momentum Condensation** in strongly interacting Bose liquids is described. This is realized as Bose-Einstein condensation into a single-particle state with a finite modulus of momentum, but not with zero momentum as in the standard case. The system demonstrates mid-range order.

15. **Scale Separation Approach** is elaborated for treating nonlinear phenomena in strongly nonequilibrium systems. The key techniques are: *Randomization of local fields* makes it possible to decouple correlation functions with the allowance for local and quantum effects. *Classification of relative quasi-invariants* extends the notion of adiabatic invariants to stochastic and partial differential equations. *Method of stochastic averaging* for such equations generalizes the multiscale averaging techniques. The approach is illustrated by a number of nonequilibrium problems dealing with the interaction of radiation with matter.

16. **Probabilistic Pattern Selection** for nonequilibrium phenomena is advanced, when nonlinear evolution equations possess multiple solutions corresponding to different spatio-temporal structures. The approach is based on ascribing to the manifold of these solutions a probability distribution of patterns. The most probable pattern corresponds to the *minimal expansion rate*.

17. **Collective Liberation of Light** is predicted to occur for an ensemble of resonant atoms with transition frequencies inside a prohibited photonic bandgap, when spontaneous emission of a single atom is suppressed. Anyway, emission becomes admissible due to collective cohereht effects.

18. **Turbulent Photon Filamentation** in resonant media and in large aperture lasers is elucidated. The theory defines the number of filaments, the distribution of their radii, and their flashing dynamics at all stages. Theoretical predictions have been confirmed by experiments.

19. **Theory of Spin Superradiance** in strongly nonequilibrium spin systems with resonator feedback fields is developed. The theory, based on microscopic Hamiltonians,

is applied to polarized nuclei, magnetic nanomolecules, ferromagnets, magnetic nanoclusters, magnetic graphene, dipolar and spinor lattices. The origins of self-organized spin superradiance are elucidated. Various relaxation regimes, both in the presence of as well as without pumping, are studied. Applications are suggested for the investigation of materials characteristics, fast repolarization of targets, construction of spin masers, creation of sensitive detectors, memory devices, and methods of information processing. Similar effects are shown to be affordable for ferroelectrics.

20. **Magnetic Semiconfinement of Atoms** is proposed, providing a mechanism for the formation of well-collimated beams from particle emitters and atom lasers.

21. **Topological Coherent Modes** of trapped Bose atoms are introduced representing *nonground-state condensates.* The theory of resonant formation of these modes is developed. This new type of matter displays several interesting effects: interference fringes, interference current, mode locking, dynamic phase transitions, critical phenomena, chaotic motion, harmonic generation, parametric conversion, atomic squeezing, entanglement production, and Ramsey fringes. The generation of coherent modes is shown to be an effective mechanism for creating quantum turbulence of trapped superfluids, triggered by counterflow instability.

22. **Inverse Kibble-Zurek scenario** is advanced, when a system with broken symmetry, being subject to external perturbations, passes through the stages of weak nonequilibrium, vortex germs, vortex rings, vortex lines, deformed vortices, vortex turbulence, grain turbulence, to wave turbulence, where the symmetry is restored. This inverse way is demonstrated by numerical simulations and confirmed by experiments with trapped Bose-condensed atoms.

23. **Classification of Nonequilibrium States** by effective temperature, Fresnel and Mach numbers is suggested. The classification is illustrated by strongly nonequilibrium trapped Bose-Einstein condensates. The results of numerical simulations are in good agreement with experiments.

24. **Negative Electric Current** in semiconductors and semimetals is shown to be feasible, when electric current turns against an applied voltage. This transient effect is due to a nonuniform distribution of charge carriers. The possibility of using this effect for measuring the mean free paths of charged particles and ions is suggested.

25. **Resonance Symmetry Breaking** is described demonstrating that resonance phenomena can be treated as nonequilibrium phase transitions accompanied by spontaneous symmetry breaking and characterized by order parameters. The concept is illustrated by helicon resonance, spin-wave resonance, and spin-reversal resonance.

26. **Measure of Entanglement Production** for arbitrary operators is introduced. This general measure can be applied for any systems, pure or mixed, binary or multiparticle, equilibrium or nonequilibrium. The notion of *evolutional entanglement production* is advanced.

27. **Generalized Adiabatic Theorem** is proved for linear as well as nonlinear

Hamiltonians. A simple and general sufficient condition for the validity of adiabatic approximation is derived, which is valid for arbitrary spectra and any kind of time variation. It is shown that in some cases the found condition is necessary and sufficient.

28. **Functional Carrying Capacity** is shown to define a novel class of dynamical models describing the evolution of complex biological and social systems, such as *nonlinear financial markets*, *symbiosis of biological species*, and *punctuated evolution of populations*.

29. **Utility Rate Equations** are formulated characterizing the coexisting dynamics of structured biological or social systems. The equation parameters are determined by the utility of a trait group for the society as a whole and by the group mutual utilities. The stationary solutions of the equations define the optimal stable fractions of society groups. The approach makes it posssible to define the distance of societies from stable states.

30. **Affective Decision Theory** is developed quantitatively taking into account emotions in the process of decision making. The theory contains no paradoxes plaguing the classical decision making. A scheme of thinking quantum systems is suggested. The new approach for describing quantum intelligence networks is advanced. The theory can be used for information processing and for the creation of affective artificial intelligence.