

The 42nd International Symposium on Lattice Field Theory (Lattice 2025)


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Seminar of department “Theory of Fundamental Interactions”
BLTP, JINR, 27 November 2025



PART 1



42nd International Symposium on Lattice Field Theory (Lattice 2025)

Nov 2–8, 2025
TIFR Mumbai
Asia/Kolkata timezone


- Overview
- Logistical Information
- Arrival information for foreign nationals
- Travel Information
- Venue
- Timetable
- Poster Session
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- Code of Conduct
- Important Dates
- Accommodation
 - Hotels
 - Instructions for Hotel Booking
- Registration

The 42nd International Symposium on Lattice Field Theory (Lattice 2025) will be held at the [Tata Institute of Fundamental Research \(TIFR\)](#), [Mumbai](#), India, November 2-8, 2025. This annual conference brings together scientists from around the world who specialize in the numerical evaluation of quantum field theories. The conference primarily focuses on the latest theoretical and algorithm developments in Lattice Gauge Theory, particularly in Quantum Chromodynamics (QCD). Given the pivotal role of high-performance computing in these simulations, discussions on software advancements, hardware innovations, and algorithmic developments have always been an integral part of the conference. The conference now also encompasses areas such as machine architectures, AI/ML applications in physics, and algorithm development for quantum computing.

The main conference venue will be the Homi Bhabha Auditorium, TIFR.

Conference website: <https://indico.global/e/lattice2025>

 **Starts** Nov 2, 2025, 4:00 PM
Ends Nov 8, 2025, 2:30 PM
Asia/Kolkata

 TIFR Mumbai
Homi Bhabha Auditorium
Tata Institute of Fundamental Research,
1, Homi Bhabha Road,
Navy Nagar, Colaba,
Mumbai, India.
Pin - 400 005

The conference website (slides of the talks are available):

<https://indico.global/e/lattice2025>

Overview of the conference



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Photos

Sun 02/11 | **Mon 03/11** | Tue 04/11 | Wed 05/11 | Thu 06/11 | Fri 07/11 | Sat 08/11 | All days

Print PDF Full screen Detailed view Filter

09:00 **Plenary session: Opening Ceremony followed by Plenary Talks** Andreas Kronfeld
Homi Bhabha Auditorium 09:00 - 10:45

10:00 **Coffee break**
10:45 - 11:15

11:00 **Plenary session** Stephen Shaper
Homi Bhabha Auditorium 11:15 - 13:00

13:00 **Lunch break**
13:00 - 14:30

Algorithms and artificial intelligence Gurtej Karwar AG65 14:30 - 16:10	Hadronic and nuclear spectrum and interactions Urs Heller AG66 14:30 - 16:10	QCD at nonzero temperature and density Simon Hands Homi Bhabha Auditorium 14:30 - 16:10	Quantum computing and quantum information Apoorva Patel AG80 14:30 - 16:10	Theoretical developments and applications beyond Standard Model Abhishek Fulaya AG77 14:30 - 16:10	Vacuum structure and confinement Avin Horsath AG77 14:30 - 16:10	Hadronic contribution to the magnetic moment of muon Finn Steiner 0406 14:30 - 16:10
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16:00 **Coffee break**
16:10 - 16:40

Algorithms and artificial intelligence	Hadronic and nuclear	QCD at nonzero	Quantum computing and	Standard Model	Theoretical developments	Hadronic contribution to
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- 6 days of presentations: Mon – Sat
- Up to 7 parallel sessions every day



Topics:

- Plenary session – 29 talks
- Algorithms and Artificial Intelligence – 31 talks
- Hadronic and Nuclear Spectrum and Interactions – 35 talks
- Hadronic contribution to the magnetic moment of muon – 14 talks
- QCD at Non-zero Temperature and Density – 32 talks
- Quantum Computing and Quantum Information – 17 talks
- Quark and Lepton Flavor Physics – 22 talks
- Software Development and Machines – 7 talks
- Standard Model Parameters – 7 talks
- Structure of Hadrons and Nuclei – 19 talks
- Theoretical Developments & Applications Beyond the Standard Model – 38 talks
- Vacuum Structure and Confinement – 12 talks

301 participants / 287 contributions (incl. 1 online talk)

Previous Lattice Conferences

- 41st: Liverpool, UK, July 28–Aug. 3, 2024 | [Website](#)
- 40th: Fermilab, USA, July 31–Aug. 4, 2023 | [Website](#)
- 39th: Bonn, Germany, Aug. 8–13, 2022 | [Proceedings](#) | [Website](#)
- 38th: Boston, MIT, USA, July 26–30, 2021 | [Proceedings](#) | [Website](#)
- 37th: Wuhan, China, June 17–22, 2019 | [Proceedings](#) | [Website](#)
- 36th: East Lansing, MI, July 22–28, 2018 | [Proceedings](#) | [Website](#)
- 35th: Granada, Spain, June 19–24, 2017 | [Proceedings](#) | [Website](#)
- 34th: Southampton, UK, July 24–31, 2016 | [Proceedings](#)
- 33rd: Kobe, Japan, July 14–18, 2015 | [Proceedings](#)
- 32nd: New York City, NY, USA, June 23–28, 2014
- 31st: Mainz, Germany, July 29–August 3, 2013
- 30th: Cairns, Australia, June 24–29, 2012
- 29th: Lake Tahoe, California, USA, July 11–16, 2011 | [Proceedings](#) | [Website](#)
- 28th: Villasimius, Sardinia, Italy, June 14–19, 2010
- 27th: Beijing, China, July 26–31, 2009
- 26th: Williamsburg, Virginia, USA, July 14–19, 2008
- 25th: Regensburg, Germany, July 30 – August 4, 2007
- 24th: Tucson, Arizona, USA, July 23–28, 2006

- 2025: 301 participants
- 2024: 496 participants
- 2023: 441 participants (incl. 85 online)
- 2022: 403 participants
- 2021: 875 participants (ONLINE)
- 2019: 355 participants
- 2018: 342 participants
- 2017: 420 participants

Noticeable part of the community didn't attend:

- unusual time (November, studying in universities are in full swing);
- difficulties in obtaining a visa (stamped visa was required; there were no in-person participants from China)

Muon $g-2$: hadronic contributions with Lattice QCD



Aida X. El-Khadra
University of Illinois Urbana-Champaign



42ND INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY

Tata Institute of Fundamental Research
Mumbai, India

02-08 November, 2025

<https://indico.global/event/14504/contributions/137039/>

Outline

- 🗨 Introduction
- 🗨 Muon g-2 Theory Initiative
- 🗨 lattice HVP
 - 🗨 introduction and set-up
 - 🗨 Windows: intermediate (W) and short-distance (SD)
 - 🗨 long-distance (LD) window
 - 🗨 QED contributions
 - 🗨 consolidated HVP average
 - 🗨 ongoing work/new results
- 🗨 HLbL
- 🗨 Summary and Outlook
- 🗨 Appendix




- 🗨 "The anomalous magnetic moment of the muon in the SM: an update"
[T. Aliberti et al, [arXiv:2505.21476](https://arxiv.org/abs/2505.21476), Phys. Repts. 1143 (2025) 0-157]
 - 🗨 "The anomalous magnetic moment of the muon in the SM":
[T. Aoyama et al, [arXiv:2006.04822](https://arxiv.org/abs/2006.04822), Phys. Repts. 887 (2020) 1-166.]
- ➡ [Muon g-2 TI workshop @ Orsay \(8-12 Sep 2025\)](#)

Anomalous magnetic moment

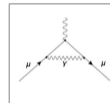
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$


Dirac (leading order): $g = 2$



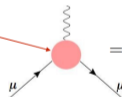
$$= (-ie) \bar{u}(p') \gamma^\mu u(p)$$

Quantum effects (loops):



$$\Rightarrow g = 2 \left(1 + \frac{\alpha}{2\pi} \right)$$


All SM particles contribute



$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2 F_2(0)$

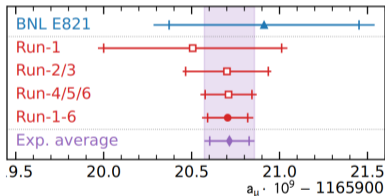
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0) = \frac{\alpha}{2\pi} + O(\alpha^2) + \dots = 0.00116\dots$

Motivation: Fermilab muon g-2 experiment

- The Fermilab experiment has released their final measurement results:
 - 07 Apr 2021: Run 1 data [B. Abi et al, *Phys. Rev. Lett.* 124, 141801 (2021)]
 - 10 Aug 2023: Run 2/3 data [D. Aguillard et al, *Phys.Rev.Lett.* 131 (2023) 16, 161802]
 - 03 Jun 2025: Run 4/5/6 data [D. Aguillard et al, *Phys.Rev.Lett.* 135 (2025) 10, 101802.]



From [Simon Corrodi @ Scientific Seminar, 03 Jun 2025](#)







▶ Martin Hoferichter talk

$$a_\mu^{\text{exp}} = 1165920715 (145) \times 10^{-11}$$

124 parts per billion!

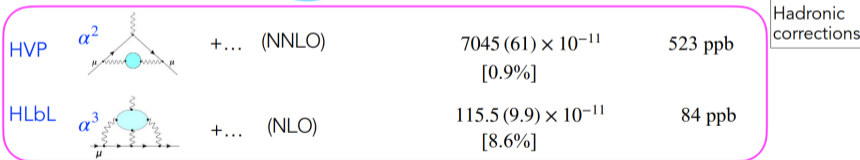
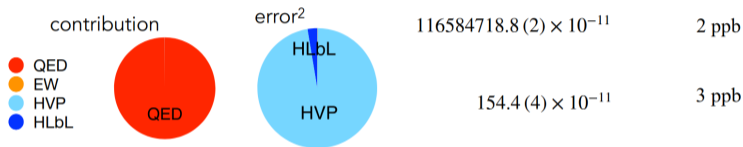
Muon $g-2$: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

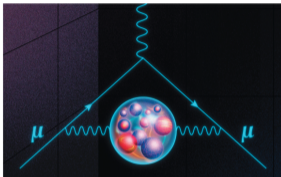
QED		+... (5 loops)	$116584718.8(2) \times 10^{-11}$	2 ppb	
EW		+... (2 loops)	$154.4(4) \times 10^{-11}$	3 ppb	
HVP	α^2 	+... (NNLO)	$7045(61) \times 10^{-11}$ [0.9%]	523 ppb	Hadronic corrections
HLbL	α^3 	+... (NLO)	$115.5(9.9) \times 10^{-11}$ [8.6%]	84 ppb	

Muon $g-2$: SM contributions

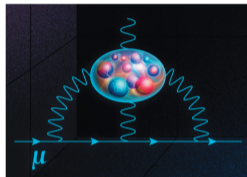
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$



Muon g-2: hadronic corrections



credit:
Samantha Koch, Fermilab



- ★ Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2 .
- ★ Cannot use perturbation theory to reliably compute the hadronic bubbles
- ★ Two-point & four-point functions:

$$\text{HVP: } \langle 0 | T \{ j_\mu j_\nu \} | 0 \rangle$$

$$\text{HLbL: } \langle 0 | T \{ j_\mu j_\nu j_\rho j_\sigma \} | 0 \rangle$$

Two independent approaches

1. Dispersive, data-driven
2. Lattice QCD

➔ Martin Hoferichter



Muon g-2 Theory Initiative

Steering Committee

- 👤 Gilberto Colangelo (Bern)
- 👤 Achim Denig (Mainz)
- 👤 Aida El-Khadra (UIUC) **chair**
- 👤 Martin Hoferichter (Bern)
- 👤 Christoph Lehner (Regensburg University) **co-chair**
- 👤 Laurent Lellouch (Marseille)
- 👤 Tsutomu Mibe (KEK)
J-PARC Muon g-2/EDM experiment
- 👤 Lee Roberts (Boston)
Fermilab g-2 experiment
- 👤 Thomas Teubner (Liverpool)
- 👤 Hartmut Wittig (Mainz)

<https://muon-gm2-theory.illinois.edu>

- 👤 Maximize the impact of the Fermilab and J-PARC experiments
 - ➡ quantify and reduce the theoretical uncertainties on the SM prediction
- 👤 assess reliability of uncertainty estimates
- 👤 summarize the theory status: White Papers
- 👤 organize workshops to bring the different communities together:
 - [First plenary workshop near Fermilab: 3-6 June 2017](#)
 - ...
 - [Seventh plenary workshop hosted by KEK/KMI \(Japan\): 9-13 Sep 2024](#)
 - [Eight plenary workshop hosted by IJCLab \(Orsay, France\): 8-12 Sep 2025](#)
 - [Ninth plenary workshop to be hosted by UConn \(US\): 3-7 Aug 2026 \(right after Lattice 2026 @UMD\)](#)



A. El-Khadra

Lattice 2025, TIFR, 3-8 Nov 2025

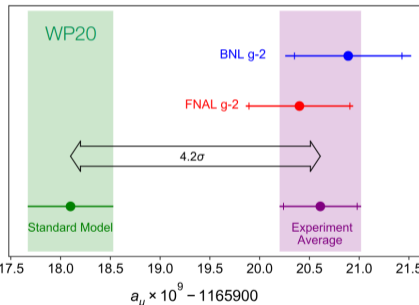
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<https://indico.global/event/14504/contributions/137039/>

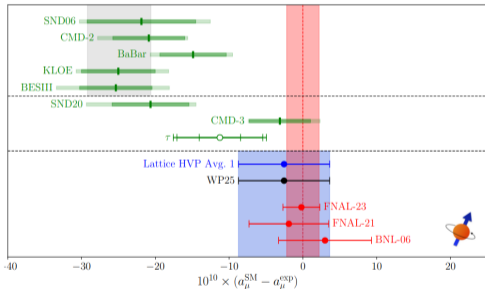
Experiment vs SM theory

$$a_\mu^{\text{SM}} = a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

🕒 2021



🕒 2025

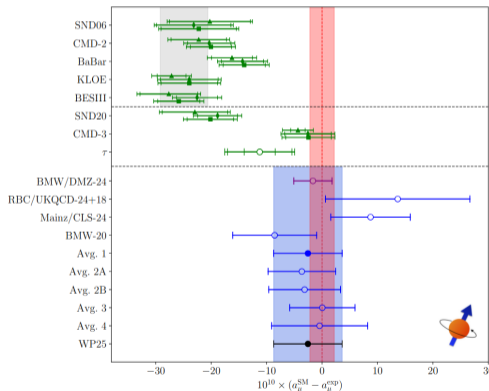
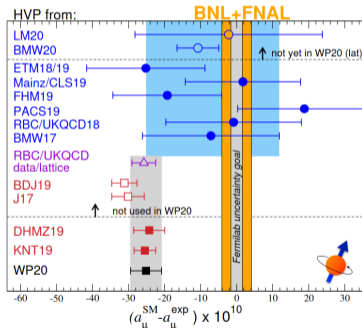


Experiment vs SM theory

$$a_\mu^{\text{SM}} = a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

2025

2021



A. El-Khadra

Latti

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<https://indico.global/event/14504/contributions/137039/>



Lattice HVP: Introduction



Leading order HVP contribution:

$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972;
E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

- Calculate $a_{\mu}^{\text{HVP,LO}}$ in Lattice QCD

Start with correlation function of EM currents: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$ $j_{\mu}^{\text{EM}} = \sum_f q_f \bar{\psi}_f(x,t) \gamma_{\mu} \psi_f(x,t)$
 $f = u, d, s, c, \dots$

Fourier transform yields $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^{\infty} dt C(t) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right) \right]$

[D. Bernecker and H. Meyer,
arXiv:1107.4388, EPJA 2011]

so that $a_{\mu}^{\text{HVP,LO}}$ can be obtained as an integral over Euclidean time, aka time momentum representation (TMR):

$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 w(Q^2) \hat{\Pi}(Q^2) = 4\alpha^2 \int_0^{\infty} dt C(t) \int_0^{\infty} dQ^2 w(Q^2) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right) \right]$$

$$\Rightarrow a_{\mu}^{\text{HVP,LO}} = 4\alpha^2 \int_0^{\infty} dt C(t) \tilde{w}(t)$$



Lattice HVP: Introduction



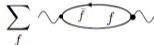
Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

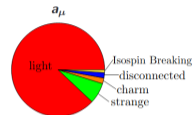
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

Note: almost always $m_u = m_d$

$f = ud, s, c, b$



- light-quark connected contribution: $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s,c,b-quark contributions $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution: $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total



- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$

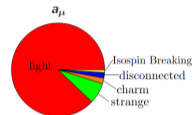


Lattice HVP: challenges



Calculate a_μ^{HVP} in Lattice QCD:
$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$



- $a_\mu^{\text{HVP,LO}}$ needed with $< 0.5\%$ precision
- subpercent statistical precision:
 - exponentially growing noise-to-signal in $C(t)$ as $t \rightarrow \infty$
 - affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects
 - Separation of $a_\mu^{\text{HVP,LO}}$ into $a_\mu^{\text{HVP,LO}}(ud)$ and $\delta a_\mu^{\text{HVP,LO}}$ is scheme dependent.
- light-quark connected contribution:
 - $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s,c,b-quark contributions
 - $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution:
 - $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
 - $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total



Lattice HVP: finite volume (FV) effects



- ★ expected size (based on ChPT) $\sim 3\%$ on typical lattice volumes with $M_\pi L \gtrsim 4$
 - ⇒ large volume studies are needed to quantify FV effects for sub-percent precision
- ★ EFT and EFT-inspired approaches:
 - NNLO ChPT [Bijnens + Relefors, arXiv:1710.04479; Aubin et al, 2019,2020, 2022,...]
 - relativistic pion EFT [Hansen & Patella, arXiv:2004.03935]
 - spectral representation using $F_\pi(k)$ + Gounaris-Sakurai parameterization (MLLGS) [Meyer 2011; Francis et al 2013; Lellouch & Lüscher, 2001]
 - use Lüscher and Lellouch-Lüscher equations to obtain FV E_n, A_n and compute $C(t, \infty) - C(t, L)$
 - Chiral Model: ChPT + ρ [Jegerlehner & Szafron, arXiv:1101.2872; HPQCD, arXiv:1601.03071]
 - For staggered fermions: modify FV EFTs to account for splittings between pions with different "tastes"
- ★ large-volume studies by PACS, BMW (2020, 2024), RBC/UKQCD, Mainz



Lattice HVP: scale setting



- $a_\mu^{\text{HVP,LO}}$ is dimensionless, but depends on dimensionful parameters (m_μ, m_q, \dots), which are expressed in terms of the lattice scale Λ :

$$a_\mu^{\text{HVP,LO}} \equiv a_\mu^{\text{HVP,LO}}(M_\mu, M_u, M_d, \dots) \quad M_\mu = \frac{m_\mu}{\Lambda}, \dots$$

- uncertainty in $a_\mu^{\text{HVP,LO}}$ due to the error in the scale determination, $\Delta\Lambda$:

[Della Morte et al, [arXiv:1705.01775](https://arxiv.org/abs/1705.01775), JHEP 2017]

$$\Delta a_\mu^{\text{HVP,LO}} = \left| \Lambda \frac{da_\mu^{\text{HVP,LO}}}{d\Lambda} \right| \times \frac{\Delta\Lambda}{\Lambda} = \left| M_\mu \frac{da_\mu^{\text{HVP,LO}}}{dM_\mu} \right| \times \frac{\Delta\Lambda}{\Lambda} \implies \frac{\Delta a_\mu^{\text{HVP,LO}}}{a_\mu^{\text{HVP,LO}}} \simeq 1.8 \frac{\Delta\Lambda}{\Lambda}$$

- \implies need to determine lattice scale to high precision ($< 0.2\%$)
- Scale setting quantities currently used in lattice QCD calculations:
 - f_π (or f_K) — depend on V_{ud} (V_{us})
radiative QED corrections $\sim 2\%$
 - Ω baryon mass — QED corrections are small ($\sim 0.1\%$)



Windows in Euclidean time



$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{w}(t) C(t)$$

- Use windows in Euclidean time to consider the different time regions separately

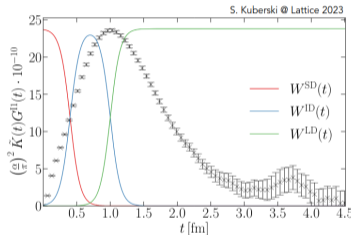
[T. Blum et al, arXiv:1801.07224, 2018 PRL]

$$t_0 = 0.4 \text{ fm}, t_1 = 1.0 \text{ fm}$$

Short Distance (SD) $t : 0 \rightarrow t_0$

Intermediate (W) $t : t_0 \rightarrow t_1$

Long Distance (LD) $t : t_1 \rightarrow \infty$



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to dispersive approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,...) and combine:

$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

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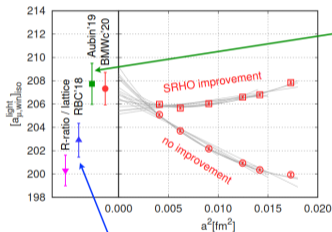


Lattice HVP: W (intermediate) window



BMW 20

[Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature]



Aubin et al -19

[C. Aubin et al, arXiv:1905.09307, PRD]

RBC/UKQCD-18

[T. Blum et al, arXiv:1801.07224, 2018 PRL]

The hadronic vacuum polarization from lattice QCD at high precision

Nov 16 – 20, 2020
Europe/Zurich timezone

Enter your search term

Virtual Muon g-2 Theory Initiative workshop (16-18 Nov 2020)

Agreement to:

- compute the 3 window observables proposed by RBC/UKQCD
- first goal: intermediate window observable
- compute individual flavor contributions + IB-breaking effects
- ...

Hadronic contribution to magnetic moment of muon

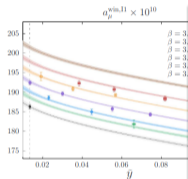


Lattice HVP: W (intermediate) window



Mainz 2022

[M. Cè et al, arXiv:2206.06582, PRD]

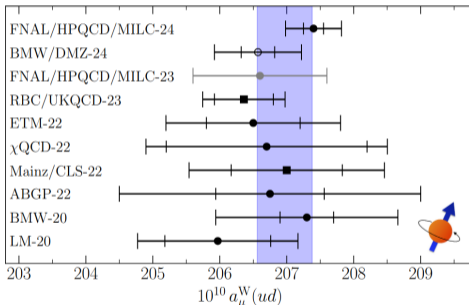


FNAL/HPQCD/MILC

[A. Bazavov et al, arXiv:2206.06582, PRD]

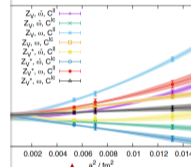
ETM 2022

[C. Alexandrou et al, arXiv:2206.15084, PRD]



RBC/UKQCD 2023

[T. Blum et al, arXiv:2301.08696, PRD]



Blind analyses



Lattice HVP: short-distance window (SD)



Log-enhanced discretization effects in the short distance region

[N. Husung et al, arXiv:1912.08498, EPJC; T. Harris et al, 2111.07948; N. Husung, 2206.03536, EPJC; ...]

- [L. Chimirri et al, arXiv:2211.15750]
proposal to alleviate by splitting Euclidean time integral and supplementing shortest distance region with continuum perturbation theory:

$$\int_0^\infty dt F(t) = \underbrace{\int_0^\infty dt [1 - \chi(t)] F(t)}_{\text{continuum PT}} + \underbrace{a \sum_{t=0}^\infty \chi(t) F(t)}_{\text{continuum limit of lattice results}}, \quad \chi(t) \sim \begin{cases} O(t^2) & t\Lambda_{\overline{\text{MS}}} \ll 1 \\ 1 & t\Lambda_{\overline{\text{MS}}} \gg 1 \end{cases}$$

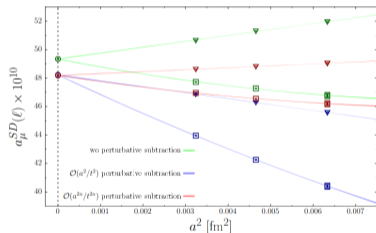


Lattice HVP: short-distance window (SD)



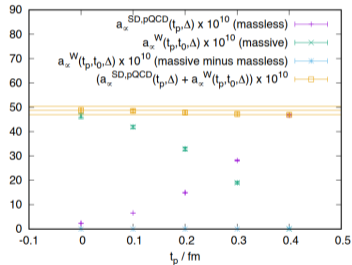
Log-enhanced discretization effects in short-distance region

ETM 2022 [C. Alexandrou et al, arXiv:2206.15084, PRD]



- subtraction of leading-order log-enhanced disc. effects

RBC/UKQCD 2023 [T. Blum et al, arXiv:2301.08696, PRD]



- stability of pert. subtraction wrt to t_p using cont. pQCD



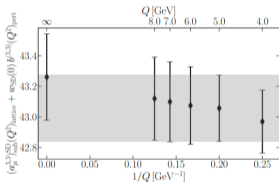
Lattice HVP: short-distance window (SD)



Log-enhanced discretization effects in short-distance region

Mainz

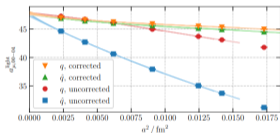
[S. Kuberski et al, arXiv:2401.11895, JHEP]



- stability with pert. subtraction at Q^2 to remove log-enhanced discretization effects

BMW/DMZ

[A. Boccaletti et al, arXiv:2407.10913]

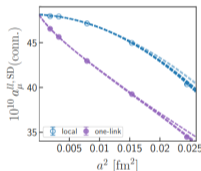


- corrected: remove log-enhanced discretization effects at tree-level

$$\hat{q}: \left[t^2 - \frac{4}{(aQ)^2} \sin^2\left(\frac{aQt}{2}\right) \right] \rightarrow \left[t^2 - \frac{4}{(aQ)^2} \sin^2\left(\frac{aQt'}{2}\right) \right]$$

FNAL/HPQCD/MILC

[A. Bazavov et al, arXiv:2411.09656, PRD]



- log-enhancement absent in local current; present and measurable in one-link current
- compare with pQCD and check stability in SD region



Lattice HVP: short-distance window (SD)



Log-enhanced discretization effects in short-distance region

● Mainz

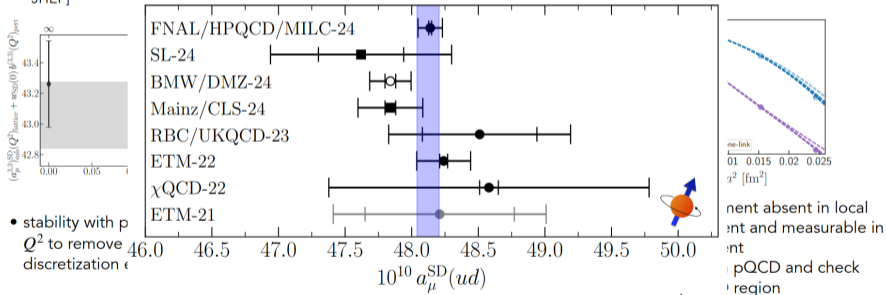
[S. Kuberski et al, arXiv:2401.11895, JHEP]

● BMW/DMZ

[A. Boccaletti et al, arXiv:2407.10913]

● FNAL/HPQCD/MILC

[A. Bazavov et al, arXiv:2411.09656, PRD]



Outline

- 🕒 Introduction
- 🕒 Muon g-2 Theory Initiative
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 - 🕒 Windows: intermediate (W) and short-distance (SD)
 - 🕒 long-distance (LD) window
 - 🕒 QED contributions
 - 🕒 consolidated HVP average
 - 🕒 ongoing work/new results
- 🕒 HLbL
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- 🕒 Appendix



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

[M. Lynch @ Lattice 2024]
(FNAL/HPQCD/MILC)

- Use improved statistical estimators

[DeGrand + Schafer, hep-lat/0401011, L. Giusti et al, hep-lat/0402002; J. Foley et al, hep-lat/0505023; T. Blum, et al, arXiv:1208.4349; ...]

Low mode improvement:

$$C_{\text{RW}}^\Gamma(t) = \sum_i \text{Tr} \left\{ S_i^{\text{RW}} \Gamma \gamma^5 S_i^{\text{RW}\dagger} \gamma^5 \Gamma \right\}, \quad S_i^{\text{RW}}(x) = M^{-1}(x;y) \eta_i(y)$$

Low-mode propagator

$$M_L^{-1} = \sum_n^{N_c} \frac{1}{\lambda_n} v_n v_n^\dagger,$$

$$\lambda_n = \begin{cases} m + i\tilde{\lambda}_n & (n \bmod 2 = 0) \\ m - i\tilde{\lambda}_n & (n \bmod 2 = 1) \end{cases}$$

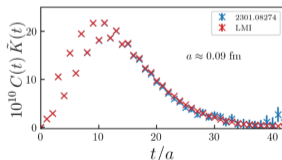
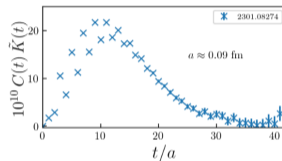
significant improvement
in statistical errors at
large Euclidean times.

Low-mode-improved random wall estimator

$$C_{\text{LMI}}^\Gamma(t) = C_{\text{RW}}^\Gamma(t) - C_{\text{RW,LL}}^\Gamma(t) + C_{\text{LL}}^\Gamma(t)$$

Exact low-mode contribution

$$C_{\text{LL}}^\Gamma(t) = \sum_i \text{Tr} \{ M_L^{-1} \Gamma M_L^{-1} \Gamma \}$$





Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

♦ bounding method:

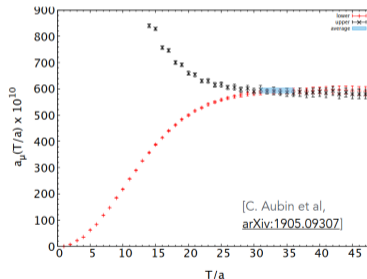
[Borsanyi et al, PRL 2018, Blum et al, PRL 2018]

for $t > t_c$: $0 \leq C(t_c) e^{-\bar{E}_{t_c}(t-t_c)} \leq C(t) \leq C(t_c) e^{-E_0(t-t_c)}$

\bar{E}_{t_c} : effective mass of $C(t)$ at t_c

E_0 : ground state = $\pi\pi$

replace $G(t > t_c)$ with upper and lower bound, vary t_c





Lattice HVP: long-distance tail (again)



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

- obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states

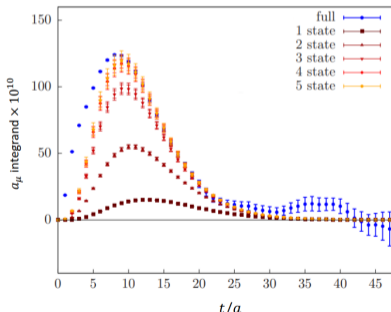
- use to reconstruct $C(t > t_c)$

- can be used to improve bounding method:

$$C(t) \rightarrow C(t) - \sum_{n=0}^N A_n^2 e^{-E_n t}$$

use E_{N+1} in upper bound

- yields big reduction in stat. errors (compared with bounding method)



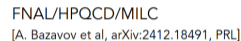
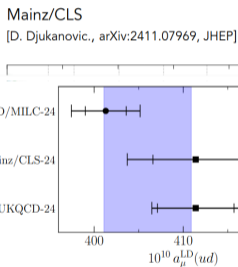
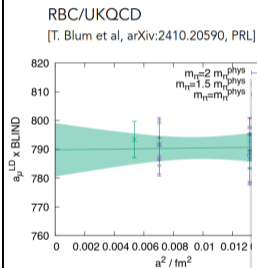
J. Mckeon @ Lattice 2024
(RBC/UKQCD)
See also:
[Bruno et al, RBC/UKQCD,
arXiv:1910.11745]



Lattice HVP: LD window



Fall 2024: RBC/UKQCD, Mainz, FNAL/HPQCD/MILC — all from blind analyses

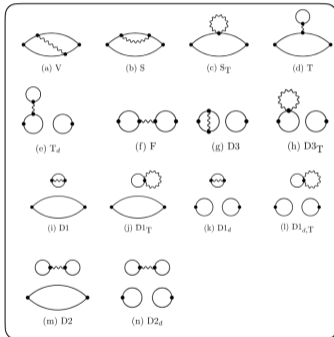




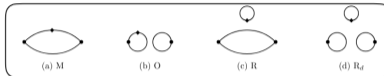
Lattice HVP: IB contributions



QED corrections:



Strong Isospin breaking (SIB) corrections



RBC/UKQCD 18 [T. Blum et al, arXiv:1801.0722, PRL]

full: W: V, S, F, ...

ETMC 19 [D. Giusti et al, arXiv:1903.07544, PRL]

full: V, S, S_T , M, ...

BMW 20 [Sz. Borsanyi et al, arXiv:1911.12137, PRL]

full, W: all diag

$t \gtrsim 1 - 1.5$ fm

Mainz/CLS 24 [M. J. Dorn et al, arXiv:2408.14001, PRL]

SD, W, LD full:

FNAL/HPQCD/MILC 24 [A. Bazavov et al, arXiv:2411.09656, PRD]

SD, W: M, O, R SIB

- ➔ Many QED + SIB effects are computed by several different groups
- ➔ sea-sea and sea-valence QED effects: only BMW 20
- ➔ long-distance QED: $\sim 0.3\%$, still a challenge

Outline

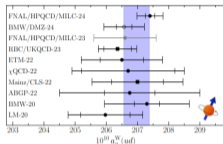
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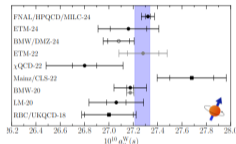
Lattice HVP: results for W



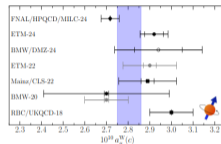
light (ud)



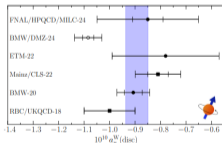
strange (s)



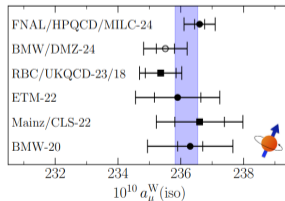
charm (c)



disconnected (disc)



iso = $ud + s + s + b + disc$



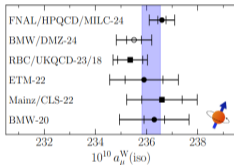
iso:
isospin
symmetric



Lattice HVP: results for W

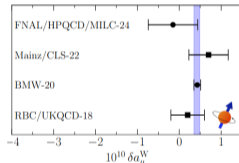


iso



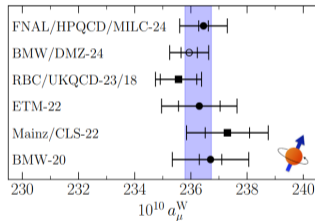
iso: isospin symmetric
 iso = $ud + s + s + b + \text{disc}$

IB

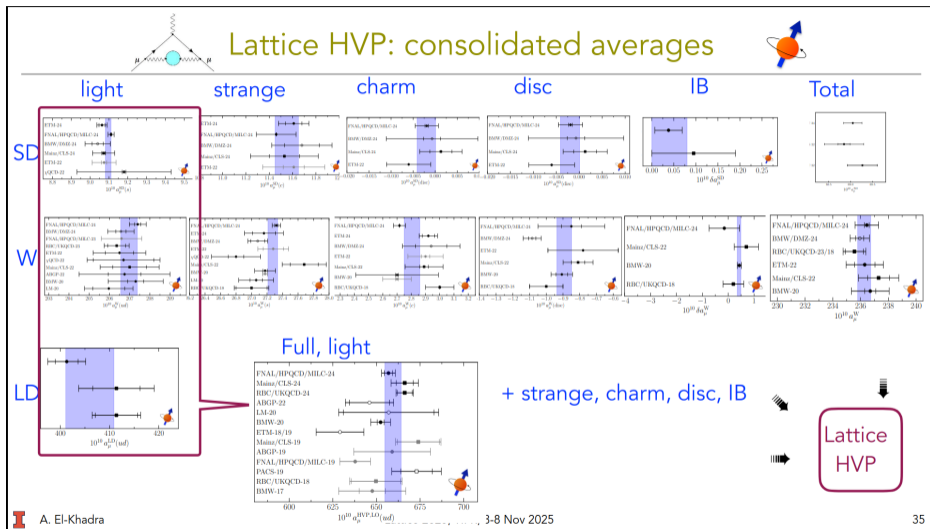


IB: isospin-breaking
 corrections

Total

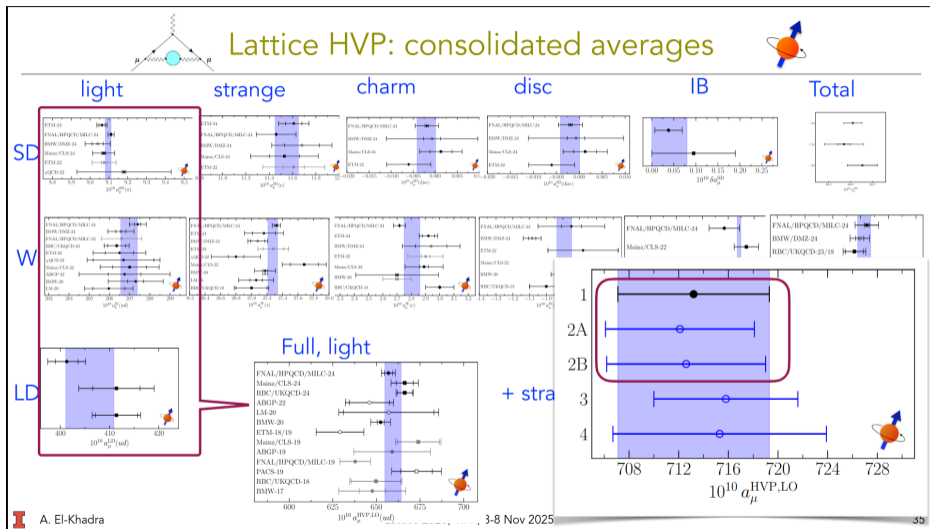


Hadronic contribution to magnetic moment of muon



<https://indico.global/event/14504/contributions/137039/>

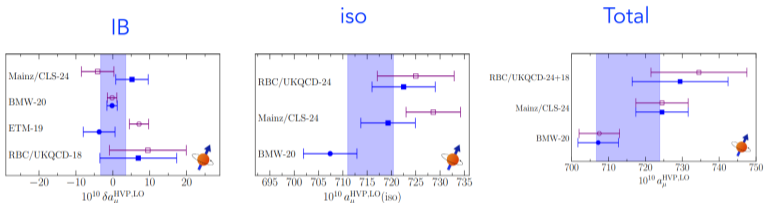
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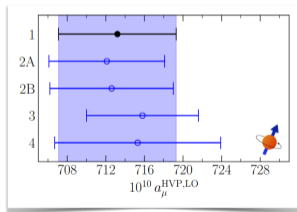


Lattice HVP: total comparisons



Averages in comparison:

Note: Averages 1, 2a, 2b maximize the number of independent lattice inputs.

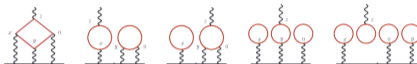
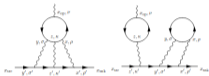




Hadronic Light-by-light: lattice



Lattice QCD+QED: Three independent and complete direct calculations of a_μ^{HLbL}



◆ **RBC/UKQCD**

[T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]

- ◆ QCD + QED_L (finite volume) stochastic

DWF ensembles at/near phys mass,
 $a \approx 0.08 - 0.2 \text{ fm}$, $L \sim 4.5 - 9.3 \text{ fm}$

◆ **Mainz group**

[E. Chao et al, arXiv:2104.02632]

- ◆ QCD + QED (infinite volume & continuum) analytic

CLS (2+1 Wilson-clover) ensembles

$m_\pi \sim 200 - 430 \text{ MeV}$, $a \approx 0.05 - 0.1 \text{ fm}$, $m_\pi L > 4$

- ◆ Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- ◆ **second result from RBC/UKQCD** [T. Blum et al, arXiv:2304.04423] using QCD + QED (inf.): $a_\mu^{\text{HLbL}} = 124.7 (11.5) (9.9) \times 10^{-11}$ consistent with previous calculations
- ◆ BMW 24 [Z. Fodor et al, arXiv:2411.11719]: Mainz approach, modified for staggered fermions

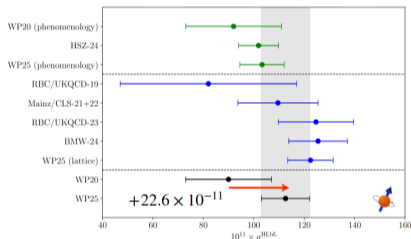


Hadronic Light-by-Light: Summary

$$a_{\mu}^{\text{HLbL}}$$

$$a_{\mu}^{\text{SM}}$$

$$a_{\mu}^{\text{HVP}} + [a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HLbL}}]$$



Dispersive approach:

[Colangelo et al, 2014; Pauk & Vanderhaegen 2014; ...; Hoferichter et al, 2024]

- ♦ model independent
- ♦ significantly more complicated than for HVP
- ♦ provides a framework for data-driven evaluations
- ♦ can also use lattice results as inputs
- ♦ ongoing work on tensors
- ♦ now ~ 10% uncertainty

Lattice QCD+QED:

- ♦ Independent calculations by three groups (RBC/UKQCD, Mainz, BMW)
- ♦ consistent with each other and with previous calculations
- ♦ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW, RBC/UKQCD]

Summary and Outlook

- ★ **HLbL**: consolidated lattice + dispersive average, now known with $\lesssim 10\%$ precision
 - ★ **HVP**: consolidated lattice average with 0.9% precision, basis for WP25 SM prediction
 - ★ ongoing work: $\Rightarrow \lesssim 0.5\%$ precision
 - 🔧 improved calculations of QED contributions, esp. at long distances;
 - 🔧 better precision for light-quark contribution at long-distances;
 - 🔧 study of scale setting and scheme dependence \Rightarrow [Christoph Lehner talk](#)
 - ★ ongoing work for data-driven HVP \Rightarrow [Martin Hoferichter talk](#)
 - ★ **Comparison between data-driven and lattice HVP: profound insights possible**
 - 🔧 **if they agree**: consolidated average for improved SM prediction
 - 🔧 **if they disagree**: detailed comparisons to identify region/source
 - ★ including τ decay data in data-driven approach:
 - requires nonperturbative evaluation of IB correction: interplay between lattice and dispersive calculation \Rightarrow [Martin Hoferichter talk](#)
 - ★ **MUonE @ CERN**: determine HVP from $e^-\mu^-$ scattering at space-like momenta
- \Rightarrow **continued coordination by Theory Initiative: another WP in $\sim 2-3$ years**

<https://indico.global/event/14504/contributions/137039/>

Hadronic Vacuum Polarisation for the muon magnetic moment

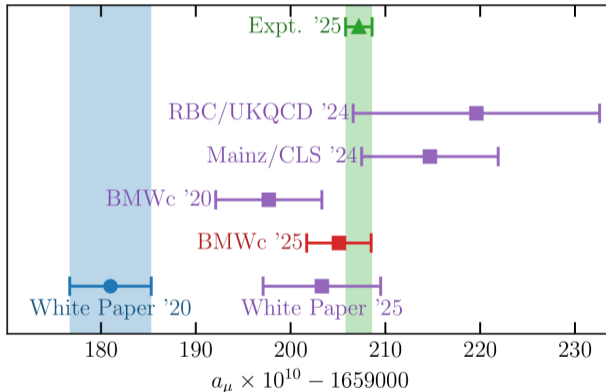
from the BMW and DMZ collaborations

Finn M. Stokes

Ramsay Fellow
Centre for the Subatomic Structure of Matter
The University of Adelaide

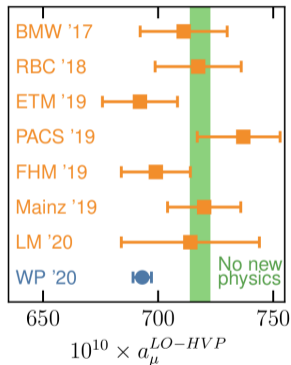
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Muon magnetic moment



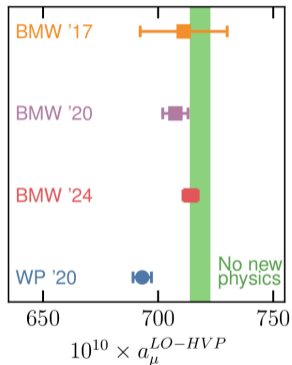
Improving precision

- Until 2020, **lattice** uncertainty larger than **data-driven**
- Lattice results mostly consistent with both data-driven and **experiment**
- **Sub-percent** lattice determination [2002.12347]: First lattice calculation with errors comparable to data-driven determinations
- New BMW result [2407.10913]: **More precise** than data-driven determinations
- Tensions in “intermediate window” already in 2020



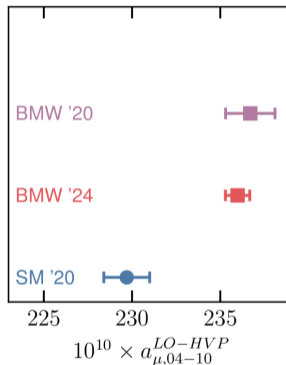
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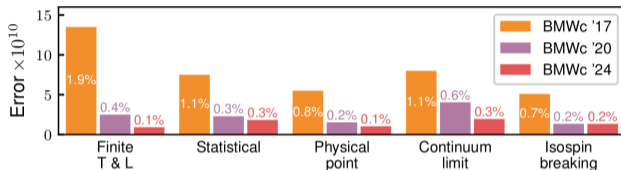


Improving precision

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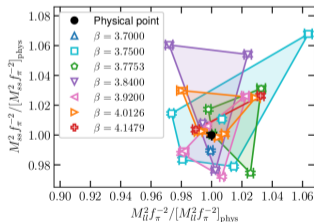
Statistical and systematic errors



- Five major sources of uncertainty in our **first work**
- Dominant error from finite-size effects
- Addressed in **2020** using dedicated large-volume simulations
- Updated in **2024**
 - Added finer lattice spacing ($a = 0.0483$ fm)
 - Broke continuum limit into pieces with different systematics
 - Hybrid approach to controlling long-distance uncertainties
- Systematic uncertainties now a similar size to statistical

Simulation parameters

- Tree-level Symanzik action
- 2 + 1 + 1 staggered fermions
- Stout smearing:
4 sweeps at $\rho = 0.125$
- $L \sim 6$ fm, $T \sim 9$ fm
- M_π & M_K around physical pt.

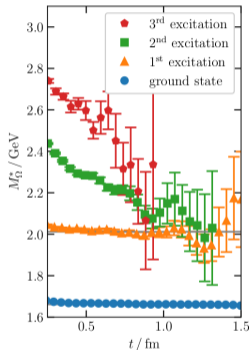


β	a/fm	$L \times T$	#conf
3.7000	0.1315	48 × 64	904
3.7500	0.1191	56 × 96	2072
3.7753	0.1116	56 × 84	1907
3.8400	0.0952	64 × 96	3139
3.9200	0.0787	80 × 128	4296
4.0126	0.0640	96 × 144	6980
4.1479	0.0483	128 × 192	4439

Ensembles for dynamical QED:

3.7000	0.1315	24 × 48	716
		48 × 64	300
3.7753	0.1116	28 × 56	887
3.8400	0.0952	32 × 64	4253

Scale setting: M_Ω



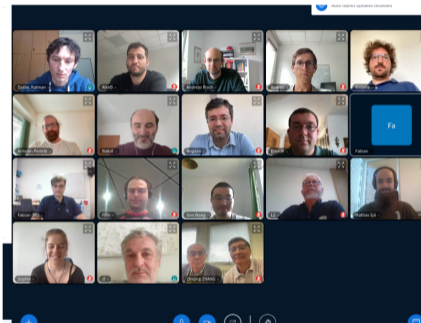
- 2020 result used mass of Omega baryon to set scale
- GEVP to isolate ground state
 - Wuppertal smearing and GPoF

$H_{t+2t_p+0}^{pp}$	$H_{t+2t_p+1}^{pp}$	$H_{t+t_p+0}^{ps}$	$H_{t+t_p+1}^{ps}$	$H_{t+t_p+2}^{ps}$	$H_{t+t_p+3}^{ps}$
$H_{t+2t_p+1}^{pp}$	$H_{t+2t_p+2}^{pp}$	$H_{t+t_p+1}^{ps}$	$H_{t+t_p+2}^{ps}$	$H_{t+t_p+3}^{ps}$	$H_{t+t_p+4}^{ps}$
$H_{t+t_p+0}^{sp}$	$H_{t+t_p+1}^{sp}$	H_{t+0}^{ss}	H_{t+1}^{ss}	H_{t+2}^{ss}	H_{t+3}^{ss}
$H_{t+t_p+1}^{sp}$	$H_{t+t_p+2}^{sp}$	H_{t+1}^{ss}	H_{t+2}^{ss}	H_{t+3}^{ss}	H_{t+4}^{ss}
$H_{t+t_p+2}^{sp}$	$H_{t+t_p+3}^{sp}$	H_{t+2}^{ss}	H_{t+3}^{ss}	H_{t+4}^{ss}	H_{t+5}^{ss}
$H_{t+t_p+3}^{sp}$	$H_{t+t_p+4}^{sp}$	H_{t+3}^{ss}	H_{t+4}^{ss}	H_{t+5}^{ss}	H_{t+6}^{ss}

- Extracted states consistent with expected resonances
- Try three different operators to test for taste-breaking effects

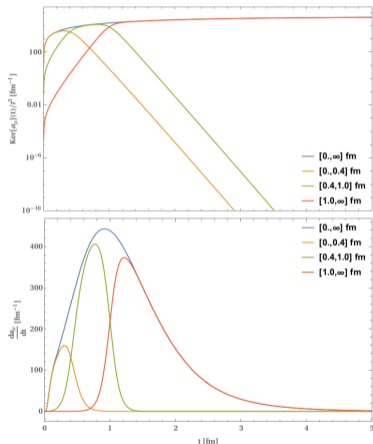
Blinding the analysis

- Analysis was performed fully **blinded**
- All a_μ contributions multiplied by unknown blinding factor
- Blinding factor only revealed when analysis was completely finalised and manuscript almost complete



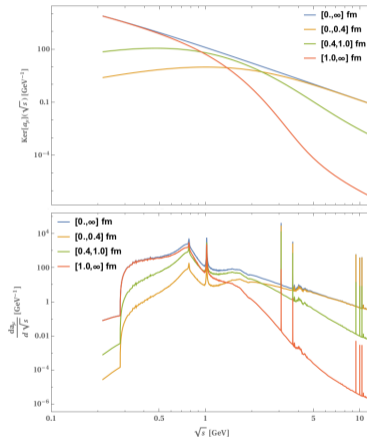
Euclidean-time windows

- Different challenges affect different time ranges
- Idea: break integral up into **windows** [RBC '18]
- Modify integration kernel to focus specific regions
- Can also modify kernel for **data-driven results**
- Intermediate window selects **rho peak**
- This is where discrepancies between data-driven inputs are evident



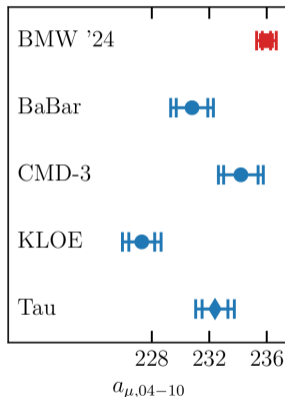
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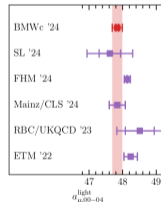
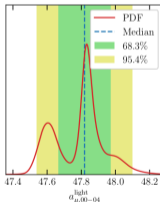
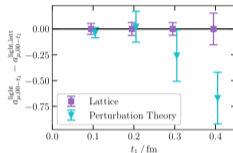
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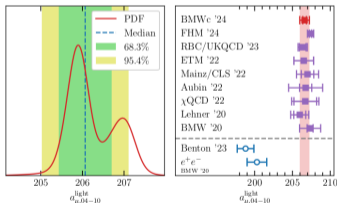
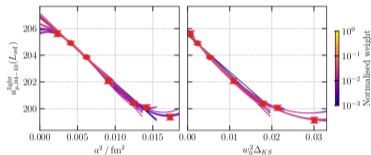
Short-distance window

- Below 0.4 fm
- Important **logarithmic** discretisation effects
- Try different momentum discretisation:
 $\hat{q} = 2 \sin(aq/2) / a$
- New test of agreement between **lattice** and **perturbation theory**
 - **Good agreement** below 0.3 fm
 - Perturbation-theory-motivated fit form justified



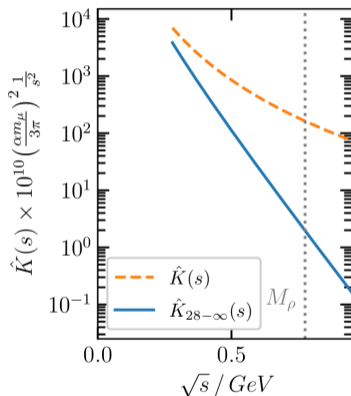
Intermediate window

- 0.4 fm to 1.0 fm
- Protected from short-range discretisation errors
- Free from long-distance noise and finite-size effects
- Many lattice groups have **consistent values**
- **Significant tension** to data-driven estimates
- The first place this tension was clearly observed



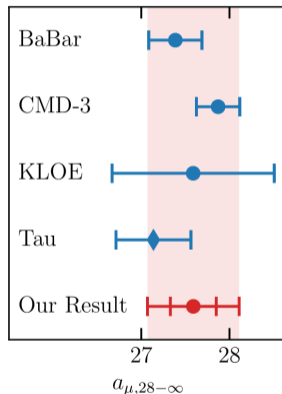
Very-long-distance tail

- Consider long-distance tail: beyond 2.8 fm
- Very sensitive to finite volume, and statistics
- Dominated by low energy part of spectrum
- Seen from the kernel $\hat{K}(s)$ that weights integral
- ρ peak strongly suppressed



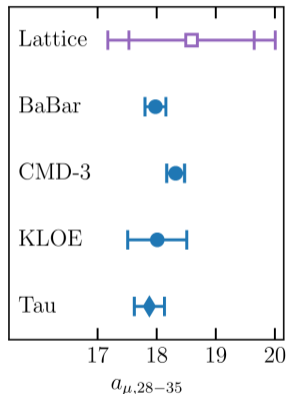
Hybrid approach

- Idea: Use data-driven result instead of lattice in tail
- Proposed in RBC '18
- What about problems with data-driven inputs?
- Thanks to ρ suppression, all experiments agree, and also agree with lattice
- Gives 4% of final value, but significant reduction in error
- May be best way to match final experimental precision



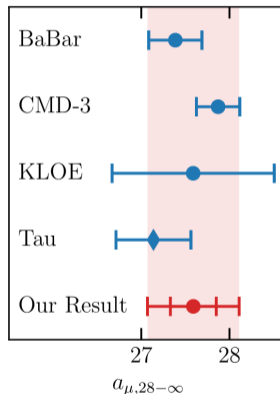
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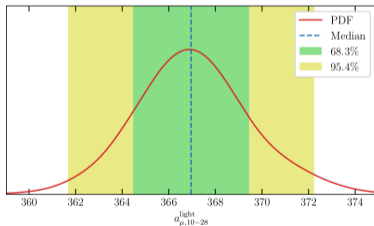
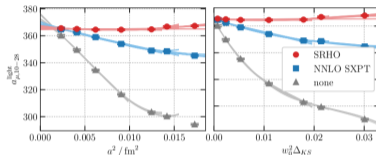
Tail uncertainty

- We include additional uncertainties associated with:
 - Ordering of averaging and integration
 - Other experiments with less coverage
 - Effect of CMD3 on the average
 - Difference between KLOE and BaBar on the rho peak
- **Negligible impact** on full result

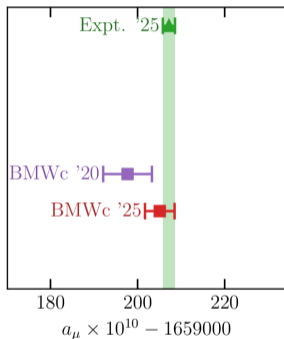


Long-distance window (Preliminary)

- Above 1.0 fm
- New result, add together:
 - Lattice calculations for all contributions to 1.0 – 2.8 fm
 - Data-driven tail for 2.8 fm to infinity
- Lattice results corrected with SRHO taste-breaking model with uncertainty from NNLO SXPT
- Uncertainty dominated by **scale setting** and **statistics**



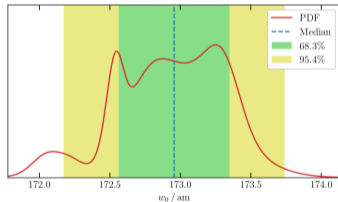
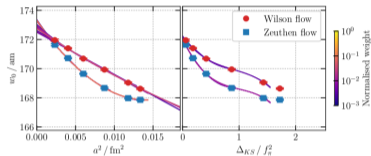
Change from 2020



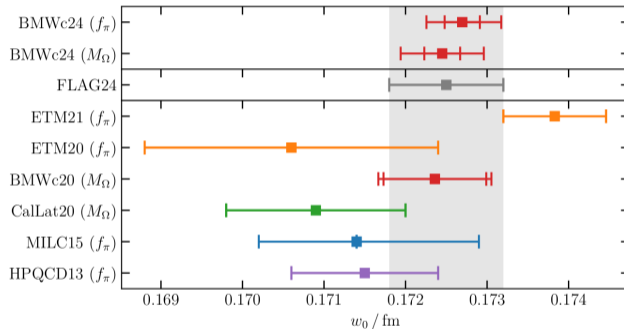
- Useful to quantify the (dis)agreement between our 2020 result, and 2024
- The major changes that have been made are
 - Finer lattice spacing
 - Division into sub-windows
 - Higher-order fits
 - Data-driven tail (removes finite-volume from 2020)
- Put together, estimate these give a difference of 7.4(5.5)
- Hence, the calculations differ by 1.3 σ

Scale setting: f_π

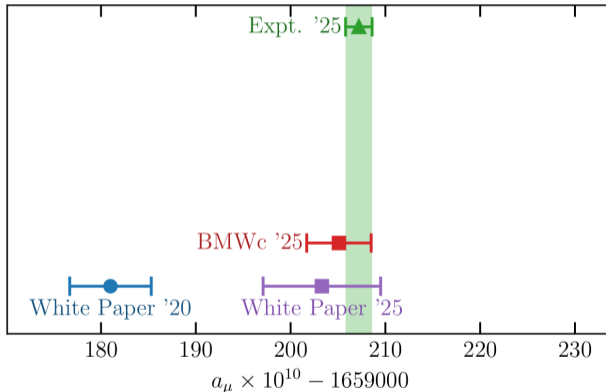
- To cross-check our analysis, implemented a **second scale setting**
- Based on the pion decay constant, f_π
- Requires radiative corrections to leptonic pion decays
[A. Cotellucci, Fri 5:20pm]
- Enters through w_0 from gradient flow
- Compare two forms of gradient flow
- Analysis once again **blinded**



Unblinding



Conclusion



RBC/UKQCD HVP status and overview

Christoph Lehner, Uni Regensburg

November 3, 2025, Lattice 2025 in Mumbai

<https://indico.global/event/14504/contributions/138066/>

The RBC & UKQCD collaborations

[BNL & RBRC](#)

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Christopher Kelly
Patrick Oare
Amarjit Soni (emeritus)
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Tianle Wang
Shuhei Yamamoto

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Sergey Syritsyn (RBRC)

<https://indico.global/event/14504/contributions/138066/>

HVP for the muon magnetic moment (RBC/UKQCD)

Main building blocks of our HVP program so far:

PHYSICAL REVIEW LETTERS 121, 022003 (2018)

Calculation of the Hadronic Vacuum Polarization Disconnected Contribution to the Muon Anomalous Magnetic Moment

T. Blum,¹ P.A. Boyle,^{1,2} T. Izubuchi,^{1,3,4} L. Liu,⁵ A. Jüttner,^{6,7} C. Lehner,^{8,9} K. Maltman,¹⁰ M. Marinkovic,¹¹ A. Penezić,¹² M. Spreng¹³

(RBC and UKQCD Collaborations)

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⁵Physics Department, Columbia University, New York, New York 10027, USA
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⁹CEPS, Physics Department, 1211 Geneva 23, Switzerland
(Received 30 December 2015; revised manuscript received 6 April 2016; published 2 June 2016)

We report the first lattice QCD calculation of the hadronic vacuum polarization (HVP) disconnected contribution to the muon anomalous magnetic moment at physical pion mass. The calculation uses a refined noise-reduction technique that enables the control of statistical uncertainties at the desired level with modest computational effort. Measurements were performed on the $40^3 \times 96$ physical-pion-mass lattice generated by the RBC and UKQCD Collaborations. We find the leading-order hadronic vacuum polarization $a_{\mu}^{\text{HVP,discon}} = -9.62(3)(2.5) \times 10^{-10}$, where the first error is statistical and the second systematic.

PHYSICAL REVIEW LETTERS 121, 022003 (2018)

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

T. Blum,¹ P.A. Boyle,^{1,2} V. Gligoski,³ T. Izubuchi,^{1,3,4} L. Liu,⁵ C. Jung,⁶ A. Jüttner,^{6,7} A. Penezić,¹² and J. T. Tsang⁸

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⁶WZNS-DM, Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA
(Received 23 January 2016; published 12 July 2016)

We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The final combination of sea, above, strange, and charm quarks including QED and strong isospin breaking effects is $a_{\mu}^{\text{HVP}} = 7.15(4)(8.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with β -state data, we significantly improve the precision to $a_{\mu}^{\text{HVP}} = 692.9(3.7) \times 10^{-10}$. This is the currently most precise determination of a_{μ}^{HVP} .

PHYSICAL REVIEW D 106, 054007 (2022)

Update of Euclidean windows of the hadronic vacuum polarization

T. Blum,¹ P.A. Boyle,^{1,2} M. Beraudo,^{3,4} D. Gaidarov,⁵ V. Gligoski,⁶ R.C. Hild,⁷ T. Izubuchi,^{1,3,4} V.C. Janke,^{8,9} L. Liu,¹⁰ C. Jung,¹¹ A. Jüttner,¹² C. Kelly,¹³ C. Lehner,¹⁴ M. Marinkovic,¹⁵ R. D. Mochel,¹⁶ A.S. Meyer,¹⁷ and J. T. Tsang¹⁸

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(Received 28 January 2021; accepted 28 August 2021; published 26 September 2021)

We compare the standard Euclidean window of the hadronic vacuum polarization using multiple independent hybrid methods. To improve the continuum and infinite-volume extrapolation of the dominant quark-connected light-quark vector-current contribution and address additional subtleties resulting from sea-quark quark and mixed chiral-currents, leading from first principles, we find $a_{\mu}^{\text{HVP}} = 235.96(5)(9) \times 10^{-10}$, which is a 1.3% increase with the recently published dispersive result $a_{\mu}^{\text{HVP}} = 229.4(1.6) \times 10^{-10}$ [G. Colangelo, A. A. Halkidis, M. Hoferichter, A. Knecht, C. Lehner, P. Marfatia, and V. Trottler, *Phys. Lett. B* **804**, 131911 (2022)] and in agreement with other recent lattice determinations. We also provide a result for the standard three-point window. The results reported here are unchanged compared to our presentation at the Hadronic Workshop of the μ 2 Theory Initiative in 2022 [C. Lehner, *Talk Presented at the 3th Plenary Meeting of the μ 2 Theory Initiative in Edinburgh (2022)*].

PHYSICAL REVIEW LETTERS 134, 201001 (2025)

Long-Distance Window of the Hadronic Vacuum Polarization for the Muon $g-2$

T. Blum,¹ P.A. Boyle,^{1,2} M. Beraudo,^{3,4} B. Chakraborty,⁵ P. Eberhard,⁶ V. Gligoski,⁷ A. Hockley,⁸ N. Hermann-Thunander,⁹ R.C. Hild,¹⁰ T. Izubuchi,^{1,3,4} L. Liu,¹¹ C. Jung,¹² C. Lehner,¹³ J. McKewen,¹⁴ A.S. Meyer,¹⁵ M. Tornik,¹⁶ J. T. Tsang,¹⁷ and X.-Y. Tan¹⁸

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(Received 30 October 2024; revised 28 March 2025; accepted 22 April 2025; published 19 May 2025)

We provide the first ab initio calculation of the Euclidean long-distance window of the isospin-symmetric light-quark connected contribution to the hadronic vacuum polarization for the muon $g-2$ and find $a_{\mu}^{\text{HVP,conn}} = 411.4(3.5)(2.4) \times 10^{-10}$. We also provide the currently most precise calculation of the total isospin-symmetric, light-quark connected contribution, $a_{\mu}^{\text{HVP,conn}} = 666.2(4.3)(2.5) \times 10^{-10}$ in the BMW20 world, which is more than 10% larger compared to the state-of-the-art results of Blum et al. [*Phys. Rev. D* **107**, 074001 (2023)] and 1.7% larger compared to the lattice QCD result of BMW20.

Next steps towards matching experimental precision:

- Update of I=0 channel (3 pion reconstruction in progress)
- High-precision QED corrections beyond electroquenched
- New finer ensemble created at physical pion mass (128l, FTHMC)

<https://indico.global/event/14504/contributions/138066/>

Long-distance reconstruction of QED corrections to the hadronic vacuum polarization for the muon $g-2$

C. Lehner,^{1,*} J. Parrino,¹ and A. Völklein¹

¹*Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*

(Dated: September 1, 2025)

The long-distance contribution of QED corrections to the hadronic vacuum polarization is particularly challenging to compute in lattice QCD+QED. Currently, it is one of the limiting factors towards matching the precision of the recent result by the Fermilab E989 experiment for the muon $g-2$. In this work, we present a method for obtaining high-precision results for this contribution by reconstructing exclusive finite-volume state contributions. We find relations between the pion-photon contributions of individual diagrams and demonstrate the reconstruction method with lattice QCD+QED data at a single lattice spacing of $a^{-1} \approx 1.73$ GeV and $m_\pi \approx 275$ MeV.

PACS numbers: 12.38.Gc

arXiv:2508.21685

<https://indico.global/event/14504/contributions/138066/>

Summary and Outlook:

- Developed a method for long-distance reconstruction of QED corrections
- Aim for complete update of 2018 result with errors significantly below 1% in 2026
- Will continue to match experimental precision of 1/4%, 128l ensemble will be crucial for this
- Next talk: Julian Parrino will give more details of QED sampling strategy and status
- Related effort: isospin breaking corrections to tau decays; paper in final proof reading stage
- Intermediate result to be published soon on scheme-dependence of LD window

<https://indico.global/event/14504/contributions/138066/>



42ND INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY

Tata Institute of Fundamental Research
Mumbai, India

02-08 November, 2025

MACHINES AND EXASCALE COMPUTING IN LQCD

A review of current hardware trends in HPC

LATTICE 2025, MUMBAI – 06/11/2025 – S. KRIEG

Member of the Helmholtz Association



<https://indico.global/event/14504/contributions/137954/>

A BIT OF HISTORY

<https://indico.global/event/14504/contributions/137954/>

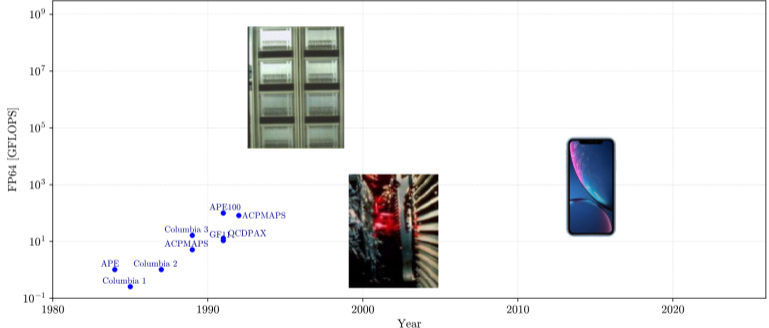
SIMULATIONS OF LATTICE QCD

A history of counting cycles

- Immense costs of LQCD calculations drove the development of HPC systems
- Lattice QCD researchers joined hardware vendors to develop hardware suitable for “the cause”
- Initially, only **quenched** calculations were possible
→ fermion determinant ignored → unquantifiable systematics
- Still, a complete quenched spectrum calculation took **20 years** and the fastest supercomputers of their time
- Status 1983: $m_N = 1000(150)$ MeV, $m_p = 800(150)$ MeV, $m_\Delta = 1300(150)$ MeV (incl. SU(2), discrete approx. etc.)

SIMULATIONS OF LATTICE QCD

A history of counting cycles



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06/11/2025

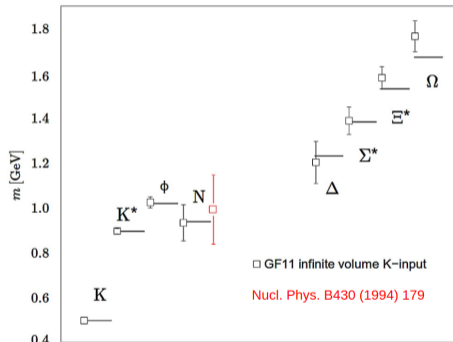
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<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

10 years of progress: early (quenched) calculations



Adapted from
hep-lat/9904003

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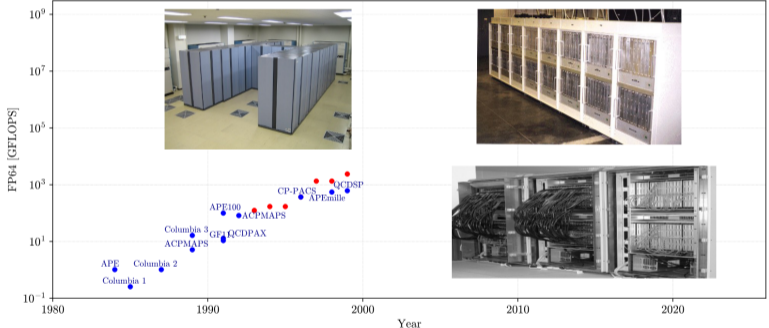
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<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

A history of counting cycles



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06/11/2025

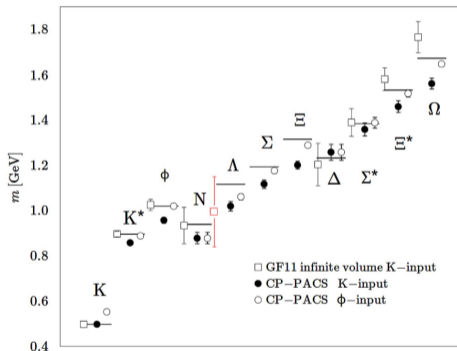
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<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

20 years of progress: end of the quenched era



- Continuum limit
- $m_\pi = 300$ MeV
- Systematics are $\approx 10\%$

Phys. Rev. Lett. 84 (2000) 238

Adapted from
hep-lat/9904003

SIMULATIONS OF LATTICE QCD

The Berlin Wall

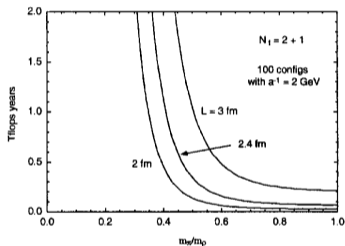


Figure 3. Cost of $N_f = 2+1$ QCD at $a^{-1} = 2$ GeV for 100 independent configurations.

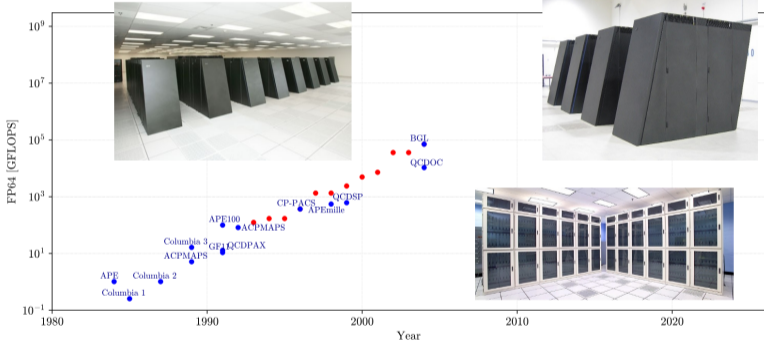
Nucl. Phys. B (PS) 106–107 (2002) 195

Algorithmic changes were needed for physical simulations. Hardware performance was not enough

- Optimized integrators
- Multigrid solvers
- Hasenbusch mass preconditioning
- Multiple integration step-sizes

SIMULATIONS OF LATTICE QCD

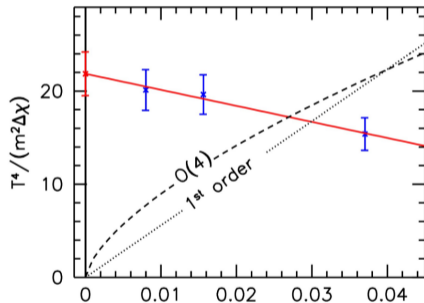
A history of counting cycles



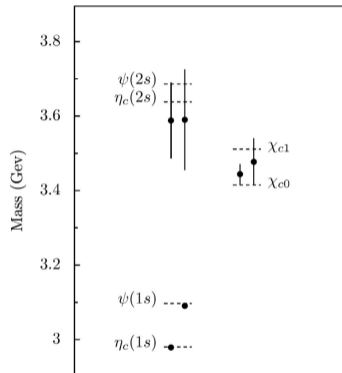
<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

Era of unquenched simulations



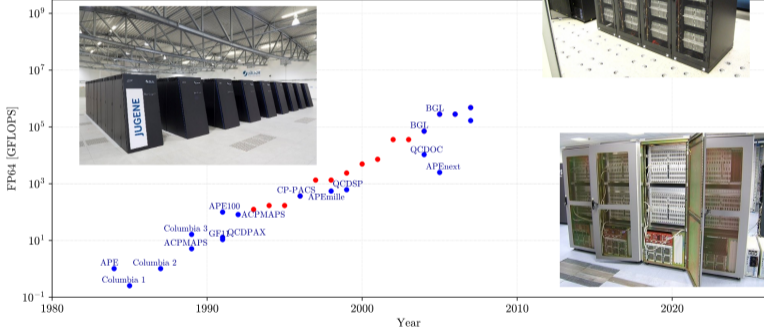
Nature 443 (2006) 675 $1/(T_c^3 V)$



Phys. Rev. D 75 (2007) 054502

SIMULATIONS OF LATTICE QCD

A history of counting cycles



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06/11/2025

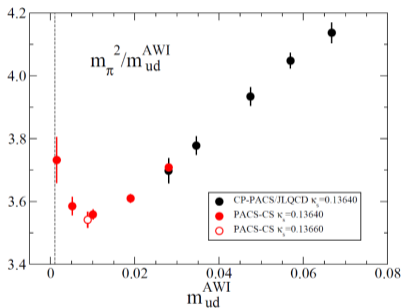
11



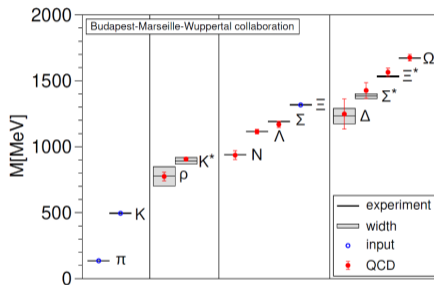
<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

30 years of progress: dynamical simulations, physical point, and spectrum



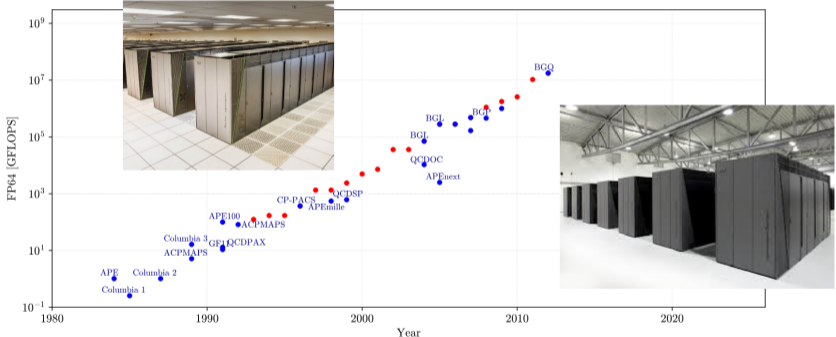
Phys. Rev. D 79 (2009) 034503



Science 322 (2008) 1224

SIMULATIONS OF LATTICE QCD

A history of counting cycles



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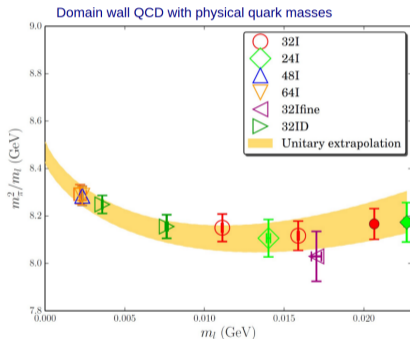
13



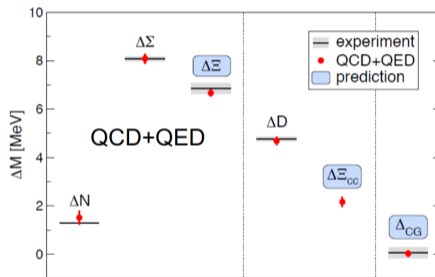
<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

New milestone: physical point with chiral fermions, dynamical QCD+QED



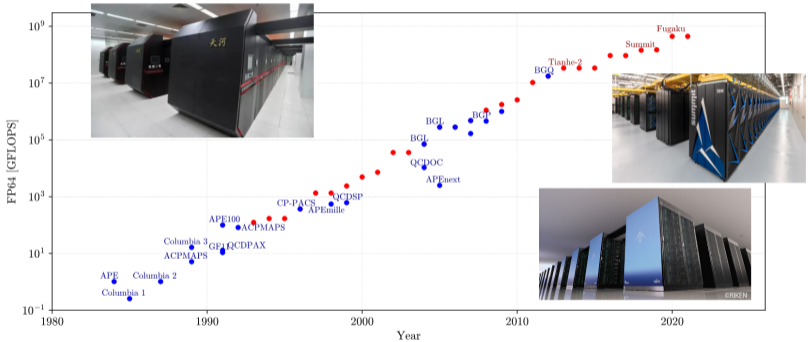
Phys. Rev. D 93 (2016) 074505



Science 347 (2015) 1452

SIMULATIONS OF LATTICE QCD

The end of specialized hardware and rise of GPUs



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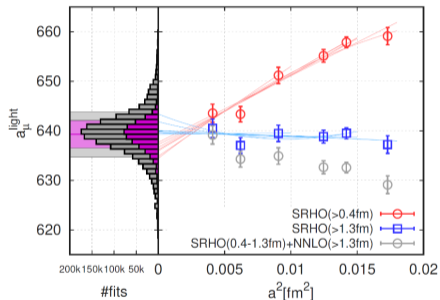
15



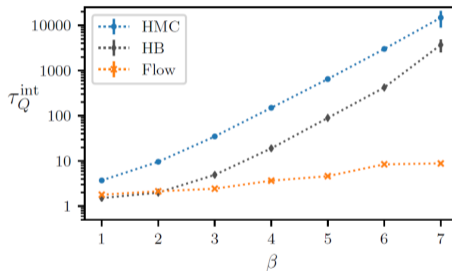
<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

Precision physics and the arrival of AI



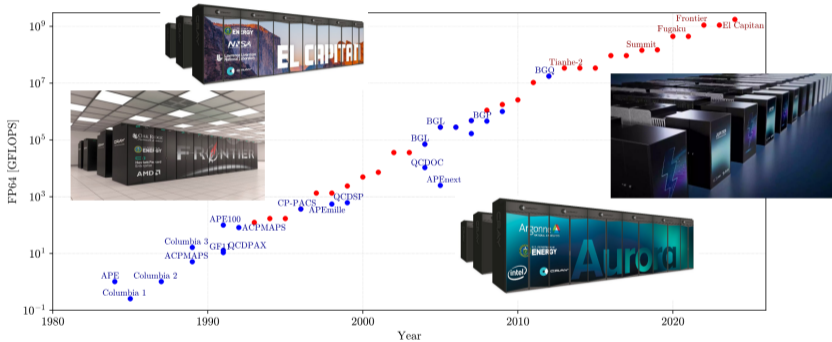
Nature 593 (2021) 51



Phys. Rev. Lett. 125 (2020) 121601

SIMULATIONS OF LATTICE QCD

The present: Exascale computing



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17



<https://indico.global/event/14504/contributions/137954/>

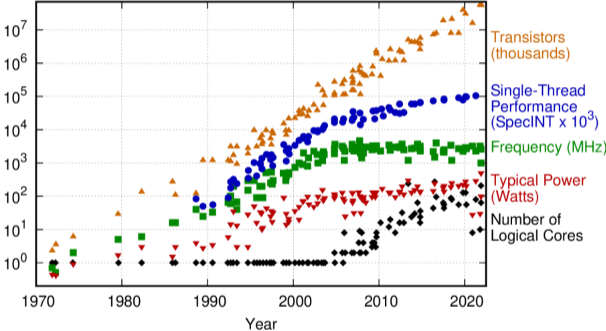
THE PRESENT

<https://indico.global/event/14504/contributions/137954/>

MOORE'S LAW...

50 Years of Microprocessor Trend Data

<https://github.com/karirupp/microprocessor-trend-data>

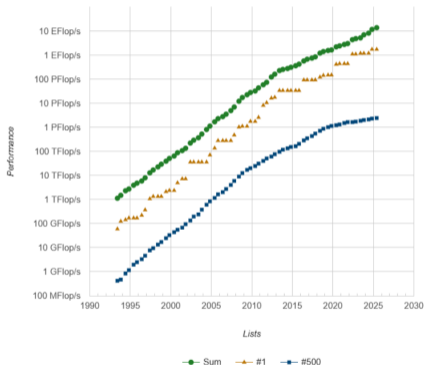


Original data up to the year 2010 collected and plotted by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten
New plot and data collected for 2010-2021 by K. Rupp

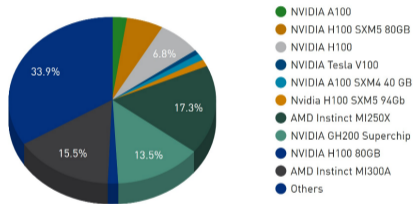
<https://indico.global/event/14504/contributions/137954/>

THE LIST (JUNE '25)

Performance Development



Accelerator/Co-Processor Performance Share



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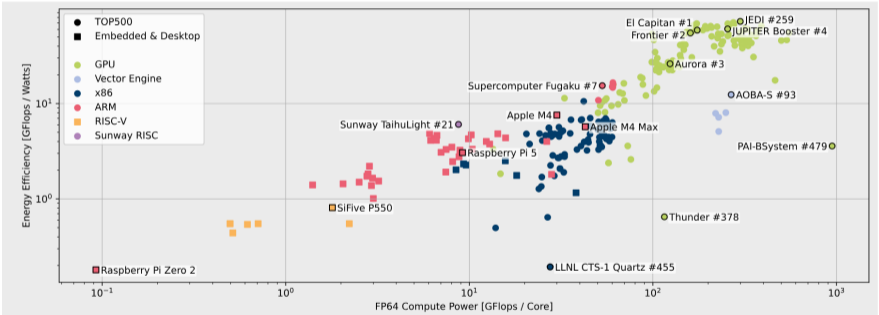
20



<https://indico.global/event/14504/contributions/137954/>

ENERGY EFFICIENCY OF VARIOUS ARCHITECTURES

High-Performance Linpack Benchmark



Sources: top500.org (2025-06) | github.com/geerlingguy/top500-benchmark

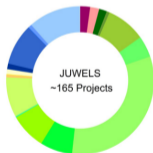


<https://indico.global/event/14504/contributions/137954/>

SIMULATIONS OF LATTICE QCD

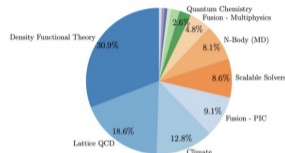
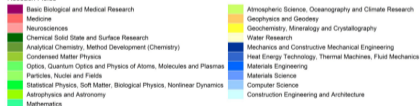
From custom hardware to “regular” users

- LQCD runs on most HPC centers in the world
- Optimized codes for different classes of target hardware to improve performance
- Consumer of 10+% of public supercomputer cycles



[JUWELS@JSC 2022]

Research Fields



[Perlmutter@NERSC 2025]

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06/11/2025

22



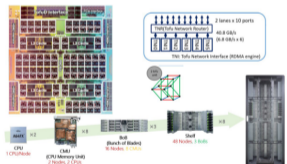
<https://indico.global/event/14504/contributions/137954/>

PRE-EXASCALE SYSTEMS

<https://indico.global/event/14504/contributions/137954/>

ARM A64FX

Fugaku @ RIKEN - #7

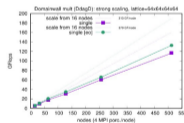
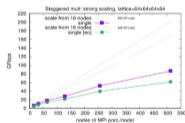
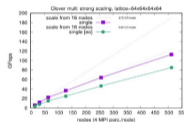
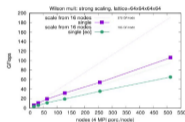


Yoshifumi Nakamura. Software development and performance of Fugaku and ARM architectures. PoS LATTICE2021 023 2022

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06/11/2025

- 158,976 nodes
 - 1x Fujitsu ARM A64FX
 - HBM2 32 GiB, 1024 GB/s
 - Tofu Interconnect D (28 Gbps x 2 lane x 10 port)
- 7,630,848 total cores
- 442.0 PFlop/s on HPL benchmark (#7 on Top500)
- 16.0 PFlop/s on HPCG benchmark (#2 on Top500)



Strong scaling of Wilson Clover, Staggered and Domain Wall operators on Fugaku with Bridge++2.0.

Aoyama et. al. Bridge++2.0: Benchmark Results on supercomputer Fugaku. PoS LATTICE2022 284 2023

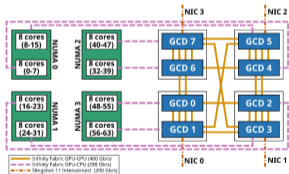


25

<https://indico.global/event/14504/contributions/137954/>

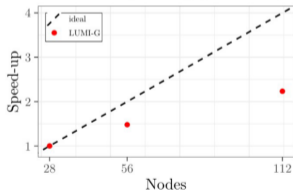
AMD MX250

LUMI-G @ CSC - #9



- 2,978 GPU nodes
 - 4x AMD MI250X
 - 64-core AMD EPYC 7A53 "Trento" CPU
 - 512 GiB CPU memory
 - Infinity Fabric Interconnect
- 2,566,080 total accelerator cores
 - 379.7 PFlop/s on HPL benchmark (#9 on Top500)
 - 4.6 PFlop/s on HPCG benchmark (#5 on Top500)

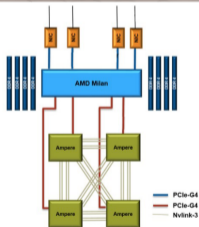
LUMI-G strong scaling ($112^3 \cdot 224$)



Strong scaling on LUMI-G with tmLQCD + QUDA. Extend Twisted Mass Collab. Status of the ETMC ensemble generation effort. LATTICE24

NVIDIA A100

Perlmutter @ NERSC - #25

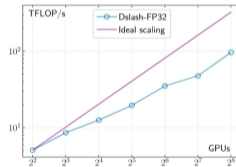
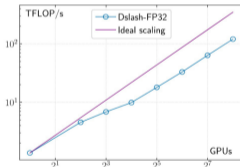


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28

- 1792 GPU nodes
 - 4x NVIDIA A100 40 GB (256 nodes with 80 GB)
 - 1x AMD EPYC 7763
 - 256 GB DDR4 DRAM
 - 4x HPE Slingshot 11
- 774,144 total accelerator cores
- 79.2 PFlop/s on HPL benchmark (#25 on Top500)
- 1.9 PFlop/s on HPCG benchmark (#9 on Top500)



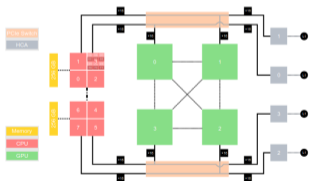
HISQ Dirac operator scaling on Perlmutter in
 SIMULATEQCD. HotQCD Collab. Computer Physics
 Communications 300 (2024) 109164 March 2024



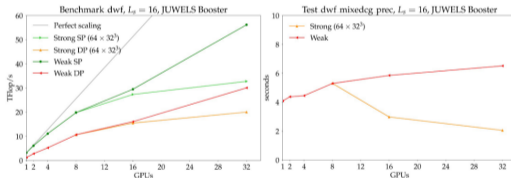
<https://indico.global/event/14504/contributions/137954/>

NVIDIA A100

JUWELS Booster @ JSC - #43



- 936 GPU nodes
- 4x NVIDIA A100 40 GB
- AMD EPYC 7402 with 2 sockets and 24 cores per socket
- 512 GB DDR4 DRAM
- 4x Mellanox HDR200 InfiniBand ConnectX 6
- 404,352 total accelerator cores
- 44.1 PFlop/s on HPL benchmark (#43 on Top500)
- 1.3 PFlop/s on HPCG benchmark (#12 on Top500)



Scaling of Domain Wall fermions on JUWELS with Grid. PUNCH4NFDI.

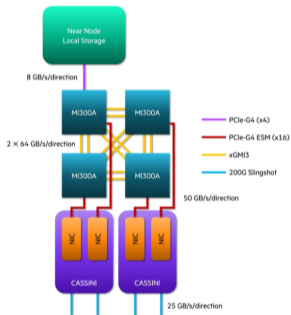


EXASCALE SYSTEMS

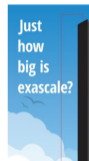
<https://indico.global/event/14504/contributions/137954/>

EL CAPITAN

LLNL, USA



CPU: AMD EPYC "Genoa"
GPU: **AMD Instinct MI300A** (4 per node)
APU (CPU GPU bundle) 128 GB HBM3 each
Performance peak: 1.74 Eflops (29.6 MW)
Deployment: 2024
#1 in TOP500



El Capitan

Tuolumne

Sierra

Sequoia

Blue Waters

Purple

White



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<https://indico.global/event/14504/contributions/137954/>



Иваньковская ГЭС



Страна	 Россия
Местоположение	Дубна ^[1]
Река	Волга
Каскад	Волжско-Камский
Собственник	ФГБУ «Канал имени Москвы»
Статус	действующая
Год начала строительства	1933
Годы ввода агрегатов	1937-1938
Основные характеристики	
Годовая выработка электроэнергии, млн кВт·ч	119
Разновидность электростанции	плотинная русловая
Расчётный напор, м	13
Электрическая мощность, МВт	28,8

<https://indico.global/event/14504/contributions/137954/>

FRONTIER

ORNL, USA

CPU: AMD EPYC "Trento", 512 GB
GPU: **AMD MI250** (4 per node, 128 GB each)
Performance peak: 1.35 Eflops (24.6 MW)
Deployment: 2022
#2 in TOP500

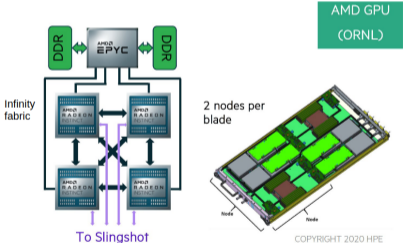
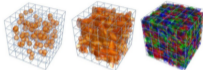


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QCD Constraints on Isospin-Dense Matter and the Nuclear Equation of State

Phys. Rev. Lett. 134, 011903 – Published 6 January, 2025



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<https://indico.global/event/14504/contributions/137954/>

AN INCREASINGLY DIVERSE LANDSCAPE



JUPITER@JSC (Germany, EU)
CPU: ARM Grace (GH200)
GPU: **H100** (GH200)
Performance peak: 0.79 Eflops
Deployment: 2025
#4 in TOP500



Fugaku @ Riken (Japan)
CPU: ARM
GPU: **none**
Performance Peak: 0.44 Eflops
Deployment: 2021
#7 in TOP500



ALPS@CSCS (Switzerland)
CPU: ARM Grace (GH200)
GPU: **H100** (GH200)
Performance peak: 0.43 Eflops
Deployment: 2024
#8 in TOP500



LUMI@CSC (Finland, EU)
CPU: AMD EPYC
GPU: **AMD MI250**
Performance peak: 0.38 Eflops
Deployment: 2023
#9 in TOP500



Leonardo@CINECA (Italy, EU)
CPU: Intel Xeon
GPU: **Nvidia A100**
Performance peak: 0.17 Eflops
Deployment: 2022
#10 in TOP500

June 2025: 4/10 top systems use Nvidia GPUs, 4/10 AMD GPUs, 1/10 Intel GPUs, 1/10 no GPUs
November 2022: 5/10 top systems use Nvidia GPUs, 3/10 no GPUs, 2/10 AMD GPUs, 0/10 Intel GPUs
November 2021: 7/10 top systems used Nvidia GPUs, 3/10 no GPUs, 0/10 AMD or Intel GPUs

<https://indico.global/event/14504/contributions/137954/>

THE FUTURE

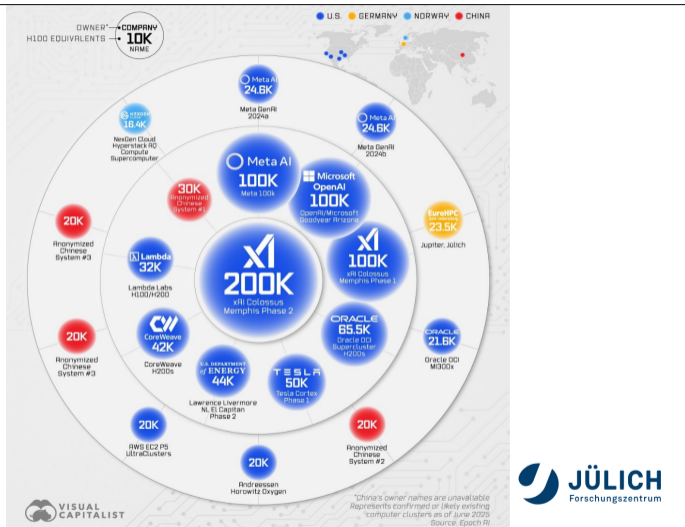
<https://indico.global/event/14504/contributions/137954/>

REALITY...

...sets in

AI takes over

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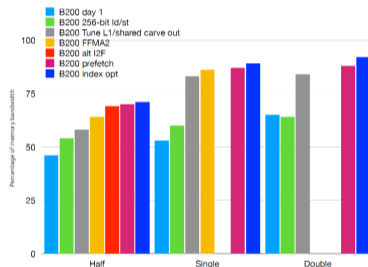
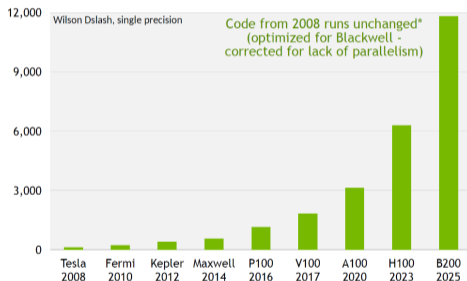


<https://indico.global/event/14504/contributions/137954/>

HARDWARE TRENDS

See talk by Mathias Wagner Tue 17:40

LQCD performance on NVIDIA



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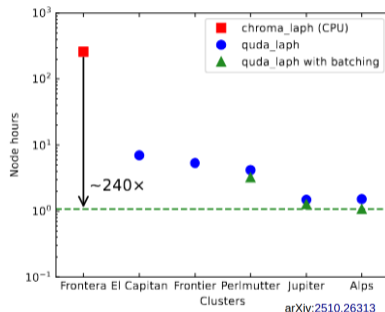


<https://indico.global/event/14504/contributions/137954/>

HARDWARE TRENDS

.. and usage consequences

- Performance portability
 - Leverage libraries that can easily handle multiple architectures well without changes to algorithms (e.g. Kokkos)
- Task scheduling / job bundling
 - Increase the average size of nodes and improve workflows to play nicely with the schedulers on very large machines. Either in code or with external tools like Flux.
 - Maximize node utilization, don't leave CPUs idling
- Better data management workflows
 - With exascale computing comes exascale data
 - Hopefully upcoming ILDG extensions to new file formats and new data objects (propagators etc...)



QUANTUM COMPUTING

A very broad landscape

Several computing paradigms



Quantum Annealer



Analog Quantum Computer



Universal Quantum Computer

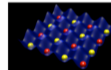
Several hardware technologies



Superconducting



Photonic



Neutral Atoms



Trapped Ions



Quantum Dots

A very active field of research for lattice gauge theories.

→ See plenaries after the break & ask the panel!

SUMMARY

- Pressure by needs of AI applications changes the vendor roadmaps
 - Mixed precision is nothing new
 - Bootstrapping from low precision shown to work (even integers)
- Energy efficiency will be a concern for scientific HPC applications (can't afford a backyard nuclear power plant)
 - A window for "specialized" hardware may open again
 - What this means for software is yet unclear

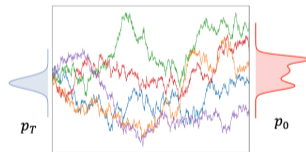
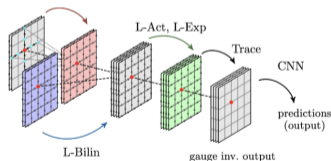
Thank you!
Christoph Lehner
Peter Boyle
Kate Clark
Mathias Wagner
Steve Gottlieb
Tilo Wettig
Maria Lombardo
Jacob Finkenrath
Bartosz Kostrzewa
Marco Garofalo
Aniket Sen
Leon Hostetler
Giovanni Pederiva
Simon Schlepphorst
Travis Whyte
Eric Gregory

Machine learning for lattice gauge theories

with a focus on 4d SU(3)

Urs Wenger

University of Bern



42nd International Lattice Conference, 6 November 2025 - TIFR Mumbai, India

<https://indico.global/event/14504/contributions/138395/>

Why machine learning in lattice gauge theories?

In lattice gauge theories we typically have:

$$Z(\beta) = \int \mathcal{D}U \exp\{-\beta A[U]\} \quad \text{with gauge coupling} \quad \beta = \frac{2N_c}{g^2}$$

and expectation values for observables:

$$\langle \mathcal{O}_\xi \rangle_\beta = \frac{1}{Z(\beta)} \int \mathcal{D}U \exp\{-\beta A[U]\} \mathcal{O}_\xi[U]$$

with a characteristic length scale ξ in units of the lattice spacing a :

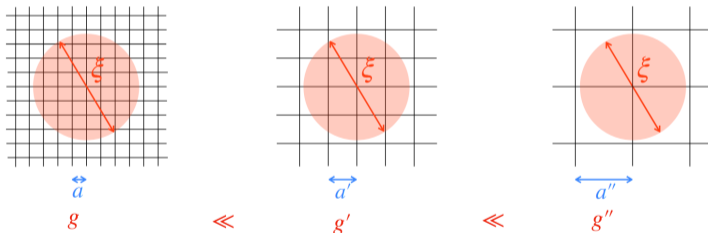
$$\frac{\xi}{a} \Rightarrow \text{dimensionless}$$

<https://indico.global/event/14504/contributions/138395/>

Why machine learning in lattice gauge theories?

The lattice spacing a is determined by the gauge coupling: $\beta = \frac{2N_c}{g^2}$

← continuum limit (2nd order phase transition $\xi/a \rightarrow \infty$)

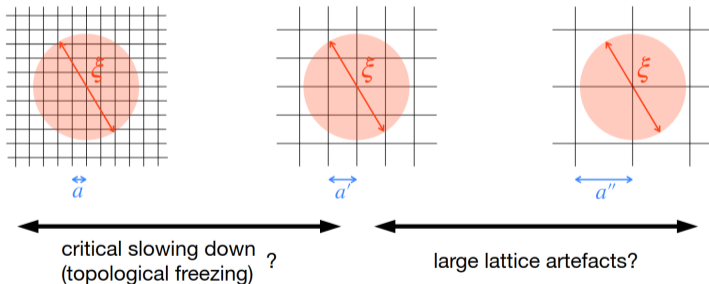


<https://indico.global/event/14504/contributions/138395/>

Why machine learning in lattice gauge theories?

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<https://indico.global/event/14504/contributions/138395/>

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and expectation values for observables:

$$\langle \mathcal{O} \rangle_\beta = \frac{1}{Z(\beta)} \int \mathcal{D}U \exp\{-\beta A[U]\} \mathcal{O}[U]$$

⇒ improve observables using ML:

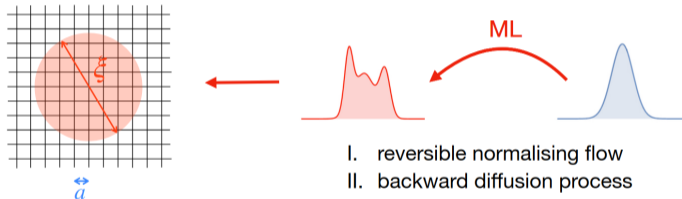
- improve signal-to-noise ratio: Lawrence (plenary talk 2024) [2502.02670]
control variates, surrogate variables,
contour deformations Detmold, Kanwar, Lin, Shanahan, Wagman [2410.03602]
- operator insertions
(Feynman-Hellman) Abbott, Boyda, Fu, Hackett, Kanwar, Romero-López, Shanahan [poster]
- optimize wave functions Mayer-Steuerte [Tue, 14:50], Romiti [2510.26904]

<https://indico.global/event/14504/contributions/138395/>

ML approaches to avoid critical slowing down

A. **Generative ML models** \Rightarrow generate **uncorrelated samples** at **fine lattice spacings**

Map a normal distribution to a target distribution



III. Stochastic normalising flows:

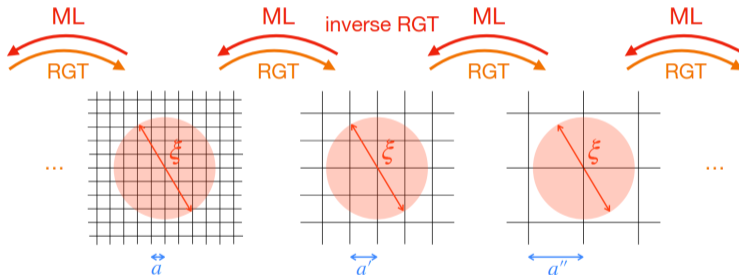
ML

\Rightarrow non-equilibrium Markov Chain Monte Carlo (NE-MCMC), Jarzynski's equality

<https://indico.global/event/14504/contributions/138395/>

ML approaches to avoid critical slowing down

B. Renormalization group transformation (RGT) relates fine to coarse lattices:



⇒ generate **uncorrelated coarse lattices** with **small lattice artefacts**

<https://indico.global/event/14504/contributions/138395/>

Approach A.I: Normalizing flows

⇒ covered by [G. Kanwar, plenary talk 2023 \[2401.01297\]](#)

<https://indico.global/event/14504/contributions/138395/>

A.I Normalising flows

Learn discrete or continuous flows
from prior distribution:

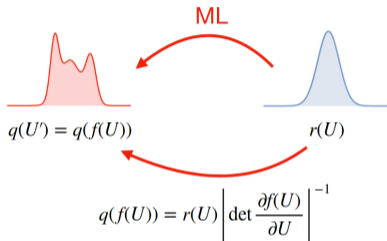
⇒ requires **invertible Jacobians**

⇒ achieved through **coupling layers**:

$$f \equiv g_m \circ \dots \circ g_1$$

with g_i equivariant and easily invertible

⇒ self-learning through
Kullback-Leibler divergence:

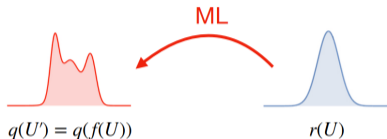


$$D_{KL}(q||p) \equiv \int \mathcal{D}U [\log q(U) - \log p(U)]$$

<https://indico.global/event/14504/contributions/138395/>

A.I Normalising flows

Learn discrete or continuous flows
from prior distribution:



⇒ for SU(N) gauge theories pioneered by the MIT group:

Albergo, Boyda, Cranmer, Hackett, Kanwar, Racanière, Jimenez Rezende, Shanahan
PRL 125 (2020) 121601 [2003.06413], PRD 103 (2021) 074504 [2008.05456]

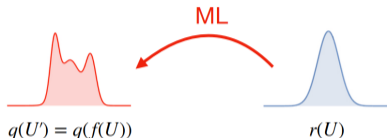
⇒ extension to 4d:

+ Abbott, Botev, Matthews, Razani, Romero-López, Urban [2305.02402, 2502.00263]

<https://indico.global/event/14504/contributions/138395/>

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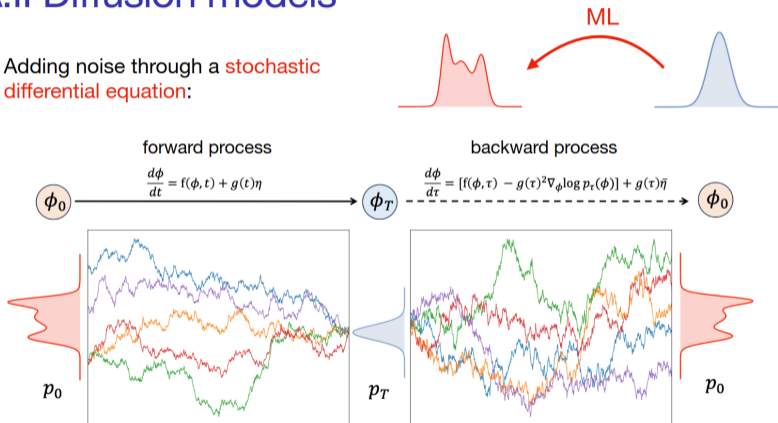
+ Abbott, Botev, Matthews, Razani, Romero-López, Urban [2305.02402, 2502.00263]

⇒ scaling to 4d, SU(N), large volumes and fine lattice spacings is challenging,
progress has slowed down

<https://indico.global/event/14504/contributions/138395/>

A.II Diffusion models

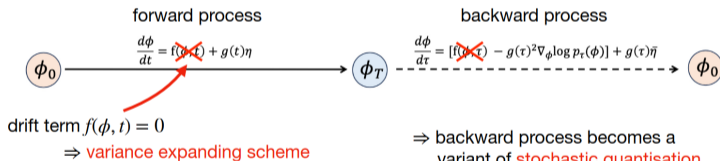
Adding noise through a **stochastic differential equation**:



<https://indico.global/event/14504/contributions/138395/>

A.II Diffusion models

Adding noise through a **stochastic differential equation**:



Aarts, W. Wang, L. Wang, Zhu, Zhou [2502.05504]

El-Khadra, Komanji, Marinkovic, Vega [2510.26081]

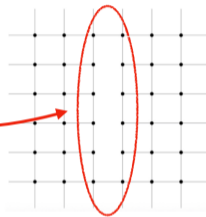
\Rightarrow so far only U(1) in 2d

but see Vega [Fri, 14:30] for extension to SU(N)

<https://indico.global/event/14504/contributions/138395/>

A.III Stochastic normalising flows

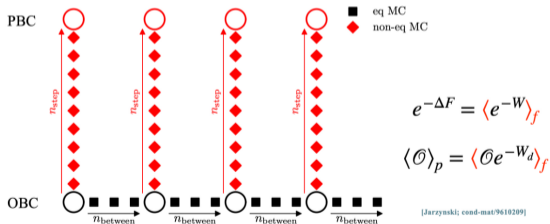
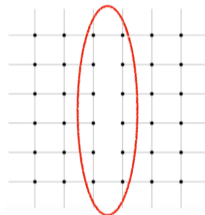
- Mitigate topological freezing by adding a **defect with open b.c.**



<https://indico.global/event/14504/contributions/138395/>

A.III Stochastic normalising flows

- Mitigate topological freezing by adding a **defect with open b.c.**
- Instead of expensive parallel tempering:
 ⇒ use non-equilibrium MCMC
 Bonanno, Nada, VDACCHINO



<https://indico.global/event/14504/contributions/138395/>

A.III Stochastic normalising flows

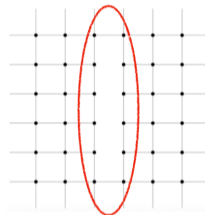
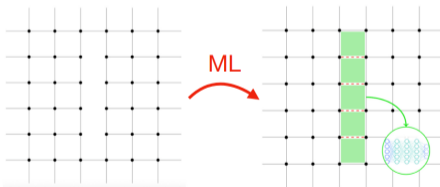
- Mitigate topological freezing by adding a **defect with open b.c.**
- Improve NE-MCMC using **stochastic normalising flows:**

Bulgarelli, Cellini, Nada

PRD 111 (2025) 074517 [2412.00200]

+ Bonanno, Panfalone, VDACCHINO, Verzichelli

[2510.25704]



⇒ speedup by factor ~ 3

⇒ scales to $\beta = 6.4, (L/a)^4 = 30^4$

$$U_0 \rightarrow g_\theta^1(U_0) \xrightarrow{P_1} U_1 \rightarrow g_\theta^2(U_1) \xrightarrow{P_2} \dots \xrightarrow{P_N} U_N = U$$

Cellini [Mon, 16:40]

<https://indico.global/event/14504/contributions/138395/>

Approach B: Inverse RGT



in collaboration with **Kieran Holland** (University of Pacific), **Andreas Ipp** and **David Müller** (TU Wien)

[Phys. Rev. D110 \(2024\) 7, 074502](#); [arXiv:2401.06481 \[hep-lat\]](#), [arXiv:2504.15870 \[hep-lat\]](#)

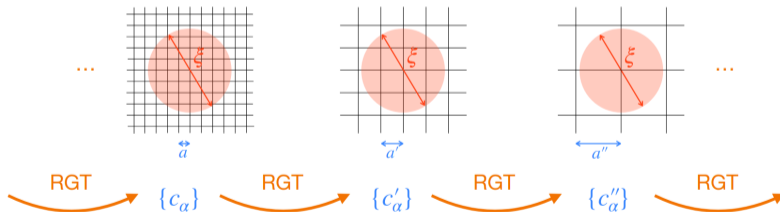
<https://indico.global/event/14504/contributions/138395/>

Renormalization group transformation

Introduce (real space) renormalization group transformation (RGT):

$$\exp \{-\beta' A[V]\} = \int \mathcal{D}U \exp \{-\beta (A[U] + T[U, V])\}$$

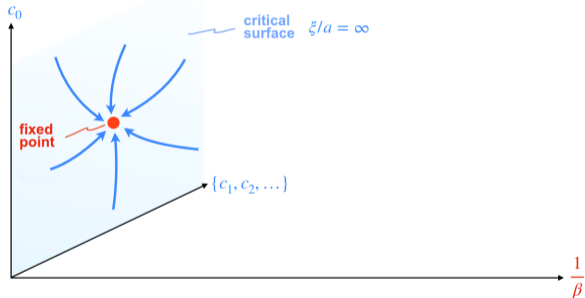
The effective action $\beta' A[V]$ is described by infinitely many couplings $\{c'_\alpha\}$:



<https://indico.global/event/14504/contributions/138395/>

Renormalization group transformation

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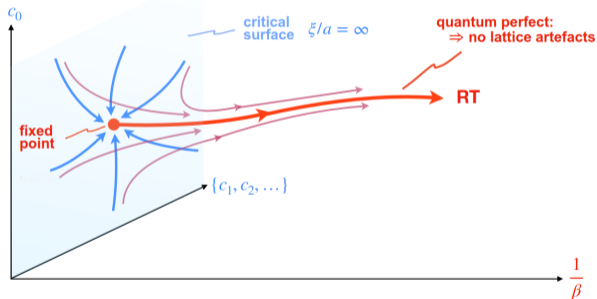


\Rightarrow fixed point of RGT iterations (when $\xi/a \rightarrow \infty$): $\{c_\alpha^{FP}\} \xrightarrow{\text{RGT}} \{c_\alpha^{FP}\}$

<https://indico.global/event/14504/contributions/138395/>

Renormalization group transformation

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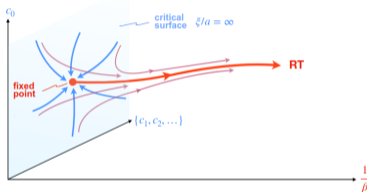


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<https://indico.global/event/14504/contributions/138395/>

Renormalization group transformation

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$$\exp\{-\beta' A[V]\} = \int \mathcal{D}U \exp\{-\beta(A[U] + T[U, V])\}$$

Two practical problems:

- how to parametrize **RT**, i.e., which set $\{c_\alpha\}$?
- how to determine $\{c_\alpha^{\text{RT}}\}$ or $\{c_\alpha^{\text{FP}}\}$?

For $\beta \rightarrow \infty$ (on critical surface) the **RG** becomes a **classical saddle point problem**:

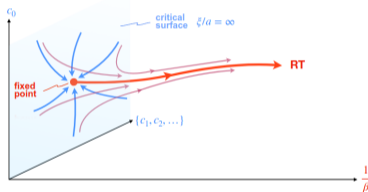
$$A^{\text{FP}}[V] = \min_{\{U\}} \{A^{\text{FP}}[U] + T[U, V]\}$$

Hasenfratz, Niedermayer [Nucl. Phys. B414 (1994) 785, hep-lat/9308004]

<https://indico.global/event/14504/contributions/138395/>

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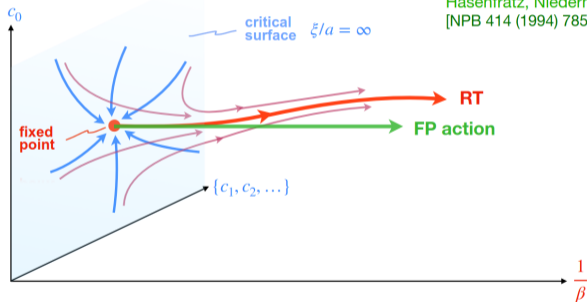
Hasenfratz, Niedermayer [Nucl. Phys. B414 (1994) 785, hep-lat/9308004]

<https://indico.global/event/14504/contributions/138395/>

Classically perfect FP actions

The classical FP action A^{FP} defines an action for all β :

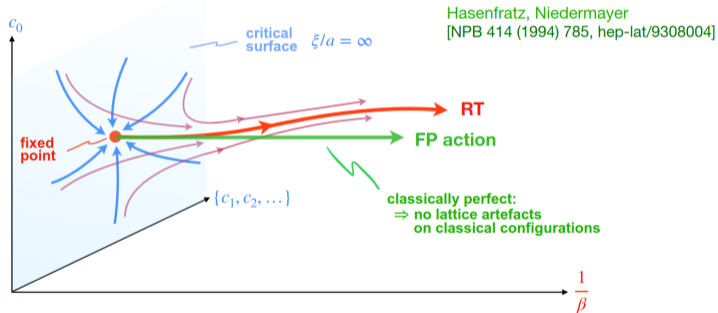
Hasenfratz, Niedermayer
[NPB 414 (1994) 785, hep-lat/9308004]



<https://indico.global/event/14504/contributions/138395/>

Classically perfect FP actions

The FP action values for coarse configurations defined through an inception procedure:



$$A^{\text{FP}}[V] = \min_{\{U\}} \{A^{\text{FP}}[U] + T[U, V]\} = \min_{\{U', U\}} \{A^{\text{FP}}[U'] + T[U', U] + T[U, V]\} = \dots$$

<https://indico.global/event/14504/contributions/138395/>

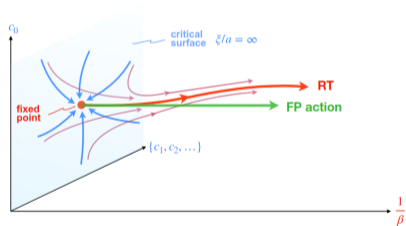
Classically perfect FP actions

Classically perfect properties:

- There are no lattice artefacts on classical configurations:
- For large β , $A^{\text{FP}}[V]$ is very close to $A^{\text{RT}}[V]$:

$$\frac{\delta A^{\text{FP}}[V]}{\delta V} = 0 \quad \Rightarrow \quad \frac{\delta A^{\text{FP}}[U]}{\delta U} = 0$$

$\Rightarrow A^{\text{FP}}[V]$ has scale invariant instanton solutions



<https://indico.global/event/14504/contributions/138395/>

Classically perfect FP actions

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$\Rightarrow A^{\text{FP}}[V]$ has scale invariant instanton solutions

\Rightarrow lattice artefacts expected to be substantially reduced:

~~$$\mathcal{O}(a^{2n}), \mathcal{O}(g^2 a^{2n}) \quad n = 1, 2, \dots$$~~

- **New property:** gradient flow with $A^{\text{FP}}[V]$ is classically perfect

\Rightarrow no lattice artefacts introduced through gradient flow

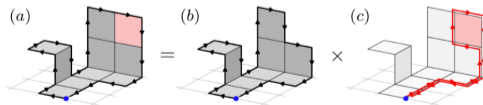
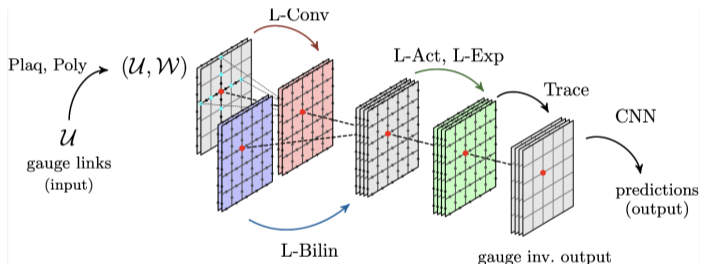
<https://indico.global/event/14504/contributions/138395/>

Machine learning the FP action

<https://indico.global/event/14504/contributions/138395/>

Machine learning the FP action

ML architecture: Lattice gauge equivariant Convolutional Neural Network (L-CNN)

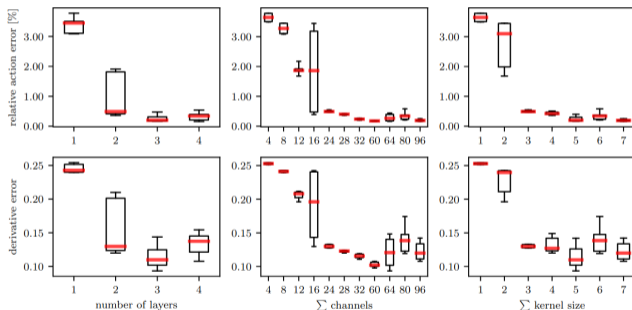


Favoni, Ipp, Müller, Schuh
PRL 128 (2022) 3, [2012.12901]

<https://indico.global/event/14504/contributions/138395/>

Machine learning the FP action: L-CNN

Architecture search:



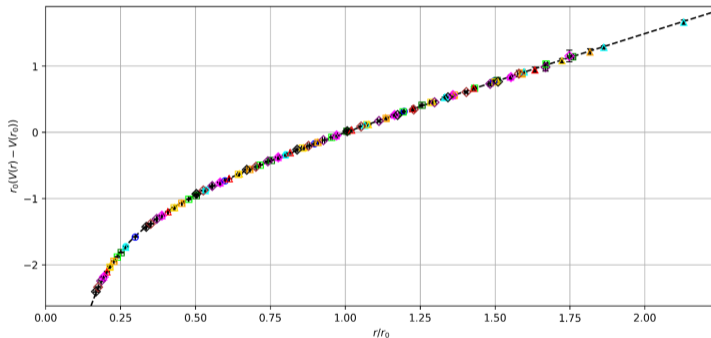
⇒ 3 L-Bilin layers, output channels {12,24,24}, kernel sizes {2,2,1} with 443k parameters

<https://indico.global/event/14504/contributions/138395/>

The FP action in action

<https://indico.global/event/14504/contributions/138395/>

Scaling of heavy-quark static potential



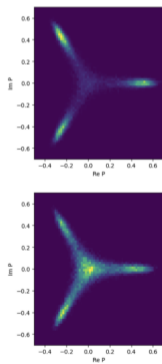
⇒ no lattice artefacts visible up to at least $a \sim 0.15$ fm

<https://indico.global/event/14504/contributions/138395/>

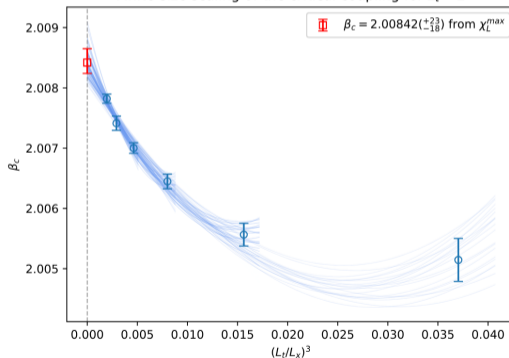
Deconfinement phase transition

Critical temperature T_c from $L_t = 2$:

\Rightarrow large L_s/L_t easily reachable



Finite-size scaling of the critical coupling for $L_t = 2$

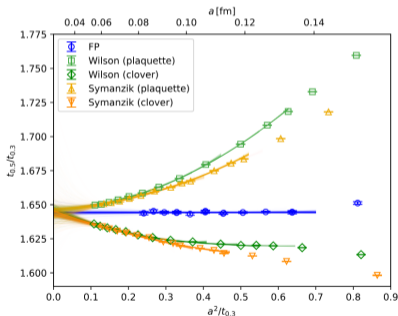


<https://indico.global/event/14504/contributions/138395/>

Scaling of gradient-flow scales

Physical reference scales defined through $t^2\langle E \rangle|_{t=t_x} = x$, $t \frac{d}{dt} (t^2\langle E \rangle) \Big|_{t=w_x^2} = x$

yielding dimensionless ratios t_x/w_x^2 or t_x/t_y as scaling quantities vs. $a^2/t_{0.3}$:



Note:

similar improvement with Zeuthen flow

[Ramos & Sint, Eur.Phys.J.C 76 (2016) 1, 15, [1508.05552](https://arxiv.org/abs/1508.05552) [hep-lat]]

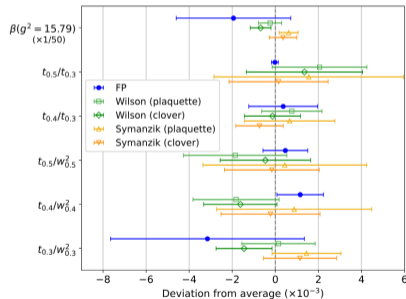
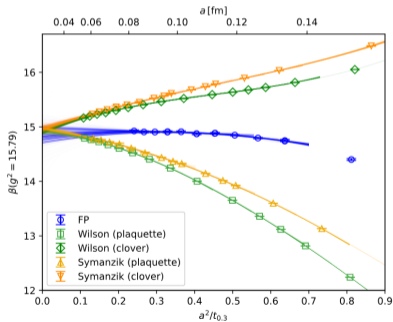
but only up to $a \lesssim 0.08$ fm.

<https://indico.global/event/14504/contributions/138395/>

Scaling of gradient-flow scales

Physical reference scales defined through $t^2 \langle E \rangle |_{t=t_x} = x$, $t \frac{d}{dt} (t^2 \langle E \rangle) |_{t=w_x^2} = x$

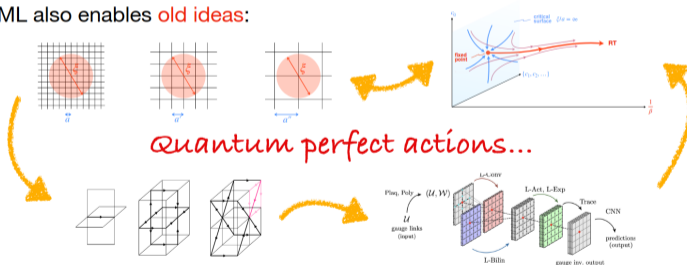
β -function at $g_{GF}^2 = 15.79$:



<https://indico.global/event/14504/contributions/138395/>

Conclusions: ML for LGT

- Scaling ML approaches to 4d SU(N) on large volumes is challenging:
 ⇒ many new ideas are available!
- ML also enables old ideas:



<https://indico.global/event/14504/contributions/138395/>

Quantum simulation of lattice gauge theories - Analogue methods and quantum complexities

Lattice 25@TIFR,
Mumbai, 6.11.25



Marcello Dalmonte
ICTP, Trieste



The Abdus Salam
International Centre
for Theoretical Physics



HEP Reviews: U. J. Wiese, Ann. Phys. 525, 777 (2013); Preskill, arXiv.1811.10085 (2018).
"Pedagogical": MD and S. Montangero, Cont. Rev. Phys. 2016 / 1602.03776.
More advanced ones: Rep. Prog. Phys. 79, 014401 (2016); 1910.00257; 1911.00003.

<https://indico.global/event/14504/contributions/137186/>

Vision



“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.”

R. P. Feynman, Int. J. Theor. Phys. (1982).

zero. But, we are going to be even more ridiculous later and consider bits written on one atom instead of the present 10^{11} atoms. Such nonsense is very entertaining to professors like me. I hope you will find it interesting and entertaining also.

R. P. Feynman,
Optics letters 1985

<https://indico.global/event/14504/contributions/137186/>

Outline

Interest in gauge theories - from QCD,
to condensed matter and quantum info



Digital and analogue quantum
simulation, and quantum hardware



Do we really need these machines?
Quantum resources and many-body



<https://indico.global/event/14504/contributions/137186/>

Why are gauge theories interesting from a quantum simulation viewpoint?

From the HEP side, a tautological question given the audience :)

- Fundamental description of nature, extremely well tested
- Despite outstanding achievements (spectrum, quark gluon plasma, BSM physics, and so many more), some aspects remain computationally challenging for classical approaches

Troyer and Wiese, PRL 2005

From condensed matter / quantum information:

- Topological matter (“quantum spin liquids”)
- Topological quantum memories

S. Sachdev “Quantum states of matter”

Moessner and Moore, “Topological matter”

<https://indico.global/event/14504/contributions/137186/>

Why is dynamics so hard?

$$\dim \mathcal{H} \propto e^N!$$

Stochastic methods: complex action problem!

$$\int \mathcal{D}\varphi e^{iS}$$

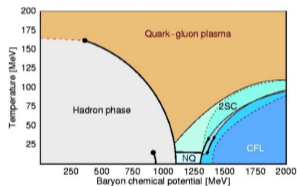
Wave function based methods
(e.g.: tensor networks, Pauli computing): quantum resources explode (see later)

$$S_{\text{ent}} \propto V$$

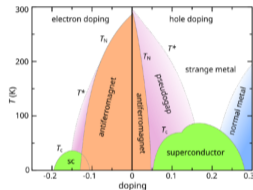
Exponential roadblocks

<https://indico.global/event/14504/contributions/137186/>

And equilibrium challenges



UniFrankfurt website



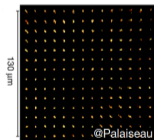
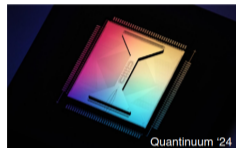
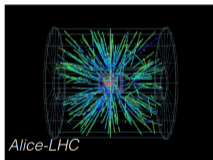
Wikimedia

Again, same technical roadblocks (*sampling and lots of quantum resources*)

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Vision

Real time dynamics / finite density regimes of strongly coupled theories



Noisy intermediate scale quantum (NISQ) devices

<https://indico.global/event/14504/contributions/137186/>

Outline

Interest in gauge theories - from QCD,
to condensed matter and quantum info



Digital and analogue quantum
simulation, and quantum hardware



Do we really need these machines?
Quantum resources and many-body



<https://indico.global/event/14504/contributions/137186/>

Classical digital and analog computation

Analog classical simulators



Analog Computation of Monte Carlo **FermiAC**

A collage of images related to the FermiAC computer, including a wooden frame structure, a hand operating a dial, and a portrait of Enrico Fermi.

The Monte Carlo tables, or FERMIAC, was an analog computer invented by physicist Enrico Fermi to aid in his studies of neutron transport.



Wind tunnel

Digital classical simulators



Pascaline



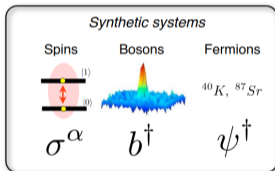
Supercomputers

<https://indico.global/event/14504/contributions/137186/>

Quantum digital and analog computation

Analog quantum simulators

Basic idea: **Emulation** - Build a *Hamiltonian* / *Lagrangian* / *Liouvillian* using the following:



Platforms: cold atoms, ions, Rydbergs, molecules, superconductors, ...

Cirac and Zoller, NatPhys. 2012; Georgescu et al., RMP2014

Digital quantum simulators

Basic idea: **Trotterize** a time-evolution

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \dots e^{-iH\Delta t_1/\hbar}$$

S. Lloyd, Science (1996)

Platforms: ions, superconductors, photons



See Zohreh's talk

Quantum Annealers



Yes, you can have one.

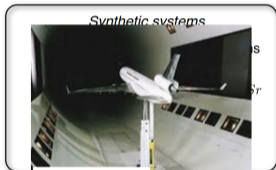
The world's most powerful quantum annealer. D-WAVE's Advantage™ quantum annealer is the most powerful quantum annealer in the world. It is the only quantum annealer that can solve real-world problems. It is the only quantum annealer that is scalable. It is the only quantum annealer that is available to researchers and developers. It is the only quantum annealer that is easy to use. It is the only quantum annealer that is affordable. It is the only quantum annealer that is reliable. It is the only quantum annealer that is secure. It is the only quantum annealer that is sustainable. It is the only quantum annealer that is... [D-WAVE](#)

<https://indico.global/event/14504/contributions/137186/>

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Wind tunnel

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Digital quantum simulators



Quantum Annealers

<https://indico.global/event/14504/contributions/137186/>

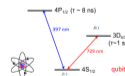
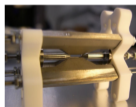
Analog quantum simulators in a nutshell

$$H = H_1 + H_2 + \dots$$



- a) (exact) mapping to microscopic degrees of freedom
- b) probing tools, protocols (e.g., state preparation)
- c) 'understanding' of errors

NB: Hamiltonian formulation is needed (see Indrakshi's talk)



<https://indico.global/event/14504/contributions/137186/>

Analog simulation: challenges

Typical challenges for quantum simulators:

- initial state preparation
- probing
- engineer the desired dynamics
- validate / control
- probing

- same as SM quantum simulators
- novel HEP challenges!



<https://indico.global/event/14504/contributions/137186/>

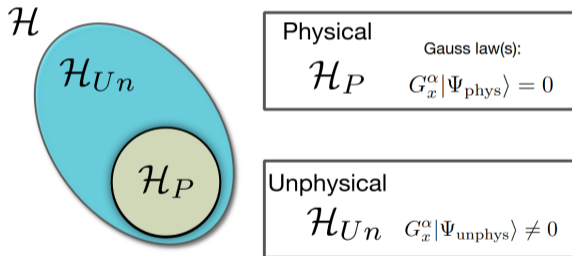
Challenge

Why is it so hard to quantum simulate *gauge* fields in analogue machines?

<https://indico.global/event/14504/contributions/137186/>

Gauge invariant Hilbert space

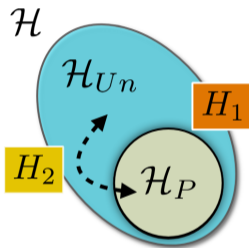
Hamiltonian formulation of gauge theories: **gauge invariance** is encoded **at the Hilbert space level**



Various formulations available - see Zohreh's talk

<https://indico.global/event/14504/contributions/137186/>

Brief summary of strategies



Work in the 'full' Hilbert space:

- energy penalties
- dissipative penalties
- exploit 'fundamental' symmetries
- other forms of dynamical constraints

Typically used in e.g. quantum magnets

<https://indico.global/event/14504/contributions/137186/>

Brief summary of strategies

$$H = H_1 + H_2 + \Delta \sum_x G_x^2$$

Work in the 'full' Hilbert space:

- energy penalties
- dissipative penalties
- exploit 'fundamental' symmetries
- other forms of dynamical constraints

$\Delta \gg f$
 Effective dynamics at low energy is **gauge invariant!**

H_1

$H_2 \simeq f$



Typically used in e.g. quantum magnets

<https://indico.global/event/14504/contributions/137186/>

State of the art: (1+1)-d, Abelian

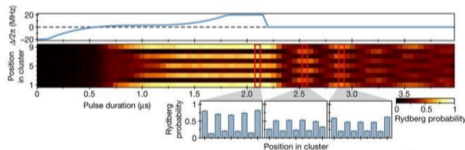
Multiple experiments with up to 100 degrees of freedom

“Exact” gauge invariance possible via matter or gauge integration in Rydberg systems

Th: Surace et al., PRX 2020. Formalisms: see Indrakshi’s talk!



Federica Surace



Bernien et al., Nature 2017 (Lukin’s group);

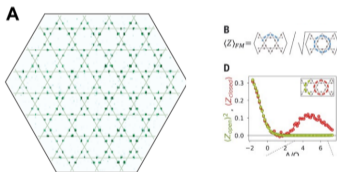
See also works at Caltech, Wuhan, Palaiseau, Tsinghua

<https://indico.global/event/14504/contributions/137186/>

State of the art: 2+1-d, Abelian

2+1D theories

Higgs+Z2 gauge structure



Exp: Semeghini et al., Science 374, 1242 (2021) [Lukin's group];
Numerics: Verresen et al., PRX '21
Gauge structure: Tarabunga et al., PRL '22

See also recent works on dynamics:

- Gonzalez-Cuadra et al., Nature 2025 (atoms)
- Cobos et al., arXiv:2507.08088 (cQED)
- Cochran et al., Nature 2025 (cQED)

<https://indico.global/event/14504/contributions/137186/>

Outline

Interest in gauge theories - from QCD,
to condensed matter and quantum info



Digital and analogue quantum
simulation, and quantum hardware



Do we really need these machines?
Quantum resources and many-body



<https://indico.global/event/14504/contributions/137186/>

Why is dynamics so hard?

$$\dim \mathcal{H} \propto e^N!$$

Stochastic methods: complex

Wave function based methods

- What is the bottleneck of error corrected quantum machines = quantum computers?
- Is it understood in gauge theories?

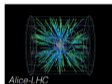
Pauli
sources

Exponential roadblocks

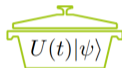
<https://indico.global/event/14504/contributions/137186/>

How does a quantum computer work?

(i) Problem of interest



(ii) Quantum algorithm



(iii) Physical gates



Most popular (surface code, etc.): Clifford + T

CNOT	Hadamard	Phase
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Bravyi and Kitaev, 2005

T gate
$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

<https://indico.global/event/14504/contributions/137186/>

What is really hard for quantum computers?

Most popular (surface code, etc.): Clifford + T

CNOT	Hadamard	Phase
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

“Easy”



- Easy fault tolerant implementation
- No quantum advantage - efficient classical computation

T gate
$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

“Hard”



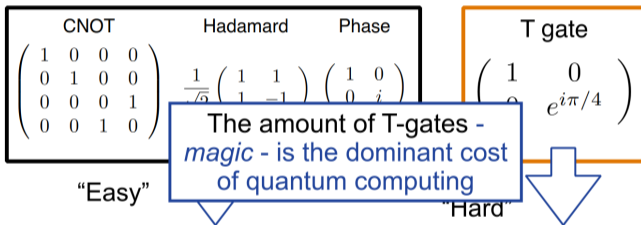
- Fault tolerance is very expensive
- Needed for universal quantum computing

Gottesman's phd thesis; Bravyi and Kitaev, 2005; Howard et al., Nature 2014

<https://indico.global/event/14504/contributions/137186/>

What is really hard for quantum computers?

Most popular (surface code, etc.): Clifford + T



- Easy fault tolerant implementation
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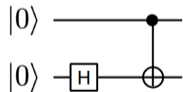
Gottesman's phd thesis; Bravyi and Kitaev, 2005; Howard et al., Nature 2014

<https://indico.global/event/14504/contributions/137186/>

A few examples

“Easy” state

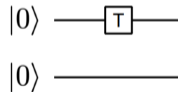
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



Only Clifford, no magic

“Hard” state

$$\frac{(|0\rangle + e^{i\pi/4}|1\rangle) \otimes |0\rangle}{\sqrt{2}}$$



Must have one T-gate,
finite magic

<https://indico.global/event/14504/contributions/137186/>

First studies in LGTs: upper bounds

Pioneering work on magic and LGTs
(Upper bounds)

x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Schwinger bosons	
							Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{12}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}
1	4	1000	10	0.001	90%	9%	19086	4.48617×10^{15}

2D SU(2), state preparation

Davoudi, Shaw, Stryker, Quantum 2024

QCD: $\sim 10^{30}$

<https://indico.global/event/14504/contributions/137186/>

Conclusions

- Real time dynamics in quantum simulators is already challenging the best available classical algorithms, also for gauge theories
- Open questions remain (scalable implementations, error estimate, probing)
- In parallel, the age of (developing) error correction motivates a combined view of many-body systems - *entanglement and magic*
- Very little is known on state complexity in quantum HEP! Tons to explore

See recent works by Robin, Savage on magic in nuclear physics!

<https://indico.global/event/14504/contributions/137186/>

For this route to succeed, key challenges that only lattice community can help solve

- New Hamiltonian formulations that are adapt to hardware (“co-design”)? (See Indrakshi’s talk)
- Complexity of gauge theories from the lens of quantum information: **entanglement and magic**
See recent works by Robin, Savage on magic in nuclear physics!
- Push boundaries of **classical computational methods for real-time dynamics** (and connections to CM)
- Learn new physics (e.g., out of equilibrium)
See 2509.03586 for a recent review

Mindset: focus on learning from eachother. True applications to QCD will come much later, but likely a lot of new physics learned in the meanwhile!

<https://indico.global/event/14504/contributions/137186/>

PART 2

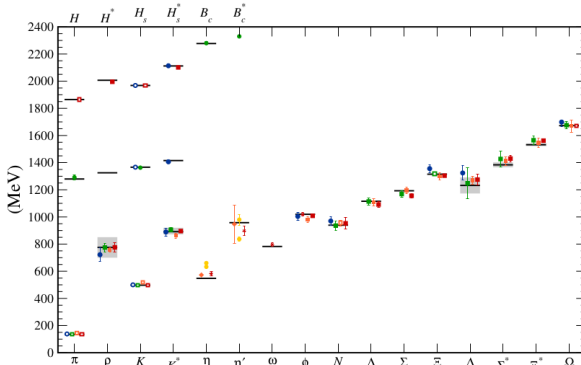
Hadron spectroscopy and interactions

Jeremy R. Green

Deutsches Elektronen-Synchrotron DESY

42nd International Symposium on Lattice Field Theory
TIFR, Mumbai, India
November 2–8, 2025

Hadron spectroscopy in 2012 (1)



A. S. Kronfeld, *Twenty-first Century Lattice Gauge Theory: Results from the QCD Lagrangian*, *Ann. Rev. Nucl. Part. Sci.* **62**, 265–284 (2012) [1203.1204]

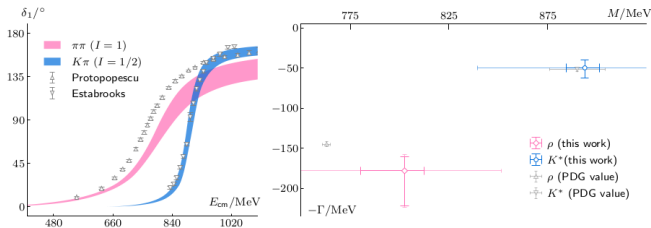
(B mesons shifted by -4 GeV.)

Calculations by different collaborations using different actions.

Some include $m_\pi \rightarrow m_\pi^{\text{phys}}$ and $a \rightarrow 0$ extrapolations.

Open symbols: inputs.

Light mesons: $\pi\pi$, πK



One ensemble with physical quark masses.

P. Boyle *et al.*, 2406.19193, 2406.19194

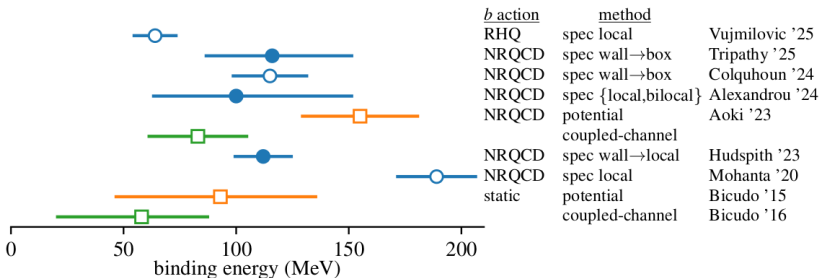
Calculations at physical pion mass. Also ETMC M. Fischer, 2006.13805 and CLQCD Z. Wang, 2502.03700.

Precision $\pi\pi$ physics?

Parallel talks:

- ▶ $I = 2$ $\pi\pi$ with staggered fermions. A. Dean M. Valois, Tue 15:30
- ▶ $K\pi$ for rare decay $B \rightarrow K^* \ell^+ \ell^-$. Felix Erben, Fri 16:40
- ▶ Spectral reconstruction above elastic region. Gabriele Morandi, Tue 15:10

T_{bb} : $I = 0$ $bb\bar{u}\bar{d}$ lattice QCD prediction of deeply bound state



open symbols: single lattice spacing or single pion mass

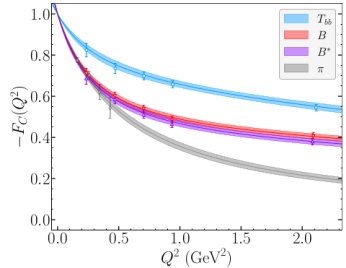
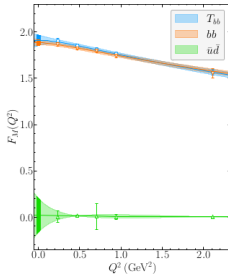
Consensus: deeply bound state. Can we make a precise prediction?

Note: left-hand cut starts 1 MeV below threshold. Can't use finite-volume quantization!

$bb\bar{s}$ bound state also predicted.

Electromagnetic form factors of T_{bb}

I. Vujmilovic *et al.*, 2510.17549; Ivan Vujmilovic, Wed 10:50
Smaller charge radius than B or B^* .



Magnetic moment is produced by b quarks.

Favours diquark-antidiquark structure:
s-wave binding of $[bb]$ 1^+ diquark with $[\bar{u}\bar{d}]$ 0^+ antidiquark.

Baryon-baryon

Consensus is emerging: no NN bound state at heavy m_π .

Amy Nicholson, next talk

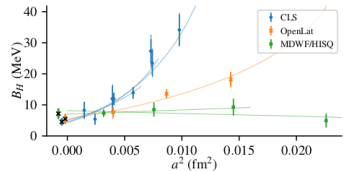
Need to compare scattering observables at similar m_π .

Caution: lattice artifacts might be significant!

JRG, 2502.15546; BaSc collaboration (in preparation)

Parallel talks:

- ▶ Lancoz applied to NN at $m_\pi = 800$ MeV Robert Perry, Fri 15:10
- ▶ Nuclear binding with varying pion mass Debsubhra Chakraborty, Fri 15:30
- ▶ central, tensor, and spin-orbit NN forces with free LapH Takuya Sugiura, Fri 15:50
- ▶ H dibaryon and $\Lambda\Lambda-N\Xi-\Sigma\Sigma$ at $m_\pi = 280$ MeV Davide Laudicina, Fri 14:50
- ▶ $N\Lambda-N\Sigma$ potentials at physical m_π Koichi Murase, Fri 14:30



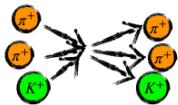
Three-particle scattering amplitudes from lattice QCD



Steve Sharpe
University of Washington



Three PGBs at maximal isospin



$$3\pi^+, \quad 3K^+, \quad \pi^+\pi^+K^+, \quad K^+K^+\pi^+$$

- Benchmark system, with simple repulsive dynamics
 - First calculation at physical quark masses
 - Study $J^P = 0^-, 1^+, 2^-$, including subchannels with $\ell = 0, 1, 2$
 - Compare to expectations from ChPT
- Use CLS ensembles ($\mathcal{O}(a)$ improved Wilson) + GEVP

	$(L/a)^3 \times (T/a)$	M_π [MeV]	M_K [MeV]	N_{cfg}	$M_\pi L$
N203	$48^3 \times 128$	340	440	771	5.41
N200	$48^3 \times 128$	280	460	1712	4.42
D200	$64^3 \times 128$	200	480	2000	4.20
E250	$96^3 \times 192$	130	500	505	4.05

[Blanton, SRS, et al.,
PRL 2020 & JHEP 2021]
[Draper, SRS, et al.,
JHEP 2023]

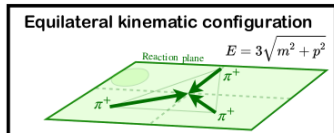
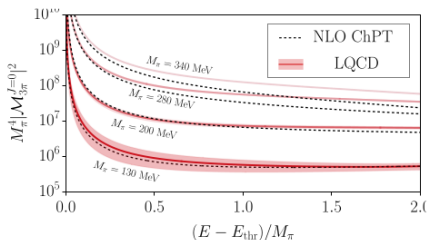
New work

$$a \simeq 0.063 \text{ fm}$$

Comparison with ChPT

$$M_\pi^4 |\text{Amplitude}|^2$$

$$J^P = 0^-$$



Observe expected breakdown of ChPT as M_π or p_π increase

Natural candidate for QC₃?

$\pi(1300)$	$I^G(J^{PC}) = 1^-(0^{-+})$	[PDGlive]		
$\pi(1300)$ MASS	[1] 1300 ± 100 MeV	▼		
$\pi(1300)$ WIDTH	[1] 200 to 600 MeV	▼		
$\pi(1300)$ DECAY MODES				
Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P(MeV/c)	
Γ_1 $\rho\pi$	seen		404	▼
Γ_2 $\pi(\pi\pi)_{s\text{-wave}}$	seen			▼
Γ_3 $\gamma\gamma$			650	▼

[1] Our estimate. See the Particle Listings for details.

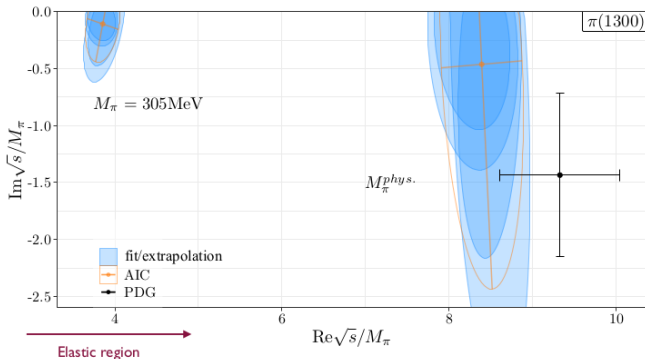
Pro: s -wave decay to 3π implies strong 3-particle coupling

Con: Lies well above the inelastic threshold for QC₃

- $M(\pi(1300)) \approx 10M_\pi \gg 5M_\pi$
- QC₃ will apply for heavier than physical quark masses

Pole positions

- Solve IVU (=infinite-volume unitarity) form of integral equations
 - Find resonance pole for $M_\pi = 305$ MeV!
- Extrapolate to physical masses using EFT-inspired form



Emergent Photons vs Confinement: roles of Vortices and Monopoles

Mendel Nguyen (Durham University)

Based on work with T. Sulejmanpasic and M. Ünsal, Phys. Rev. Lett. 134, 141902 (2025), arXiv:2401.04800

Understanding confinement

Mechanisms: what configurations dominate in the path integral?

Magnetic monopoles—condensation results in “dual” (magnetic) superconductivity

Center vortices

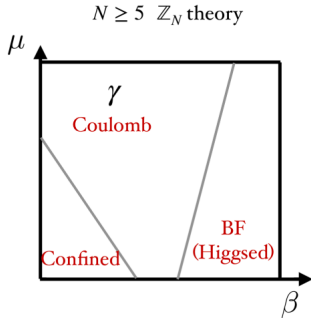
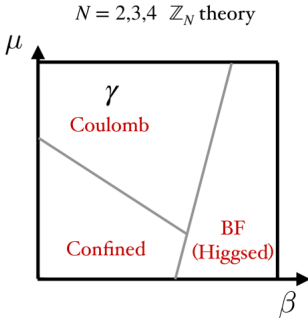
Reintroduce monopoles

$$S = \frac{1}{e_0^2} \sum_{\text{plaqs}} (\Delta_{[\mu} a_{\nu]} + 2\pi m_{\mu\nu})^2 + \mu_0^2 \sum_{\text{links}} (M_\mu)^2 \quad (M_\mu \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \Delta_\nu m_{\rho\sigma})$$

Dual action describes $U(1)$ gauge field coupled to monopoles and *heavy* charges (of mass $\sim \mu_0$).

Coulomb phase persists for finite sufficiently large μ_0 .

Phase diagram



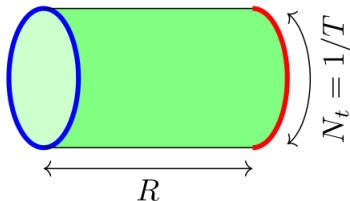
Nambu-Gotō string

Nambu-Gotō (NG) string model:

$$S_{NG} = \sigma \int_{\Sigma} d^2\xi \sqrt{g}$$

The Polyakov loop correlator at large distance is dominated by:

$$G(R) \sim K_0(E_0 R), \quad E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$



EST corrections beyond Nambu-Gotō

To obtain the correct EST describing the gauge theory, it is essential to go **beyond the NG approximation...**

$$S_{EST} = \int d^2\xi [\sigma + \gamma_1 \mathcal{R} + \gamma_2 \mathcal{K}^2 + \gamma_3 \mathcal{K}^4 + \dots]$$

- \mathcal{R} is topological invariant
- \mathcal{K}^2 is proportional to EoM \rightarrow can be eliminated

Low-energy universality of the EST \implies First correction BNG can
[O. Aharony, Z. Komargodski, 1906.08098] only appear at the order $1/N_t^7$

S-matrix approach \implies First two correction ($1/N_t^7$ and $1/N_t^9$ terms) are
[J.E. Miró controlled by the same parameter, next parameter
et al, 1906.08098] appears at $1/N_t^{11}$ order.

Effective string theory predictions for closed strings in (2+1)D

Why closed? Avoids extra contributions due to endpoints.

$$E_0(N_t) = \sigma N_t + \dots$$

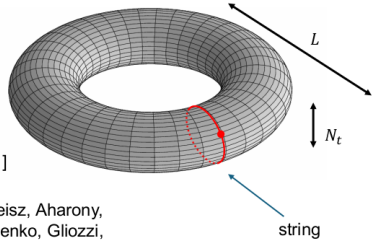
$$E_0(N_t) = \sigma N_t - \frac{\pi}{6N_t} + \dots \quad [\text{Luscher '81}]$$

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} + \dots \quad [\text{Luscher, Weisz, Aharony, Mayer, Gorbenko, Gliozzi, Meineri, etc}]$$

Nambu-Goto

$$E_0(N_t) = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2} - \frac{32\pi^6 \gamma_3}{225\sigma^3 N_t^7} - \frac{64\pi^7 \gamma_3}{675\sigma^4 N_t^9} - \frac{\frac{2\pi^8 \gamma_3}{45} + \frac{32768\pi^{10} \gamma_5}{3969}}{\sigma^5 N_t^{11}}} .$$

[Elias Mirò et al '19]



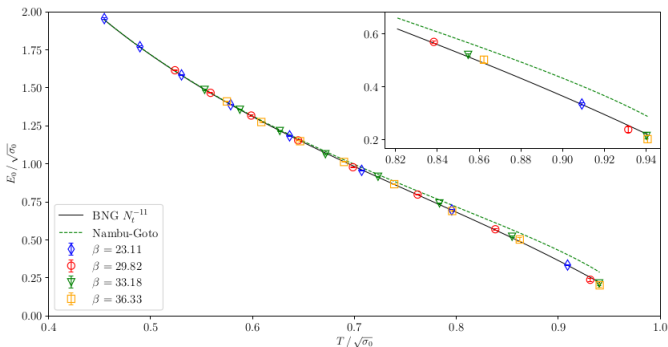
$$\tilde{\gamma}_n = \gamma_n + (-1)^{(n+1)/2} \frac{1}{n^{2n-1}}$$

$$\tilde{\gamma}_3 \geq 0 ,$$

$$\tilde{\gamma}_5 \geq 4\tilde{\gamma}_3^2 - \frac{1}{64}\tilde{\gamma}_3 .$$

SU(3) numerical results

[M. Caselle, N. Magnoli, A. Nada, M. Panero, D. Panfalone and L. Verzhicelli,
JHEP 08 (2024) 198, 2407.10678]



Combined best fits of the SU(3) data including all terms up to $1/N_t^{11}$

Finite density lattice QCD as an inverse problem

(aka analytical continuation from imaginary to real chemical potential via Cauchy integral formula)

Francesco Di Renzo (University of Parma and INFN)

Lattice 2025

Tata Institute of Fundamental research, Mumbai, 03/11/2025

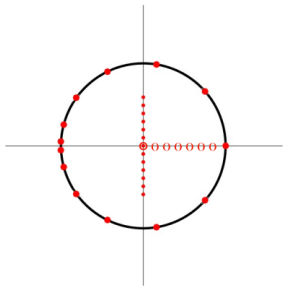


In collaboration with P. Dimopoulos, M. Aliberti and D. Gavriel (Parma)
(and *Bielefeld Parma Collaboration*)



UNIVERSITÀ
DI PARMA





With your favourite **QUADRATURE** method ... you can go **numeric!**

De facto, you would like to think of **Legendre quadrature**

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} \frac{f(Re^{i\theta}) Re^{i\theta}}{Re^{i\theta} - z_0} d\theta \simeq \frac{1}{2\pi} \sum_{k=1}^n w_k \frac{f(Re^{i\theta_k}) Re^{i\theta_k}}{Re^{i\theta_k} - z_0}$$

$$y_i = \frac{1}{2\pi} \sum_{k=1}^n w_k \frac{Re^{i\theta_k}}{Re^{i\theta_k} - z_i} \hat{f}_k, \quad i = 1, 2, \dots, n$$

$$A \mathbf{x} = \mathbf{b}$$

SOLVE for the \hat{f}_k !

Not the end of the story ... Obviously, it is a good (CHEAP) idea to also remember **Cauchy formula for derivatives**

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z - z_0)^{n+1}} dz = \frac{n!}{2\pi i} \int_0^{2\pi} \frac{f(R \exp(i\theta)) R \exp(i\theta)}{(R \exp(i\theta) - z_0)^{n+1}} d\theta$$

IT WORKS! results are very much consistent

As a(n independent) cross-check, we compare the derivatives we compute with results from the literature (HotQCD)



Our results ($N_\tau = 6$)

$\chi_2(0)$	0.1096 (2) (4)
$\chi_4(0)$	0.0862 (12) (17)
$\chi_6(0)$	0.0018 (44) (37)
$\chi_8(0)$	-0.185 (16) (10)
$\chi_{10}(0)$	-0.149 (21) (46)

HotQCD ($N_\tau = 6 - 8$)

0.1087 (4)
0.0844 (41)
-0.0410 (342) [*]
-0.71 (92) [!]

[*] $N_\tau = 8$

[!] $N_\tau = 8$ differ.def.

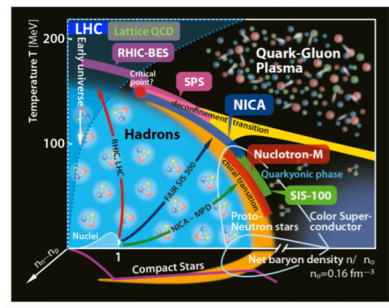
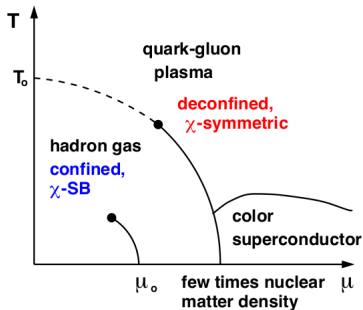
Quarkonium in Non-zero Isospin Chemical Potential Environment

At $T \simeq 0$

Seyong Kim
Sejong University

Work in progress with B. Brandt (Bielefeld U.) and G. Endrodi (Eotvos U.)

Motivation



Equation of state and speed of sound of isospin-asymmetric QCD on the lattice

B.B. Brandt,^a F. Cuteri^b and G. Endrödi^a

^a *Fakultät für Physik, Universität Bielefeld, Universitätsstraße 25, 33615 Bielefeld, Germany*

^b *Institut für Theoretische Physik, Goethe-Universität Frankfurt am Main, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany*

E-mail: brandt@physik.uni-bielefeld.de, cuteri@itp.uni-frankfurt.de, endrodi@physik.uni-bielefeld.de

ABSTRACT: We determine the QCD equation of state at nonzero temperature in the presence of an isospin asymmetry between the light quark chemical potentials on the lattice.

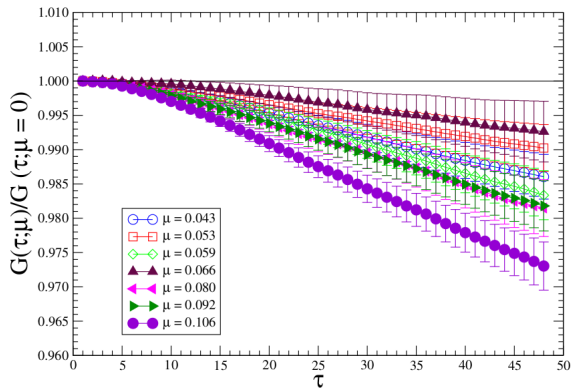
$$a = 0.1535 \text{ fm}, m_\pi = 135 \text{ MeV}, \frac{1}{2}m_\pi \simeq 0.053a^{-1}$$

JHEP07(2023)1

Lattice ensemble

μ	j	No. of config.
0.000	0.0000	102
0.048	0.0010	198
	0.0018	
	0.0036	
0.053	0.0010	200
	0.0018	
	0.0036	
0.059	0.0010	196
	0.0018	
	0.0036	
0.066	0.0010	196
	0.0018	
	0.0036	
0.080	0.0010	196
	0.0018	
	0.0036	
0.092	0.0010	197
	0.0018	
	0.0036	
0.106	0.0010	196
	0.0018	
	0.0036	

Upsilon correlator ratio ($j = 0.0036$)



Spectral reconstruction and dimensional reduction in high-temperature gauge theories

(to appear on ArXiv soon)

Pavel Buividovich¹ Ben Hind¹

¹Department of Mathematical Sciences
University of Liverpool, UK

Lattice 2025 Conference, November 2025



Real-time correlators and spectral functions

- ▶ We live in real (Minkowski) time and are interested in **real-time correlators**: ($\beta \equiv T^{-1}$)

$$G_R(t) = i\theta(t) \mathcal{Z}^{-1} \text{Tr} \left(e^{-\beta\hat{H}} \left[\hat{O}, e^{i\hat{H}t} \hat{O} e^{-i\hat{H}t} \right] \right) \quad (1)$$

- ▶ ... and spectral functions

$$S(\omega) = \frac{1}{\pi} \text{Im} G_R(\omega), \quad G_R(\omega) = \int dt e^{-i\omega t} G_R(t) \quad (2)$$

- ▶ Central objects in linear response theory, give access to **transport coefficients** and **excitation spectra**
- ▶ Continuous functions of t (or ω), contain **infinite amount of information**

Imaginary-time correlators

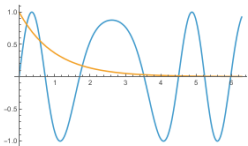
- ▶ Conventional Lattice QCD simulations: Euclidean correlators

$$G(\tau) = \text{Tr} \left(\hat{O} e^{-\tau \hat{H}} \hat{O} e^{-(\beta-\tau) \hat{H}} \right) \quad (3)$$

- ▶ $N_\tau/2 \sim O(10)$ independent data points for $G(\tau)$ in conventional lattice QCD simulations, very **limited information**
- ▶ Green-Kubo relations - challenging to invert numerically:

$$G(\tau) = \int_0^{+\infty} dw \frac{\cosh(w(\tau - \beta/2))}{\sinh(w\beta/2)} S(w) \quad (4)$$

- ▶ Numerical inversion of Green-Kubo relations ([Backus-Gilbert](#), [MaxEnt](#)): **limited frequency resolution** $\Delta w \sim \pi T$



Spectral reconstruction at high temperatures

- ▶ At high T , gauge fields **almost independent of** τ - higher Matsubara modes suppressed as $\sim \exp\left(- (2\pi k)^2 T \vec{A}^2(w_k)\right)$
- ▶ Effectively 3D, quasi-classical gauge fields [Appelquist, Pisarski'81]
- ▶ Fermionic correlators

$$G(\tau) = \langle (\bar{q}Oq)(0) (\bar{q}Oq)(\tau) \rangle \quad (5)$$

still exhibit nontrivial dependence on τ

- ▶ **Quantum fermions** in the background of **static** (τ -independent) **gauge fields**
- ▶ Solved in terms of **Dirac Hamiltonian** $h(\vec{A})$:

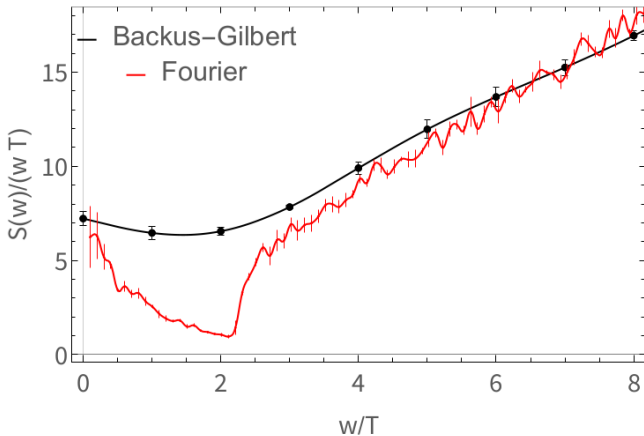
$$\mathcal{D} = \gamma_0 \left(\partial_\tau - iA_0 - h[\vec{A}] \right) \quad (6)$$

Euclidean correlators in static gauge fields: analytic expressions

$$\begin{aligned} G_s(\tau) &= \left\langle \left\langle \text{Tr} \left(O \frac{e^{-\tau h}}{1 + e^{-\beta h} \Omega} O \frac{e^{-(\beta-\tau)h} \Omega}{1 + e^{-\beta h} \Omega} \right) \right\rangle \right\rangle_{\vec{A}(\vec{x}), \Omega} = \\ &= \left\langle \left\langle \sum_{m,n} |O_{mn}|^2 \frac{e^{-\tau \epsilon_m}}{1 + e^{-\beta \epsilon_m} \Omega} \frac{e^{-(\beta-\tau) \epsilon_n} \Omega}{1 + e^{-\beta \epsilon_n} \Omega} \right\rangle \right\rangle_{\vec{A}(\vec{x}), \Omega} \quad (7) \end{aligned}$$

- ▶ Fermionic Green's functions in terms of Hamiltonian (related work for WD fermions: [Nagata, Nakamura, ArXiv:1009.2149](#), [Alexandru, Wenger, ArXiv:1009.2197](#))
- ▶ Explicit analytic control of τ dependence term-by-term
- ▶ Ω is the global center element (Polyakov loop phase)

Spectral function $\bar{q}\gamma_i q$, FASTSUM Gen2, $T/T_c = 1.90$



- ▶ Finite- m_q threshold + transport peak broadening

Understanding dimensional reduction in finite
temperature QCD from spatial string tension and
its consequences

Sayantana Sharma

The Institute of Mathematical Sciences

November 5, 2025

LATTICE 2025, Tata Institute of Fundamental Research, Mumbai

Dimensional reduction from the Lüscher term

- The pseudo-potential can thus be parametrized as,

$$V_s(r, T) = A + \sigma_s \cdot r - \frac{\alpha}{r}.$$

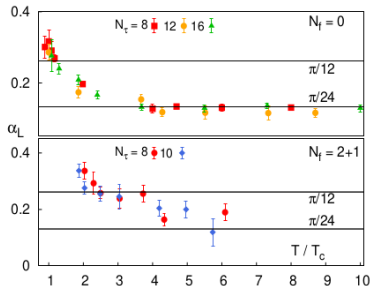
- At distances $r \leq (\pi T)^{-1}$ perturbative gluon exchange dominates.
- At intermediate distances $(\pi T)^{-1} < r < (g^2 T / \pi)^{-1}$ the non-perturbative **Lüscher term** dominantly contributes as α_L / r .

Lüscher et al, Nucl. Phys. B 173 (1980) 365, Nucl. Phys. B 180 (1981) 317.

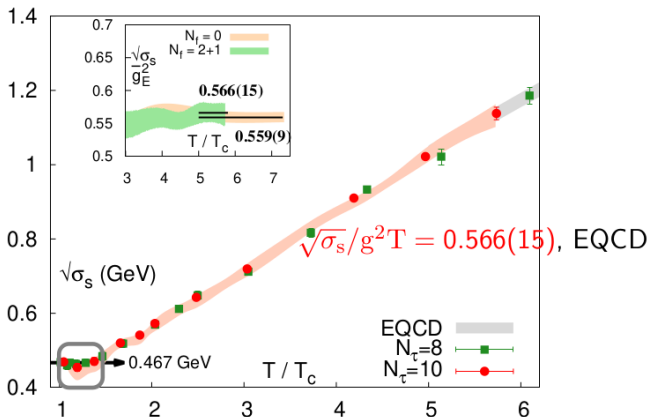
- Coefficient is a universal constant

$$\alpha_L = (D - 2) \frac{\pi}{24}.$$

- Clearly shows **onset of dim. reduction** at $T > 750$ MeV.



Understanding the Temperature dependence of σ_s



The scales πT and gT starts to separate out!

Consistent with earlier results by M. Cheng et. al., Phys. Rev. D 78 (2008) 034506.

Meson spectrum and low-energy constants in large- N QCD

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42ND INTERNATIONAL SYMPOSIUM
ON LATTICE FIELD THEORY

Tata Institute of Fundamental Research
Mumbai, India, 02-08 November, 2025

Based on:

Non-perturbative determination of meson masses and low-energy constants in large- N QCD

CB, M. García Pérez, A. González-Arroyo, K.-I. Ishikawa, M. Okawa

to appear in JHEP (2025) [arXiv:2508.05446]

Introduction: large- N volume independence

't Hooft large- N limit of QCD:

$N \rightarrow \infty$ with fixed 't Hooft coupling $\lambda = g^2 N$ and N_f ($N_f/N \rightarrow 0$).

Theoretical framework with fundamental phenomenological implications.

Standard approach: extrapolation towards $N = \infty$ from $N \lesssim \mathcal{O}(10)$.

Our approach: large- N volume independence.

Large- N equivalence of space-time and color degrees of freedom

[Eguchi–Kawai PRL 48 (1982) 1063]

$V_{\text{eff}} = N^2 V \implies N \rightarrow \infty$ is a thermodynamic limit, since $V_{\text{eff}} \rightarrow \infty$.

This idea suggests the possibility of studying $SU(\infty)$ Yang–Mills theory on **a one-point lattice**. Reducing $V = 1$ allowed us to reach $N = 841$.

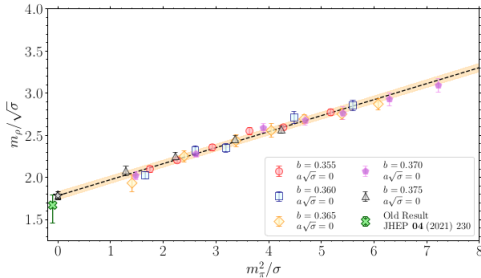
Our setup: **one-point lattice** with **twisted boundary conditions** to avoid center symmetry breaking \implies **Twisted Eguchi–Kawai (TEK)** model

[A. González-Arroyo & M. Okawa PRD 27 (1983) 2397; JHEP 07 (2010) 043]

Example: the ρ meson

Meson masses extracted for 5 lattice spacings, $0.05 \text{ fm} \lesssim a \lesssim 0.10 \text{ fm}$, and 4–7 pion masses with $m_\pi \ell \sim 5\text{--}12$. Joint fit (no chiral logs at large N):

$$\frac{1}{\sqrt{\sigma}} m_\rho(a, m_\pi) = c_0 + c_1 a \sqrt{\sigma} + c_2 \frac{m_\pi^2}{\sigma}$$



Points
 $\frac{1}{\sqrt{\sigma}} m_\rho(a, m_\pi) - c_1 a \sqrt{\sigma}$

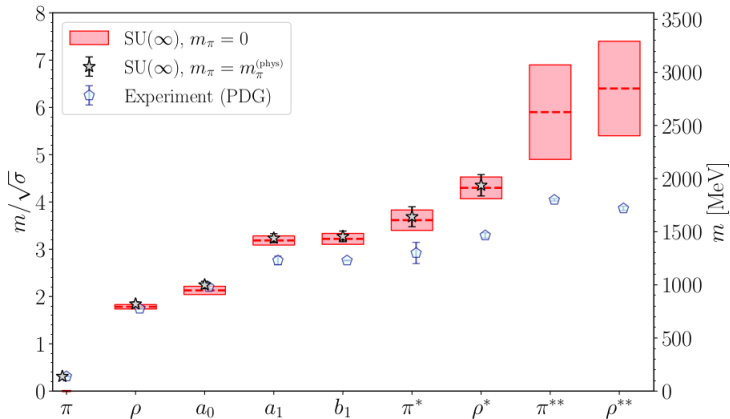
Curve (shaded area)
 $c_0 + c_2 \frac{m_\pi^2}{\sigma}$

Old TEK result (cross point): $N_{\max} = 361$. Now: $N_{\max} = 841$.

Meson state tower

$$m_{\pi}^{(\text{phys})} = 138.04 \text{ MeV [PDG]}$$

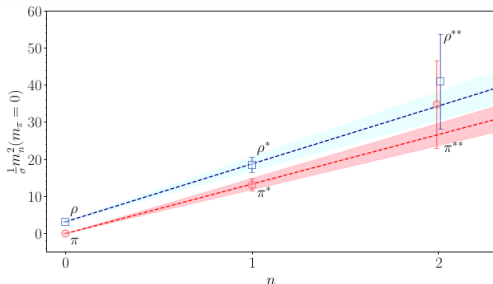
$$\sqrt{\sigma}|_{\text{phys}} = 445 \text{ MeV [Bulava et al. 2403.00754]}$$



This comparison gives an idea of the magnitude of higher-order corrections in $1/N$

Radial Regge trajectories

$$\text{Radial Regge trajectory} \quad \longrightarrow \quad \frac{m_n^2}{\sigma} \Big|_{m_\pi=0} = \frac{m_0^2}{\sigma} + \frac{\mu_r}{\sigma} n$$



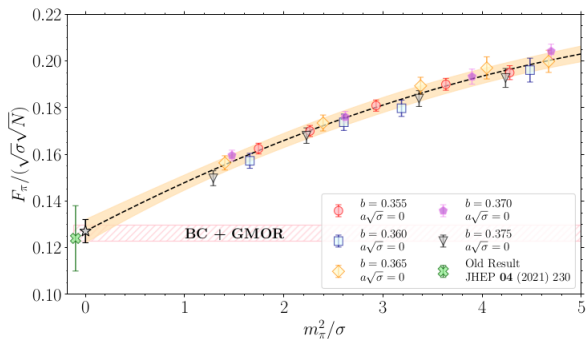
$$\frac{\mu_r}{\sqrt{\sigma}} \simeq 2.65 \text{ (from experimental masses) [Anisovich PRD 62 (2000) 051502]}$$

$$\frac{\mu_r}{\sqrt{\sigma}} \simeq \sqrt{4\pi} \simeq 3.55 \text{ (large-}N \text{ Polyakov effective model) [Dubin et al. PLB 323 (1994) 41]}$$

$$\frac{\mu_r}{\sqrt{\sigma}} \simeq 3.65(21) \text{ (TEK, } \pi\text{-chan.)} \qquad \frac{\mu_r}{\sqrt{\sigma}} \simeq 3.95(24) \text{ (TEK, } \rho\text{-chan.)}$$

F_π from pion correlator

Direct determination of F_π/\sqrt{N} agrees with combination of B_R and Σ_R/N
 Old TEK result (cross point): $N_{\max} = 361$. Now: $N_{\max} = 841$.



$$F_\pi(m_\pi) = F_\pi \left[1 + \frac{\bar{\ell}_4}{16\pi^2 F_\pi^2} m_\pi^2 + \mathcal{O}(m_\pi^4) \right] \quad \frac{\bar{\ell}_4}{N} \sim \mathcal{O}(N^0) \text{ (NLO LEC)}$$

Lattice QCD at finite T and density

On selected topics

Heng-Tong Ding(丁亨通)
Central China Normal University

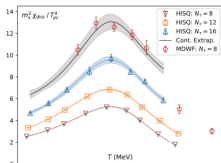


The 42nd International Symposium
on Lattice Field Theory
Nov 2 – 8, 2025 @TIFR Mumbai

Chiral crossover transition at the physical point in $N_f=2+1$ QCD

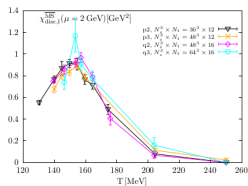
Consistent results among HISQ, MDWF & Overlap

Möbius domain-wall fermions
Nt=8 lattices



Rajiv V. Gavai, Mischa E. Jaensch, Olaf Kaczmarek, Frithjof Karsch, Mugdha Sarkar, Ravi Shanker, Sayantan Sharma, Sipaz Sharma, Tristan Ueding,
Phys. Rev. D 111 (2025) 3, 034507

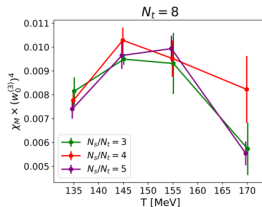
Möbius domain-wall fermions
Nt=12 & 16 lattices



Yu Zhang et al., [JLQCD]

Issaku Kanamori
Wed 11:30 AM

Overlap fermions
Nt = 8

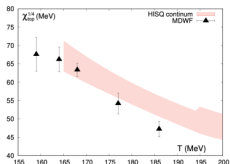


Andrey Kotov
Mon 3:10 PM

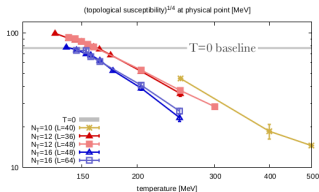
F

Topological susceptibilities at physical point

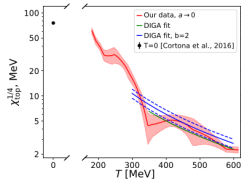
$N_f=2+1$, $N_t=8$
Mobius Domain wall fermions



$N_f=2+1$, $N_t=10,12 \& 16$
Mobius Domain wall fermions



$N_f=2+1+1$, Wilson twisted mass fermions
Continuum limit



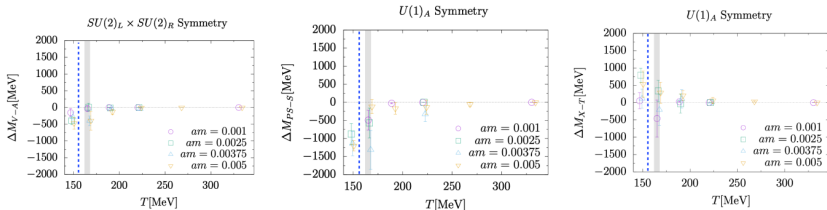
R. V. Gavai, M. E. Jaensch, O. Kaczmarek et al.,
Phys. Rev. D 111 (2025) 3, 034507

Issaku Kanamori
Wed 11:30 AM

A. Yu. Kotov, M.P. Lombardo, A. Trunin
JHEP 09 (2025) 045, arXiv: 2502.15407

Screening mass in Nf=2 QCD with MDWF toward chiral limit

$a \sim 0.075$ fm, $am=0.001$ (other 3 quark masses) smaller (larger) than the physical quark mass



Grey band: $T_{pc} \sim 165(3)$ MeV at the physical point estimated from chiral susceptibility
 Vertical blue line: Chiral phase transition $T_c \sim 153$ MeV: from chiral extrapolation to vanishing quark mass

[JLQCD] arXiv:2401.06459

Y. Aoki, H. Fukaya, S. Hashimoto et al. [JLQCD], Phys. Rev. D 111(2025) 114506

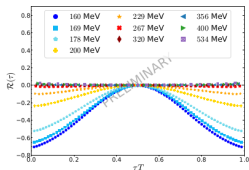
At $T \gtrsim 190$ MeV $\sim 1.2T_c$ differences in screening masses with $am=0.001$ are absent

Fate of Axial U1 anomaly: dependences on N_f and M_π

O(a) improved Wilson fermion

$N_f=2+1$, anisotropic lattice ($\xi = 7$),
 $32^3 \times N_t$; $N_t \in [80,24]$ for
 $T \in [160,534]$ MeV with $M_\pi=378(1)$ MeV

$$U_A(1): R(\tau) = \frac{G_{PS}(\tau) - G_S(\tau)}{G_{PS}(\tau) + G_S(\tau)}$$

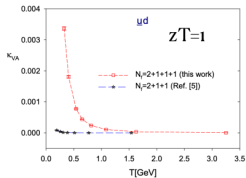


Ryan Bignell
Wed 10:50 AM

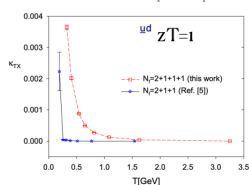
Optimal Domain Wall fermions

$N_f=2+1+1: 32^3 \times N_t$; $N_t \in [16,2]$ for $T \in [192,1540]$ MeV at physical point
 $N_f=2+1+1+1: 40^3 \times N_t$; $N_t \in [20,2]$ for $T \in [325,3252]$ MeV with $M_\pi \sim 700$ MeV

$$SU(2)_L \times SU(2)_R: \kappa_{VA}(zT) = \frac{|C_V(zT) - C_A(zT)|}{C_V(zT) + C_A(zT)}$$



$$U_A(1): \kappa_{TX}(zT) = \frac{|C_T(zT) - C_X(zT)|}{C_T(zT) + C_X(zT)}$$

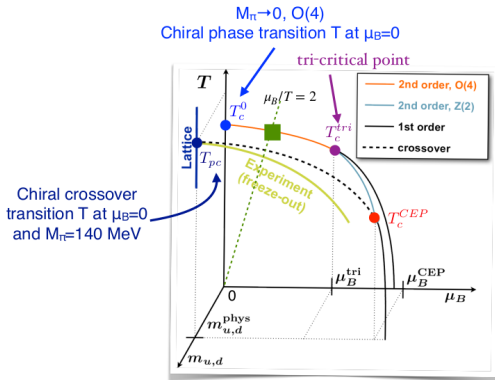


Ting-Wai Chiu, Symmetry 17 (2025) 5, 700

This work: Ting-Wai Chiu, Symmetry 17 (2025) 5, 700; [5]: Ting-Wai Chiu, Phys.Rev.D 110 (2024) 1, 014502

10

Upper bound for T_c^{CEP} at Critical End Point (CEP)



Continuum and chiral extrapolated

$$T_c^0(\mu_B = 0) = 132_{-6}^{+3} \text{ MeV}$$

HTD et al. [HotQCD], PRL 123 (2019) 062002
 See consistent results using Wilson fermions from
 Kotov et al., PLB 823 (2021) 136749

Chiral curvature line on $Nt=8$ lattices

$$T_c^0(\mu_B) = (1 - 0.015(1) \hat{\mu}_B^2) T_c^0(0)$$

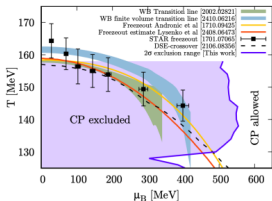
HTD et al. [HotQCD], PRD 109 (2024) 114516

Updated upper bound with $\kappa_2(Nt=8)$

$$T_c^{CEP} \lesssim T_c^0(\hat{\mu}_B = 2) \sim 125 \text{ MeV}$$

Updates on the CEP search in Lattice QCD

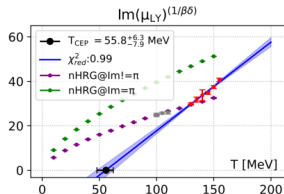
Contour of constant entropy
strangeness neutrality
& corrections to the linear μ_B^2 dep.



No CEP at $\mu_B < 450$ MeV at the 2 σ level

Borsányi et al. [Wuppertal-Budapest],
arXiv:2502.10267

Lee-Yang edge singularity
on $16^3 \times 8$ lattices



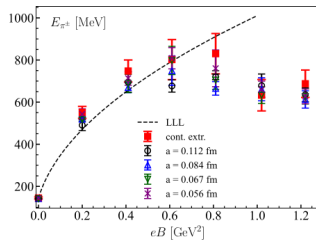
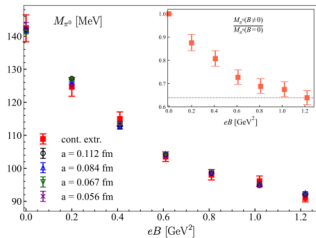
$T^{\text{CEP}} < 103$ MeV
with 84% probability or
CEP does not exit

Adam et al., [Wuppertal-Budapest]
arXiv:2507.13254

Analyses are done based on
extrapolations from LQCD
simulations at $T \gtrsim 120 - 130$ MeV

Recall: Previous Predictions
 $T_c^{\text{CEP}} \approx 83 - 120$ MeV

Pions in external magnetic fields in the vacuum in the continuum limit with physical pion mass



- ☞ Mass of neutral pion: Monotonic decrease as eB increases
- ☞ Energy of charged pion: non-monotonic behavior in eB , not observed in quenched QCD

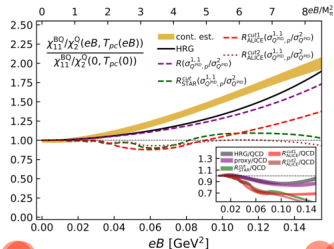
HTD & Dan Zhang, work in progress

See similar results with larger than physical pion mass at one single lattice cutoff:
HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

QCD Magnetometer: Lattice v.s. EXP

Arpith Kumar
Fri 2:50 PM

Baryon-electric charge correlation over electric charge fluctuations

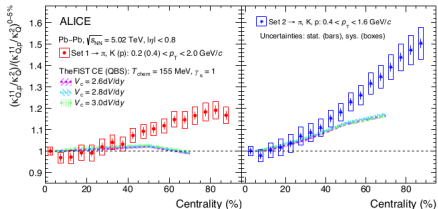


Central collision



Peripheral collision

HTD, Jin-Biao Gu, Arpith Kumar, Sheng-Tai Li,
Phys.Rev.D 111 (2025) 11, 114522, Phys.Rev.Lett. 132 (2024) 20, 201903

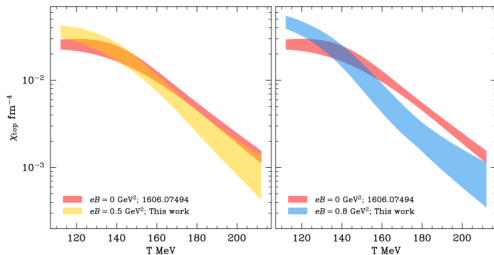


ALICE Collaboration, *JHEP* 08 (2025) 210

Similar trend observed in EXP
Better probe: double ratio of $\chi_{11}^{BQ}/\chi_{11}^{QS}$

34

First LQCD results of Topological susceptibility in external magnetic fields



At $eB = 0.8 \text{ GeV}^2$:
Enhanced anomaly at $T \lesssim 130 \text{ MeV}$
but suppressed at higher T

Axial inverse magnetic catalysis?

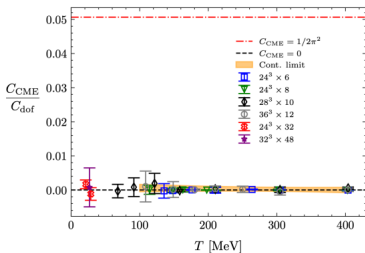
Yuanyuan Wang, Shinya Matsuzaki,
Phys. Rev. D 105 (2022) 074015

B.B. Brandt, G. Endrődi, J.J. Hernández Hernández and G. Markó, JHEP 12(2024)028

Chiral magnetic effect (CME)

Indicator of vanishing 'Global' CME

CME conductivity coefficient in uniform magnetic field

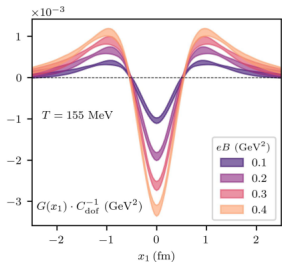


$$\langle J_3 \rangle = C_{\text{CME}} \mu_5 eB + \mathcal{O}(\mu_5^3, B^3)$$

B. Brandt et al., JHEP 09 (2024) 092

Localized CME in equilibrium QCD

CME correlator in non-uniform magnetic field



$$G(x_1) \equiv \left. \frac{\partial \langle J_3(x_1) \rangle}{\partial \mu_5} \right|_{\mu_5=0}$$

B. Brandt et al., Phys. Rev. D 112 (2025) 034508

Non-perturbative QCD

thermodynamics with $N_f=3$ up to the electroweak scale

MICHELE PEPE

INFN Sez. Milano – Bicocca
Milan (Italy)

Based on

M. Bresciani, M. Dalla Brida, **L. Giusti** and M. Pepe

Phys. Rev. Lett. 134 (2025) 20, 201904

The numerical study

- QCD on the lattice with $N_f = 3$ quarks in the chiral limit
- 9 values of the temperature in the range 3 – 165 GeV
- large spatial volumes to have finite volume effects under control (checked explicitly): $g^2 \sim 1$; $LT \sim 10 - 25$
- shifted boundary conditions $\xi = (1, 0, 0)$
- reduced lattice artifacts: $O(a)$ - improved Wilson fermions
- continuum limit extrapolation: $L_0/a = 4, 6, 8, 10$; $L/a = 144$

T	T (GeV)
T_0	164.6(5.6)
T_1	82.3(2.8)
T_2	51.4(1.7)
T_3	32.8(1.0)
T_4	20.63(63)
T_5	12.77(37)
T_6	8.03(22)
T_7	4.91(13)
T_8	3.040(78)

The QCD Equation of State

- $\hat{g}^2(T)$: 5-loop \overline{MS} gauge coupling at $\mu = 2\pi T$

M. Baikov et al. PRL 118, (2017) 082002

$$\frac{1}{\hat{g}^2(T)} = \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{MS}}} + \dots$$

$$\Lambda_{\overline{MS}} = 341 \text{ MeV}$$

M. Bruno et al. PRL 119 (2017) 10, 102001

it is a function of T; simpler to compare with Perturbation Theory

- phenomenological approach:
linear fit works well and
compatible with SB limit

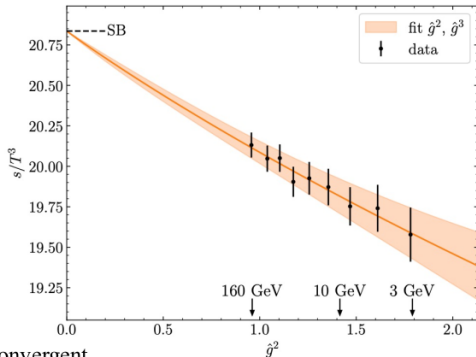
- enforcing exact SB limit

$$\frac{s(T)}{T^3} = \frac{32\pi^2}{45} \left[s_0 + s_2 \left(\frac{\hat{g}}{2\pi} \right)^2 + s_3 \left(\frac{\hat{g}}{2\pi} \right)^3 \right]$$

$$s_2 = -5.1(9); \quad s_3 = 5(5)$$

in PT: $s_2 = -8.438$

but PT is very poorly/slowly convergent...

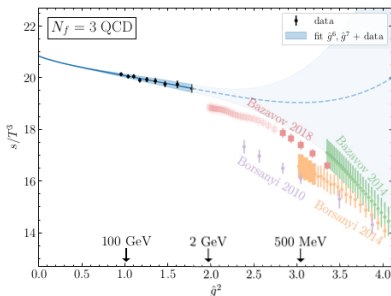
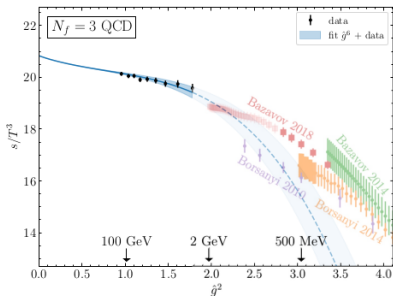


The QCD Equation of State: PT or not PT?

- $s(T)/T^3$: fit unknown contributions with the data

$$\frac{s(T)}{T^3} = \frac{1}{T^3} \frac{dp(T)}{dT} = \frac{32\pi^2}{45} \left[\sum_{k=0}^6 s_k \left(\frac{\hat{g}(T)}{2\pi} \right)^k + q_c \left(\frac{\hat{g}(T)}{2\pi} \right)^6 \right]$$

$$\frac{s(T)}{T^3} = \frac{1}{T^3} \frac{dp(T)}{dT} = \frac{32\pi^2}{45} \left[\sum_{k=0}^7 s_k \left(\frac{\hat{g}(T)}{2\pi} \right)^k + q_c \left(\frac{\hat{g}(T)}{2\pi} \right)^6 \right]$$



- higher order effects, including non-perturbative contributions, are necessary

Towards a determination of Thermal Static Potential at Finite Density from Lattice QCD

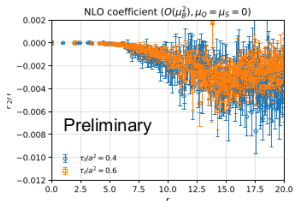
Jishnu Goswami with
Dibyendu Bala and Olaf Kaczmarek

Bielefeld University

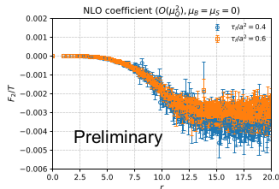


- ▶ Method : static free energy, $F/T = F_0 + F_2(\mu/T)^2$ from Taylor expanded Wilson line correlator at $\tau = N_t = 16, m_l = m_s/27$.
- ▶ Result : $F_2(r, T) < 0$, for μ_B and μ_Q channels. $\mu_B \rightarrow$ probes response to net baryon density and $\mu_Q \rightarrow$ probes electric charge imbalance.
- ▶ They weight quark flavors differently, so comparing both isolates how flavor charges influence screening. In both cases, increased density \longrightarrow signature of more screening.
- ▶ Outlook : Towards LO + NLO thermal static potential.

Free energy NLO co-efficient

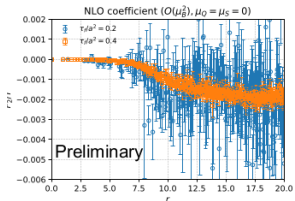


$T = 153$ MeV

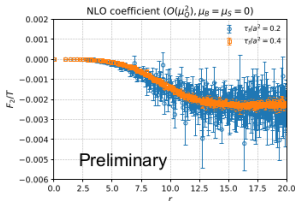


We thank F. Karsch for useful comments and suggestions !!

First Signature of screening at finite chemical potential with physical quark masses from static free energy !!

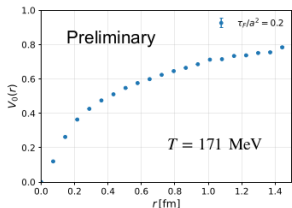
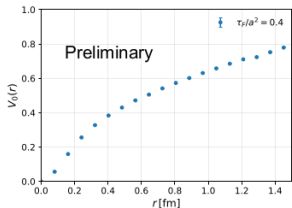


$T = 171$ MeV



LO + NLO potential at finite temperature

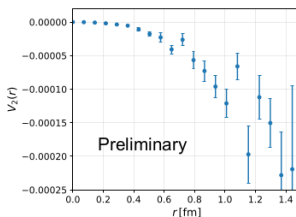
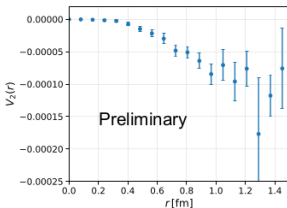
$T = 153 \text{ MeV}$



$$V(r, \mu) = V_0(r) + V_2(r)\mu_Q^2$$

First Signature of screening at finite chemical potential with physical quark masses from static thermal potential !! LO part needs proper renormalisation.

$T = 153 \text{ MeV}$



$T = 171 \text{ MeV}$

Dibyendu Bala, Saumen Datta, Phys. Rev. D 101, 034507 (2020)



Thank you for your attention!