Spin oscillations of neutrinos scattered by the supermassive black hole

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and

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13th November, 2025

References

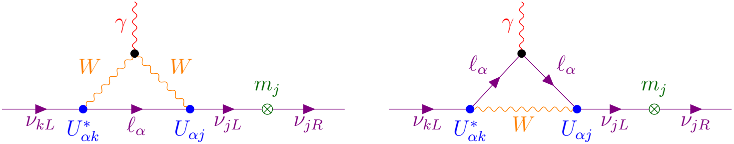
Published	2025	M. Deka and M. Dvornikov, Review Article Mod. Phys. Lett. A, Vol. 40, No. 36 (2025) 2530013. M. Deka and M. Dvornikov, Phys.Atom.Nucl. 88 (2025) 3, 513-519.
	earlier	M. Deka and M. Dvornikov, Phys.Atom.Nucl. 87 (2024) 4, 483-488. M. Dvornikov, JCAP 09 (2023) 039. M. Dvornikov, Class.Quant.Grav. 40 (2023) 1, 015002. M. Dvornikov, JCAP 06 (2013) 015.
Unpublished	2025	M. Deka and M. Dvornikov, arXiv:2510.26621. M. Deka and M. Dvornikov, arXiv:2504.07816.

Neutrino Magnetic Moment

- In minimally extended Standard Model, neutrinos acquire electromagnetic properties through quantum loops effects. For detailed review, see C. Giunti, A. I. Studenikin, 2014; C. Giunti et. al, 2024.
- Finite neutrino masses and mixing leads to neutrinos having nonzero magnetic moments.
- Upper bounds from experimental and astrophysical measurements, e.g.

$$\begin{aligned}
\text{GEMMA} &: \quad \mu_{\nu_e} < 2.9\times 10^{-11}\mu_{\text{B}}\\
\text{CONUS} &: \quad \mu_{\nu_e} < 7.5\times 10^{-11}\mu_{\text{B}}\\
\text{Dresden-II} &: \quad \mu_{\nu_e} < 2.1\times 10^{-10}\mu_{\text{B}}\\
\text{Super-Kamiokande} &: \quad \mu_{\text{S}}^{\text{HE}} < 1.1\times 10^{-10}\mu_{\text{B}}\\
\text{Borexino} &: \quad \mu_{\nu_e} < 3.9\times 10^{-11}\mu_{\text{B}}\\
\text{XMASS-I} &: \quad \mu_{\text{S}}^{\text{LE}} < 1.8\times 10^{-10}\mu_{\text{B}}
\end{aligned}$$

Neutrino Magnetic Moment



- Non-zero μ leads to helicity flipping interaction with the electromagnetic fields.

- Change of neutrino spin direction with respect to its momentum within the same flavor

$\Rightarrow$ Neutrino Spin Oscillations.

Fujikawa and Shrock, 1980.

Dirac or Majorana Neutrino

- Magnetic moment of Dirac neutrino:

$$\mu_{ii}^{\mathrm{D}} \simeq \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19} \left(\frac{m_i}{eV}\right) \mu_{\mathrm{B}} \tag{1}$$

$$\mu_{ij}^{\mathrm{D}} \simeq -3.9 \times 10^{-23} \mu_{\mathrm{B}} \left(\frac{m_i + m_j}{eV}\right) \times \sum_{\ell=e,\mu,\tau} U_{\ell i}^{*} U_{\ell j} \left(\frac{m_\ell}{m_\tau}\right)^2 \tag{2}$$

- U is the mixing matrix.
- Magnetic moment of Majorana neutrino:

$$\mu_{ii}^{\mathrm{M}} = 0 \tag{3}$$

$$\mu_{ij}^{\mathrm{M}} = 0 \quad \text{if } \nu_i \text{ and } \nu_j \text{ have same CP phase} \tag{4}$$

$$\mu_{ij}^{\mathrm{M}} \simeq 2\mu_{ij}^{\mathrm{D}} \quad \text{if } \nu_i \text{ and } \nu_j \text{ have opposite CP phase} \tag{5}$$

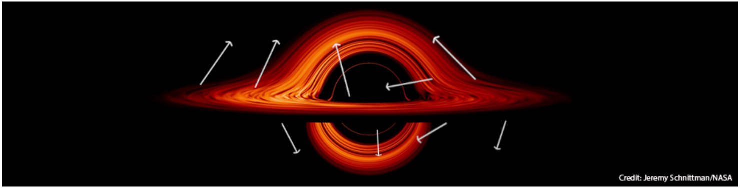
R. E. Shrock, 1982.

Dirac or Majorana Neutrino

- To study spin oscillations, we shall consider only the diagonal elements of the magnetic moment matrix in order to avoid flavor oscillations.
- Hence, we can consider Dirac neutrinos only.
- Since in our work we deal with the ultra-relativistic neutrinos only, i.e. neutrinos with $m \ll E$, we assume the diagonal magnetic moments to be equal.
- For computational purpose, we choose μ to be closer to the experimental upper bound.

Testing Ground

- Accretion disks in SMBH in some galaxies can be sources of both photons and high energy neutrinos. Chen & Beloborodov, 2006.



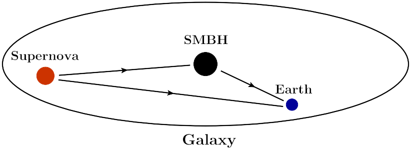
Credit: NASA's Goddard Space Flight Center/Jeremy Schnittman (Image modified by M. Deka).

- White arrows represent neutrino emissions.

Testing Ground

- Before arriving at the observer, these neutrinos move in strong gravitational field near BH.
- Their spins can precess in the presence of external fields of the accretion disk, and they become right handed.
- Right-handed neutrinos are considered to be sterile.
- We shall observe an effective reduction of the initial neutrino flux.

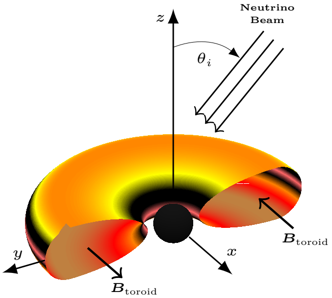
This Work

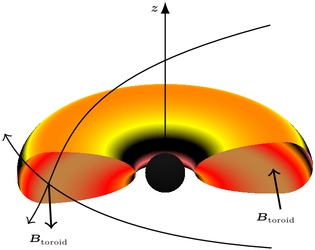


- However, it is computationally complex and expensive to directly deal with the realistic situation.
- Instead, we consider a uniform flux of ultra-relativistic left-polarized Dirac neutrinos coming from a far object, such as a distant core-collapsing supernova.
- They approach a BH at an angle, θ_i, w.r.t. to the BH spin, so that $(r, \theta, \phi)_{\text{source}} = (\infty, \theta_i, 0)$.
- They are either captured or scattered by the BH.

This Work

- We are interested only in the scattered neutrinos.
- We consider a thick accretion disk surrounding the BH with only the toroidal magnetic field.
- The scattered Neutrinos undergo interactions with the matter and magnetic fields in the disk resulting spin precession.
- Some of the left handed neutrinos can become right handed.
- We finally look at the probability distributions of the handedness of the neutrinos at the observer position $(\theta, \phi)_{\text{obs}}$.


Neutrino Beam
z
θ_i
y
x
$\boldsymbol{B}_{\text{toroid}}$
$\boldsymbol{B}_{\text{toroid}}$


z
$\boldsymbol{B}_{\text{toroid}}$
$\boldsymbol{B}_{\text{toroid}}$

Kerr Metric

- We describe the spacetime of a spinning black hole in Kerr metric.
- Boyer-Lindquist coordinates, $x = (t, r, \theta, \phi)$:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{rr_g}{\Sigma}\right)dt^2 + 2\frac{rr_g a \sin^2\theta}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 \\ &- \Sigma d\theta^2 - \frac{\Xi}{\Sigma}\sin^2\theta d\phi^2 \end{aligned} \tag{6}$$

$$\Delta = r^2 - rr_g + a^2,\ \Sigma = r^2 + a^2\cos^2\theta,\ \Xi = (r^2 + a^2)\Sigma + rr_g a^2 \sin^2\theta \tag{7}$$

- BH mass: $M = r_g/2$.
- BH spin: $J = Ma(0 < a < M)$.

Equations of Motion

- Hamilton-Jacobi equation:

$$H\left(x^{\mu}, \frac{\partial S}{\partial x^{\mu}}\right) + \frac{\partial S}{\partial \lambda} = 0 \tag{8}$$

- λ is the curve parameter.
- Four constants of motion:

$$m \quad : \quad \text{mass of the particle} \quad \left(\frac{1}{2}g^{\mu\nu}\partial_{\mu}S\partial_{\nu}S = -\frac{1}{2}m^2\right) \tag{9}$$

$$E \quad : \quad \text{Particle energy.} \tag{10}$$

$$L \quad : \quad \text{Angular momentum.} \tag{11}$$

$$Q \quad : \quad \text{Carter constant.} \tag{12}$$

- If S is a solution of the Hamilton-Jacobi equation:

$$S = -\frac{1}{2}m^2\lambda - Et + L\phi + S_r(r) + S_{\theta}(\theta) \tag{13}$$

- The trajectory of an ultra-relativistic neutrino:

$$0 = \frac{\partial S}{\partial Q} \Rightarrow \int \frac{dr}{\pm\sqrt{R}} = \int \frac{d\theta}{\pm\sqrt{\Theta}} \tag{14}$$

$$0 = \frac{\partial S}{\partial L} \Rightarrow$$

$$\phi = a \int \frac{dr}{\Delta\sqrt{R}} \left[(r^2 + a^2)E - aL\right] + \int \frac{d\theta}{\sqrt{\Theta}} \left[\frac{L}{\sin^2\theta} - aE\right] \tag{15}$$

- We define the radial and polar potentials:

$$R = \left[(r^2 + a^2)E - aL\right]^2 - \Delta\left[Q + (L - aE)^2\right] \tag{16}$$

$$\Theta = Q + \cos^2\theta \left(a^2E^2 - \frac{L^2}{\sin^2\theta}\right) \tag{17}$$

- Dimensionless variables:

$$r = x r_g,\ L = y r_g E,\ Q = w r_g^2 E^2,\ a = z r_g,\ \tilde{\mathfrak{t}} = \cos\theta \tag{18}$$

- Discrete grid in radial direction.
- For an incoming neutrino

$$I_x = z\sqrt{\tilde{\mathfrak{t}}_+^2 + \tilde{\mathfrak{t}}_-^2} \int_x^\infty \frac{\mathrm{d}x'}{\sqrt{R(x')}} \tag{19}$$

$$\tilde{\mathfrak{t}}_\pm^2 = \frac{1}{2z^2}\left[\sqrt{(z^2 - y^2 - w)^2 + 4z^2 w} \pm (z^2 - y^2 - w)\right] \tag{20}$$

- Similar definition for an outgoing neutrino.
- x_{tp} is the turn point: the maximal real root of the equation $R(x) = 0 \Rightarrow$ Minimum value of x.

- Neutrinos approaching the BH from infinity go through $(N-1)$ oscillations between $\pm\tilde{\mathfrak{t}}_+$ before reaching $\tilde{\mathfrak{t}}$ at the turn point.

$$\tilde{\mathfrak{t}} \in [\tilde{\mathfrak{t}}_i, \pm\tilde{\mathfrak{t}}_+] \cup \underbrace{[+\tilde{\mathfrak{t}}_+, -\tilde{\mathfrak{t}}_+] \cup \cdots \cup [-\tilde{\mathfrak{t}}_+, +\tilde{\mathfrak{t}}_+]}_{(N-1)\text{ times}} \cup [+\tilde{\mathfrak{t}}_+, \tilde{\mathfrak{t}}]. \qquad (21)$$

  - Upper neutrinos: $\tilde{\mathfrak{t}}$ is increasing to $+\tilde{\mathfrak{t}}_+$ in the first segment.
  - Lower neutrinos: $\tilde{\mathfrak{t}}$ is decreasing to $-\tilde{\mathfrak{t}}_+$ in the first segment.
- For an incoming upper neutrino

$$\tilde{\mathfrak{t}}(x) = \cos\theta(x) = \tilde{\mathfrak{t}}_+ \mathrm{cn}\left((-1)^N \left\{ F - I_x + 4K \left\lfloor \frac{N}{2} \right\rfloor \right\} \middle| \frac{\tilde{\mathfrak{t}}_+^2}{\tilde{\mathfrak{t}}_-^2 + \tilde{\mathfrak{t}}_+^2} \right) \qquad (22)$$

$$N = \left\lfloor \frac{I_x - F}{2K} \right\rfloor + 1, F \equiv F\left(\arccos\frac{\tilde{\mathfrak{t}}_i}{\tilde{\mathfrak{t}}_+}, \frac{\tilde{\mathfrak{t}}_+^2}{\tilde{\mathfrak{t}}_+^2 + \tilde{\mathfrak{t}}_-^2} \right), K \equiv \left(\frac{\tilde{\mathfrak{t}}_+^2}{\tilde{\mathfrak{t}}_+^2 + \tilde{\mathfrak{t}}_-^2} \right) \qquad (23)$$

- Similarly for incoming lower neutrinos, and outgoing upper and lower neutrinos.
- Computation of ϕ involves (In)complete elliptic integrals of third kind.

Black Hole Shadow Curve

- We are interested in scattered neutrinos only.
- The edge between the scattered and captured neutrinos is given by $R(\tilde{x}) = R'(\tilde{x}) = 0 \Rightarrow$ BH Shadow curve. Gralla et. al, 2018.

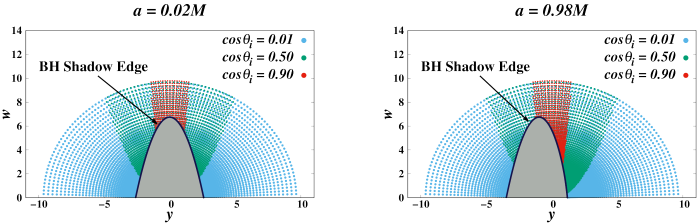
$$y = -\frac{1}{z(2x-1)}\left[x^2(2x-3) + z^2(2x+1)\right], \tag{24}$$

$$w = \frac{x^3}{z^2(2x-1)^2}\left[8z^2 - x(2x-3)^2\right], \tag{25}$$

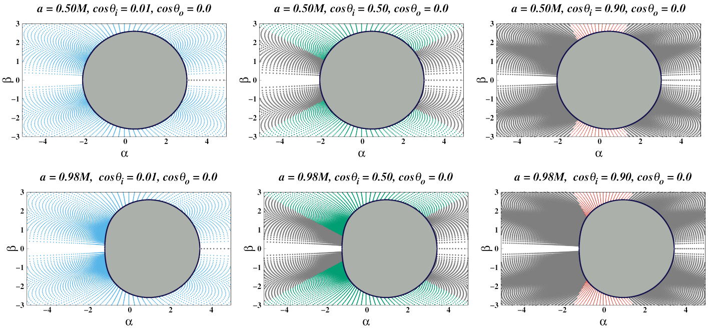
$$x_\pm = 1 + \cos\left[\frac{2}{3}\arccos(\pm 2z)\right], \quad x_- < x < x_+. \tag{26}$$

- Additional condition for $\tilde{\mathfrak{t}}_i \neq 0$

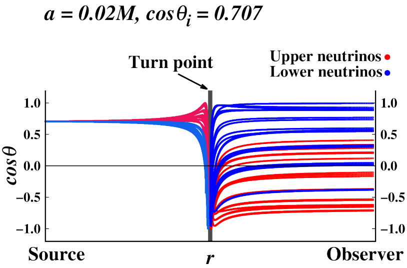
$$\tilde{\mathfrak{t}}_i \leq \tilde{\mathfrak{t}}_+ \Rightarrow w \geq -z^2\tilde{\mathfrak{t}}_i^2 + y^2\frac{\tilde{\mathfrak{t}}_i^2}{1-\tilde{\mathfrak{t}}_i^2} \tag{27}$$

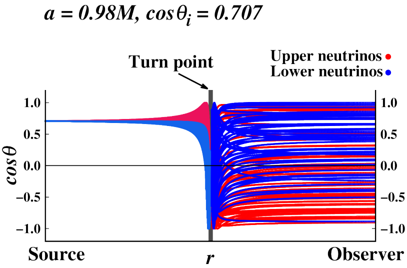

a = 0.02M
a = 0.98M
BH Shadow Edge
BH Shadow Edge
cosθ_i = 0.01
cosθ_i = 0.50
cosθ_i = 0.90
w
y
14
12
10
8
6
4
2
0
−10
−5
0
5
10

- -


a = 0.50M, cosθ_i = 0.01, cosθ_o = 0.0
a = 0.50M, cosθ_i = 0.50, cosθ_o = 0.0
a = 0.50M, cosθ_i = 0.90, cosθ_o = 0.0
a = 0.98M, cosθ_i = 0.01, cosθ_o = 0.0
a = 0.98M, cosθ_i = 0.50, cosθ_o = 0.0
a = 0.98M, cosθ_i = 0.90, cosθ_o = 0.0
β
α
3
2
1
0
−1
−2
−3
−4
−2
0
2
4

Trajectories of Scattered Neutrinos


$a = 0.02M, \cos\theta_i = 0.707$


$a = 0.98M, \cos\theta_i = 0.707$

Neutrino spin evolution in flat space-time

- The Bargmann-Michel-Telegdi (BMT) equation for a charged particle moving in an electromagnetic field, Bargmann et. al, 1959.

$$\frac{\mathrm{d}S^{\mu}}{\mathrm{d}\tau} = 2\mu F^{\mu\nu} S_{\nu} - 2\mu' U^{\mu} F^{\nu\lambda} U_{\nu} S_{\lambda}, \tag{28}$$

- Neutrino is an electrically neutral particle. Hence $\mu = \mu'$, and

$$m\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau} = eF^{\mu\nu} U_{\nu} = 0 \tag{29}$$

- A neutrino moves along a straight line in the flat space-time.
- BMT equation is generalized to include the electroweak interactions of neutrinos with the background matter

$$\frac{\mathrm{d}S^{\mu}}{\mathrm{d}\tau} = 2\mu(F^{\mu\nu} S_{\nu} - U^{\mu} F^{\nu\lambda} U_{\nu} S_{\lambda}) + \sqrt{2} G_{\mathrm{F}} \varepsilon^{\mu\nu\lambda\rho} G_{\nu} U_{\lambda} S_{\rho} \tag{30}$$

Dvornikov and Studenikin, 2012.

Neutrino spin evolution in curved space-time

- For a spinning particle in curved spacetime Papapetrou, 1951; Wald, 1972

$$\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} = p^{\mu}U^{\nu} - U^{\mu}p^{\nu} \tag{31}$$

$$\frac{\mathrm{D}p^{\mu}}{\mathrm{d}\tau} = -\frac{1}{2}R^{\mu}{}_{\nu\rho\lambda}U^{\nu}S^{\rho\lambda}, \quad S^{\mu} = -\frac{1}{m^2}\frac{\varepsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}p_{\nu}S_{\lambda\rho} \tag{32}$$

- $\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} \neq 0$ and $\mathrm{D}p^{\mu} \neq 0$ since U^{μ} and p^{μ} are not collinear in general.
- The motion of a spinning particle deviates from the geodesics.
- For a point-like particle, this deviation is negligible. Rietdijk and Van Holten, 1992.
- Proposed in Pomeransky and Khriplovich, 1998 that the spin of a point-like particle is parallel transported along the geodesics.
- Later confirmed in Sorge and Zilio, 2007; Obukhov et. al, 2017.

Neutrino spin evolution in curved space-time

- The covariant equation for the neutrino spin four-vector,

$$\frac{DS^\mu}{D\tau} = 2\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_\nu U_\lambda S_\rho, \frac{DU^\mu}{D\tau} = 0. \tag{33}$$

  M. Dvornikov, 2013.

- We make a transformation to a local Minkowskian frame.

$$x_a = e_a^\mu x_\mu, \quad \eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu}, \quad \eta_{ab} = (1, -1, -1, -1) \tag{34}$$

- After making a boost to the particle rest frame, the neutrino invariant 3-spin vector can then be defined as

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{em}} + \boldsymbol{\Omega}_{\mathrm{matter}}. \tag{35}$$

  M. Dvornikov, 2023.

- $\boldsymbol{\Omega}$ can be explicitly calculated in a given metric.

Effective Schrödinger Equation

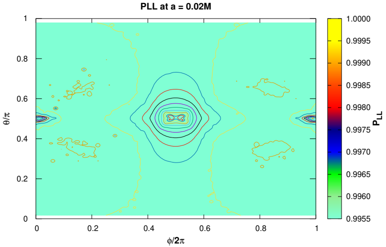
- Instead, we solve the effective Schrödinger equation for the neutrino polarization,

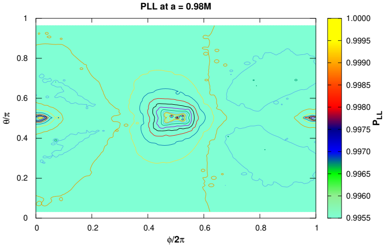
$$i\,\frac{d\psi}{dx} = H_x\,\psi \tag{36}$$

$$\hat{H}_x = -\mathcal{U}_2(\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_x)\mathcal{U}_2^{\dagger}, \quad \boldsymbol{\Omega}_x = r_g\boldsymbol{\Omega}\frac{\mathrm{d}t}{\mathrm{d}r}, \quad \mathcal{U}_2 = \exp(\mathrm{i}\pi\sigma_2/4) \tag{37}$$

- We use four-step Adams-Bashforth and Adams-Moulton predictor-corrector method to solve for ψ.
- For an incoming left polarized neutrino, $\psi^T_{-\infty} = (1, 0)$.
- For an outgoing neutrino, it becomes, $\psi^T_{+\infty} = (\psi^{(R)}_{+\infty}, \psi^{(L)}_{+\infty})$.
- The probability of a neutrino remaining left polarized: $P_{\mathrm{LL}} = |\psi^{(L)}_{+\infty}|^2$.

$$\Omega = \Omega_{\mathrm{g}} \cancel{+ \Omega_{\mathrm{matter}} + \Omega_{\mathrm{em}}}$$





ODE method

$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}}$ ~~$+ \boldsymbol{\Omega}_{\mathrm{matter}} + \boldsymbol{\Omega}_{\mathrm{em}}$~~ arXiv:2510.26621

- One has to set up an appropriate grid along particle position, r. This is not well-defined,
- The evaluation of trajectories depends on the numerical computations of (In)complete elliptic integrals of first kind.
- The determination of the polar angle, θ, depends on the number of turns that the neutrino makes around the BH as well as the numerical evaluation of elliptic Jacobi functions.
- In many cases, these determinations are approximate.
- Desirable to have an alternative method.

ODE method

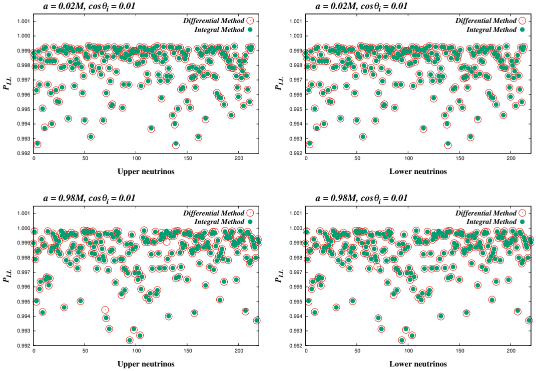
$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}}$ ~~$+ \boldsymbol{\Omega}_{\mathrm{matter}} + \boldsymbol{\Omega}_{\mathrm{em}}$~~

arXiv:2510.26621

- System of equations:

$$\frac{1}{z}\frac{\mathrm{d}\tilde{t}}{\mathrm{d}x} = \frac{\pm\sqrt{\Theta(\tilde{t})}}{\pm\sqrt{R(x)}}, \qquad \mathrm{i}\frac{\mathrm{d}\psi}{\mathrm{d}x} = \hat{H}_x\psi, \tag{38}$$

- Adaptive Runge–Kutta–Fehlberg 7(8) method. $\theta_i = 90°$.



Accretion Disk

- We choose a thick “Polish doughnut” disk surrounding the BH. Abramowicz et al., 1978.
- When a neutrino crosses the disk, its path is long enough for the spin to rotate on a sizable angle with respect to the neutrino velocity.
- The accretion disk rotates around BH with relativistic velocities.
- Made up of relativistic Hydrogen plasma such that it is electrically neutral, i.e. $n_e = n_p$.
- There is no differential rotation between the components of the plasma, i.e. $U_e^\mu = U_p^\mu$.
- The four potential for the neutrino electroweak interactions with the background matter in the locally Minkowskian frame, $g^a = e^a{}_\mu G^\mu = (g^0, 0, 0, g^3)$.

- We consider only toroidal magnetic field inside the disk (Komissarov, 2006).

$$B^{\phi} = \sqrt{\frac{2p_m^{(\mathrm{tor})}}{|g_{\phi\phi} + 2l_0 g_{t\phi} + l_0^2 g_{tt}|}}, \quad B^t = l_0 B^{\phi} \tag{39}$$

$$p_m^{(\mathrm{tor})} = K_m \mathcal{L}^{\kappa-1} \left[\frac{\kappa-1}{\kappa} \frac{W_{\mathrm{in}} - W}{K + K_m \mathcal{L}^{\kappa-1}} \right]^{\frac{\kappa}{\kappa-1}} \tag{40}$$

$$\rho = \left[\frac{\kappa-1}{\kappa} \frac{W_{\mathrm{in}} - W}{K + K_m \mathcal{L}^{\kappa-1}} \right]^{\frac{1}{\kappa-1}} \tag{41}$$

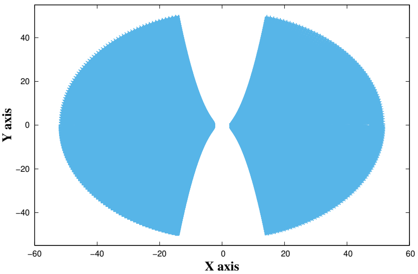
$$\mathcal{L} = g_{tt} g_{\phi\phi} - g_{t\phi}^2, \quad |\mathbf{B}|_{\max}^{(\mathrm{tor})} = \sqrt{2p_m^{(\mathrm{tor})}}, \quad \kappa = \frac{4}{3} \tag{42}$$

- The form of the disk depends on the potential,

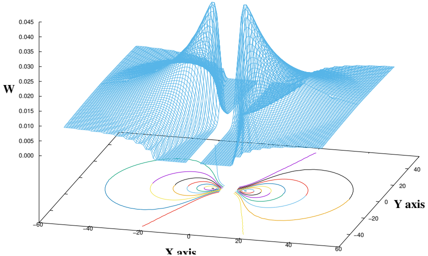
$$W(r,\theta) = \frac{1}{2} \ln \left| \frac{g_{tt} g_{\phi\phi} - g_{t\phi}^2}{g_{\phi\phi} + 2l_0 g_{t\phi} + l_0^2 g_{tt}} \right| \tag{43}$$

- We consider both co-rotating and counter-rotating disks.

Accretion Disk Cross−section at a = 0.50M

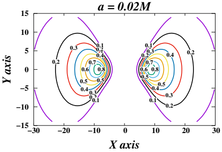


Accretion Disk Potential at a = 0.50M



Density Profile

Magnetic Field Profile



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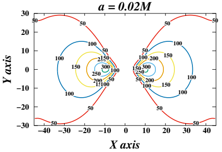
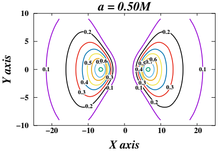
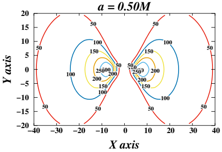


(b)



(c)



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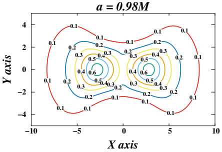
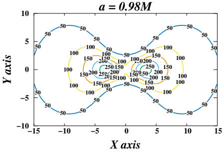




Numerical Parameters

- The mass of SMBH is $10^8 M_\odot$. The BH spin is $0 < a < 0.98M$.
- The maximal strength of the toroidal fields is 320 G. It is 1% of the Eddington limit for this BH mass Beskin, 2010.
- The maximal matter density of hydrogen plasma is 10^{18} cm^{-3}. Such density can be found in some AGN Jiang et al., 2019.
- We consider Neutrino magnetic moment, $\mu = 10^{-13}\mu_\mathrm{B}$. It is below the best astrophysical constraint Viaux et al., 2013.
- The number of scattered neutrinos for each combination of a and θ_i is more than 2 million.
- All the computations have been carried out at Govorun Supercluster of JINR. We have used more than 2000 SkyLake and IceLake processors continuously for several weeks.

$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{matter}} \cancel{+ \boldsymbol{\Omega}_{\mathrm{em}}}$ arXiv:2504.07816

- The matter interactions can be decomposed into transversal and longitudinal components:

  (A. I. Studenikin, 2004; A. I. Studenikin et al, 2018)

$$\begin{aligned} \boldsymbol{\Omega}^{\mathrm{matter}} &= \frac{1}{\gamma}(\gamma\, \boldsymbol{\Omega}_{\|}^{\mathrm{matter}} + \boldsymbol{\Omega}_{\perp}^{\mathrm{matter}}) \qquad (44) \\ \gamma &= (1-\beta^2)^{-1/2} \\ \boldsymbol{\beta} &: \text{ neutrino velocity.} \end{aligned}$$

- $\boldsymbol{\Omega}_{\|}^{\mathrm{matter}}$ has no impact on spin oscillations.
- For ultra-relativistic neutrinos, $\gamma \to \infty$ in flat spacetime at the source and observer positions.
- However, the analogue of γ as a function of the neutrino velocity, is not well defined in the curved spacetime near a spinning BH.

$\mathbf{\Omega} = \mathbf{\Omega}_{\mathrm{g}} + \mathbf{\Omega}_{\mathrm{matter}} \cancel{+ \mathbf{\Omega}_{\mathrm{em}}}$ arXiv:2504.07816

- Thus, the quantity $\mathbf{\Omega}_{\perp}^{\mathrm{matter}}/\gamma$ can become non-zero and may introduce spin oscillations.

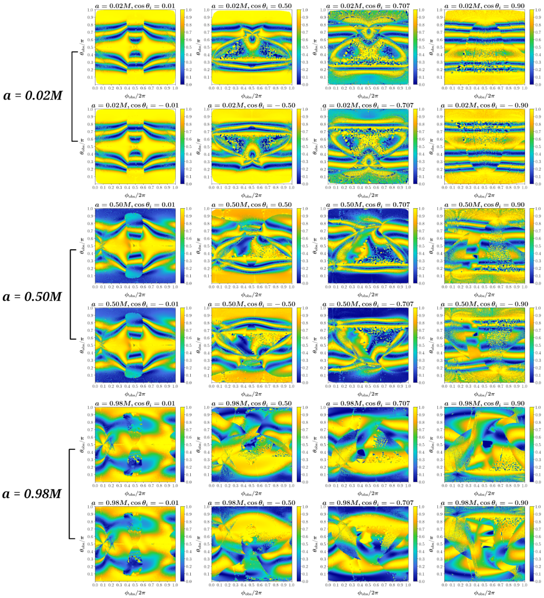
- We can probe this phenomena only numerically.

- Our study finds that $P_{\mathrm{LL}} \approx 1$ for various angles with all cases of BH spin for both co-rotating and counter-rotating disks.

$\Rightarrow$ No spin oscillation in the presence of only gravity and background matter.

$$\mathbf{\Omega} = \mathbf{\Omega}_{\mathrm{g}} + \mathbf{\Omega}_{\mathrm{matter}} + \mathbf{\Omega}_{\mathrm{em}}$$

Co-rotating Disk



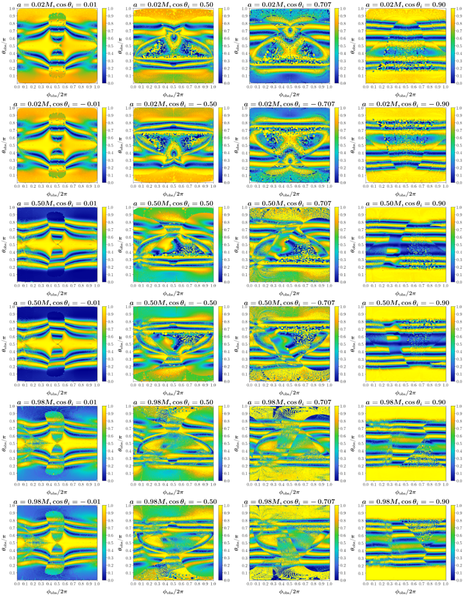
Counter-rotating Disk

$$\mathbf{\Omega} = \mathbf{\Omega}_{\mathrm{g}} + \mathbf{\Omega}_{\mathrm{matter}} + \mathbf{\Omega}_{\mathrm{em}}$$

a = 0.02M

a = 0.50M

a = 0.98M



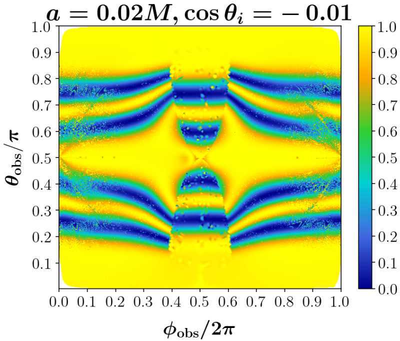
Conclusion

- Only toroidal magnetic field is sufficient enough for spin oscillations to occur.
- We investigate P_{LL} for a number of different θ_i's. This is important since the relative position between a neutrino source and Earth is not known during the observation.

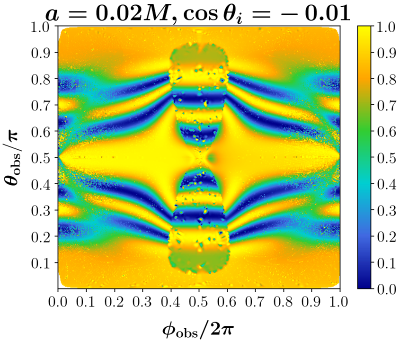
Conclusion

- There is a clear difference between P_{LL}'s for the co-rotating and counter-rotating disks even for a slowly rotating BH.

Co-rotating Disk



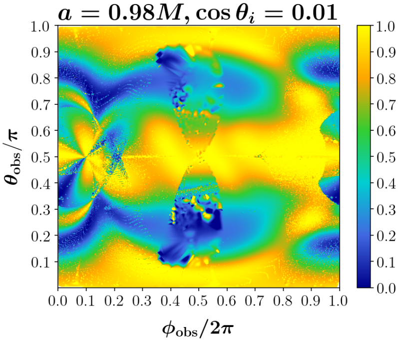
Counter-rotating Disk



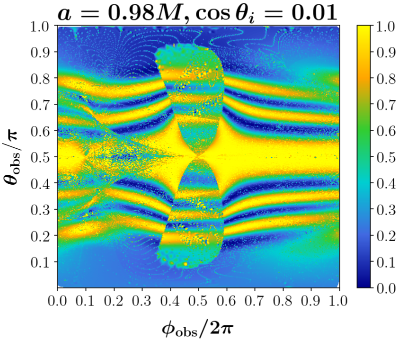
Conclusion

- No symmetric distributions of P_{LL} w.r.t. the $\theta_{\mathrm{obs}} = \pi/2$ plane can be seen for a rotating BH at lower $\cos\theta_i$'s.

Co-rotating Disk



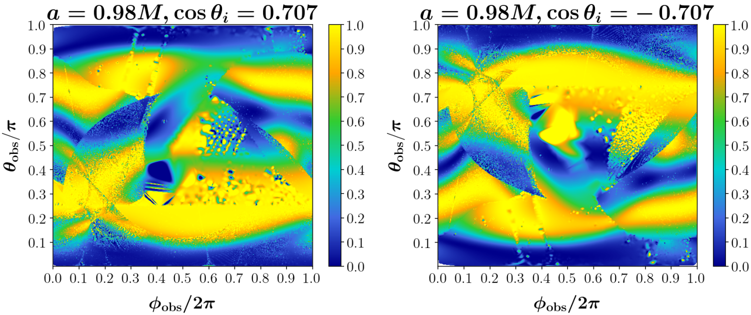
Counter-rotating Disk



Conclusion

- No inverse symmetry of P_{LL} for the opposite values of $\cos\theta_i$ with the same BH spin is exhibited for a rotating BH.

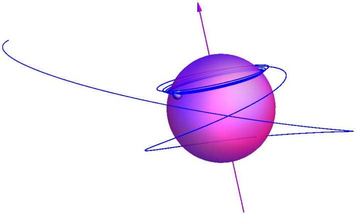
Co-rotaing Disk

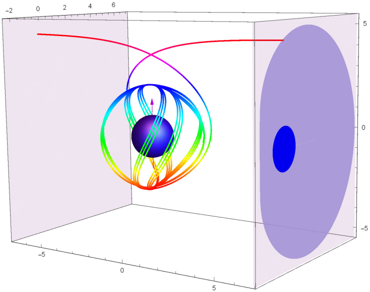


Thank you!

Extras

Dokuchaev and Nazarova, 2020





Neutrino spin evolution in curved spacetime

- We consider neutrino as a Dirac particle with nonzero magnetic moment, μ.
- Weakly interacts with the background matter.
- Four velocity of a neutrino is parallel transported along geodesics.
- The covariant equation for the neutrino spin four vector in curved spacetime (Pomeransky and Khriplovich, 1998; Dvornikov, 2013; Dvornikov, 2023),

$$\frac{DS^\mu}{D\tau} = 2\,\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_\nu U_\lambda S_\rho, \quad \frac{DU^\mu}{D\tau} = 0.$$

$$\begin{aligned} DS^\mu &= dS^\mu + \Gamma^\mu_{\alpha\beta}S^\alpha dx^\beta \\ G_F &= 1.17\times 10^{-5}\mathrm{GeV}^{-2} : \text{Fermi constant} \\ G_\mu &: \text{covariant effective potential.} \end{aligned}$$

We introduce a locally Minkowskian coodinates,

$$x_a = e_a^\mu x_\mu, \tag{45}$$

where $e_a^\mu (a = 0 \cdots 3)$ are the vierbein vectors satisfying the relations

$$e_a^\mu e_b^\nu g_{\mu\nu} = \eta_{ab}, \;\; e_\mu^a e_\nu^b \eta_{ab} \; = \; g_{\mu\nu} \tag{46}$$

Here $e_\mu^a e_\mu^a$ are the inverse vierbein vectors ($e_a^\mu e_\nu^a = \delta_\nu^\mu$ and $e_a^\mu e_\mu^b = \delta_a^b$) and $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$.

$$\begin{aligned} e_0^\mu &= \left(\sqrt{\frac{\Xi}{\Sigma\Delta}}, 0, 0, \frac{arr_g}{\sqrt{\Sigma\Delta\Xi}}\right), \;\; e_1^\mu = \left(0, \sqrt{\frac{\Delta}{\Sigma}}, 0, 0\right), \\ e_2^\mu &= \left(0, 0, \frac{1}{\sqrt{\Sigma}}, 0\right), \;\; e_3^\mu = \left(0, 0, 0, \frac{1}{\sin\theta}\sqrt{\frac{\Sigma}{\Xi}}\right) \end{aligned} \tag{47}$$

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{em}} + \boldsymbol{\Omega}_{\mathrm{matt}} \tag{48}$$

$$\begin{aligned} \boldsymbol{\Omega}_{\mathrm{g}} &= \frac{1}{2U^t}\left[\boldsymbol{b}_g + \frac{1}{1+u^0}(\boldsymbol{e}_g \times \boldsymbol{u})\right] \\ \boldsymbol{\Omega}_{\mathrm{em}} &= \frac{\mu}{U^t}\left[u^0\boldsymbol{b} - \frac{\boldsymbol{u}(\boldsymbol{u}\boldsymbol{b})}{1+u^0} + (\boldsymbol{e} \times \boldsymbol{u})\right] \\ \boldsymbol{\Omega}_{\mathrm{matt}} &= \frac{G_{\mathrm{F}}}{\sqrt{2}U^t}\left[\boldsymbol{u}\left(g^0 - \frac{(\boldsymbol{g}\boldsymbol{u})}{1+u^0}\right) - \boldsymbol{g}\right] \end{aligned} \tag{49}$$

Here $u^a = (u^0, \boldsymbol{u}) = e^a_\mu U^\mu$, $U^\mu = \frac{dx^\mu}{d\tau}$ is the four velocity in the world co-ordinates and τ is the proper time. $G_{ab} = (\boldsymbol{e}_g, \boldsymbol{b}_g) = \gamma_{abc}u^c$, $\gamma_{abc} = \eta_{ad}e^d_{\mu;\nu}e^\mu_b e^\nu_c$ are the Ricci rotation coefficients, the semicolon stays for the covariant derivative, and $f_{ab} = e^\mu_a e^\nu_b F_{\mu\nu} = (\boldsymbol{e}, \boldsymbol{b})$ is the electromagnetic field tensor in the locally Minkowskian frame, and $F_{\mu\nu}$ is an external electromagnetic field tensor. μ is the neutrino magnetic moment, and $G_{\mathrm{F}} = 1.17 \times 10^{-5}$ Gev^{-2} is the Fermi constant. $g^a = (g^0, \boldsymbol{g}) = e^a_\mu G^\mu$, G^μ is the contravariant effective potential of the neutrino electroweak interaction with a background matter.

Toroidal Fields

- The electromagnetic field tensor

$$F_{\mu\nu} = E_{\mu\nu\alpha\beta} U_f^{\alpha} B^{\beta}, \quad E^{\mu\nu\alpha\beta} = \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} \tag{50}$$

- The four vector fluid velocity in the disk and toroidal magnetic field are

$$U_f^{\mu} = (U_f^t, 0, 0, U_f^{\phi}), \qquad U_f^t = \sqrt{\left|\frac{\mathcal{A}}{\mathcal{L}}\right| \frac{1}{1 - l_0\Omega}}, \quad U_f^{\phi} = \Omega U_f^t \tag{51}$$

$$B^{\mu} = (B^t, 0, 0, B^{\phi}), \qquad B^{\phi} = \sqrt{\frac{2p^{(\mathrm{tor})_m}}{|\mathcal{A}|}}, \quad B^t = l_0 B^{\phi} \tag{52}$$

- The angular velocity in the disk

$$\Omega = -\frac{g_{t\phi} + l_0 g_{tt}}{g_{\phi\phi} + l_0 g_{t\phi}} \tag{53}$$

and

$$\mathcal{L} = g_{tt} g_{\phi\phi} - g_{t\phi}^2, \quad \mathcal{A} = g_{\phi\phi} + 2 l_0 g_{t\phi} + l_0^2 g_{tt} \tag{54}$$