Dubna April,24 2025

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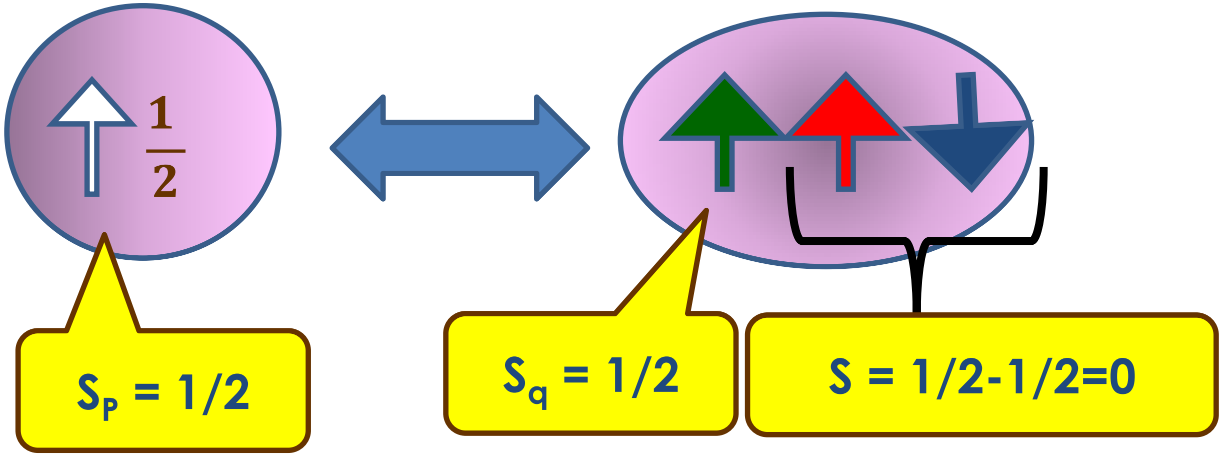
Обзор современного состояния проблемы спина протона при высоких энергиях

Proton is a compound particle, so its spin is made out of spins of the proton constituents

Proton spin $S_p = 1/2$. In the simplest model, proton consists of three quarks of different colours:



No spin problem with the proton spin description if proton is non-relativistic and consists of 3 quarks only

experiments on e^+e^--annihilation in hadrons demonstrated that nucleons (protons) consist of partons, i.e. quarks and gluons

So, Angular Moment conservation relates the hadron spin to the parton (quarks and gluon) spins

Proton spin =1/2. Proton consists of quarks (quark spin = 1/2) and gluons (gluon spin = 1)

Expectation:

$$\frac{1}{2} = S_q \quad + \quad S_g$$

quarks

gluons

First experimental investigation of the nucleon spin in DIS was carried out by European Muon Collaboration (EMC) in 1987 at CERN

$$\boxed{S_P{}^{exp} = 0.126 \pm 0.010_{syst} \pm 0.015_{stat}}$$

This was named **Proton Spin Puzzle/ Spin Crisis**

$$S_q = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x)$$

Quark helicity distribution

$$S_g = \int_0^1 dx\, \Delta G(x)$$

Gluon helicity distribution

x is a fraction of the proton momentum carried by the parton: $\vec{k} = x\vec{P}$

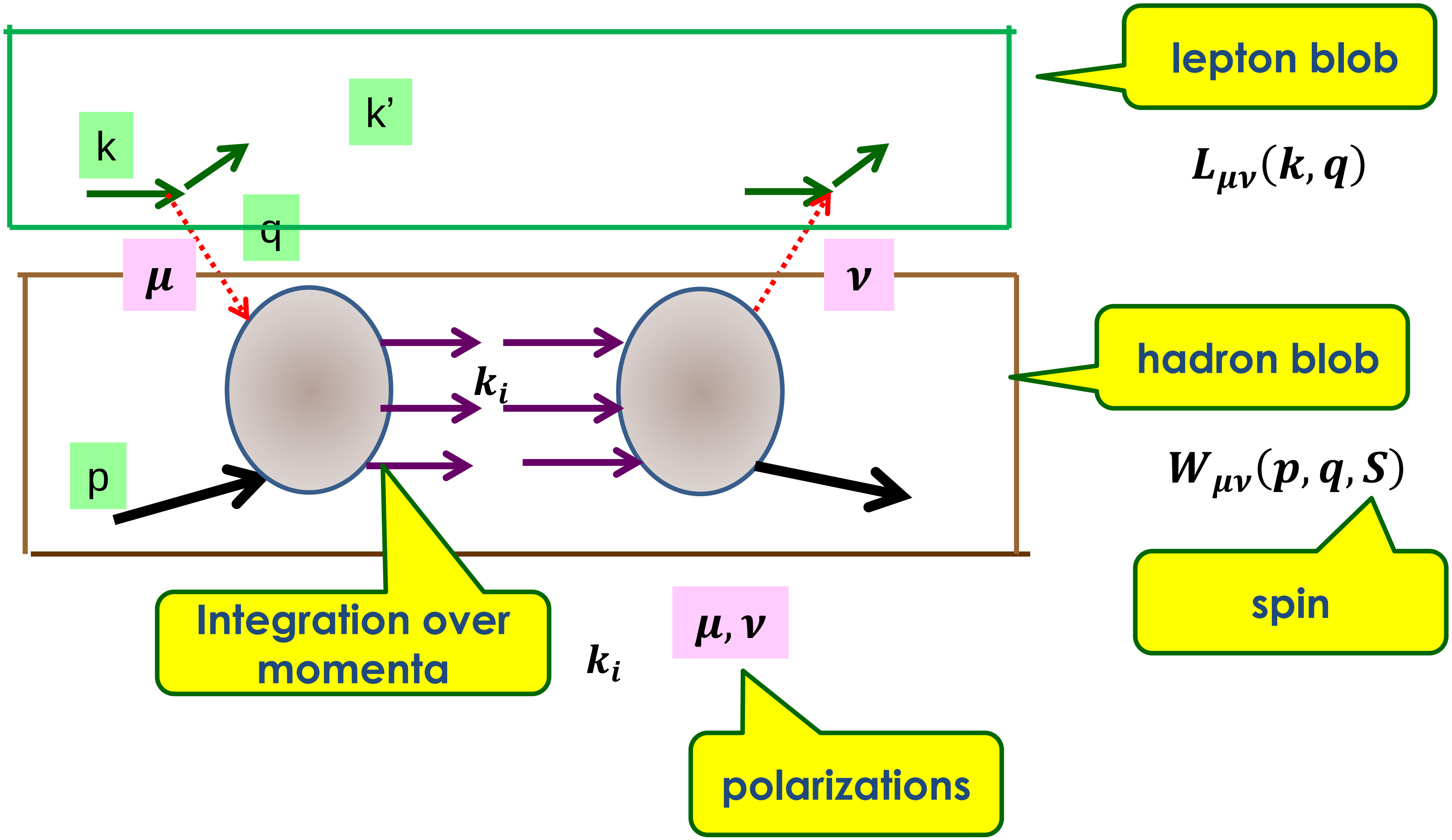
Angular momentum conservation violated: $S_q + S_g < 1/2$

To explain Puzzle, there were introduced additional contributions: Angular Orbital Moments of quarks and gluons, L_q and L_g Jaffe-Manohar (1990), Ji (1997)
Nevertheless, accounting for them in the Born approximation did not solve the problem:

$$S_q + S_g + L_q + L_g \quad < 1/2$$

Experimental data on proton spin at high energies have arrived from lepton-hadron Deep-Inelastic Scattering (DIS)

Deeply Inelastic Scattering: Inclusive cross-section



lepton blob

$L_{\mu\nu}(k, q)$

hadron blob

$W_{\mu\nu}(p, q, S)$

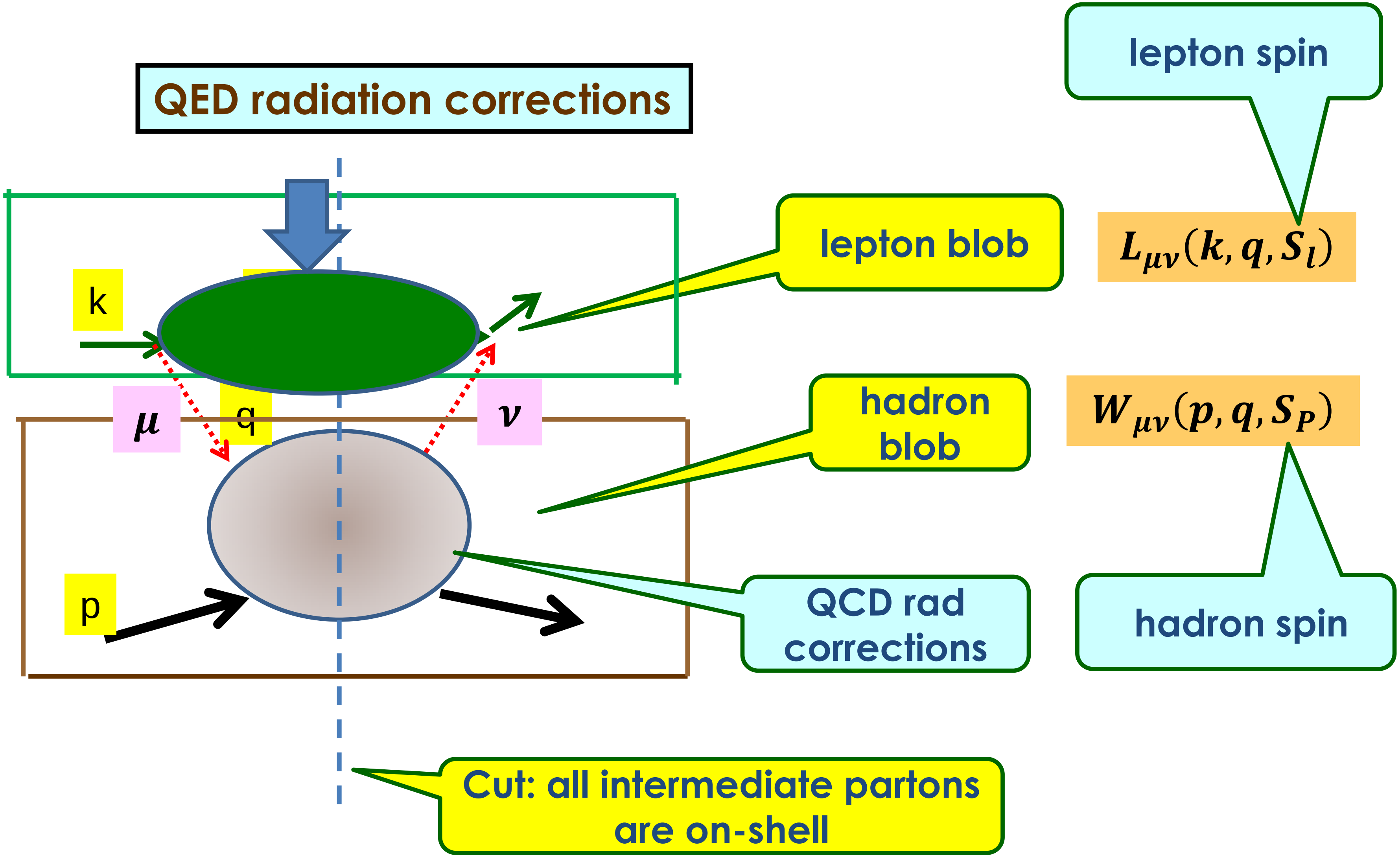
spin

Integration over momenta

k_i

μ, ν

polarizations



Inclusive cross section ~ $L_{\mu\nu}(k, q)$ $W_{\mu\nu}(p, q, S)$

Standard parametrization of $W^{spin}{}_{\mu\upsilon}$

Bose

$$W^{spin}{}_{\mu\nu} = W^{spin}{}_{\nu\mu}$$

Current conservation

$$W^{spin}{}_{\mu\nu}q_\mu = W^{spin}{}_{\mu\nu}q_\nu = 0$$

Hadron mass

Longitudinal component of spin

transverse component of spin

$$W^{spin}{}_{\mu\upsilon}= i\,\epsilon_{\mu\nu\lambda\rho}\;\; m_H q_\lambda/pq\left[S^{||}{}_\rho\, g_1\left(x, Q^2\right) + S^{\perp}{}_\rho\, g_\perp\left(x, Q^2\right)\right]$$

Spin-dependent structure functions

$$g_\perp = g_1 + g_2$$

$$S^{||}{}_\rho \approx p_\rho/m_H$$

Each structure function depends on the invariant energy *w = 2pq* and virtuality of the photon *Q*²

$$x = \; Q^2/2pq, \qquad 0 < x < 1$$

Spin structure functions are asymmetries:

$$\boxed{g_1 \sim \sigma_L(\uparrow\uparrow) - \sigma_L(\uparrow\downarrow)}$$

$$\boxed{g_\perp = g_1 + g_2 \sim \sigma_T(\uparrow\uparrow) - \sigma_T(\uparrow\downarrow)}$$

Spins are longitudinal

subscripts:

L –longitudinal
T - transverse

Spins are transverse

At high energies, when masses are neglected, $S_L \leftrightarrow h$

helicity

Experimental data on S_L and S_T
come from investigation of structure function g_1
of Deep-Inelastic Scattering at COMPASS and RHIC

Taken from wwwcompass.cern.ch

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.

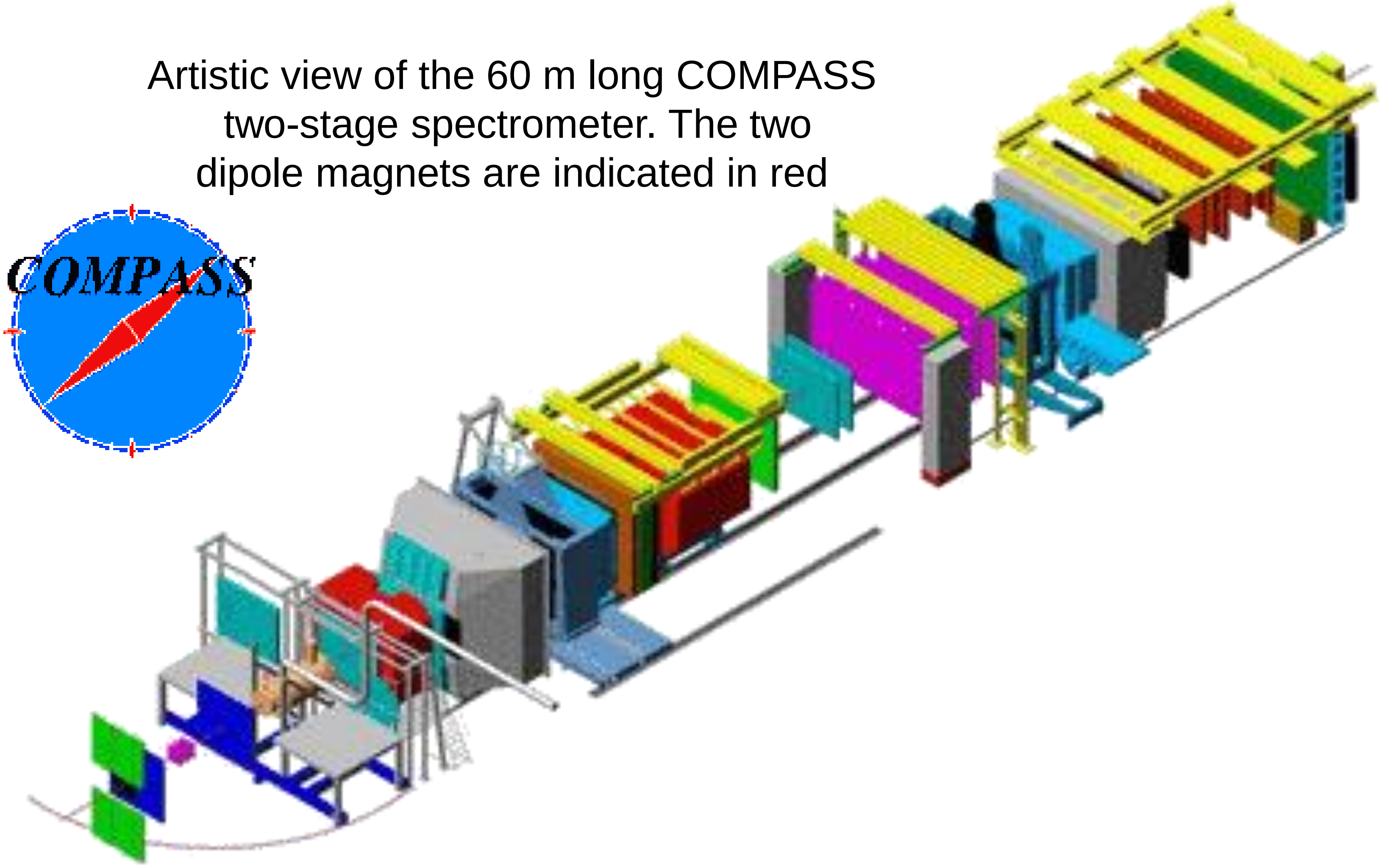
On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.

Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

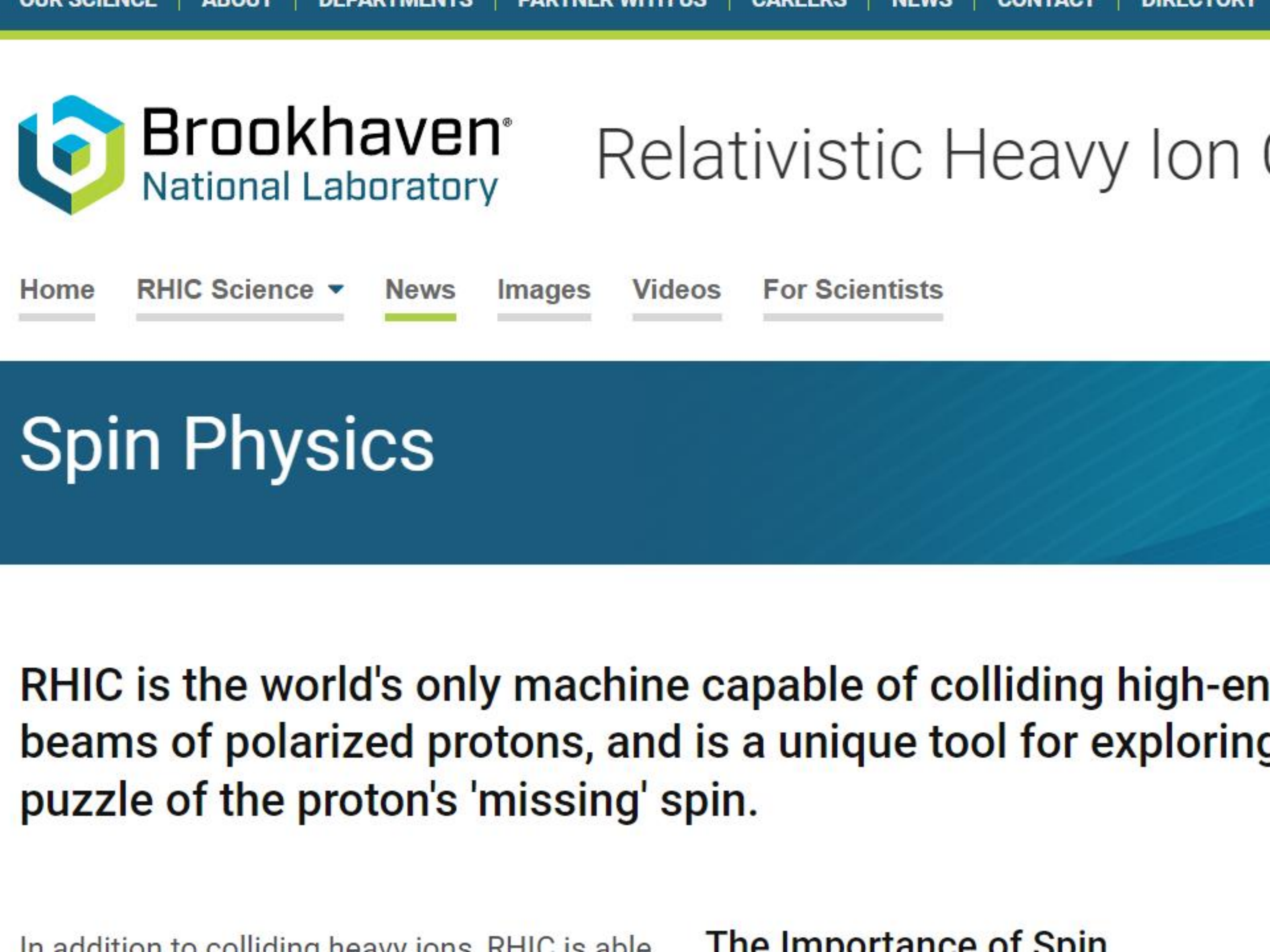
Taken from wwwcompass.cern.ch

COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy



Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red



Brookhaven®
National Laboratory

Relativistic Heavy Ion

Spin Physics

RHIC is the world's only machine capable of colliding high-en
beams of polarized protons, and is a unique tool for explorin
puzzle of the proton's 'missing' spin.

In addition to colliding heavy ions, RHIC is able

The Importance of Spin

Aim of the RHIC experiments: measuring S_q and S_g

$$S_q = \frac{1}{2}\int_0^1 dx\, h_q(x, Q^2)$$

Quark helicity distribution

$$S_g = \int_0^1 dx\, h_g(x, Q^2)$$

Gluon helicity distribution

Q^2 is fixed at RHIC: Q^2=10 GeV^2 so I will skip Q^2

In literature: $h_q \equiv \Delta\Sigma \quad h_g \equiv \Delta G$

Actually they obtained $\bar{S}_q$ and $\bar{S}_g$ at RHIC

$$\bar{S}_q = \frac{1}{2}\int_{x_1}^1 dx\, h_q(x)$$

$$\bar{S}_g = \int_{x_2}^1 dx\, h_q(x)$$

$x_1 = 0.001$

$x_2 = 0.05$

RHIC data (2015):

$S_q = 0.15 \div 0.20$ at $0.001 < x < 1$

$S_g = 0.13 \div 0.26$ at $0.05 < x < 1$

$Q^2 = 10\ GeV^2$

knowledge of $h_q(x)$ and $h_g(x)$ at smaller **x** is out of the RHIC reach

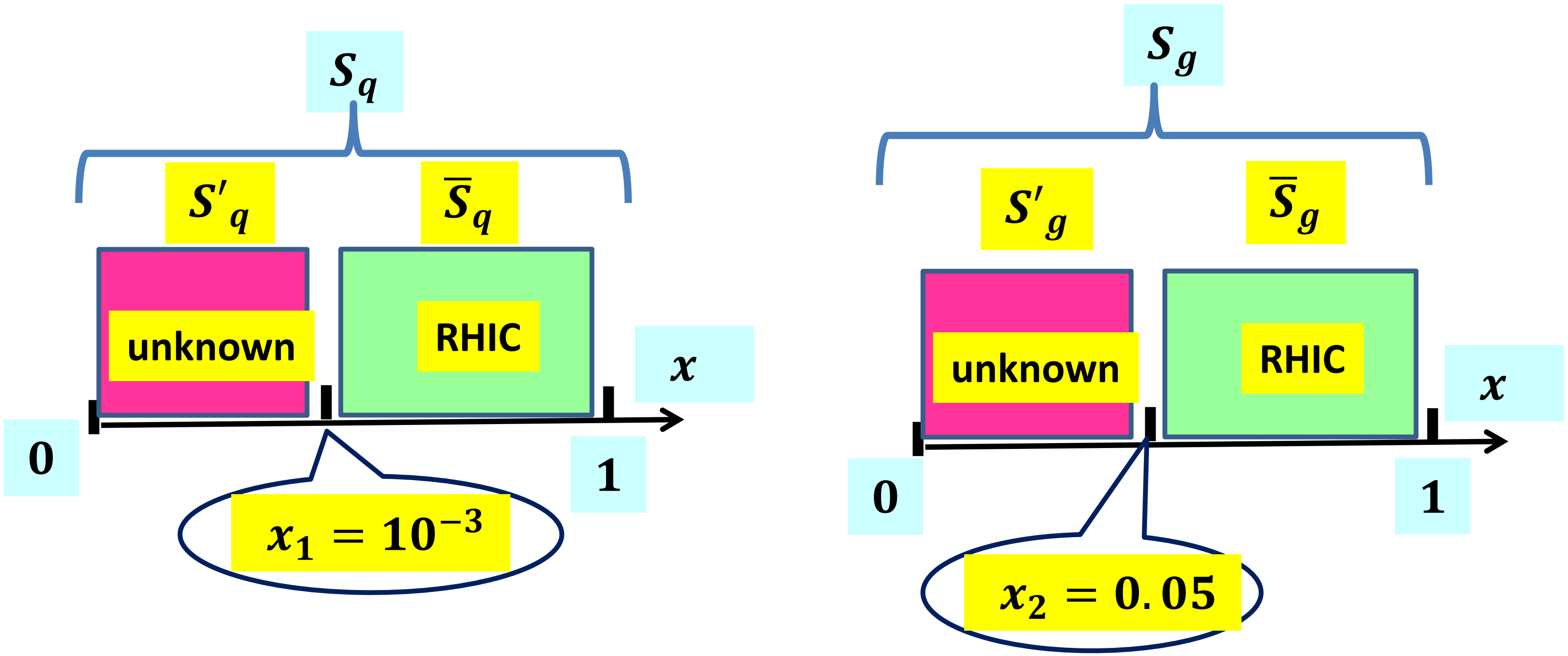
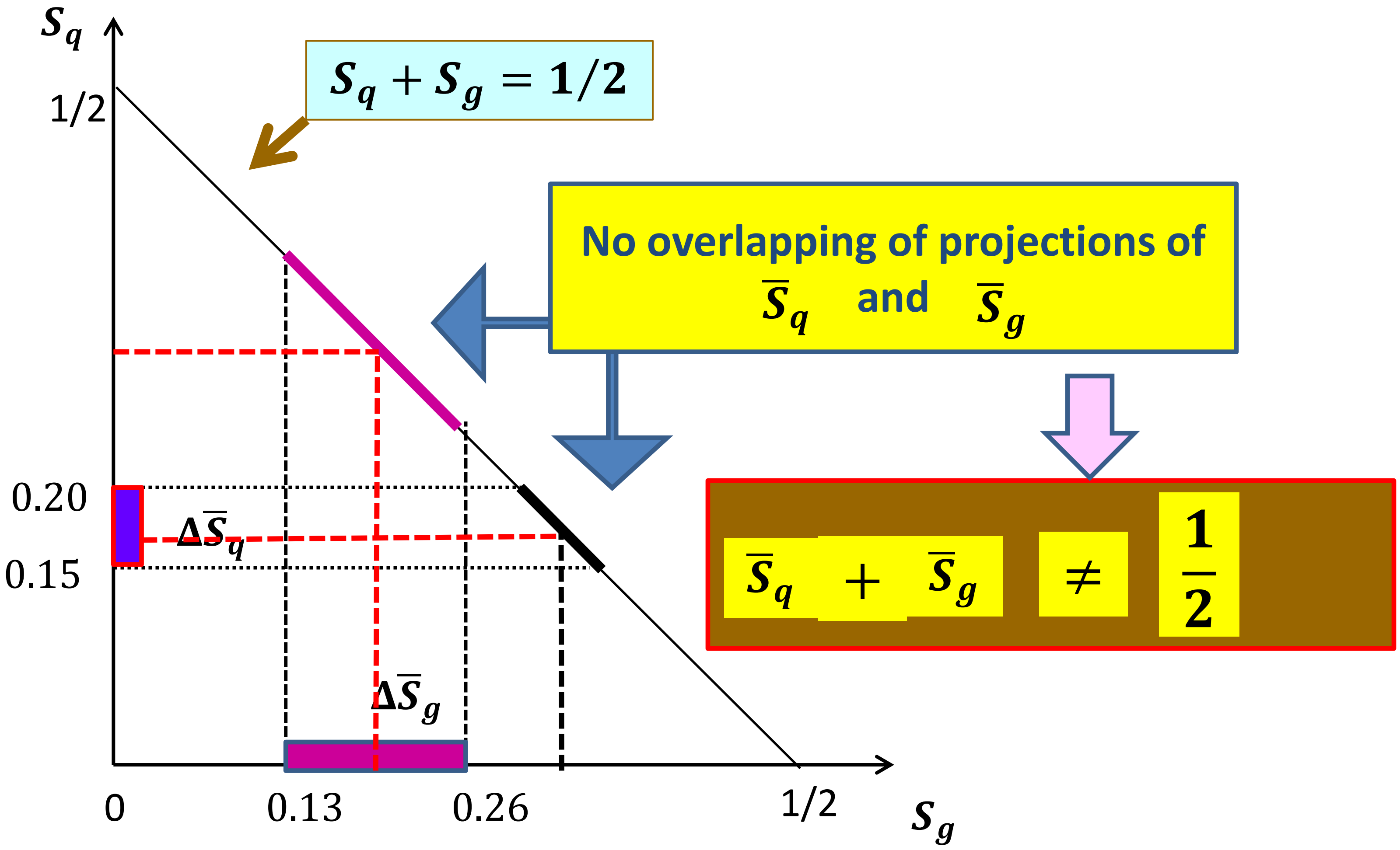


Illustration of the RHIC data:



$S_q + S_g = 1/2$

No overlapping of projections of $\bar{S}_q$ and $\bar{S}_g$

$$\bar{S}_q + \bar{S}_g \neq \frac{1}{2}$$

Missing contributions to the proton spin:

$$S'_q = \frac{1}{2}\int_0^{x_1} dx\, h_q(x)$$

$$S'_g = \int_0^{x_2} dx\, h_q(x)$$

$$x_1 = 0.001$$

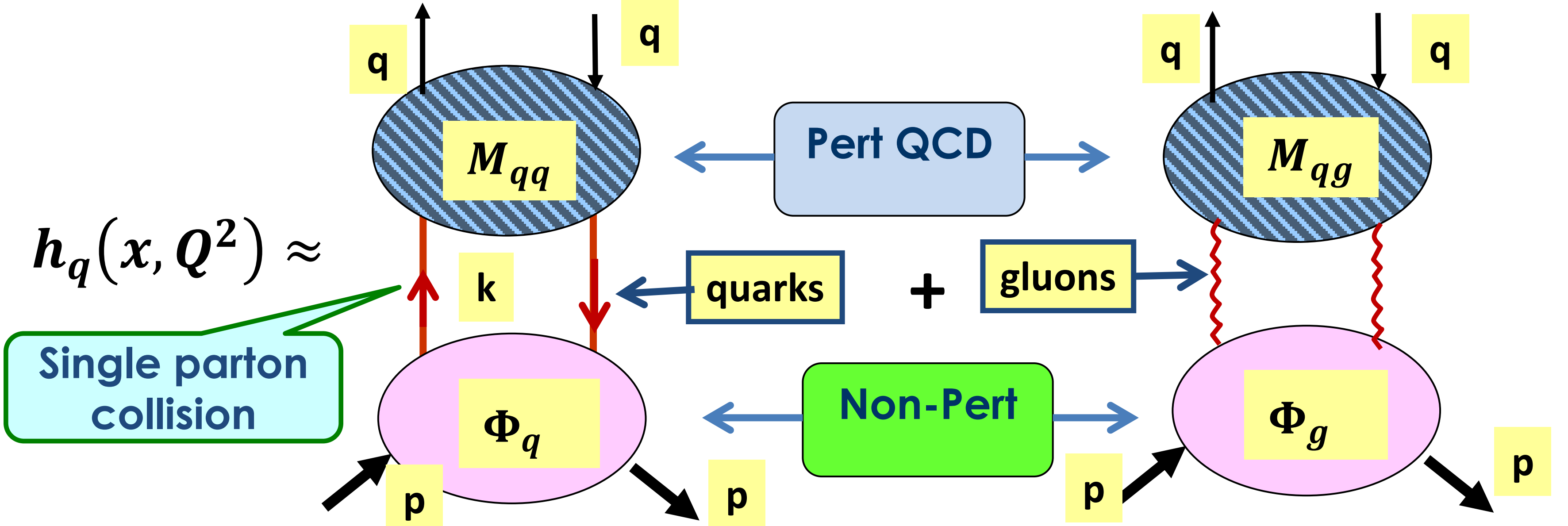
$$x_2 = 0.05$$

They cannot be registered at RHIC, so they should be calculated. Available theoretical instrument is QCD but QCD is a regular technical means at large momenta only.

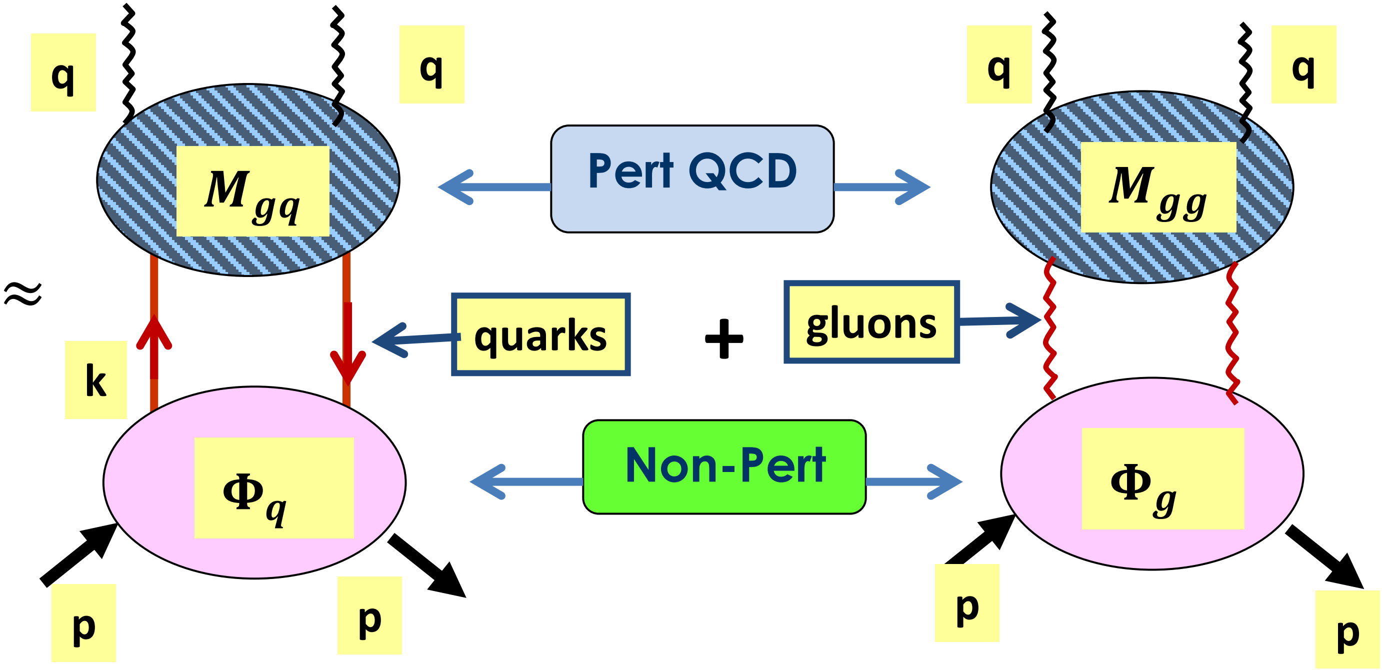
In order to describe an impact of the small momenta region, the QCD Factorization concept is used.

QCD Factorization for helicities:

$h_q(x, Q^2) \approx$



$h_g(x, Q^2) \approx$



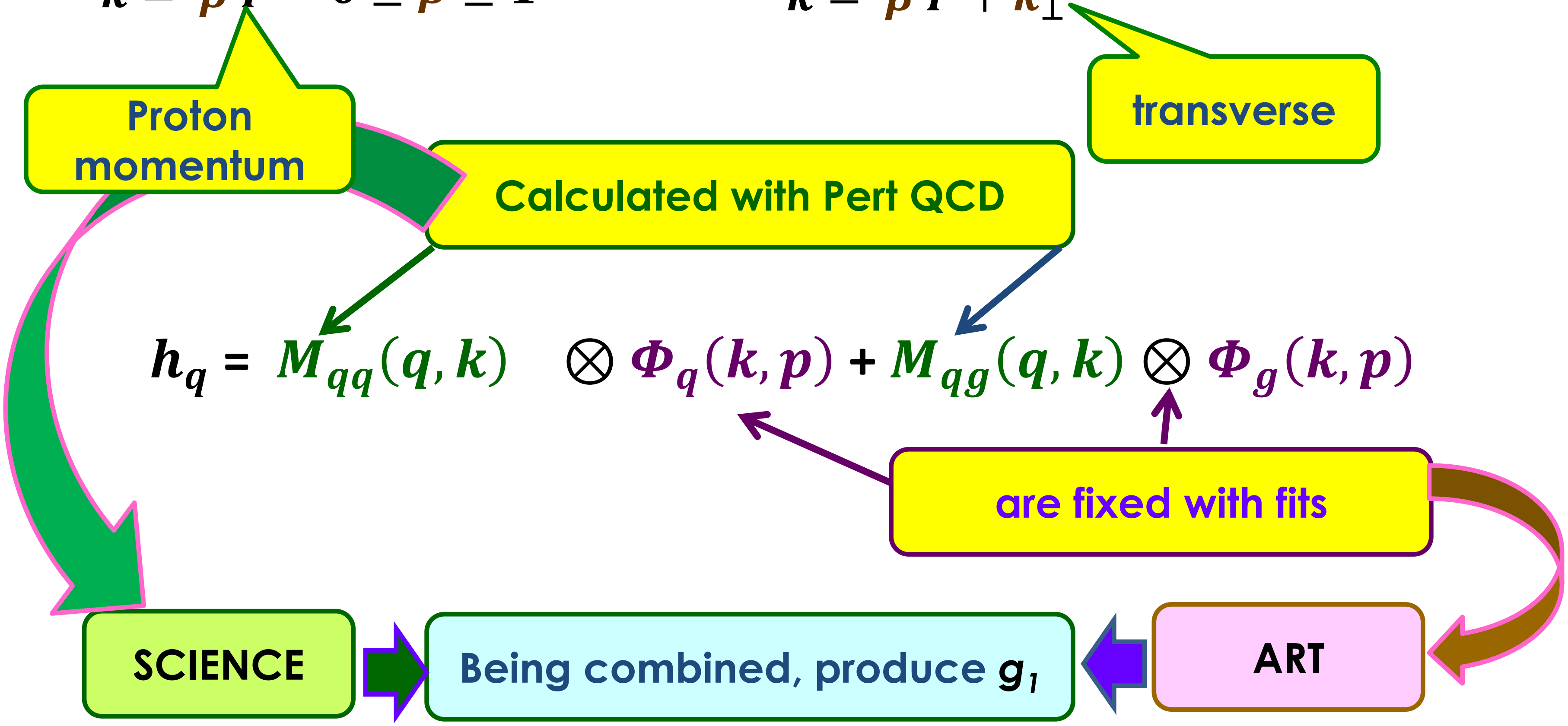
Forms of QCD factorization:

Collinear Factorization:

$$\vec{k} = \beta \vec{P} \qquad 0 \le \beta \le 1$$

KT Factorization:

$$\vec{k} = \beta \vec{P} + \vec{k}_{\perp}$$



Proton momentum

transverse

Calculated with Pert QCD

$$h_q = M_{qq}(q,k) \otimes \Phi_q(k,p) + M_{qg}(q,k) \otimes \Phi_g(k,p)$$

are fixed with fits

SCIENCE

Being combined, produce g_1

ART

similarly $h_g = M_{gq}(q,k) \otimes \Phi_q(k,p) + M_{gg}(q,k) \otimes \Phi_g(k,p)$

Any reaction in HEP includes combinations of Art and Science

No theory for non-perturbative contributions is available

Mellin transform: $M_{ij}(x, Q^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\,\pi\,i}\, x^{-\omega}\, f_{ij}(\omega, Q^2)$ i,j=q,g

Perturbative components of g_1 look very differently: For example, at Q^2 neglected

$$g_1{}^{(q)}(x) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\,\pi\,i}\, x^{-\omega}\, \omega\, F_q(\omega) \qquad F_q = \frac{\omega - f_{gg}}{\omega^2 - \omega(f_{qq} + f_{gg}) - (f_{qq}f_{gg} - f_{qg}f_{gq})}$$

Coefficient functions

$$g_1{}^{(g)}(x) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\,\pi\,i}\, x^{-\omega}\, \omega\, F_g(\omega) \qquad F_g = \frac{\omega - f_{qg}}{\omega^2 - \omega(f_{qq} + f_{gg}) - (f_{qq}f_{gg} - f_{qg}f_{gq})}$$

However, their small-x asymptotics are identical save unimportant factors. Asymptotics are of Regge type and the intercepts are identical

Most known method to calculating in QCD is DGLAP

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

It was suggested for operating at large x

All structure functions in DGLAP are calculated in several orders in the coupling

DGLAP expressions contain terms calculated in LO, NLO and NNLO

Most important contributions at small x are ~ $\underline{{\alpha_s}^n/\omega^{2n}}$

$$C(\omega) \approx \underbrace{1}_{\text{LO}} + \underbrace{\alpha_s\left[\frac{c_1}{\omega^2} + \frac{c'_1}{\omega}\right]}_{\text{NLO}} + \underbrace{{\alpha_s}^2\left[\frac{c_2}{\omega^4} + \frac{c'_2}{\omega^3} + \frac{c''_2}{\omega^2} + \frac{c'''_2}{\omega}\right]}_{\text{NNLO}}$$

$$\int_{-i\infty+\sigma}^{i\,\infty+\sigma} \frac{d\,\omega}{2\,\pi\,i}\, e^{\omega \ln(1/x)}\, \frac{1}{\omega^{1+n}} = \frac{1}{n!} \ln^n(1/x)$$

NB: Small x ⟺ small ω

The most singular terms are most important at small x

$$C(x) = 1 + \alpha_s ln(1/x)\underbrace{\left[c_1 + \alpha_s c_2\, ln^2(1/x) + \alpha_s{}^2 c_3\, ln^4(1/x)+..\right]}$$

Double – Logarithmic (DL) contributions

V.V. Sudakov, V.G. Gorshkov-V.N. Gribov-L.N. Lipatov-G.V. Frolov

Total resummation of DL contributions is called Double-Logarithmic Approximation

$$C(\omega) \approx 2\omega/\left[\omega + \sqrt{\omega^2 - b^2}\right]$$

Comes from Born terms

Resummation of pole terms leads to the branching singularity

One of the most effective methods of calculations in DLA is Infra-Red Evolution Equations L.N. Lipatov

Small-x asymptotics in DLA is obtained with applying Saddle-Point Method at $x \to 0$

small-x asymptotics

Bartels-Ermolaev-Manaenkov-Ryskin

$$h_q \sim h_g \sim g_1(x, Q^2) \sim \frac{\overset{\text{Numerical factor}}{\kappa}}{ln^{3/2}(1/x)} \underbrace{x^{-\overset{\text{intercept}}{\Delta}}}_{\text{Regge behavior}} \left(Q^2/\mu^2\right)^{\Delta/2}$$

grows at $x \to 0$ much steeper than the N..NLO DGLAP asymptotics:

$$g_1(x, Q^2) \sim e^{\alpha_s \sqrt[2n]{ln^{2n-1}(1/x) ln(Q^2/\mu^2)}}$$

Applicability region of Regge asymptotics

Ermolaev-Greco-Troyan

Asymptotics

We introduce $R_{as}(x, Q^2) = \overline{g}_1(x, Q^2) / g_1(x, Q^2)$

and numerically study its **x**-dependence at fixed ***Q***^{2}

Asymptotics reliably represent g_1 when R_{as} is close to 1.
Numerical analysis at Q^2 = 1 GeV^2 yields

$x = 10^{-3}$ $R_{AS} \approx 0.5$

$x = 10^{-4}$ $R_{AS} \approx 0.7$

$x = 10^{-6}$ $R_{AS} \approx 0.9$ → $\mathbf{x} < x_0 = 10^{-6}$

Appicability region for asymptotics

The more **Q**^{2} , the less **x**$_0$

There are two most known approaches in the literature to solve Proton Spin Puzzle

1) BFSSW: Borsa-de Florian-Sassot-Stratmann-Wogelsang

Perturbative evolution equations: NLO and NNLO DGLAP

2) KSPCTTABHFYBM

Kovchegov-Sivert-Pitonyak-Tarasov-Tawaburt-Cougoulic-Adamiak-Boussarie-Hatta- Fen Yuan-Baldonado-Melnitchouk

Perturbative evolution equations: KPSCTT

Kovchegov-Sivert-Pitonyak-Tarasov-Tawaburt-Cougoulic

KPSCTT operates with the small-x asymptotics $g_1 \sim x^{-a}$, using them at any x. However, the asymptotics should not have been used outside its applicability region.

It turned out that KPSCTT predicts $S_q + S_g < 1/2$

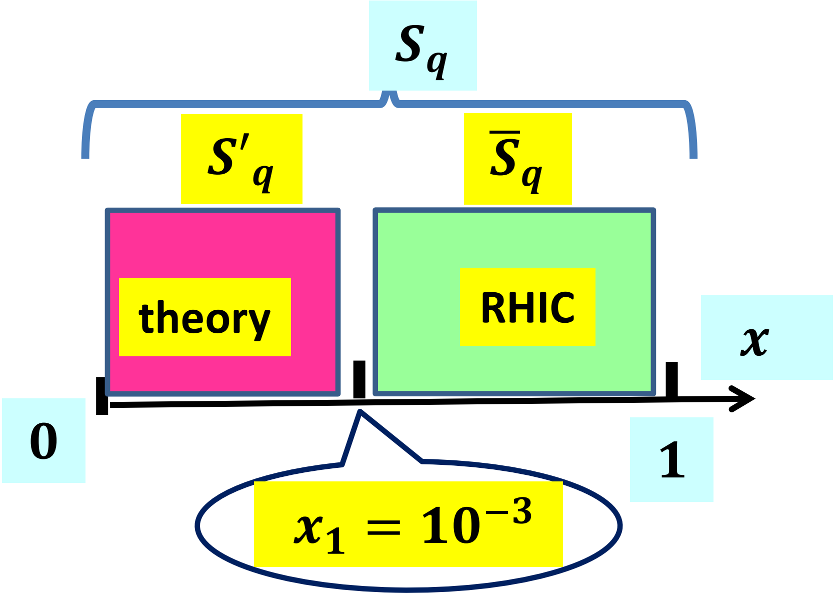
In order to get **1/2**, parton Angular Orbital Momentum (OAM) contribution was added to S_q, S_g

Boussarie- Hatta – Yuan, 2019; Kovchegov- Manley, 2023

This leads to

$$S_q + S_g + (L_q + L_g) = \frac{1}{2}$$

with L_q , L_g described by the same asymptotic formulae as S_q , S_g and used at any x



S_q

S'_q

$\bar{S}_q$

theory

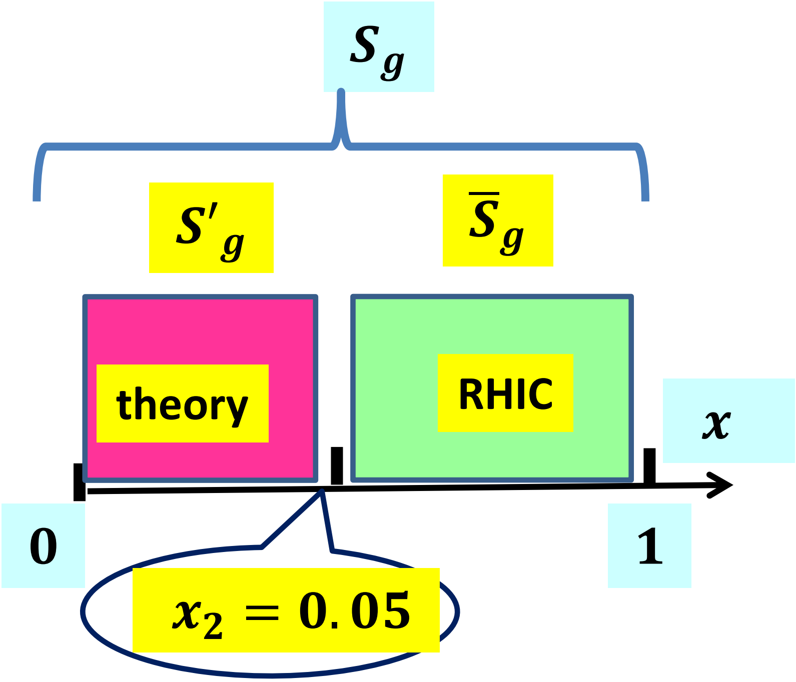
RHIC

x

0

1

$x_1 = 10^{-3}$



S_g

S'_g

$\bar{S}_g$

theory

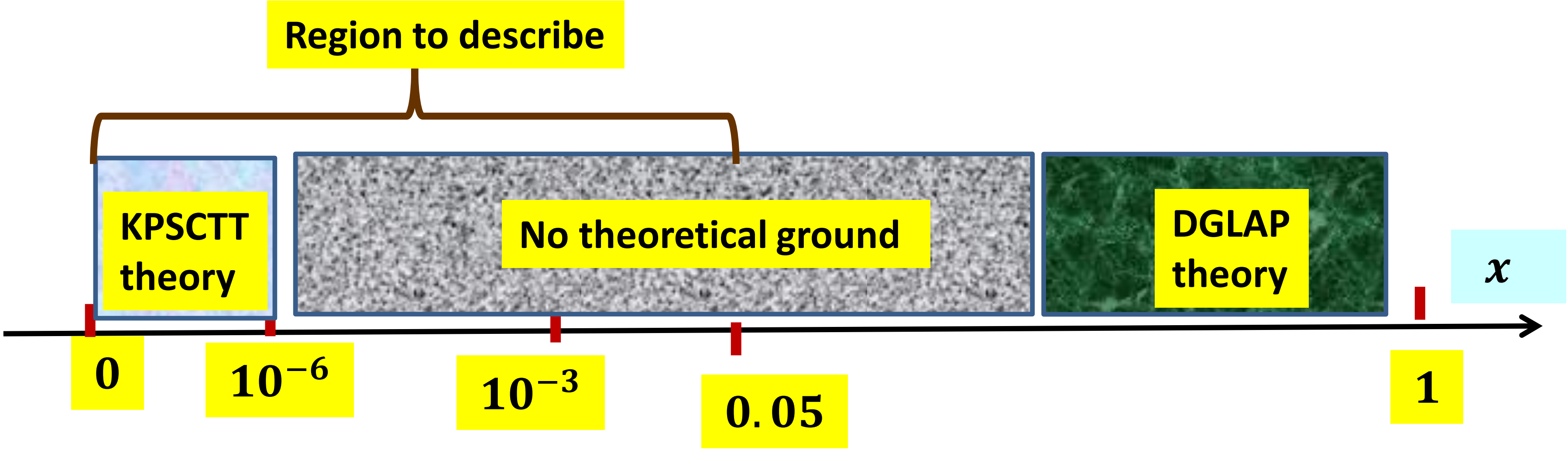
RHIC

x

0

1

$x_2 = 0.05$



Region to describe

KPSCTT theory

No theoretical ground

DGLAP theory

x

0

10^{-6}

10^{-3}

0.05

1

PUZZLE: the both perturbative methods work at widely different and quite restricted regions but nevertheless each of them brings excellent results and solves the proton spin problem

EXPLANATION: the leading impact is made by the fits for the initial parton distributions

1) DGLAP fit $\Phi = N \; x^{-a} \; (1-x)^{b} \; \left(1 + c x^{d}\right)$

N, a, b, c, d – free parameters, all of them are positive

Regge asymptotics at small **x**,
Much steeper than DGLAP itself

LO DGLAP $g_1 \sim e^{\sqrt{ln(1/x)}}$

2) KPSCTT fit: the same expression as DGLAP fit save the factor x^{-a} because this factor is included in perturbative component

We suggest a new approach to the proton spin problem, which is free of these drawbacks:

We account for the total summation of DL contributions to helicities and then modify it:

(i) Account for the running QCD coupling
(ii) Account for non-DL contributions, borrowing them from DGLAP

By doing so, we obtain the formulae which can be used at any **x**
Form of the fits becomes much simpler:

$$\Phi_{q,g} = N \quad x^{-a} \quad (1-x)^{b} \left(1 + c x^{d}\right)$$

can be dropped when DLA and DGLAP are combined,

Mimics resummation of DL and should be dropped when the resummation is taken into account

As a result,

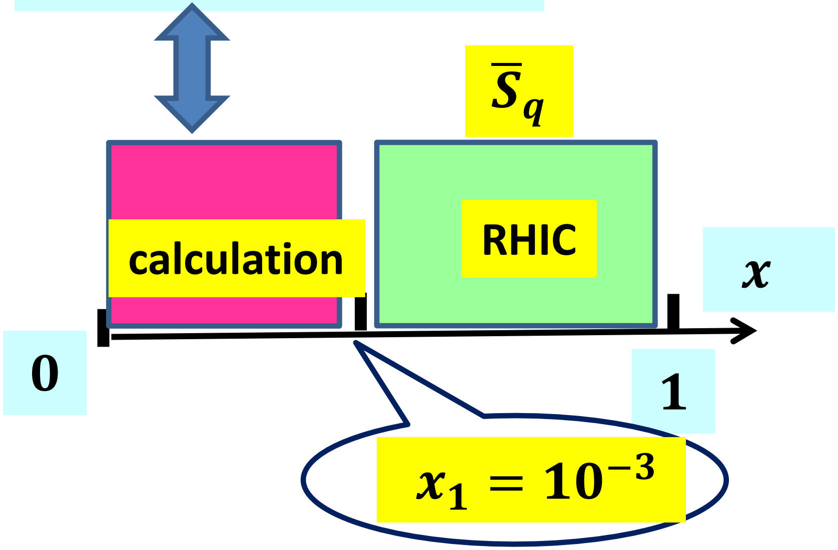
$$\Phi_q \approx N_q \qquad \Phi_g \approx N_g$$

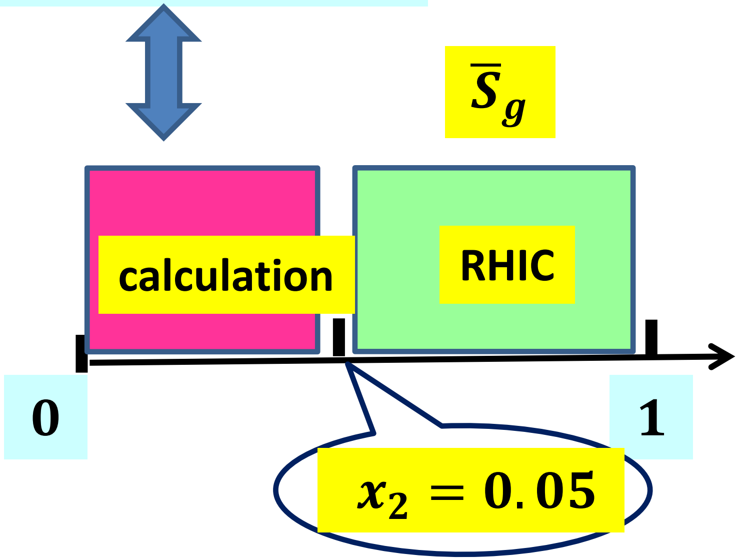
Unknown and cannot be fixed from theoretical grounds

Objects to calculate in DLA:

$$S'_q = \frac{1}{2}\int_0^{x_1} dx\, h_q(x)$$

$$S'_g = \int_0^{x_2} dx\, h_q(x)$$





Remind: $h_{q,g}$ are parton helicities

Each of them includes both perturbative and non-perturbative contributions

QCD Factorization:

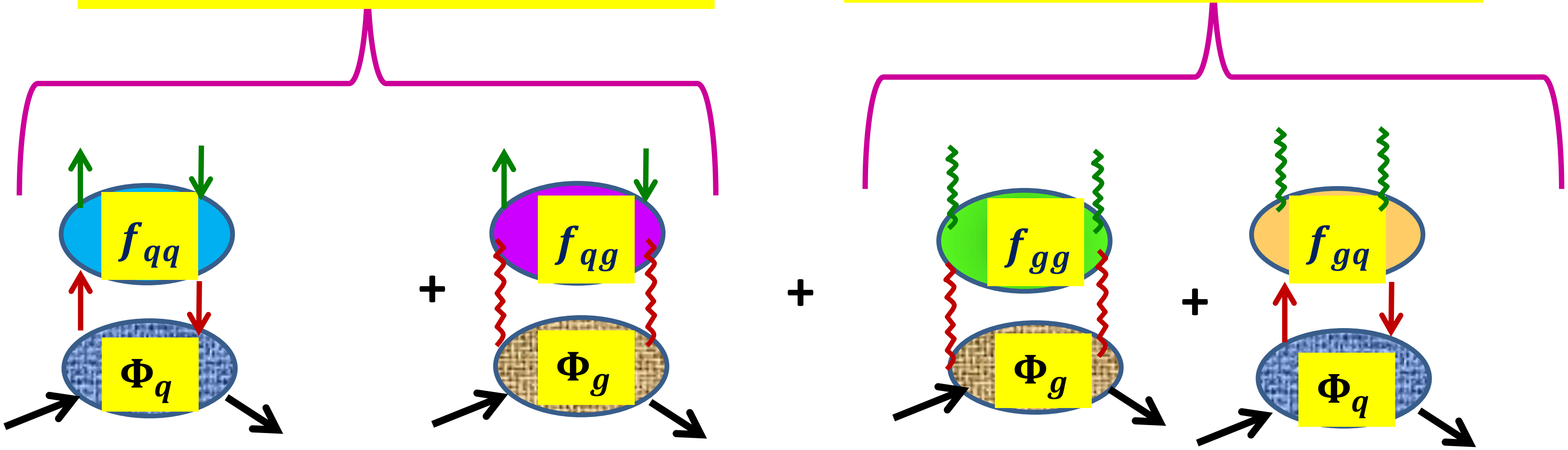
Perturbative components

$$h_q = f_{qq} \otimes \Phi_q + f_{qg} \otimes \Phi_g \qquad h_g = f_{gq} \otimes \Phi_q + f_{gg} \otimes \Phi_g$$

parton distributions

QCD Factorization for $h_q(x)$

QCD Factorization for $h_g(x)$



$$\Phi_q \approx N_q \qquad \Phi_g \approx N_g$$

Each intermediate state consists of 2 partons : Single Parton Collisions

Fortunately, this is OK for DLA calculations

Expressions for f_{ik} are known

Ermolaev-Greco-Troyan

On the contrary, N_q and N_g are unknown. Fix them from the RHIC data on $\overline{S}_q$ and $\overline{S}_g$ respectively

$$\left.\begin{aligned}\overline{S}_q &= \frac{1}{2} N_q \int_{x_1}^{1} dx\, f_{qq}(x) + N_g \frac{1}{2} \int_{x_1}^{1} dx\, f_{qg}(x) \\ \overline{S}_g &= N_q \int_{x_2}^{1} dx\, f_{gq}(x) + N_g \int_{x_2}^{1} dx\, f_{gg}(x)\end{aligned}\right\}$$

algebraic equations for $N_{q,g}$

Solving this system, express $N_{q,g}$ through $\overline{S}_{q,g}$

$$\left.\begin{aligned} S'_q &= \frac{1}{2} N_q \int_0^{x_1} dx\, f_{qq}(x) + N_g \frac{1}{2} \int_0^{x_1} dx\, f_{qg}(x) \\ S'_g &= N_q \int_0^{x_2} dx\, f_{gq}(x) + N_g \int_0^{x_2} dx\, f_{gg}(x) \end{aligned}\right\}$$

All terms in the r.h.s., are known, so it is possible to perform the integrations

This is program of straightforward calculation of parton contributions to the nucleon spin. However, its implementation is technically difficult because exact expressions for $f_{ik}(x)$ are quite complicated

Instead, we present an approximation way to obtain a tentative solution to the proton spin puzzle

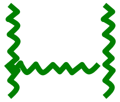
Main contributions come from gluons, so skip all quark contributions

$$f_{gg}(x) \approx \int_{-i\,\infty}^{i\,\infty} \frac{d\,\omega}{2\,\pi\,i}\;\omega\,x^{-\omega}\;F_{gg}(\omega)$$

with

Very simple expression

$F_{gg}(\omega) =$



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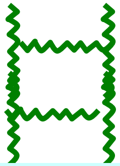

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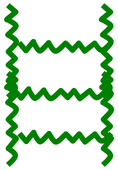


$+\ldots$

$$= 4\pi^2\left[\omega - \sqrt{\omega^2 - a_g}\right]$$

Poles at $\omega = 0$

$\frac{a_g}{\omega}$ $\quad \frac{{a_g}^2}{\omega^3}$ $\quad \frac{{a_g}^3}{\omega^5}$

where

$a_g = 4\alpha_s N/\pi$

comes from the Born graph

$$f_{gg}(x) = \quad 4\pi^2 \frac{d}{d\,\xi}\left(\frac{C}{\xi} I_1\left(\xi\sqrt{a_g}\right)\right) \qquad \xi = ln(1/x)$$

modified Bessel function

Small-x asymptotics is of the Regge type:

$$f_{gg} \sim \frac{c}{\xi^{3/2}} e^{\xi\sqrt{a_g}} = \frac{c}{\xi^{3/2}} x^{-\sqrt{a_g}}$$

NOTE: Coincides with KPSCTT result

Kovchegov-Sivert-Pitonyak-Tarasov-Tawaburt-Cougoulic

There was a kind of polemics in 2016- 2023

With quarks neglected, we obtained (Bartels-Ermolaev-Ryskin, 1996)

$a_g = z_g \left(\alpha_s N/2\pi\right)^{1/2}$ with $z_g = 3.66$

In 2016 there was published KPS-result (Kovchegov-Pitonyak-Sievert) where it was claimed that

$z_h = 2.45$

KPS 2016

In 2023 this result was seriously corrected by Kovchegov- Pitonyak - Sievert – Cougoulic- Tarasov- Tawabutr when they constructed KSPTT evolution equation instead of KPS. Their estimate of 2023 is

$z_h = 3.66$

KPSCTT 2023

which coincides with BER result of 1996

How to include quarks in the simplest way?

intercepts of the helicities and g_1 are known in DLA.
They account for both gluon and quark contributions and also account for the running coupling effects

Ermolaev-Greco-Troyan

$\omega_0 = 0.86$

Both virtual quarks and gluons contribute

Perfectly agrees with the estimate

$\omega_0 = 0.88 \pm 0.14$

obtained by fitting HERA results

Kochelev-Lipka-Nowak-Vento-Vinnikov

Hybrid model:

Replace the purely gluonic intercept a_g by the genuine intercept ω_0
Therefore, we get a simple interpolation formula. It coincides with the exact expression in the Born approximation and predicts the correct asymptotics

So, we obtain approximate expressions for the quark and gluon helicities

$h_q = C_q \frac{I_2(z)}{z}$ $h_g = C_g \frac{I_2(z)}{z}$ with $\boxed{z = \omega_0 ln(1/x)}$

Unknown, include non-perturbative contributions

Fix them, using the RHIC data

RHIC data

$$\boxed{\bar{S}_q = [0.15 \div 0.20]}$$

$$\bar{S}_q = \frac{1}{2} C_q A_q$$

$$\boxed{A_q = \int_{x_1}^{1} dx \frac{I_2(z)}{z}} \boxed{= 0.138}$$

$$\boxed{\bar{S}_g = [0.13 \div 0.26]}$$

$$\bar{S}_g = C_g A_g$$

$$\boxed{A_g = \int_{x_2}^{1} dx \frac{I_2(z)}{z}} \boxed{= 0.874}$$

$C_{q,g}$ are known, so we can calculate S'_q and S'_g

$$S'_q = \frac{1}{2} C_q B_q \qquad\qquad S'_g = C_g B_g$$

where

$$B_q = \int_0^{x_1} dx \frac{I_2(z)}{z} = 0.0243 \qquad\qquad B_g = \int_0^{x_2} dx \frac{I_2(z)}{z} = 0.0747$$

Obtain

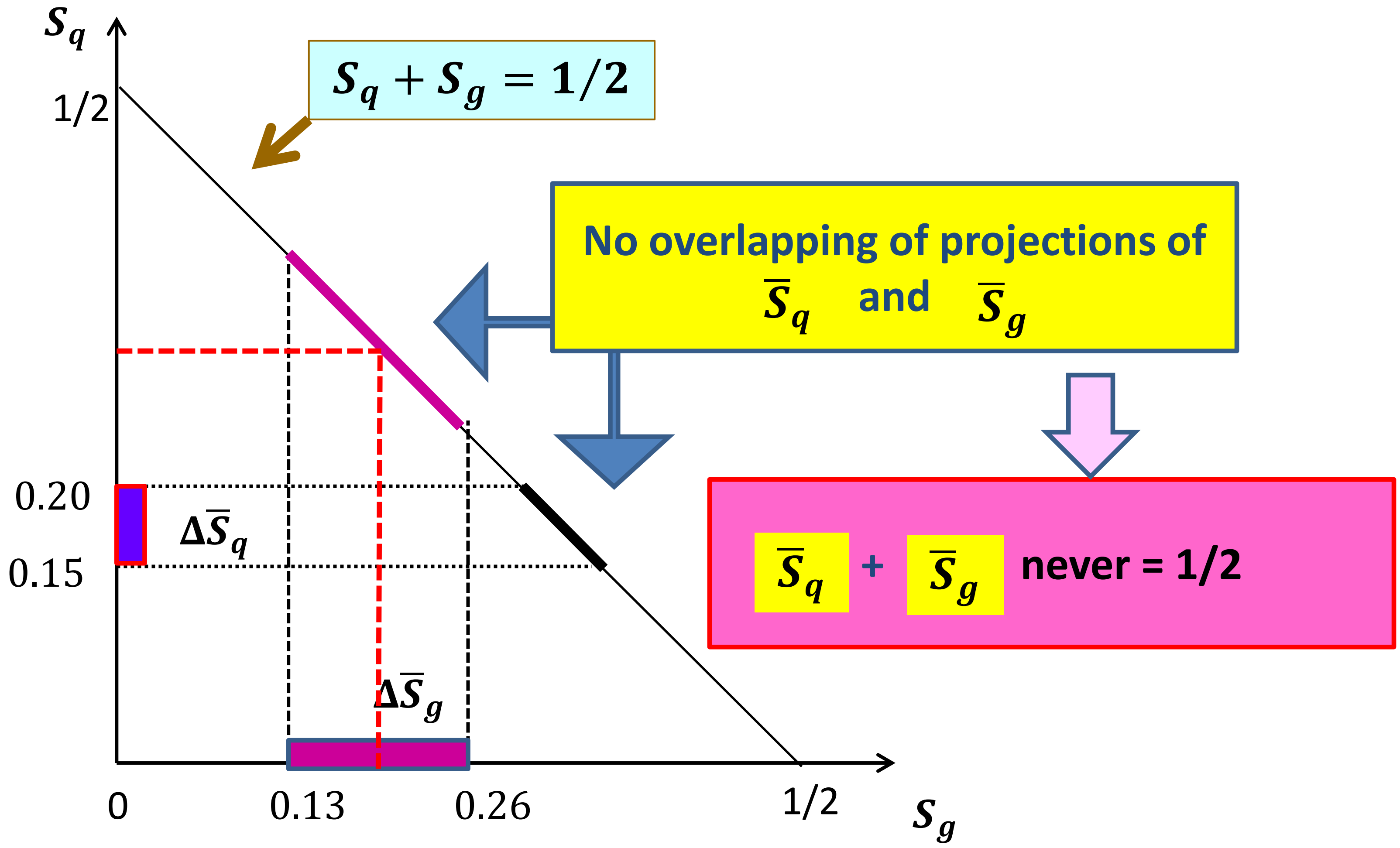
$$S_q = \bar{S}_q + S'_q = \bar{S}_q [1 + B_q/A_q] = \bar{S}_q [1 + 0.18]$$

$$S_g = \bar{S}_g + S'_g = \bar{S}_g [1 + B_g/A_g] = \bar{S}_g [1 + 0.85]$$

$$0.18 \leq S_q \leq 0.24 \qquad\qquad 0.24 \leq S_g \leq 0.72$$

$$0.42 \leq S_P \leq 0.72$$

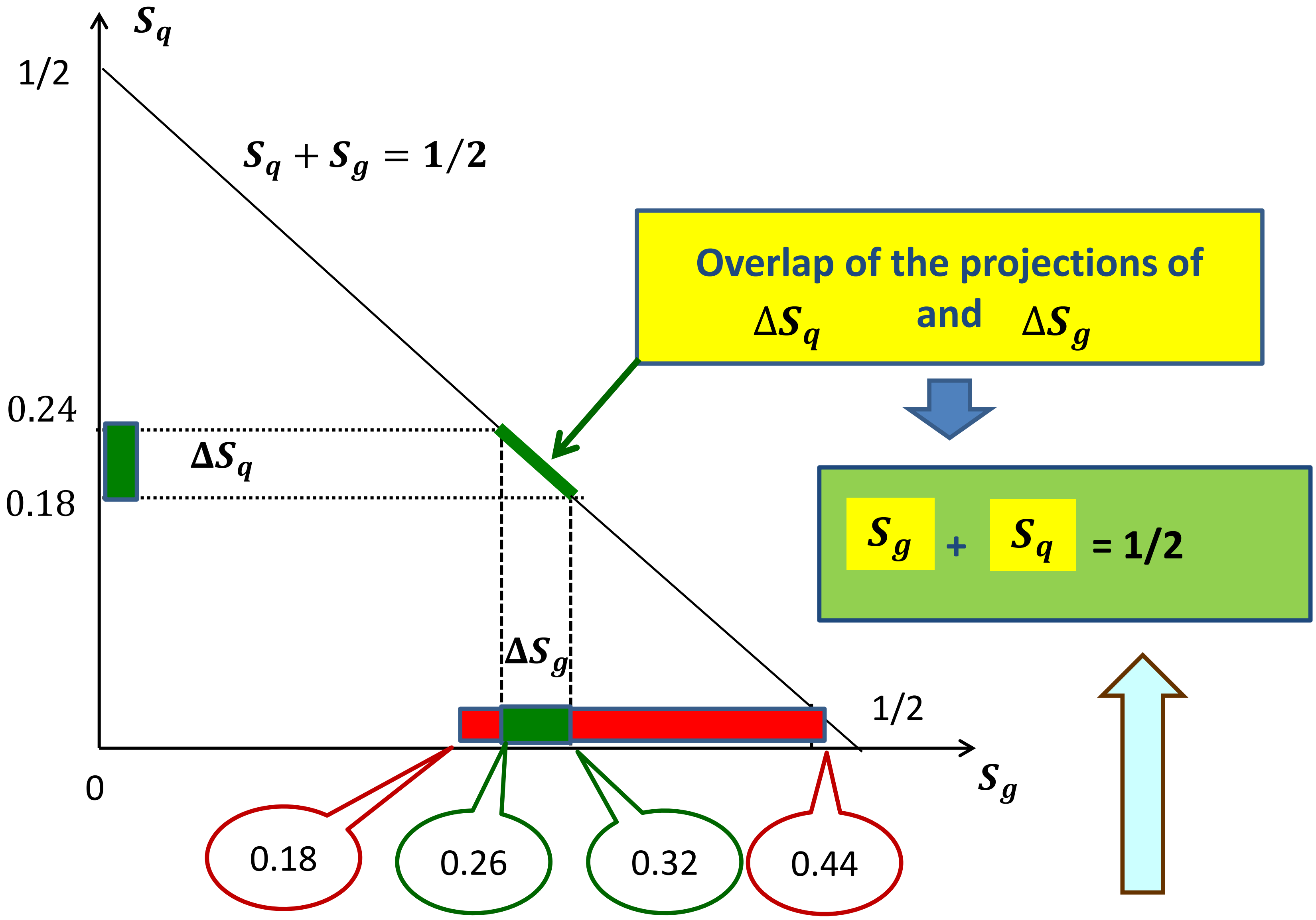
Once more the RHIC data:



$S_q + S_g = 1/2$

No overlapping of projections of $\bar{S}_q$ and $\bar{S}_g$

$\bar{S}_q$ + $\bar{S}_g$ never = 1/2



Overlap of the projections of
ΔS_q and ΔS_g

$S_g + S_q = 1/2$

No contribution of Angular Orbital Momenta is required though accounting for AOM is obligatory in the literature

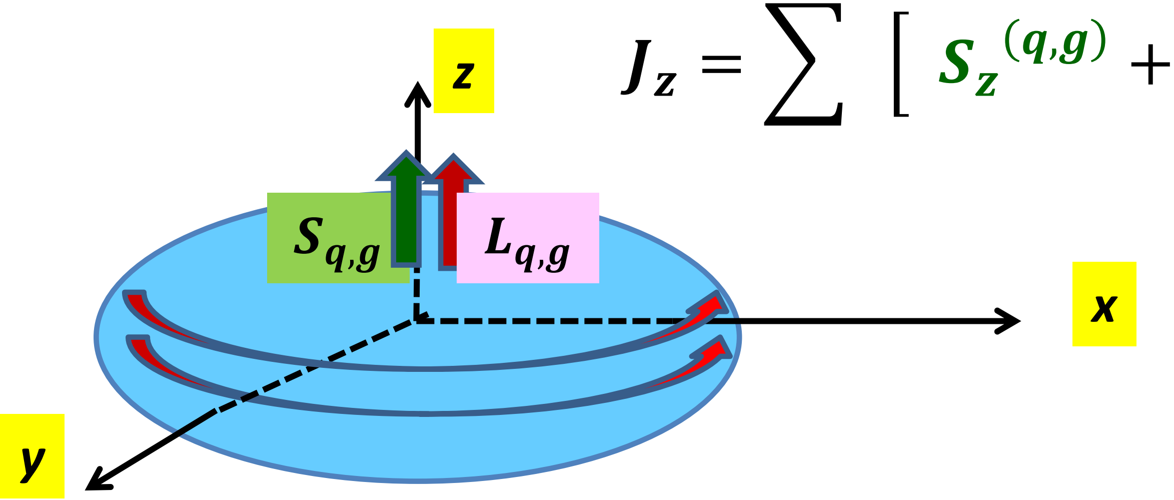
Remark on additional contributions to the proton spin

Axial Anomaly contribution

Efremov-Teryaev, Altarelli-Ross, Carlitz-Collins-Mueller

Orbital Angular Momentum

Jaffe-Manohar, Ji



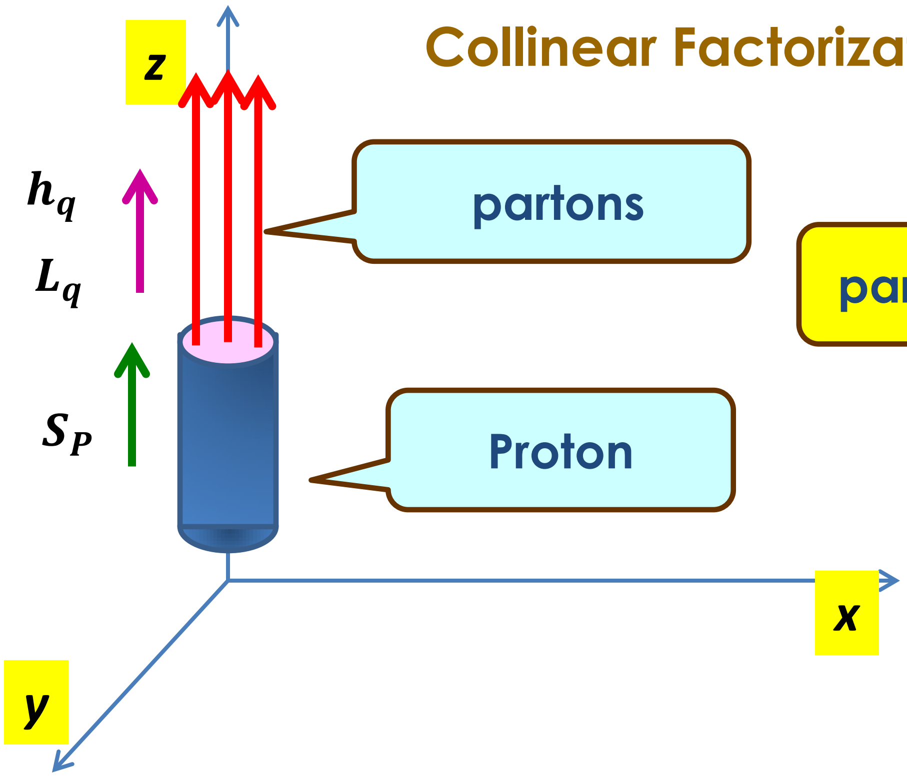
$$J_z = \sum \left[S_z{}^{(q,g)} + L_z{}^{(q,g)} \right] = \frac{1}{2}$$

The goal

Classic physics $L_z = xk_y - yk_x$

Components of parton momenta

Remark on Angular Orbital Momentum



Collinear Factorization:

$$k_i = \beta_i \, P$$

proton momentum

parton momentum

Absent in Collinear Factorization

Angular Orbital Momenta of partons:

$$L_z = x k_y - y k_x$$

both DGLAP and KPSCTT involve Collinear Factorization, so the use of them together in combination with AOM is not grounded theoretically

AOM is theoretically incompatible with Collinear Factorization
So, all numerous publications where inclusion of AOM is crucial should be revised

There are no objections about applying AOM in KT Factorization

where $$k_i = \beta_i P + k^{\perp}{}_i$$

However, all involved formulae and fits have to be remade so as to include dependence on $k^{\perp}$

Twisted partons ??

CONCLUSIONS

Using DLA for calculation of the parton contributions S_q and S_g leads to perfect agreement with the value 1/2 of the proton spin.

In contrast to KPSCTT, we do not use asymptotics for the parton contributions because the asymptotics should not have been used outside their applicability region $x < x_0$
Neither we use DGLAP because there is no theoretical grounds to apply it at small x; all success of DGLAP at small x heavily depends on the fits for the initial parton distributions

On the contrary, DLA contributions are the most important, leading ones at small x.
In order to simplify calculations, we start with accounting for the gluon contribution to the parton helicities and then implicitly add quark contributions through the intercept value. Non-perturbative contributions to the helicities cannot be calculated with QCD methods, so we fix them with using the RHIC data. As a result, the sum of the parton helicities in DLA proved to be in agreement with the value =1/2 of the proton spin.

Including into consideration Orbital Angular Momenta of quarks and gluons is not crucial for solving the Proton Spin Puzzle

Moreover, introducing OAM seems to be inconsistent with Collinear Factorization, where the initial partons move collinearly with the hadron: $\vec{k} = \beta\vec{p} \quad (0 < \beta < 1)$ while transverse momenta $\vec{k}_{\perp}$ of the partons are neglected