Accounting for the bound states' motion in the chiral effective field theory, "Modern theory of the nuclear forces"

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Introduction

Reasons for choosing the subject of the talk:

- Concerns the only missing part for completing the extremely popular, developed in more than three decades systematic approaches to the effective field theory (EFT) proposed by Steven Weinberg
- Directly related to the current high-accuracy calculations in this approaches
- A part of the work has been done in Dubna >10 years prior this EFT was proposed,

  That time our PT was applied to high energy physics.

Nuclear forces from chiral Lagrangians

- S. Weinberg: theory of "Nuclear forces from chiral Lagrangians", Phys.Lett.B 251 (1990) 288,
- small energy denominators due to pure-nucleon intermediate states, $\frac{1}{E-2\sqrt{m^2+p^2}} = \frac{1}{2\sqrt{m^2+k^2}-2\sqrt{m^2+p^2}} = \frac{m}{k^2-p^2}$, shallow resonances $\rightarrow$ too low convergence energy for amplitudes.
- Use Schroedinger equation where effective potential is defined as the sum of old-fashioned perturbation theory (PT) graphs, excluding those with pure-nucleon intermediate states.
- Nowadays these potentials are calculated to the higher than 5-th order of the ChPT.
- PT is necessary for the corresponding ab initio high accuracy nuclear studies to systematically account for the motion of the bound states involved in scattering processes

On moving bound states

- In the study of amplitudes involving bound states (e.g. em and other currents) it is important to express them in terms of the bound state wave functions at rest, because just the latter are much more convenient to model/calculate, than those for the moving bound state. Wave functions just at rest are also important for the study of the bound states' structure (size, shape etc.).
- In the covariant approaches the wave functions in the different reference frames are related to each other by kinematic Lorentz transformations.

Dynamical boost

- Unfortunately in the most convenient approaches to the extensively used Weinberg's EFT, the TOPT and so called "Unitary transformation" (UT) method, the boost transformations are dynamical, i.e. they depend on the interaction.
- To account for this interaction we construct the PT for the boost operator, which is the only missing part for completing the developed in more than three decades systematic approaches to EFT.
- Such PT is especially timely in view of the current high-accuracy calculations of the nuclei form factors in chiral EFT, E. Epelbaum et al. Phys.Rev.C 103 (2021) 2, 024313.

Examples of high-accuracy calculations

- Corresponding examples of high-accuracy calculations, Few Body Syst. 65 (2024) 2, 39, J.Phys.Conf.Ser. 2586 (2023) 1, 012006:

  (i) deuteron structure radius, $r = 1.9729 + 0.0015 - 0.0012$ fm,

  (ii) deuteron quadrupole moment, $Q_d = 0.2854 + 0.00380.1700 fm^2$, at N^4LO in agreement with very precise experimental datum $Q_d = 0.285699 fm^2$

  (iii) deuteron structure radius was used to extract neutron charge radius $r_n^2 = -0.105 + 0.005 - 0.006 fm^2$.

Previous attempts

- Previous ad hoc attempts, though irrelevant for ab initio studies:
  J. Phys. G 28, R37 (2002), Phys. Rev. C 61, 034002 (2000), Phys. Rev. D 10, 1777 (1974), Annals Phys. 104, 380 (1977), Phys. Rev. C 55, 2214 (1997), Phys. Rev. Lett. 87, 180401 (2001), Phys. Rev. C 65, 064009 (2002)

A part of work done in Dubna

- Our ab initio PT for boost operator
  A.K., A. Khvedelidze, B. Magradze, V. Matveev,
  Theor.Math.Phys. 72, 710 (1988); 78 (1989) 162; 78 (1989) 227; 78, 252 (1989)
  That time PT was applied to high energy physics using asymptotic freedom.
- The same PT is just what is needed in the low energy physics based on the chiral EFT.
  Recent high precision nuclear studies necessitate missed so far such PT to complete the low energy nucleon EFT, often called "the modern theory of nuclear forces"
- Here we present such PT for practitioners of the two most extensively used approaches to "chiral effective field theory", one based on the TOPT another on the UT method

PT for boost operator in QFT

Boost generator: $\bar{M}^{i0} = M_0^{i0} + \int d^4x x^i H_I(x)\delta(x^0)$,
M_0^{i0} and $M_I^{i0} = \int d^4x x^i H_I(x)$ are its free and interaction parts.
Boost transformation operator:
$\bar{T}_\lambda = \exp(i\omega n^i \bar{M}^{i0}) = \exp\left(i\omega n^i M_0^{i0} + i\omega \int d^4x n^i x^i H_I(x)\delta(x^0)\right)$
where λ is the velocity 4-vector, $\lambda^2 = 1$, $\omega = arcsh|\vec{\lambda}|$, $n^i = \frac{\lambda^i}{|\vec{\lambda}|}$.
The operator $\bar{T}_\lambda$ relates the bound state vector $|P, I\rangle$ with 4-momentum P to that at rest, $|\vec{0}, I\rangle$, with spin variables I,
$|P, i\rangle = \sum_j \bar{T}_\lambda |\vec{0}, j\rangle \mathcal{D}(\lambda, \lambda_0)_i^j$,
where $\lambda = P/M$, $\lambda_0 = (1, \vec{0})$ and $\mathcal{D}(\lambda, \lambda_0)_i^j$ is Wigner rotation matrix.

PT for boost operator in QFT

To construct the PT for the boost operator we proceed in direct analogy with the construction of the S-matrix, i.e. we consider the "evolution equation" for the combination $\bar{A}(\lambda) = T^{+}_{\lambda 0}\bar{T}_{\lambda}$,

$$\frac{\partial \bar{A}(\lambda)}{\partial \omega} = T^{+}_{\lambda 0}\left(-i\int d^4x \vec{n}\vec{x} H_I(x)\delta(x^0)\right)\bar{T}_{\lambda} = i\bar{B}(\lambda)\bar{A}(\lambda)$$

$$\bar{B}(\lambda) = T^{+}_{\lambda 0}\left(-\int d^4x \vec{n}\vec{x} H_I(x)\delta(x^0)\right)T_{\lambda 0} =$$

$$-\frac{1}{|\vec{\lambda}|}\int d^4x (\Lambda^{-1}_{\lambda}x)^0 H_I(\Lambda^{-1}_{\lambda}x)\delta(x^0)$$

$$= -\frac{1}{|\vec{\lambda}|}\int d^4x x^0 H_I(x)\delta([\Lambda_{\lambda}x]^0) = -\frac{1}{|\vec{\lambda}|}\int d^4x x^0 H_I(x)\delta(\lambda x)$$

$$= -\frac{1}{\lambda^0}\int d^4x \vec{\lambda}\vec{x} H_I(x)\delta(\lambda x) = -\frac{\partial}{\partial \omega}\int d^4x H_I(x)\theta(\lambda x)$$

The solution to this eq can be written in the three forms presented in (16,18,19) of Theor.Math.Phys. 72, 710 (1988), all of them applicable for perturbative calculations,

PT for boost operator in QFT

$$\bar{A}(\lambda) = T_\omega \exp\left(i\int_0^\omega d\omega' B(\omega')\right)$$
$$= T_{\frac{x^0}{xn}} \exp\left(i\int_\Omega d^4x\epsilon(x^0)H_I(x)\right)$$
$$= \left\{T\exp\left(i\int_{\Omega_1} d^4xH_I(x)\right)\right\}\left\{T\exp\left(i\int_{\Omega_2} d^4xH_I(x)\right)\right\}^\dagger =$$

$$\left\{T\exp\left(i\int dx\theta(x^0)\theta(-\lambda x)H_I(x)\right)\right\}\left\{T\exp\left(i\int dx\theta(-x^0)\theta(\lambda x)H_I(x)\right)\right\}^\dagger$$

$$WRONG = \left\{T\exp\left(i\int d^4x\theta(\lambda x)H_I(x)\right)\right\}\left\{T\exp\left(i\int d^4x\theta(x^0)H_I(x)\right)\right\}^\dagger$$

because the boundary condition $\bar{A}(\vec{\lambda}=0)=1$ is not satisfied

Here the orderings T_ω and $T_{\frac{x^0}{xn}}$ are defined as

$$T_\omega B(\omega_1)B(\omega_2) = \theta(\omega_1-\omega_2)B(\omega_1)B(\omega_2)+\theta(\omega_2-\omega_1)B(\omega_2)B(\omega_1),$$

$$T_{\frac{x^0}{xn}} H_I(x')H_I(x) = \theta\left(\frac{x'_0}{x'n}-\frac{x_0}{xn}\right)H_I(x')H_I(x)+(x'\leftrightarrow x)$$

But "T" stands for ordinary time-ordering!

Boost operator in TOPT

- All three forms given above imply expansion in powers of the product of velocity $\vec{\lambda}$ by couplings involved in the chiral interaction Lagrangian therefore it is interesting how do they interplay in practical calculations
- $|\mathrm{P}\rangle = \bar{T}_\lambda |\vec{0}\rangle$ can be written in terms of the two nucleon wave functions, $\Psi_P(\vec{p}_1, \vec{p}_2) = \langle p_1, p_2 | P \rangle$
  $\Psi_P(\vec{p}_1, \vec{p}_2) =$
  $\int \langle p_1, p_2 | \bar{A}(\lambda) T_{\lambda 0} (1 + G_M \mathcal{Q} T^{ir}_M) | k_1, k_2 \rangle (dk) \Psi_0(k_1, k_2)$.,
  T^{ir} encodes all but two-nucleon intermediate states
- In the LO $A(\lambda) \sim 1$, $T^{ir}_M) = 0$ therefore the boost transformation in the LO approximation is $\langle p_1, p_2 | T_\lambda | k_1, k_2 \rangle \sim \langle p_1, p_2 | T_{\lambda 0} | k_1, k_2 \rangle = \delta^3(\vec{\Lambda}^\lambda \bar{p}_1 - \vec{k}_1) \delta^3(\vec{\Lambda}^\lambda \bar{p}_2 - \vec{k}_2)$, which corresponds to the kinematic Lorentz transformation given by $\Psi_P(\vec{p}_1, \vec{p}_2) \sim \Psi_0(\vec{\Lambda}^\lambda \bar{p}_1, \vec{\Lambda}^\lambda \bar{p}_2)$,

PT in UT method

- Unitary transformation U diagonalises Poincare generators $\bar{P}_\alpha$
  $P_\alpha = U\bar{P}_\alpha U^+$ is diagonal in number of nucleons
  UT boost operator:
  $T_\lambda = U\bar{T}_\lambda U^+ = UT_{\lambda 0}\bar{A}(\lambda)U^+ = (UT_{\lambda 0}U^+)(U\bar{A}(\lambda)U^+)$ (A)
- $|{}^u\mathrm{P}\rangle = U|\mathrm{P}\rangle$ is bound state vector in UT method
  Then bound state current: $\langle \mathrm{P}'|J^\mu|\mathrm{P}\rangle = \langle {}^u\mathrm{P}'|UJ^\mu U^+|{}^u\mathrm{P}\rangle$
- Boosting bound state $|\mathrm{P}\rangle^u = T_{\vec{V}}|\vec{0}\rangle^u$, explicitely, in variables:
  $\Psi_\mathrm{P}(\mathrm{p}_1', \mathrm{p}_2') = \int \langle \mathrm{p}_1', \mathrm{p}_2'|T_\mathrm{V}|\mathrm{k}_1, \mathrm{k}_2\rangle \frac{d^3\mathrm{k}_1}{2\omega_{k_1}} \frac{d^3\mathrm{k}_2}{2\omega_{k_2}} \Psi_0(\mathrm{k}_1, \mathrm{k}_2)$
  $\langle \mathrm{p}_1', \mathrm{p}_2'|T_\mathrm{V}|\mathrm{k}_1, \mathrm{k}_2\rangle$ is calculable perturbatively to any precision with the help of Eq.(A).

PT from the two-body Schroedinger equation

- The corresponding PT based only on the two-body Schroedinger equation.
  This PT is formulated in terms of NN potentials developed in great details and extensively used for the analysis of the scattering amplitudes etc. in both, TOPT and UT method approaches, rather than in terms of the quantised interaction Hamiltonian density H_I used above.
- It is based on the Schroedinger eq for moving bound state with momentum P, $\left(P^0 - \sqrt{m_1^2 + \mathrm{p}_1^2} - \sqrt{m_2^2 + \mathrm{p}_2^2}\right) \Psi_P(\vec{p}) =$
  $\int d^3\mathrm{k} V(P, \mathrm{p}, \mathrm{k}) \Psi_P(\vec{k})$ $\qquad (P)$

PT from the two-body Schroedinger equation

- The proposed PT represents the expansion of the solution to the Eq(P) around its solution at the rest frame
  $\Psi_P(\vec{p}) = \Psi_0(\Lambda^{\vec{\lambda}}\bar{p}) + \delta\Psi_P(\vec{p})$, where $\bar{p}$ is "on-mass-shell" 4D relative momentum of the constituents, $\bar{p} = \left(\frac{\vec{P}\vec{p}}{P^0}, \vec{p}\right)$
  because $(\frac{P}{2} \pm p)^2 = m^2 \rightarrow Pp = 0$,
  Λ^λ is the boost transformation reducing the 4-vector $\lambda = P/M$ to its rest frame, $\Lambda^\lambda\lambda = (1, \vec{0})$,
- Leading order (LO) solution to Eq(P), Ψ_0(p), satisfies Eq(P) for the total 3-momentum set to zero, $\vec{P} = 0$,
  $\left(M - 2\sqrt{m^2 + \mathrm{p}^2}\right)\Psi_0(\vec{p}) = \int d\vec{k} V(M, 0, \vec{p}, \vec{k})\Psi_0(\vec{k})$

Extension of the standard time independent PT, 2303.14862 [quant-ph]

- $[G_0^{-1}(E_n) - K_0(E_n) - K_1(E_n)]|\psi_n\rangle = 0$ where $K_1(E)$ is the perturbation potential with arbitrary E dependence, NOT just linear.

- Green function of the unperturbed eq, $G_u(E)$, is known $[G_0^{-1}(E) - K_0(E)]G_u(E) = 1$

- Unperturbed potential $K_0(E)$ gives rise to a bound state at energy $\mathcal{E}_n$, then $G_u(E)$ has a pole at $\mathcal{E}_n$, $\langle x|G_u(E)|y\rangle = \frac{\phi_n(x)\bar{\phi}_n(y)}{E-\mathcal{E}_n} + G_u^b(E)$

- Use the factorised form of the pole contribution, $|\phi_n\rangle\langle\phi_n|$, to solve the perturbed eq algebraically, $|\psi_n\rangle = [1 - G_u^b(E_n)K_1(E_n)]^{-1}|\phi_n\rangle$

Boost directly in terms of nuclear forces

- Use our extension of the standard PT for $\delta\Psi_P(\vec{p})$,
  $\Psi'_P(\underline{p}) = \langle \underline{p}|[1 - G_u^b K_1]^{-1}|\underline{k}\rangle d\underline{k}\Psi_0(\underline{k}), \quad (PT)$
  where

- $G_u^b(E, p', p) = G_u(E, p', p) - \frac{\Psi_0(p')\bar{\Psi}_0(p)}{E-M}$,
  $\langle p'|G_u(E)|p\rangle = G_u(E, p', p)$ is the Green function for the NN system at rest,
  $\left(E - 2\sqrt{m^2 + p'^2}\right) G_u(E, p', p) -$
  $\int dk V(E, 0, p', k) G_u(E, k, p) = \delta^3(p' - p)$,

- $\langle p'|K_1|p\rangle = V'(P, \underline{p}', \underline{p}) I(P, \underline{p}) - V(M, 0, p', p)$
  $+\delta(p' - p)\left(M - 2\sqrt{m^2 + p^2} - P^0 + \sqrt{\vec{P}^2 + 4(m^2 + \underline{p}^2)}\right)$
  $P^0 = \sqrt{\vec{P}^2 + M^2}$

Boost directly in terms of nuclear forces

$$\Psi'_P(\underline{p}) = \langle \underline{p}|[1 - G_u^b K_1]^{-1}|\underline{k}\rangle d\underline{k}\Psi_0(\underline{k}), \qquad (PT)$$

The crucial property of (PT): it is expressed in terms of the NN potential V, just that which is studied in more than three decades in great details, to high precision, in both approaches (TOPT and UT method) and is being extensively used for the systematic analysis of NN scattering.

This is in contrast to our first version which is expressed in terms of the interaction hamiltonian density $H_I(x)$.

Thanks

THANK YOU!