Quasielastic neutrino scattering in a scaling model with relativistic effective mass (SuSAM*)

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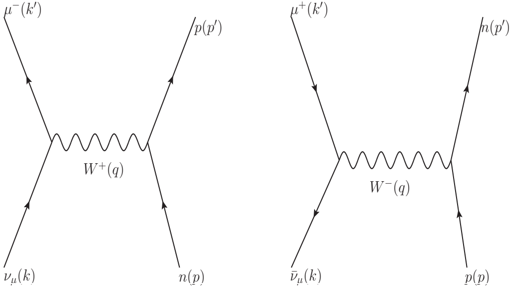
Contents

Charged Current Quasielastic (CCQE) reactions



In the standard model (SM), the CC lepton currents couple to the $W^{\pm}$ bosons through the well-known $V - A$ form through the interaction lagrangian

$$\mathcal{L} = \frac{-g}{2\sqrt{2}} \left[\bar{l}(x)\gamma^\mu \left(\mathbf{1} - \gamma_5\right) \nu_l(x) W_\mu(x) + \bar{\nu}_l(x)\gamma^\mu \left(\mathbf{1} - \gamma_5\right) l(x) W_\mu^\dagger(x) \right]$$

When applying the Feynman rules to obtain the reduced matrix element for the transition between initial and final states

$$-i\mathcal{M}_{fi} = \left(\frac{-i\,g}{2\sqrt{2}}\right)^2 \cos\theta_c \; \bar{u}_{r'}(\mathbf{k}')\gamma^\mu \left(\mathbf{1} - \gamma_5\right) u_r(\mathbf{k}) \; i\,D^W_{\mu\nu}(q) \left\langle N'(p')\right| J^\nu_N(q) \left|N(p)\right\rangle$$

Charged Current Quasielastic (CCQE) reactions

where $\langle N'(p')|\, J_N^{\nu}(q)\, |N(p)\rangle$ is the transition matrix element between nucleon (or nuclear) states driven by the weak CC hadron current. The weak CC $J_N^{\nu}(q)$ current operator is also of the form $V - A$, but as the nucleons are not elementary particles, its structure is more involved than for leptons. It can be shown that invoking several symmetries that hold for strong interactions, like invariance under G-parity, time-reversal and the absence of second class currents in the SM, the vector and axial-vector parts of the current can be written as

$$V^{\mu}_{s's}(\mathbf{p}',\mathbf{p}) = \bar{u}_{s'}(\mathbf{p}') \left[2F_1^V(q^2)\gamma^{\mu} + i\frac{2F_2^V(q^2)}{2m_N}\sigma^{\mu\nu}q_{\nu} \right] u_s(\mathbf{p}) \qquad (1)$$

$$A^{\mu}_{s's}(\mathbf{p}',\mathbf{p}) = \bar{u}_{s'}(\mathbf{p}') \left[G_A(q^2)\gamma^{\mu}\gamma_5 + \frac{G_P(q^2)}{2m_N}q^{\mu}\gamma_5 \right] u_s(\mathbf{p}) \qquad (2)$$

where the Dirac and Pauli isovector form factors $F_i^V = \frac{1}{2}(F_i^p - F_i^n)$ can be related to the electromagnetic ones by assuming that the vector part of the weak CC currents $V^{\mu}_{cc\pm} = V^{\mu}_1 \pm i\, V^{\mu}_2$ belongs to the same multiplet of conserved vector currents as the third isospin current V_3^{μ} entering into the electromagnetic current $J^{\mu}_{\rm em} = V_3^{\mu} + \frac{1}{2}V_Y^{\mu}$, where the triplet of isospin vector currents is written as $V_i^{\mu} = \mathcal{V}^{\mu}\frac{\tau_i}{2}$, where $\mathcal{V}^{\mu}$ carries the Lorentz-Dirac structure and τ_i the flavor (isospin) structure allowing transitions among different nucleon states.

Low four-momentum transfer regime=effective Fermi theory

In the expression for the reduced matrix element $\mathcal{M}$, the $W^{\pm}$-boson propagator appears, but this can be made to disappear in the low four-momentum transfer regime, transferring its contribution to the effective Fermi coupling constant G_F.

$$D^{W}_{\mu\nu}(q) = \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \underbrace{\longrightarrow}_{|q^2| \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$

Within this approximation the matrix element for the transition reduces to the Fermi point-like interaction

$$-i\mathcal{M}_{fi} = -i\frac{G_F \cos\theta_c}{\sqrt{2}}\, l^{\mu}_{r\,r'}(\mathbf{k},\mathbf{k}')\,\langle N'(p')|\, J^{N}_{\mu}(q)\,|N(p)\rangle\;;\ \text{with}\quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \tag{3}$$

This limit is widely valid for most of the current and past scattering experiments, as far as $|q^2| \ll M_W^2 \approx 6400$ GeV^2.

Calculating cross sections: lepton and hadron tensors

The S-matrix transition matrix element can be written as

$$S_{fi} = -i\,(2\pi)^4\,\delta^4(p+q-p')\,\mathcal{M}_{fi}$$

To calculate the differential cross section it is necessary to raise to the square the probability amplitude for the transition, S_{fi}, and to sum (or integrate) over unobserved discrete quantum numbers (or continuum) of the final state and average over those of the initial state. In this way, when calculating $|\mathcal{M}_{fi}|^2$ and summing over final fermion polarizations and averaging over the initial ones, we obtain

$$\begin{aligned}
\overline{|\mathcal{M}_{fi}|^2} &= \frac{1}{2}\sum_{r\,r'}\sum_{s\,s'}\frac{G_F^2\cos^2\theta_c}{2}\left(l^{\mu}_{r\,r'}(\mathbf{k},\mathbf{k}')J^N_{\mu}(q)\right)^*\left(l^{\nu}_{r\,r'}(\mathbf{k},\mathbf{k}')J^N_{\nu}(q)\right)\\
&= \frac{G_F^2\cos^2\theta_c}{2}\underbrace{\left(\sum_{r\,r'} l^{\mu *}_{r\,r'}(\mathbf{k},\mathbf{k}')\,l^{\nu}_{r\,r'}(\mathbf{k},\mathbf{k}')\right)}_{L^{\mu\nu}(\mathbf{k},\mathbf{k}')}\underbrace{\left(\frac{1}{2}\sum_{s\,s'} J^{N*}_{\mu}(q)J^N_{\nu}(q)\right)}_{w^{\mathrm{s.n.}}_{\mu\nu}(\mathbf{p},\mathbf{p}')}
\end{aligned}$$

Lepton and hadron tensors

When summing over lepton polarizations, the following trace appears

$$
\begin{aligned}
L^{\mu\nu}(\mathbf{k},\mathbf{k}') &= \mathrm{Tr}\left[\gamma^\mu(\mathbf{1}-\gamma_5)(\not{k}'+m')\gamma^\nu(\mathbf{1}-\gamma_5)\not{k}\right] \\
&= 8\left[\underbrace{k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k'}_{s^{\mu\nu}} - i\underbrace{\epsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta}_{a^{\mu\nu}}\right] \qquad (4)
\end{aligned}
$$

The single nucleon (and hadron or nuclear) tensor has one property we will use later. It is an hermitic Lorentz tensor. Changing the Lorentz indices is equivalent to take the complex conjugate.

$$
w^{\mathrm{s.n.}*}_{\mu\nu}(\mathbf{p},\mathbf{p}') = w^{\mathrm{s.n.}}_{\nu\mu}(\mathbf{p},\mathbf{p}') \qquad (5)
$$

The above equation implies that the symmetric part of the tensor is real and the antisymmetric part of it is purely imaginary. And also in the opposite direction, i.e, the imaginary part of the tensor is antisymmetric and the real part of the tensor is symmetric.

Properties of hadron and lepton tensors

$$\begin{aligned}\mathrm{Sym}\left(w_{\mu\nu}^{\mathrm{s.n.}}\right) &= \frac{1}{2}\left(w_{\mu\nu}^{\mathrm{s.n.}} + w_{\nu\mu}^{\mathrm{s.n.}}\right) = \frac{1}{2}\left(w_{\mu\nu}^{\mathrm{s.n.}} + w_{\mu\nu}^{\mathrm{s.n.}*}\right) = \mathrm{Re}\left(w_{\mu\nu}^{\mathrm{s.n.}}\right) \\ \mathrm{Antisym}\left(w_{\mu\nu}^{\mathrm{s.n.}}\right) &= \frac{1}{2}\left(w_{\mu\nu}^{\mathrm{s.n.}} - w_{\nu\mu}^{\mathrm{s.n.}}\right) = \frac{1}{2}\left(w_{\mu\nu}^{\mathrm{s.n.}} - w_{\mu\nu}^{\mathrm{s.n.}*}\right) = i\,\mathrm{Im}\left(w_{\mu\nu}^{\mathrm{s.n.}}\right)\end{aligned}$$

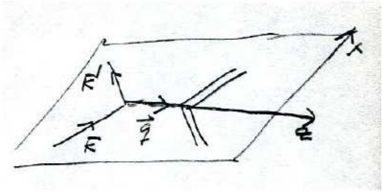
These properties mean that when calculating the contraction of the lepton and hadron tensor, $s^{\mu\nu}$ will only appear multiplying real parts of the hadron tensor and $a^{\mu\nu}$ only multiplying imaginary parts of the hadron tensor.

Finally, writing the final one-body phase space (for the final lepton) we can write the inclusive (integrated and summed over the initial and final hadronic states) cross section:

$$\frac{d\sigma}{d\Omega_{\hat{k}'}dE'} = \frac{G_F^2 \cos^2\theta_c}{4\pi^2}\frac{k'}{E_\nu}\left(s^{\mu\nu} - i\,a^{\mu\nu}\right)W_{\mu\nu}^{\mathrm{had}} \tag{6}$$

where $W_{\mu\nu}^{\mathrm{had}}$ is the inclusive nuclear hadron tensor for a given lepton kinematics $(E', \hat{k}')$, where the initial and final nucleon momenta will be fully integrated and summed over polarizations to obtain the 1p-1h hadron tensor, or relevant nuclear responses in a particular frame of reference.

Reference frame used to simplify the calculations



We assume that the initial nucleus is at rest and, either it has no spin or, if it has, it is unpolarized. If the nucleus were polarized, there would be another privileged direction (that of the spin of the nucleus) and more response functions would enter in the description of the cross section. The only information the nucleus has about the neutrino interaction is the energy and momentum transfer it receives.

For each final lepton kinematics $(E', \hat{k}')$, we choose $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ to define the Z-axis. The X-axis to fix the scattering plane is defined by the transverse component of the neutrino momentum with respect to the momentum transfer.

$$\mathbf{k}_T = \mathbf{k} - \frac{\mathbf{k} \cdot \mathbf{q}}{\mathbf{q}^2}\mathbf{q} \quad \text{lies} \quad \| \quad \text{X} - \text{axis}$$

Reference frame used to simplify the calculations

This can be done because it represents a global rotation of the system as a whole in the scattering plane. This rotation will not change the initial or final lepton energies at all, nor will change the values of the scalar products involved in the contraction of the lepton tensor $L^{\mu\nu}(\mathbf{k}, \mathbf{k}')$ and the **inclusive** nuclear hadron one $W^{\rm had}_{\mu\nu}(q)$. All the involved scalar products between four-momenta appearing in the contraction can be reduced to the combination k^2, k'^2 and $k \cdot k'$. And in addition, this rotation leaves an initial nucleus at rest, remaining at rest.
With these previous assumptions, we can assume that the spatial components of the hadron tensor $W^{ij}_{\rm had}$ can be expanded in the tensorial basis composed by

$$\delta^{ij}, \qquad q^i q^j \quad \text{(symmetric)}, \qquad \epsilon^{ijk} q^k \quad \text{(antisymmetric)}$$

The time-space part of the tensor $W^{0i}_{\rm had}$ should be proportional to q^i and the time-time component will be one of the other additional nuclear response functions. We can write for the above components of the tensor the explicit expansion:

Reference frame used to simplify the calculations

$$W^{ij} = \frac{1}{2} R_T \left(\delta^{ij} - \frac{q^i q^j}{\mathbf{q}^2} \right) + R_{LL} \frac{q^i q^j}{\mathbf{q}^2} + i \, R_{T'} \, \epsilon^{ijk} \frac{q^k}{|\mathbf{q}|} \tag{7}$$

$$W^{0i} = W^{03} \frac{q^i}{|\mathbf{q}|}; \qquad W^{i0} = W^{30} \frac{q^i}{|\mathbf{q}|} \tag{8}$$

$$W^{00} \equiv R_{CC} \tag{9}$$

Using the properties of the hadron tensor, $W^{\mu\nu} = W^{\nu\mu *}$, obviously the diagonal parts of it are real and the W^{12} and W^{21} components are purely imaginary satisfying $W^{12} = -W^{21}$ in the frame where $\mathbf{q}$ defines the Z-axis. Normally, the symmetric transverse part to $\mathbf{q}$ is called the T response function

$$R_T = W^{11} + W^{22},$$

the antisymmetric transverse part is called the T' response

$$R_{T'} = \frac{-i}{2} \left(W^{12} - W^{21} \right)$$

and the symmetric longitudinal-longitudinal part is called the LL response

$$R_{LL} = W^{33}$$

Clarifying note

In other approaches, one writes the nuclear hadron tensor in fully covariant fashion, where the tensor basis for the expansion has 5 relativistically invariant structure functions ($W_i(\nu, Q^2), i = 1, \dots 5$) contributing to the inclusive cross section (the $W_6(\nu, Q^2)$ does not contribute). At the end, in both approaches we have the same algebraically independent structure functions or responses, namely 5. But these invariant structure functions can be written as appropriate combinations of the components of the hadron tensor (the 5 response functions) in a reference frame where $\mathbf{q}$ defines the Z-axis.

This apparent paradox, that components of a tensor (which of course are not invariant under any Lorentz transformation, in particular rotations) in a particular frame can be used to construct invariant quantities (the structure functions or the response functions) is somewhat similar to construct the s-Mandelstam variable from the total energy in the CM frame of two colliding particles. Of course, individual energies of the colliding particles are not invariant under Lorentz transformations, but their sum in a privileged (the CM one) frame is an invariant quantity, $\sqrt{s}$.

Reference frame used to simplify the calculations

In this frame, the neutrino and the final lepton have the following four-momenta

$$\begin{aligned} k^\mu &= (E_\nu, E_\nu \sin\theta_{\nu q}, 0, E_\nu \cos\theta_{\nu q}) \\ k'^\mu &= (E', \mathbf{k}') \end{aligned}$$

where $\mathbf{k}'$ obviously also lies on the scattering plane and can be obtained by performing a rotation around the Y-axis of angle θ_l over the direction defined by the neutrino, this θ_l is the scattering angle of the final lepton with respect to the neutrino.

$$\begin{aligned} \mathbf{k}' &= \frac{|\mathbf{k}'|}{E_\nu}\mathcal{R}_y(\theta_l)\mathbf{k} = |\mathbf{k}'| \begin{pmatrix} \cos\theta_l & 0 & \sin\theta_l \\ 0 & 1 & 0 \\ -\sin\theta_l & 0 & \cos\theta_l \end{pmatrix} \begin{pmatrix} \sin\theta_{\nu q} \\ 0 \\ \cos\theta_{\nu q} \end{pmatrix} \\ &= \begin{pmatrix} |\mathbf{k}'| \sin(\theta_{\nu q} + \theta_l) \\ 0 \\ |\mathbf{k}'| \cos(\theta_{\nu q} + \theta_l) \end{pmatrix}; \qquad \text{with} \quad \mathbf{k}\cdot\mathbf{k}' = E_\nu |\mathbf{k}'| \cos\theta_l \end{aligned}$$

Some contractions

The contraction between the symmetric parts of the lepton and hadron tensors make appear only the symmetric components of the hadron tensor, or, for those components which have not definite symmetry, like W^{03} or W^{30}, the contraction makes them to appear in symmetrized form

$$s_{\mu\nu} W^{\mu\nu} = V_{CC} R_{CC} + 2 V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T \tag{10}$$

with $R_{CL} = -\frac{1}{2}(W^{03} + W^{30})$, and the kinematical factors that come from the symmetric part of the lepton tensor can be found, for instance, in PRD 97, 116006 (2018), among other papers.
For the contraction between the antisymmetric parts of lepton and hadron tensor, only the T' response survives to the contraction. Nor even appears the antisymmetrized part of the W^{03} hadron tensor component.

$$a_{\mu\nu} W^{\mu\nu} = (W^{03} - W^{30}) \epsilon_{0123} \underbrace{(k_x k'_y - k_y k'_x)}_{k_y = k'_y = 0} + 2\, i\, R_{T'} (E_\nu k'_z - E' k_z) \tag{11}$$

Cross section

So at the end, after performing the contractions, the cross section can be finally written in terms of the five response functions as

$$\frac{d\sigma}{d\Omega_{\hat{k}'}dE'} = \frac{G_F^2 \cos^2\theta_c}{4\pi^2}\frac{k'}{E_\nu}\left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm 2V_{T'}R_{T'}\right) \tag{12}$$

where the $\pm$ sign in the contribution of the T' response is for neutrinos/antineutrinos, respectively.

This is because in the antineutrinos case, the role of the $\mathbf{k}$ and $\mathbf{k}'$ vectors gets interchanged in the reduced matrix element $\mathcal{M}_{fi}$. The symmetric part of the lepton tensor is also symmetric under the interchange $k \leftrightarrow k'$, but the antisymmetric part of the lepton tensor is precisely antisymmetric under that interchange of four-momenta and gets a minus sign with respect to the neutrinos case. This sign gets finally reflected in the contraction with the antisymmetric part of the hadron tensor.

QE responses in the RFG

Up to now we have been discussing in general the formalism for inclusive (from the hadron point of view) neutrino scattering cross sections, where the response functions R_K are the nuclear ones. The QE responses in the RFG model can be obtained assuming that in the impulse approximation (IA) the $W^{\pm}$-boson is absorbed (and therefore the energy and momentum transfer $(\omega, |\mathbf{q}|)$) individually and incoherently by a single nucleon in the RFG picture of the nucleus.
If all nucleons contribute incoherently to the total cross section, we have to perform a sum over all of them.
The nuclear one-body electroweak current operator will induce transitions from the ground-state of the RFG to excited states with one particle above the Fermi surface and leaving a hole below the Fermi momentum. In this way we obtain what is usually called the 1p-1h contribution. In this approach we can write (with our normalizations for spinors) for the nuclear hadron tensor the following expression

$$W_{\mu\nu} = \sum_{\mathbf{p}<k_F} \sum_{s,s'} \frac{m_N^2}{E_{\mathbf{p}} E'_{\mathbf{p}+\mathbf{q}}} \delta(E' - E - \omega) J_\mu^*(\mathbf{p}', \mathbf{p}) J_\nu(\mathbf{p}', \mathbf{p}) \theta(|\mathbf{p}'| - k_F) \quad (13)$$

where the electroweak current matrix element between plane wave states is

$$J^\mu(\mathbf{p}', \mathbf{p}) = V^\mu_{s's}(\mathbf{p}', \mathbf{p}) - A^\mu_{s's}(\mathbf{p}', \mathbf{p}) \quad (14)$$

QE responses in the RFG

Again the sum over polarizations gives traces over Dirac matrices that, at the end, will be written in terms of the nucleon vector and axial form factors and on-shell four-momenta. To be more specific

$$2w^{\text{s.n.}}_{\mu\nu}(\mathbf{p}', \mathbf{p}) = \sum_{s,s'} J^*_\mu(\mathbf{p}', \mathbf{p}) J_\nu(\mathbf{p}', \mathbf{p}) =$$

$$\text{Tr}\left[\gamma^0 (V_\mu(q) - A_\mu(q))^\dagger \gamma^0 \frac{(\not{p}' + m_N)}{2m_N} (V_\nu(q) - A_\nu(q)) \frac{(\not{p} + m_N)}{2m_N}\right] \tag{15}$$

where

$$V^\mu(q) = \left[2F_1^V(q^2)\gamma^\mu + i\frac{2F_2^V(q^2)}{2m_N}\sigma^{\mu\nu}q_\nu\right] \tag{16}$$

$$A^\mu(q) = \left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{G_P(q^2)}{2m_N}q^\mu\gamma_5\right] \tag{17}$$

Finally, the sum over initial nucleon momenta $|\mathbf{p}| < k_F$ can be transformed, with appropriate normalization for the number of nucleon states inside the Fermi sphere, to give:

QE responses in the RFG

$$W_{\mu\nu} = \frac{3Nm_N^2}{4\pi k_F^3} \int d^3p \, \delta(E' - E - \omega) \, \theta(E'_{\mathbf{p}+\mathbf{q}} - E_F) \frac{w^{\text{s.n.}}_{\mu\nu}(\mathbf{p}+\mathbf{q}, \mathbf{p})}{E_{\mathbf{p}} E'_{\mathbf{p}+\mathbf{q}}} \tag{18}$$

where N is the number of neutrons because for neutrinos, the W^+ can only be absorbed by neutron states. For antineutrinos, $N \to Z$. The above integral can be analytically computed for the relevant nuclear response functions entering in the cross section, $R_K(K = CC, CL, LL, T, T')$. The final result, instead of being written in terms of $(\omega, |\mathbf{q}|)$ can be written in terms of a single variable $\psi(\omega, |\mathbf{q}|)$, called the scaling variable [1,2]. This can be defined as

$$\psi \equiv \pm \sqrt{\frac{\epsilon_0 - 1}{\epsilon_F - 1}} \tag{19}$$

where all the kinematic variables are written dimensionless by normalizing to the nucleon mass,

$$\epsilon_0 = \max \left[\gamma_- \equiv \kappa \sqrt{1 + 1/\tau} - \lambda, \epsilon_F - 2\lambda \right] \tag{20}$$

[1] W.M. Alberico et al, **Phys. Rev. C 38**, 1801 (1988)

[2] M.B. Barbaro et al, **Nucl. Phys. A643**, 137 (1998)

QE responses in the RFG

In the above slide the kinematic variables ω and $q \equiv |\mathbf{q}|$ are changed to the dimensionless ones and combinations of them

$$\left.\begin{array}{l} \kappa \equiv \frac{|\mathbf{q}|}{2m_N} \\ \lambda \equiv \frac{\omega}{2m_N} \end{array}\right\} \rightarrow \tau = \kappa^2 - \lambda^2 = \frac{Q^2}{4m_N^2}, \tag{21}$$

$$\eta_F \equiv \frac{k_F}{m_N}, \qquad \epsilon_F = \frac{E_F}{m_N} = \sqrt{1+\eta_F^2} \tag{22}$$

And the $+$ sign in the scaling variable applies if $\lambda > \tau$ (what means to be at energy transfers larger than that at the QE peak position at $\lambda = \tau$), while the $-$ sign applies for $\lambda < \tau$. The meaning of the ϵ_0 variable can be understood in the next plot, where we place ourselves in two situations, one where there can be Pauli blocking and other one where there cannot be.

QE responses in the RFG

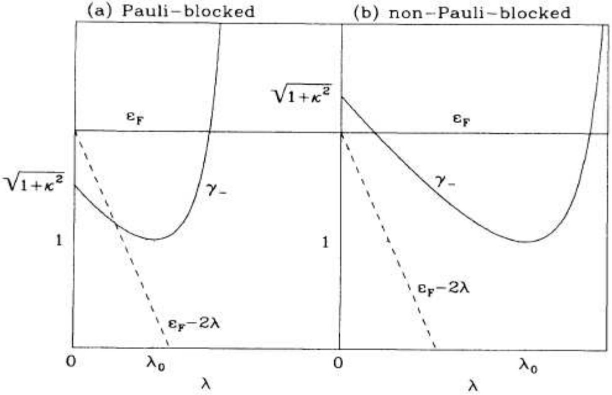


Figure: Behavior of γ_- as a function of λ in two regimes: (a) Pauli-blocked region ($\kappa < \eta_F$) and non-Pauli-blocked region ($\kappa > \eta_F$). It is also shown the line $\Gamma = \epsilon_F - 2\lambda$, which can be larger than γ_- for small λ in the Pauli-blocked regime.

QE responses in the RFG

Finally, with the aid of this scaling variable, which is a single function of two independent ones (λ, κ), one can write the nuclear response functions in the RFG after having integrated the relevant single-nucleon responses with the momentum distribution of the Fermi gas. The expressions can be also found, for instance, in PRD 97, 116006 (2018). I write them now for completeness. All of them can be written in factorized form as

$$R_K = \frac{N\xi_F}{m_N \eta_F^3 \kappa} U_K \, f_{\mathrm{RFG}}(\psi) \tag{23}$$

where $f_{\mathrm{RFG}} = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$ and $\xi_F = \sqrt{1+\eta_F^2} - 1$. The step function appears because ϵ_0 is the lower limit in the final integration over the initial nucleon energy E, and this value has to be always lower than the Fermi energy E_F for the integral to give a non-zero contribution. This restricts the scaling variable in the RFG model to be between -1 and 1. The U_K can be considered as "RFG-integrated single-nucleon response functions", and their expressions are:

QE responses in the RFG in the scaling formalism

$$
\begin{aligned}
U_{CC} &= \frac{\kappa^2}{\tau}\left[(2G_E^V)^2 + \frac{(2G_E^V)^2+\tau(2G_M^V)^2}{1+\tau}\Delta + G_A^2\Delta\right] + \frac{\lambda^2}{\tau}\left(G_A - \tau G_P\right)^2 \\
U_{CL} &= -\frac{\lambda\kappa}{\tau}\left[(2G_E^V)^2 + \frac{(2G_E^V)^2+\tau(2G_M^V)^2}{1+\tau}\Delta + G_A^2\Delta + \left(G_A - \tau G_P\right)^2\right] \\
U_{LL} &= \frac{\lambda^2}{\tau}\left[(2G_E^V)^2 + \frac{(2G_E^V)^2+\tau(2G_M^V)^2}{1+\tau}\Delta + G_A^2\Delta\right] + \frac{\kappa^2}{\tau}\left(G_A - \tau G_P\right)^2 \\
U_T &= 2\tau(2G_M^V)^2 + \frac{(2G_E^V)^2+\tau(2G_M^V)^2}{1+\tau}\Delta + G_A^2\left[2(1+\tau)+\Delta\right] \\
U_{T'} &= 2G_A(2G_M^V)\sqrt{\tau(1+\tau)}\left[1+\tilde{\Delta}\right]
\end{aligned}
\tag{24}
$$

with

$$
\begin{aligned}
G_E^V &= F_1^V - \tau F_2^V; \qquad \Delta = \frac{\tau}{\kappa^2}\xi_F(1-\psi^2)\left[\kappa\sqrt{1+\frac{1}{\tau}} + \frac{\xi_F}{3}(1-\psi^2)\right] \\
G_M^V &= F_1^V + F_2^V; \qquad \tilde{\Delta} = \sqrt{\frac{\tau}{1+\tau}}\frac{\xi_F(1-\psi^2)}{2\kappa}
\end{aligned}
$$

QE responses in the RFG in the scaling formalism

Notice that, despite the factorization form of the nuclear responses, the scaling variable ψ not only appears in the scaling function, but also in the Δ and $\tilde{\Delta}$, so the factorization is not complete in the sense of "an integrated single-nucleon response independent of ψ" multiplied by a scaling function only dependent on ψ. But the quantities Δ and $\tilde{\Delta}$ are proportional to $\xi_F = \sqrt{1+\eta_F^2} - 1 \approx \frac{k_F^2}{2m_N^2} \ll 1$, and represent a small correction.

In any case, the "factorized" expression for the nuclear response functions

$$R_K = \frac{N\xi_F}{m_N \eta_F^3 \kappa} U_K \, f_{\mathrm{RFG}}(\psi)$$

is still useful because one can substitute "by hand" the RFG scaling function by suitable, either phenomenological or theoretically-based in more sophisticated models (as the RMF for finite nuclei with scalar and vector potentials), other ones that describe better the QE electron scattering data than the RFG model.

Hints on scaling properties of QE electron scattering data [3]

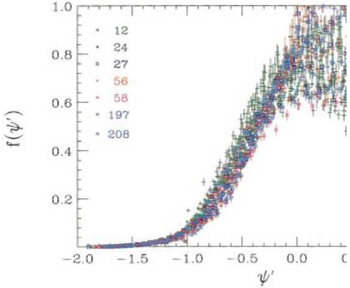


Figure: Scaling function $f(\psi')$ for all nuclei with $A \geqslant 12$ and all available kinematics.

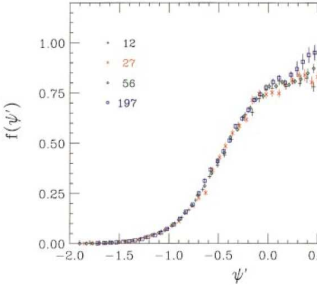


Figure: Scaling function $f(\psi')$ for ^{12}C, ^{27}Al, ^{56}Fe and ^{197}Au at the same kinematics ($q \approx 1$ GeV/c)

[3] Figures taken from reference T.W. Donnelly and I. Sick, **Phys. Rev. C 60**, 065502 (1999)

Some warnings

- The RFG model with the scaling variable defined above does not describe properly the QE electron scattering data. Therefore, in the literature many different modifications have been done.
- In some studies a modified scaling variable ψ', obtained from ψ by making a shift in λ (like a separation energy), is used to reproduce the data (as in the previous figure) and fulfill scaling. Problem with this approach? The shift in omega breaks gauge invariance and normally people try to restore it at the end.
- In other studies, different scaling functions are used for the longitudinal and transverse responses in electron scattering, $f_L(\psi')$ and $f_T(\psi')$. These functions are adjusted to data when a Rosenbluth separation between L and T responses can be done. From these studies it is known that $f_T > f_L$. This phenomenon has been always suggested to be due to the transverse enhancement induced in the T response due to Meson-exchange currents (MEC) and N-N correlations.
- The good point of RMF models with scalar and vector potentials in finite nuclei, based on the QHD-I model of Walecka, Horowitz and B.D. Serot is that these theoretical models predict that $f_T > f_L$, due to the enhancement of the lower-components of the relativistic spinors calculated within the theory and produce the tail seen in the data for $\psi > 1$. Bad point? The scaling functions do not scale, i.e, they are not functions alone of ψ. They also depend on q. Solution? to make again a shift in λ to restore scaling property, at the price of losing gauge invariance again...

SuSAM* = Super Scaling Analysis with M^*

The SuSAM* collaboration:

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- V.L. Martinez Consentino
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- J.E. Amaro

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It is not the first time that somebody tries to describe the QE electron scattering cross section using a relativistic effective mass. Pioneering works on this subject are those of Rosenfelder[4] and Wehrberger[5]. But as far as we know, this is the first attempt to find a scaling function and the relativistic effective masses by fitting the QE responses in electron scattering data for the whole database of nuclei we have at our disposal.

[4] Annals of Physics 128, 188-240 (1980).

[5] Phys.Rept. 225 (1993), 273-362.

SuSAM* literature

- **Global Superscaling Analysis of Quasielastic Electron Scattering with Relativistic Effective Mass.** J.E. Amaro, V.L. Martinez-Consentino, E. Ruiz Arriola, I. Ruiz Simo, **Phys.Rev. C98 (2018),** 024627
- **Quasielastic charged-current neutrino scattering in the scaling model with relativistic effective mass.** I. Ruiz Simo, V.L. Martinez-Consentino, J.E. Amaro, E. Ruiz Arriola, **Phys.Rev. D97 (2018)** 116006
- **Fermi-momentum dependence of relativistic effective mass below saturation from superscaling of quasielastic electron scattering**, V.L. Martinez-Consentino, I. Ruiz Simo, J.E. Amaro, E. Ruiz Arriola. Oct 13, 2017. **Phys.Rev. C96 (2017)** 064612
- **Superscaling analysis of quasielastic electron scattering with relativistic effective mass**, J.E. Amaro, E. Ruiz Arriola, I. Ruiz Simo. **Phys.Rev. D95 (2017)** 076009
- **Scaling violation and relativistic effective mass from quasi-elastic electron scattering: Implications for neutrino reactions** J.E. Amaro, E. Ruiz Arriola, I. Ruiz Simo, **Phys.Rev. C92 (2015)** 054607

Walecka model QHD-I (Quantum Hadrodynamics)

The building blocks of this model are the nucleon doublet field $\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$ and two neutral and isoscalar mesons, one of them is scalar (σ) and the other one is vector (ω^μ). The Lagrangian density for this model is given by:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N\right)\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\,\omega_\mu\,\omega^\mu \\ &- g_\omega\bar{\psi}\gamma^\mu\psi\,\omega_\mu + g_\sigma\bar{\psi}\psi\,\sigma \qquad (25)\end{aligned}$$

where $F^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$.
The field equations for this model can be obtained from the Euler-Lagrange ones and these are:

$$\left(\partial^\mu\partial_\mu + m_\sigma^2\right)\sigma = g_\sigma\,\bar{\psi}\psi \qquad (26)$$

$$\partial_\nu F^{\nu\mu} + m_\omega^2\,\omega^\mu = g_\omega\,\bar{\psi}\gamma^\mu\psi \qquad (27)$$

$$\left[\gamma^\mu\left(i\partial_\mu - g_\omega\,\omega_\mu\right) - \left(m_N - g_\sigma\,\sigma\right)\right]\psi = 0 \qquad (28)$$

Mean Field Theory (MFT) description of the model

If we consider the situation in which we have a system of B baryons in a large box of volume V and we are in the rest frame of the matter, i.e, the baryon current $B^\mu = (\rho_B, \mathbf{B}) = \bar{\psi}\gamma^\mu\psi$ has $\mathbf{B} = 0$. If the baryon density B/V increases, the sources increase as well; and if these are large enough, one would expect to substitute the meson fields by their expectation values:

$$\sigma \rightarrow \langle\sigma\rangle \equiv \sigma_0, \qquad \omega^\mu \rightarrow \langle\omega^\mu\rangle \equiv (\omega_0, 0) \tag{29}$$

Since we are restricting ourselves to stationary situation and uniform system, σ_0 and ω_0 are constants completely independent of space and time. And since the matter is at rest, the three-vector field $\vec{\omega} = 0$.

Mean Field Theory (MFT) description of the model

We can substitute these meson fields on the Lagrangian density to obtain the mean-field Lagrangian:

$$\mathcal{L}_{\rm MFT} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m_N)\,\psi - \frac{1}{2} m_\sigma^2\,\sigma_0^2 + \frac{1}{2} m_\omega^2\,\omega_0^2 - g_\omega\,\bar{\psi}\gamma^0\psi\,\omega_0 + g_\sigma\,\bar{\psi}\psi\,\sigma_0 \tag{30}$$

Only the fermion field has to be quantized, and we can particularize the previous Dirac equation to our MFT problem:

$$\left[i\gamma^\mu \partial_\mu - g_\omega\,\gamma^0\,\omega_0 - (m_N - g_\sigma\,\sigma_0)\right]\psi(t,\mathbf{x}) = 0 \tag{31}$$

We can see here that the effect of the scalar field is a shift in the baryon mass from m_N to $m_N^* \equiv m_N - g_\sigma\,\sigma_0$, and that of the vector field is a shift in the energy spectrum.

Saturation of nuclear matter

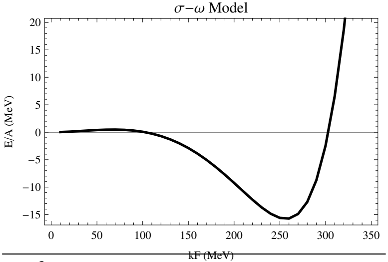
σ_0 and m_N^* need to be solved self-consistently for each k_F (or alternatively for each baryon density)[6]

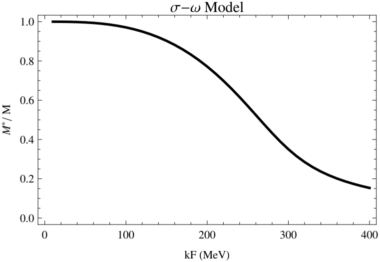
$$\sigma_0 = \frac{g_\sigma}{m_\sigma^2} \left\langle : \bar{\psi}\psi : \right\rangle \qquad \left\langle : \bar{\psi}\psi : \right\rangle = \frac{4}{(2\pi)^3} \int_0^{k_F(\rho_B)} d^3p \, \frac{m_N^*}{E^*(\mathbf{p})}$$

where k_F and ρ_B, the baryon density, are connected through the well-known expression

$$\rho_B = \frac{2k_F^3}{3\pi^2}$$

The saturation curve for nuclear matter is obtained (binding energy per particle)





[6] B.D. Serot and J.D. Walecka, Advances in Nuclear Physics, 16 (1986), 1-338.

Mean Field Theory (MFT) description of the model

We can look for plane-wave solutions of the Dirac equation in this MFT approximation:

$$\psi^{(+)}_{\mathbf{k}\lambda}(t,\mathbf{x}) = U(\mathbf{k},\lambda)e^{i\mathbf{k}\cdot\mathbf{x}-i\epsilon_+(\mathbf{k})t}, \qquad \psi^{(-)}_{\mathbf{k}\lambda}(t,\mathbf{x}) = V(\mathbf{k},\lambda)e^{-i\mathbf{k}\cdot\mathbf{x}-i\epsilon_-(-\mathbf{k})t} \tag{32}$$

Substituting these possible solutions in the Dirac equation, we can obtain the corresponding Dirac equations in momentum representation:

$$[\mathbf{k}\cdot\vec{\alpha} + m_N^*\beta]\,U(\mathbf{k},\lambda) = [\epsilon_+(\mathbf{k}) - g_\omega\,\omega_0]\,U(\mathbf{k},\lambda) \tag{33}$$

$$[\mathbf{k}\cdot\vec{\alpha} - m_N^*\beta]\,V(\mathbf{k},\lambda) = -[\epsilon_-(-\mathbf{k}) - g_\omega\,\omega_0]\,V(\mathbf{k},\lambda) \tag{34}$$

with $\vec{\alpha} = \gamma^0\vec{\gamma}$ and $\beta = \gamma^0$ being the usual Dirac matrices.
Eqs. (33) and (34) look like the free Dirac equation of a fermion of mass $m_N^* = m_N - g_\sigma\,\sigma_0$ with “energy” eigenvalues
$E^*(\mathbf{k}) \equiv \sqrt{\mathbf{k}^2 + m_N^{*2}} = \begin{cases} \epsilon_+(\mathbf{k}) - g_\omega\,\omega_0 \\ -[\epsilon_-(-\mathbf{k}) - g_\omega\,\omega_0] \end{cases}$

Mean Field Theory (MFT) description of the model

Therefore, we can interpret $U(\mathbf{k},\lambda)$ and $V(\mathbf{k},\lambda)$ as free spinors of a fermion of mass m_N^* satisfying the Dirac equation with "energy" eigenvalues $E^*(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_N^{*2}}$. And these m_N^* and $E^*(\mathbf{k})$ are the new "masses" and "on-shell" energies that enter in the normalization of the new nucleon spinors.

So we can write the nuclear hadron tensor for this model as it if were that of a RFG with energies E^* and masses m_N^*:

$$\begin{aligned} W^{\mu\nu} &= \frac{3N}{4\pi k_F^3}\int d^3p\, \frac{(m_N^*)^2}{E^*(\mathbf{p})E^*(\mathbf{p}+\mathbf{q})}\theta(k_F - |\mathbf{p}|)\theta(|\mathbf{p}+\mathbf{q}| - k_F) \\ &\times \delta\left(\omega - [E^*(\mathbf{p}+\mathbf{q}) - E^*(\mathbf{p})]\right) w^{\mu\nu}_{s.n.}(\mathbf{p}',\mathbf{p}) \end{aligned} \quad (35)$$

where now the single-nucleon tensor is given by:

$$2w^{\mu\nu}_{s.n.}(\mathbf{p}',\mathbf{p}) = \sum_{s\,s'} J^{\mu *}(\mathbf{p}',\mathbf{p})J^{\nu}(\mathbf{p}',\mathbf{p}) =$$

$$\mathrm{Tr}\left[\gamma^0(V^\mu(q) - A^\mu(q))^\dagger\gamma^0\frac{(\not{p}' + m_N^*)}{2m_N^*}(V^\nu(q) - A^\nu(q))\frac{(\not{p} + m_N^*)}{2m_N^*}\right] \quad (36)$$

And the assumption of the SuSAM* model is that the relativistic nucleon effective mass m_N^* only enters in the model through the nucleon spinors, while the vector and axial currents are the same as for free nucleons.

How to take advantage from the RFG formulae?

In principle, one can take the above eq. (36) and explicitly calculate the trace. One would obtain a single-nucleon tensor which will explicitly depend both on m_N^* (coming from the spinors) and on m_N (coming from the vector and axial-vector currents). These new expressions can again be integrated with the momentum distribution and Pauli-blocking in the Fermi gas model and the result would be right, but the final expressions for the nuclear responses R_K will contain explicitly the two masses.

Can we take the expressions obtained for the RFG model and use them with minimal changes? The answer is yes, but you have to be skilled. For example, if the vector and axial vector currents were

$$
\begin{aligned}
V^\mu(q) &= \left[2F_1^V(q^2)\gamma^\mu + i\frac{2F_2^V(q^2)}{2m_N^*}\sigma^{\mu\nu}q_\nu\right] \\
A^\mu(q) &= \left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{G_P(q^2)}{2m_N^*}q^\mu\gamma_5\right]
\end{aligned}
$$

then you can take the final expressions for the nuclear responses in the RFG model and use them with the only replacement $m_N \to m_N^*$ **everywhere**. BUT these are not the currents we are assuming.

How to take advantage from the RFG formulae?

Then, how to use the RFG formulae without changing the vector and axial currents? With this trick, we write the currents with an explicit m_N^* but hiding the free mass m_N in a redefinition (or re-scaling) of the F_2^V and G_P form factors.

$$\begin{aligned} V^\mu(q) &= \left[2F_1^V(q^2)\gamma^\mu + i\frac{2F_2^V(q^2)\frac{m_N^*}{m_N}}{2m_N^*}\sigma^{\mu\nu}q_\nu \right] \\ A^\mu(q) &= \left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{G_P(q^2)\frac{m_N^*}{m_N}}{2m_N^*}q^\mu\gamma_5 \right] \end{aligned}$$

And now we redefine $F_2^{V*}(q^2) = F_2^V(q^2)\frac{m_N^*}{m_N}$ and $G_P^*(q^2) = G_P(q^2)\frac{m_N^*}{m_N}$. Now we can use the final RFG-based expressions with the replacements

$$\begin{aligned} m_N &\rightarrow m_N^* \\ F_2^V &\rightarrow F_2^{V*} \\ G_P &\rightarrow G_P^* \end{aligned} \qquad (37)$$

How to take advantage from the RFG formulae?

The above replacements imply for the Sachs form factors in terms of which the RFG formulae are written that now these get modified in the following form

$$G_E^V \quad \rightarrow \quad G_E^{V*} \equiv F_1^V - \tau^* F_2^{V*} = F_1^V - \tau^* F_2^V \frac{m_N^*}{m_N} \tag{38}$$

$$G_M^V \quad \rightarrow \quad G_M^{V*} \equiv F_1^V + F_2^{V*} = F_1^V + F_2^V \frac{m_N^*}{m_N} \tag{39}$$

where all the kinematic variables, λ, κ, ξ_F, η_F... now have to be defined, of course, in terms of the effective mass m_N^*. In particular, $\tau^* = \kappa^{*2} - \lambda^{*2} = \frac{Q^2}{4m_N^{*2}}$.
Something similar happens for the pseudoscalar form factor when related to the axial form factor via PCAC. In the free case, the relation between them is

$$G_P = \frac{4m_N^2}{Q^2 + m_\pi^2} G_A \rightarrow G_P^* = \frac{m_N^*}{m_N} G_P = \frac{4\, m_N^*\, m_N}{Q^2 + m_\pi^2} G_A \tag{40}$$

It would seem from the above discussion that, at the end, one can take the RFG formulae in the scaling approach and substitute all the variables with those with $*$ and obtaining then a new scaling variable ψ^*. BUT, where is then the free nucleon mass m_N? OK, it will not appear explicitly in the formulae if we make the above replacements, but it is not absent at all. It is implicitly hidden in the G_E^{V*}, G_M^{V*} and G_P^* redefinitions.

Important clarification

Nonetheless, it is worth pointing out that the form factors appearing in the currents, either if you write them in terms of Pauli and Dirac form factors or directly from the very beginning in terms of the Sachs isovector electric and magnetic form factors, **are not changed at all in the SuSAM* model**. This is one of the assumptions of the model, that the currents are the free ones with the same form factors as if the interaction occurred on a free nucleon.
The above replacements are just a trick to avoid calculating again the traces and to use the RFG-formulae with the minimal changes as possible.
Therefore, at the end we finish with the expression

$$R_K = \frac{N\xi_F^*}{m_N^* \eta_F^{*3} \kappa^*} U_K \, f_{\mathrm{RFG}}(\psi^*) \tag{41}$$

Now, our next goal is to find a suitable scaling function (instead of the RFG one) $f^*(\psi^*)$ of the new scaling variable ψ^* (defined with the relativistic effective mass) in such a way that the experimental electron scattering data scale as better as possible.
At the end, the best thing we find for the electron scattering data is a dense cloud of experimental "QE" points that scale within an uncertainty band that we parametrize as the sum of two Gaussians.

The SuSAM* approach

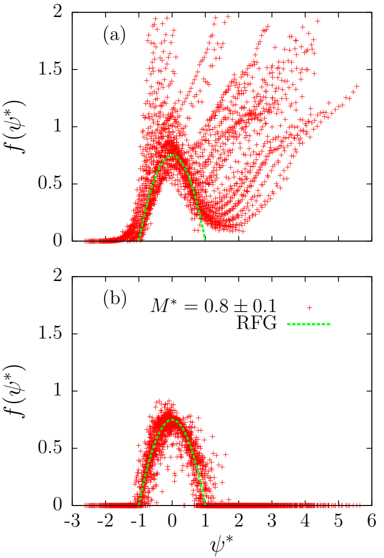
- **The cross section is computed by using the RFG equations with the above stated replacements to take into account that the currents do not change by assumption of the model.**
- **But using a phenomenological scaling function fitted to experimental data.**
- The experimental scaling function $f^*_{\rm exp}$ is computed from the data by dividing the experimental cross section by the "integrated single-nucleon" contribution

$$f^*_{\rm exp} = \frac{\left(\frac{d\sigma}{d\Omega' d\epsilon'}\right)_{\rm exp}}{\sigma_{\rm Mott}\left[v_L(ZU^p_L + NU^n_L) + v_T(ZU^p_T + NU^n_T)\right]\frac{\xi^*_F}{m^*_N \eta^{*3}_F \kappa^*}}$$

- We tune $M^* \equiv \frac{m^*_N}{m_N}$ and k_F to find the best scaling of data

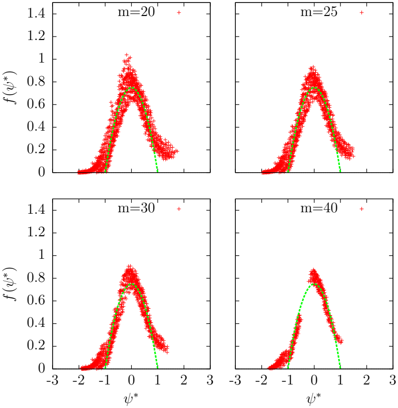
SuSAM* analysis of ^{12}C

- (a) $M*$ scaling analysis of the experimental data of ^{12}C compared to the RFG parabola.
- $M^* = 0.8$ and $k_F = 225$ MeV/c
- Scaling is violated but a large fraction of the data collapse into a data cloud surrounding the RFG parabola
- (b): RFG Monte Carlo simulation of QE data with relativistic effective mass $M^* = 0.8 \pm 0.1$.



The SuSAM* phenomenological quasielastic peak

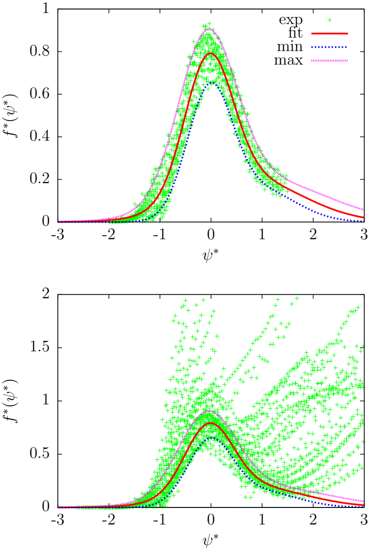
- Data selection by computing the data density
- n = number of points inside a ($r = 0.1$) circle
- All selected points with $n > m$ are considered "quasielastic" within an uncertainty band
- For the SuSAM* we choose the case $m = 25$, where a well defined data band is obtained



Phenomenological M^*-scaling function for ^{12}C.

$$f^*(\psi^*) = a_3 e^{-(\psi^*-a_1)^2/(2a_2^2)} + b_3 e^{-(\psi^*-b_1)^2/(2b_2^2)} \qquad \text{(Band A)}$$

- $f^*(\psi^*)$ and the uncertainty band, $f^*_{min} < f^* < f^*_{max}$, are fitted to experimental data.
- Well described as sum of two Gaussians
- Only data with density $n \geq 25$ inside a $(r = 0.1)$ circle are included.
- $\simeq$ 1000 QE data / 2500 are described by the band



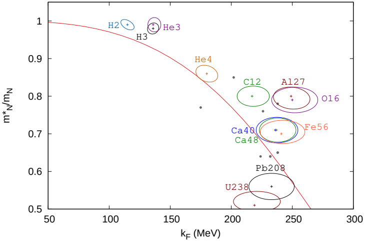
Data are from
O. Benhar, D. Day and I. Sick, arXiv:nucl-ex/0603032.
http://faculty.virginia.edu/qes-archive/

Global fits of SuSAM* parameters

Fits to (e,e') data for nuclei: ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{12}C, ^{6}Li, ^{9}Be, ^{24}Mg, ^{59}Ni, ^{89}Y, ^{119}Sn, ^{181}Ta, ^{186}W, ^{197}Au, ^{16}O, ^{27}Al, ^{40}Ca, ^{48}Ca, ^{56}Fe, ^{208}Pb, and ^{238}U.

- Separate fits for each nucleus to the ^{12}C scaling function
- Global fit of all the data including the scaling function parameters.

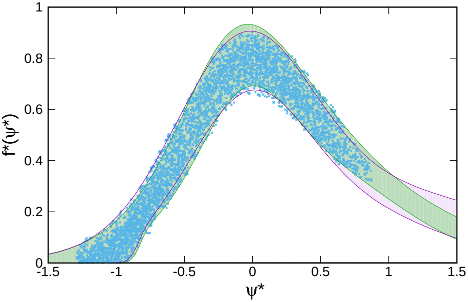
- Errors ΔM^* and Δk_F are computed in a χ^2 fit.
- Results are compared to the $\sigma - \omega$ model of Serot and Walecka, Adv.Nucl.Phys.16(1986)1.



SuSAM* scaling bands

4230 QE data for ALL nuclei: ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{12}C, ^{6}Li, ^{9}Be, ^{24}Mg, ^{59}Ni, ^{89}Y, ^{119}Sn, ^{181}Ta, ^{186}W, ^{197}Au, ^{16}O, ^{27}Al, ^{40}Ca, ^{48}Ca, ^{56}Fe, ^{208}Pb, and ^{238}U.

- (e, e') data are scaled with the best parameters of the global fit and selected with the density criterion.
- band C (in pink): global fit
- band B (in green): ^{12}C band



Data are from Benhar, Day and Sick http://faculty.virginia.edu/qes-archive/

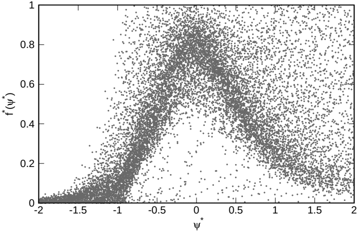
Parametrization of SuSAM* bands B, C:

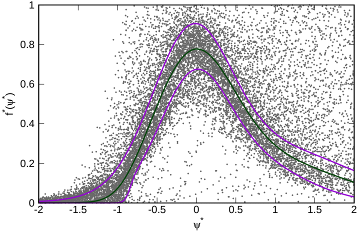
$$f^*(\psi^*) = \frac{a_3 e^{-(\psi^*-a_1)^2/(2a_2^2)} + b_3 e^{-(\psi^*-b_1)^2/(2b_2^2)}}{1 + e^{-\frac{\psi^*-c_1}{c_2}}}$$

Scaling of world data with SuSAM* parameters

^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{12}C, ^{6}Li, ^{9}Be, ^{24}Mg, ^{59}Ni, ^{89}Y, ^{119}Sn, ^{181}Ta, ^{186}W, ^{197}Au, ^{16}O, ^{27}Al, ^{40}Ca, ^{48}Ca, ^{56}Fe, ^{208}Pb, and ^{238}U.

- (e, e') world data scaled with the best parameters of the global fit
- $\simeq 9000$ / 20000 data inside band C
- 4230 data are true quasielastic
- Points outside of the band are non-quasielastic (delta-peak, inelastic, or low-energy regions)
- The scaling band estimates the theoretical uncertainty of the QE peak description in the SuSAM* model.

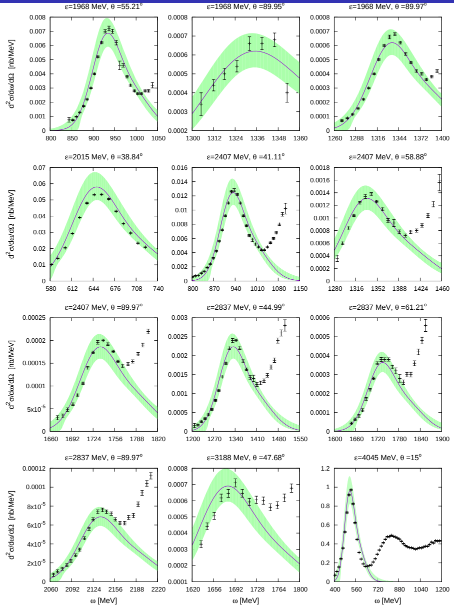




Data are from Benhar, Day and Sick http://faculty.virginia.edu/qes-archive/

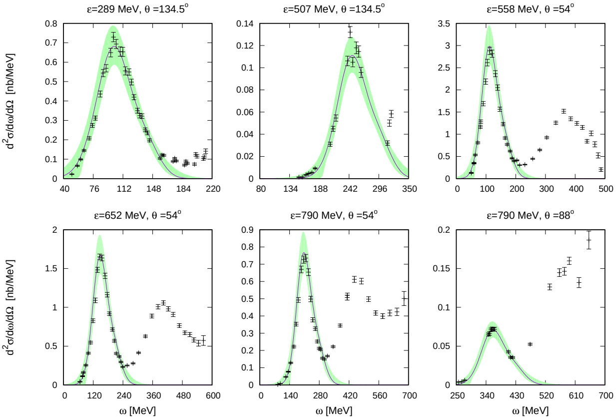
SuSAM* quasielastic (e, e') results for ^{2}H

- (e, e') cross section data for ^{2}H
- Compared to the SuSAM* QE model
- band B
- $k_F = 82$ MeV/c
- $M^* = 1$



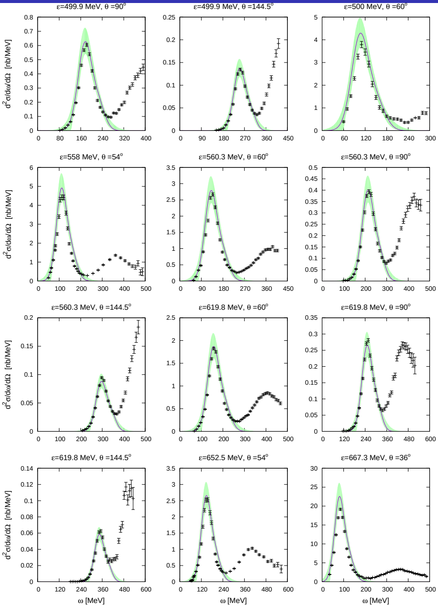
SuSAM* (e, e') results for ^{3}H

- $k_F = 136$ MeV/c
- $M^* = 0.98$



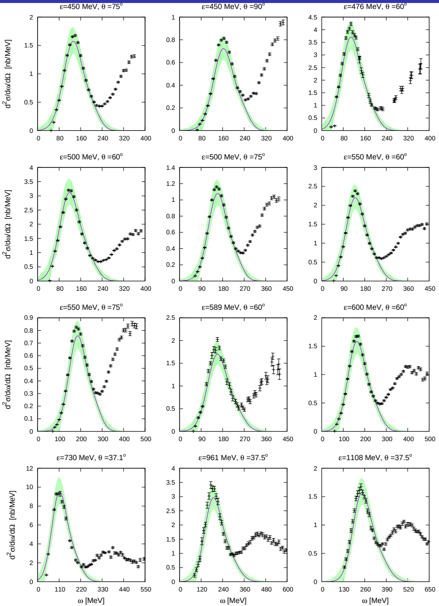
SuSAM* quasielastic (e, e') results for ^{3}He

- $k_F = 130$ MeV/c
- $M^* = 0.98$



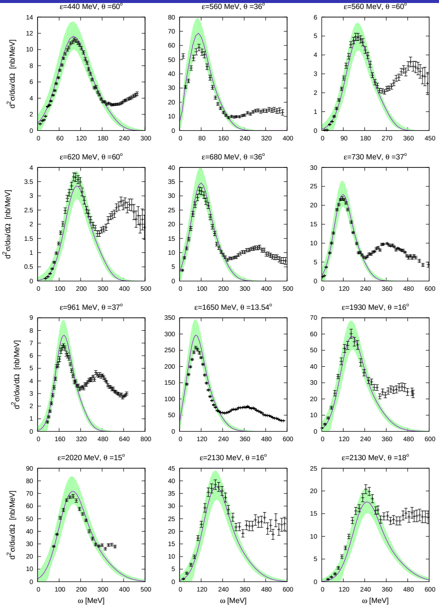
SuSAM* (e, e') results for ^{4}He

- $k_F = 180$ MeV/c
- $M^* = 0.86$



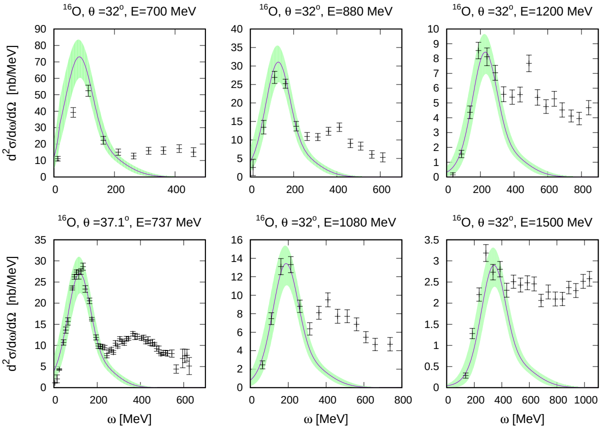
SuSAM* (e, e') results for ^{12}C

- $k_F = 217$ MeV/c
- $M^* = 0.8$

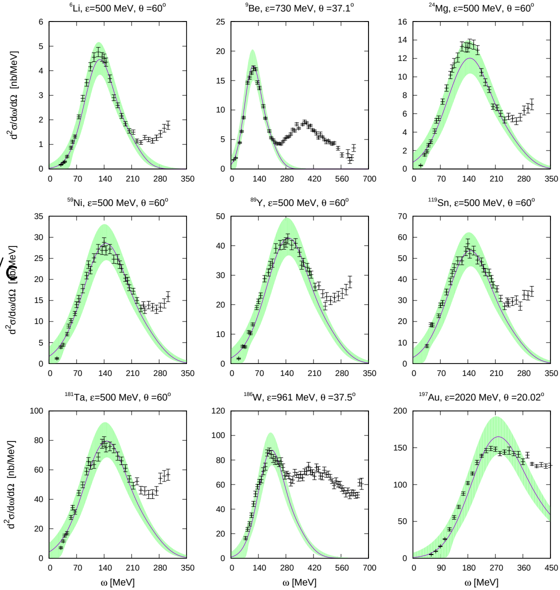


SuSAM* (e, e') results for ^{16}O

- $k_F = 230$ MeV/c
- $M^* = 0.8$

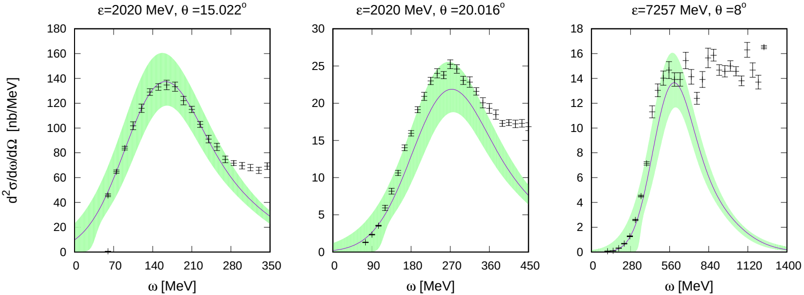


- $k_F = 175$ — 238 MeV/c
- $M^* = 0.77$ — 0.78



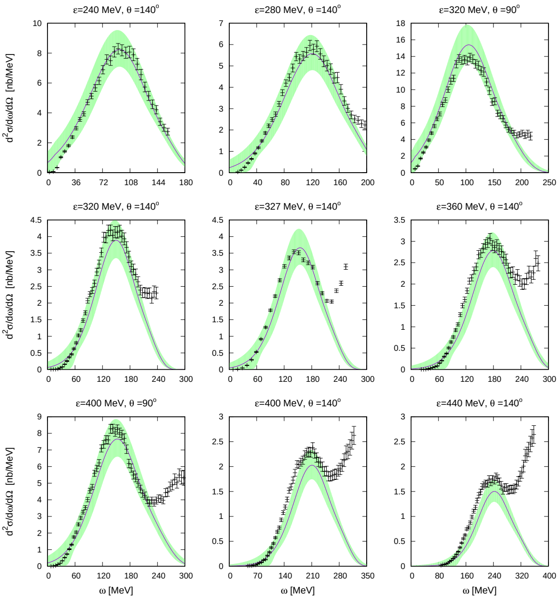
SuSAM* (e, e') results for ^{27}Al

- $k_F = 249$ MeV/c
- $M^* = 0.8$



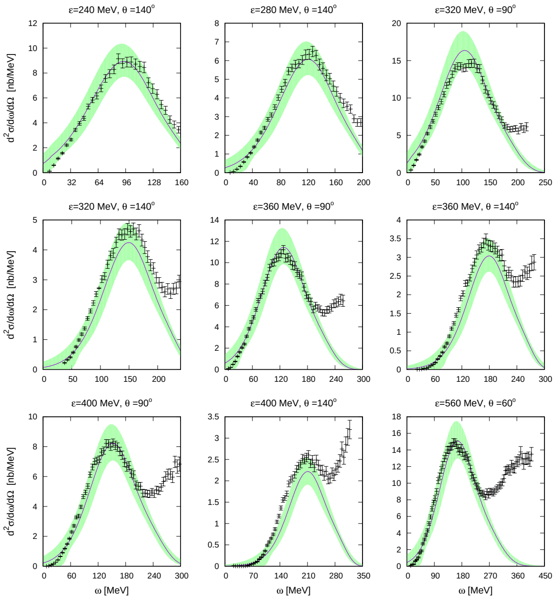
SuSAM* (e, e') results for ^{40}Ca

- $k_F = 236\text{MeV}/c$
- $M^* = 0.8$



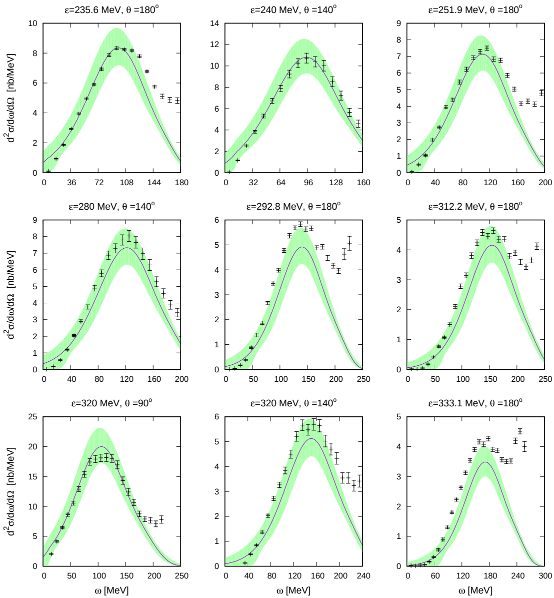
SuSAM* (e, e') results for ^{48}Ca

- $k_F = 236\text{MeV}/\text{c}$
- $M^* = 0.8$



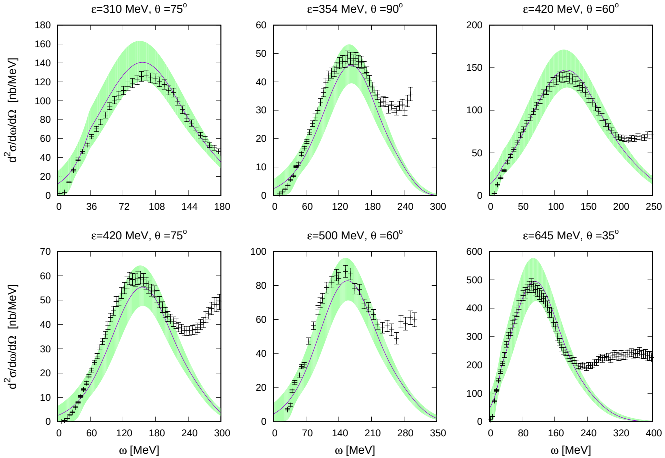
SuSAM* (e, e') results for ^{56}Fe

- $k_F = 240\text{MeV}/c$
- $M^* = 0.7$



SuSAM* (e, e') results for ^{208}Pb

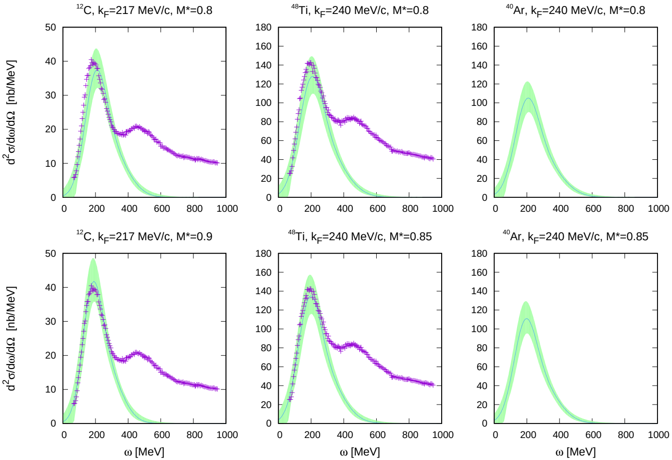
- $k_F = 233$ MeV/c
- $M^* = 0.56$



SuSAM* (e, e') results for recent new data and predictions for ^{40}Ar

- $k_F = 217$ MeV/c for $A = 12$
- $k_F = 240$ MeV/c for $A = 48, 40$

Inclusive (e, e') for ^{12}C, ^{48}Ti and ^{40}Ar
$\epsilon = 2222$ MeV, $\theta = 15.541^{\text{o}}$,



Data from the recent JLab experiment H. Dai et al., (JLab Hall A Collaboration), **Phys. Rev. C98 (2018)** 014617.

SuSAM* predictions for CCQE neutrino scattering

Flux-averaged double differential cross section

$$\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu} = \frac{1}{\Phi_{tot}} \int dE_\nu \Phi(E_\nu) \frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}(E_\nu)\,, \qquad (42)$$

$\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}(E_\nu)$: the SuSAM* cross section

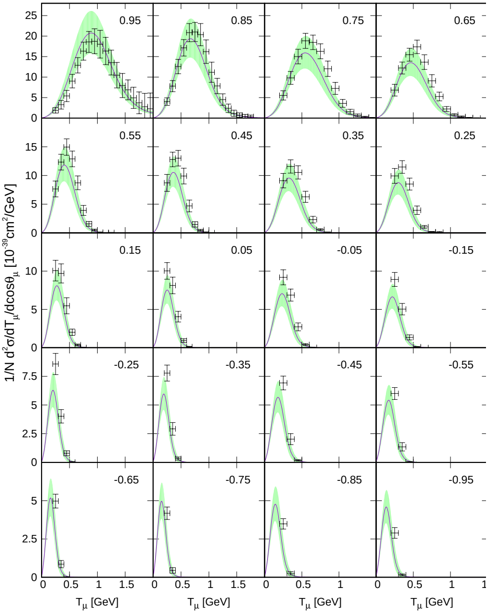
neutrino energy E_ν.

Neutrino flux: $\Phi(E_\nu)$

SuSAM* predictions for MiniBooNE, (ν_μ, μ^-)

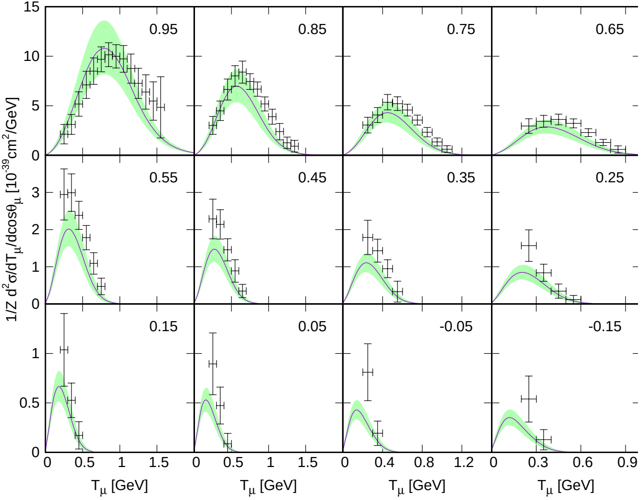
Each panel is labeled by the mean value of $\cos\theta_\mu$ in the experimental bin.

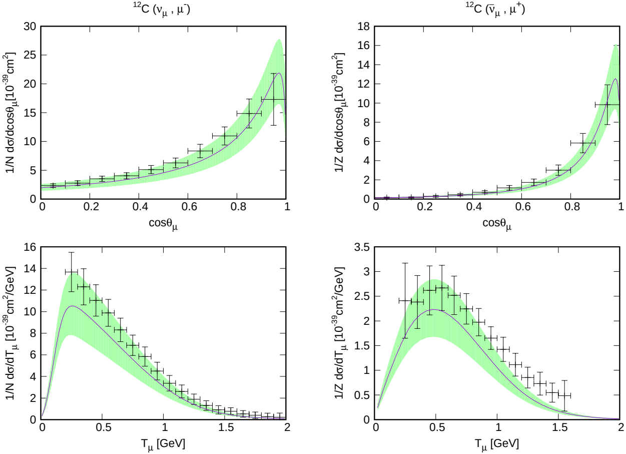
Experimental data are from MiniBooNE



SuSAM* predictions for MiniBooNE, $(\bar{\nu}_\mu, \mu^+)$

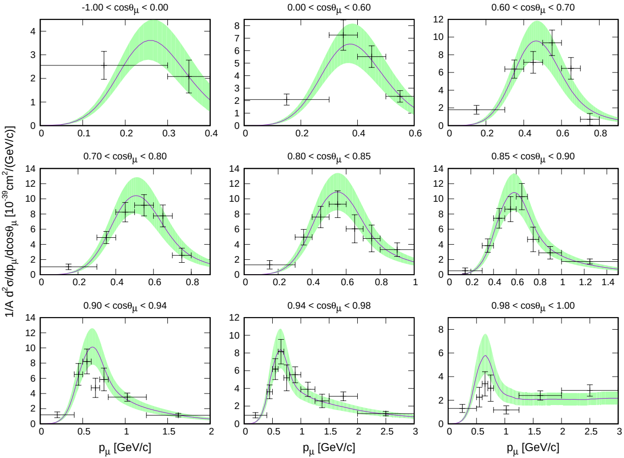
Each panel is labeled by the mean value of $\cos\theta_\mu$ in the experimental bin.





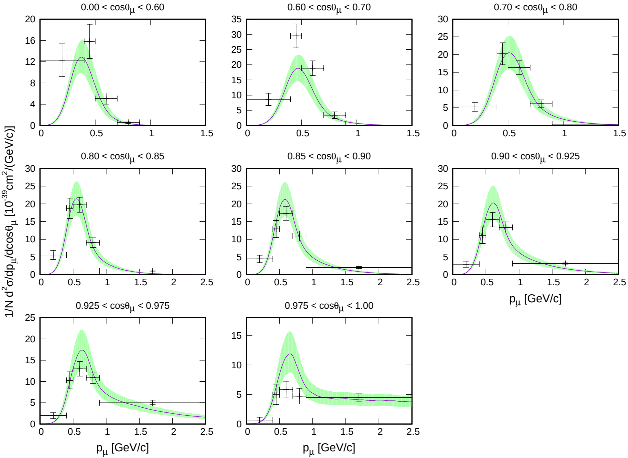
Flux-integrated single-differential cross sections per target neutron (proton) for the CCQE neutrino (antineutrino) reactions on ^{12}C in the SuSAM* model.
Left panels are for neutrinos and right ones for antineutrinos.
The experimental data are from MiniBooNE

SuSAM* predictions for T2K ^{12}C (ν_μ, μ^-)



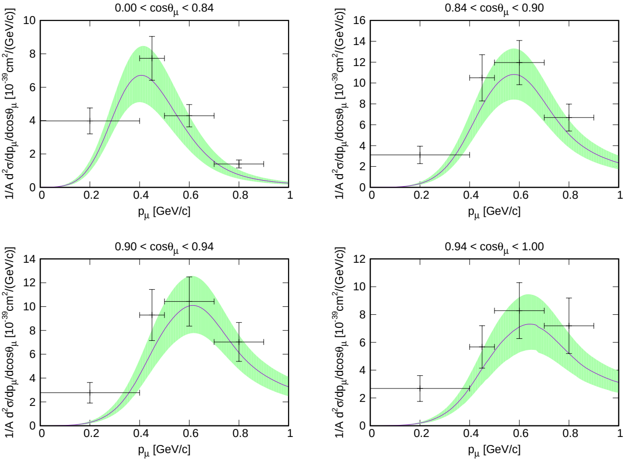
T2K flux-folded double differential CCQE cross section per nucleon for ν_μ scattering on ^{12}C in the SuSAM* model.
Experimental data are from T2K

SuSAM* predictions for T2K ^{16}O (ν_μ, μ^-)



T2K flux-folded double differential CCQE cross section per nucleon for ν_μ scattering on ^{16}O in the SuSAM* model.
Experimental data are from T2K

SuSAM* predictions for T2K ^{12}C (ν_μ, μ^-)

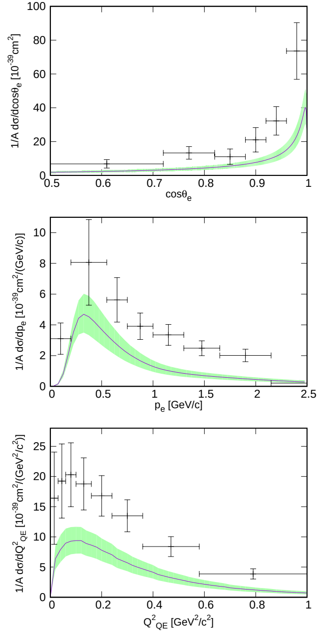


T2K flux-folded double differential CC inclusive cross section per nucleon for ν_μ scattering on ^{12}C in the SuSAM* model.
Experimental data are from T2K

SuSAM* predictions for T2K ^{12}C (ν_e, e^-)

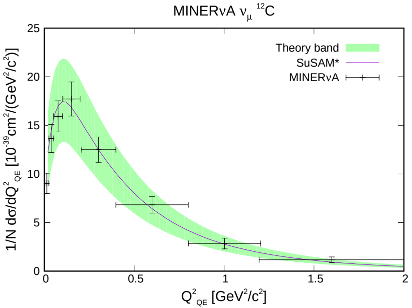
T2K flux-folded single differential CC inclusive cross section per nucleon for ν_e scattering

The neutron binding energy in Q^2_{QE} is $E_B = 25$ MeV.



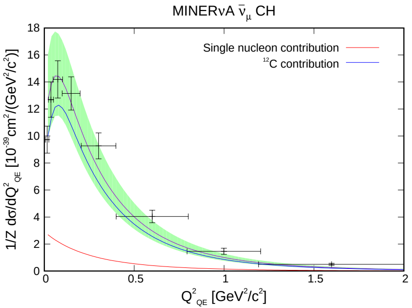
SuSAM* predictions for MINERvA Q^2_{QE} distributions

Flux-folded CCQE
(ν_μ, μ^-) from ^{12}C
$(\bar{\nu}_\mu, \mu^+)$ from CH



The data are from MINERvA

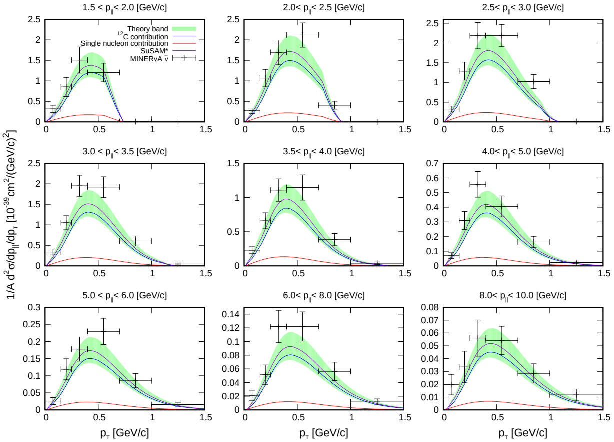
The H contribution is obtained from the elastic antineutrino-proton cross section divided by $Z = 7$.



SuSAM* predictions for MINERvA CCQE $(\overline{\nu}_\mu, \mu^+)$

Flux-folded double-differential cross section $\dfrac{d^2\sigma}{dp_{||}\,dp_\perp}$

- Antineutrino CCQE scattering from CH
- Compared to the MINERvA experiment.
- The $\overline{\nu}_\mu - H$ cross section is divided by $A = 13$.
- $\theta_\mu < 20^o$ cut



Conclusions

- SuSAM* is a **new phenomenological scaling approach based on the RMF** theory of nuclear matter. It depends on M^* and k_F
- The **phenomenological scaling function** is extracted from a selection of (e, e') QE data that approximately **scale inside a band**.
- The SuSAM* band has been parametrized and it provides a **global description of the QE** (e, e') **cross section** for all the nuclei considered.
- The width of the SuSAM* band represents the theoretical uncertainty of the model from effects breaking the factorization of the cross section (such as **MEC, FSI, long and short-range correlations**)
- **SuSAM* has so far been applied to predict CCQE-like neutrino cross sections together with the theoretical error within the model, as extracted from the analysis to electron scattering data, but without adjusting any neutrino cross section parameter (axial mass...).**
- The model can be improved in several ways: it can be used as it stands now to be applied to describe CCQE neutrino/antineutrino cross sections and try to fit additionally the nucleon axial mass. This could provide a global fit to the world electron and neutrino QE scattering data. We want to collaborate in this line with the group of JINR.

Conclusions (continuation)

- We also expect to collaborate in the implementation of this SuSAM* model within the neutrino event generator GENIE.
- We can also improve the model by substituting the Fermi momentum distribution (step function) by a low temperature Fermi momentum distribution. If the integrals can be done analytically, perfect. If not, the final integral over the initial nucleon energy can be done numerically and a new scaling function (different of that of RFG) would appear. In fact, probably the temperature of the momentum distribution could be also fitted to reproduce electron and/or neutrino QE scattering data.

Comparisons with other scaling approaches: advantages of SuSAM* and weak points

- Advantages: We only have a single scaling function to describe a really large quantity of experimental data.
- All the nuclear response functions have analytical expressions in terms of the "integrated single-nucleon responses" and the scaling function.
- The quality in the description of data, especially for the case of neutrino/antineutrino scattering is similar to other scaling approaches as SuSAv2-MEC[7,8], currently being also implemented (or already) in GENIE. But this SuSAM* approach is far much simpler and does not need to rely on RMF calculations in finite nuclei like SuSAv2. The code is much faster and we don't need to resort to pre-generated tables or parameterizations for the MEC response functions as in the SuSAv2-MEC model.

[7] G.D. Megias et al, Physical Review D 94, (2016) 013012 (for electron scattering)

[8] G.D. Megias et al, Physical Review D 94, (2016) 093004 (for neutrino/antineutrino scattering)

Conclusions (continuation)

Weak points of the SuSAM* as compared with other scaling approaches

- As this model is purely phenomenological, it is not possible to obtain the nucleon momentum distributions to generate hadron events within the Monte Carlo, just because the scaling function appears after integrating over the initial nucleon momenta and at that point you have lost all the information about the momentum distribution. From the phenomenological scaling function it is not possible to obtain the momentum distribution that hypothetically would generate it. However, in the SuSAv2 and all other RMF-based models (in finite nuclei), as they calculate the single-particle wave functions, they can produce the momentum distributions from these single-particle wave functions.
- The SuSAM* band, although it seems quite narrow to describe properly QE electron scattering cross section data, gets much enhanced when applied directly to predict neutrino/antineutrino CCQE-like cross sections. This means that it almost covers every data, just because the uncertainty of the model fitted to electron scattering gets magnified when translated to neutrino/antineutrino observables.

Thank you for your attention!