Conformal Anomalies, Quantum Entanglement, Boundaries, and Distributional Geometry

D.V. Fursaev
Dubna University & BLTP JINR

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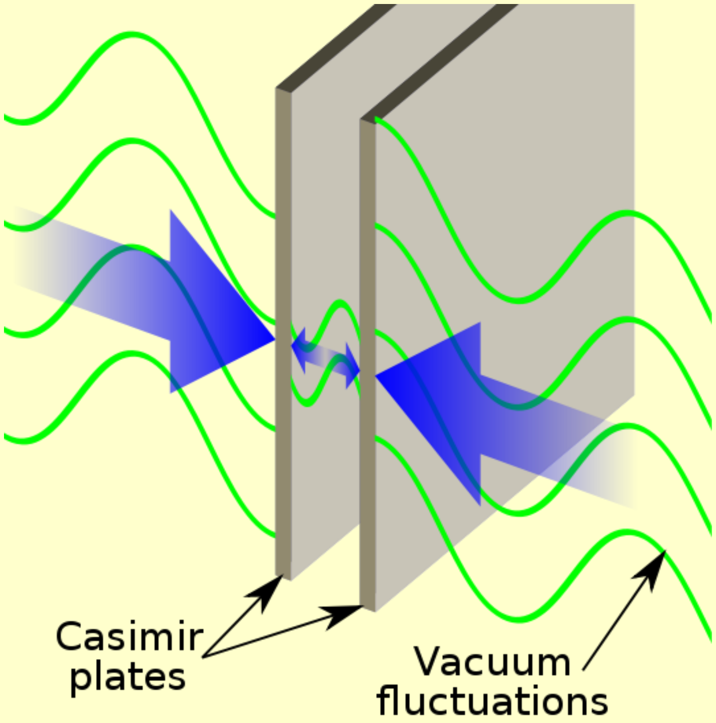
Recent publications:

D.V. Fursaev, “Quantum entanglement on boundaries”, JHEP 1307 (2013) 119, e-Print: arXiv:1305.2334 [hep-th]

D.V. Fursaev, “Conformal anomalies of CFT’s with boundaries”, JHEP 1512 (2015) 112, e-Print: arXiv:1510.01427 [hep-th]

D.V. Fursaev, S.N. Solodukhin, “Anomalies, entropy and boundaries” Phys. Rev. D93 (2016) no.8, 084021, e-Print: arXiv:1601.06418 [hep-th]



Motivations:

● Boundaries result in observable effects in QFT (the Casimir forces);

● Boundaries change single-particle spectra, we expect that the entangle-ment entropy (EE) is sensitive to the boundaries;

● EE carries a new piece of information about physics of boundaries in QFT (how states are entangled across the boundary): importance for condensed matter

We consider EE when an entangling surface crosses the boundary

Finite size effects of EE in 2D CFT’s

J. L. Cardy, “Boundary Conditions, Fusion Rules and the Verlinde Formula," Nucl. Phys. B 324, 581 (1989);

I. Affleck and A. W. W. Ludwig, “Universal non-integer 'ground state degeneracy' in critical quantum systems," Phys. Rev. Lett. 67, 161 (1991);

and other works

Entanglement (Renyi) Entropy

$\rho_1 = \mathrm{Tr}_2\, \rho$ – reduced density matrix

$$S_1^{(\alpha)} = \frac{\ln \mathrm{Tr}_1\, \rho_1{}^{\alpha}}{1-\alpha}$$ – entanglement Renyi entropy

In general, $\alpha > 0,$ and $\alpha \neq 1$

Next we consider integer values $\alpha = n = 2, 3, 4, \ldots.$

I. **Basic properties**

$S_1^{(\alpha)} \geq 0, \quad (S_1^{(\alpha)} = 0,$ if and only if ρ_1 is pure state)

Different limits:

$$S_1^{(\alpha)} \to S_1 \quad , \quad \alpha \to 1$$

$$S_1 \equiv -\mathrm{Tr}_1\, \rho_1 \ln \rho_1 \quad - \quad \text{entanglement entropy}$$

$$\lim_{\alpha \to 0} S_1^{(\alpha)} = \ln D,$$

where D is the # of nonvanishing eigenvalues of reduced density matrix ρ_1

$$\lim_{\alpha \to \infty} S_1^{(\alpha)} = -\ln \lambda_1$$

where λ_1 is the largest eigenvalue of ρ_1

Basic properties

II. "Symmetry" in a pure state

$$S_1^{(\alpha)} = S_2^{(\alpha)}$$

sketch of the proof: let $\rho = |\psi\rangle\langle\psi|$, then $|\psi\rangle = \sum_{aA} C_{Aa} |A\rangle |a\rangle$

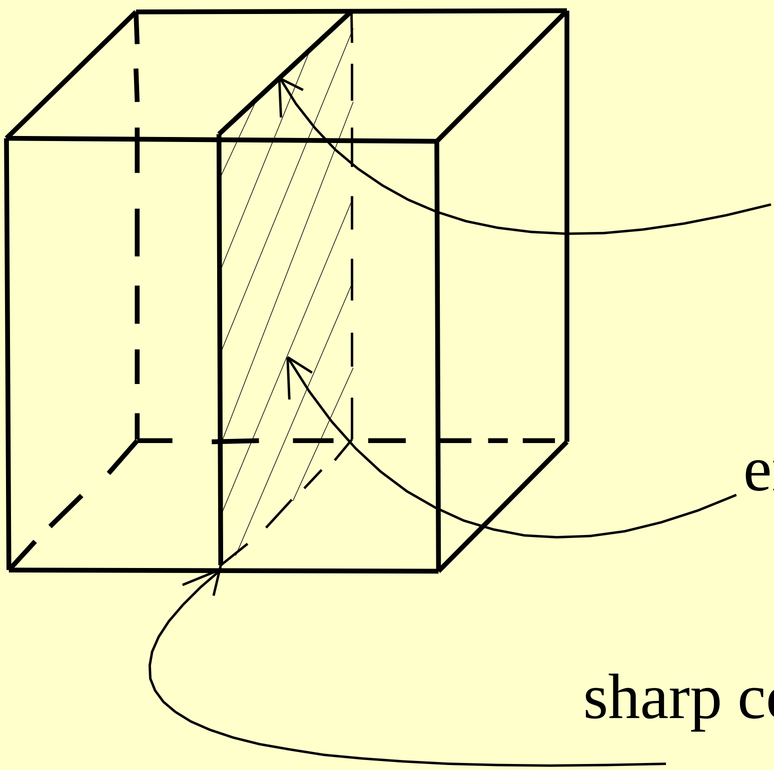
$$\rho_1(A|B) = \sum_a C_{Aa} C^*{}_{Ba} \quad \rightarrow \quad \rho_1 = CC^+$$

$$\rho_2(a|b) = \sum_A C_{Aa} C^*{}_{Ab}, \quad \rightarrow \quad \rho_2 = C^T C^*$$

non-vanishing eigenvalues of ρ_1 and ρ_2 coincide;

in general, $\rho = \dfrac{e^{-H/T}}{Tr\, e^{-H/T}} \quad \rightarrow S_1^{(\alpha)} \neq S_2^{(\alpha)}$

first studies of boundary effects in 4D QFT's



$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon, \qquad s_{\log} = C(\alpha_i),$$

ε is UV cutoff

Fursaev, PRD73, 124025 (2006)

Wilczek, Hertzberg, PRL 106, 050404 (2011)

Boundary terms appear in

$s_{\log}$ - the 'logarithmic part' of EE

$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon,$$

This may be important:

we expect that the logarithmic part of EE is related to the conformal anomaly and may have a holographic description

EE and trace anomaly in d=4:

local conformal anomaly

$$\langle T^{\mu}_{\mu} \rangle = -2aE - cI - \frac{c'}{24\pi^2}\nabla^2 R$$

$$E = \frac{1}{16\pi^2}\left(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$$ – "density" of the Euler n.

$$I = -\frac{1}{16\pi^2}C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}, \quad C_{\mu\nu\lambda\rho}$$ – the Weyl tensor

"bulk charges" a, c

a- monotonically decreases under RG flow from UV to IR

suggested by J. Crardy, PLB 215, 749-752 (1988),

proved by Z.Komargodski and A.Schwimmer, JHEP 12 (2011)099

3 invariants on a smooth entangling surface *B* in d=4 (no boundaries)

$$F_a = -\frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, R(B) \quad , \qquad R(B) - \text{scalar curvature of } B$$

$$F_c = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, C_{\mu\nu\lambda\rho} n_i^\mu n_j^\nu n_i^\lambda n_j^\rho \quad , \quad C_{\mu\nu\lambda\rho} - \text{Weyl tensor of } M \text{ at } B,$$

$$F_b = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x \left(\frac{1}{2}\mathrm{Tr}(k_i)\mathrm{Tr}(k_i) - \mathrm{Tr}(k_i k_i) \right) ,$$

$(k_i)_{\mu\nu}$ – extrinsic curvatures of B, n_i – normal vectors

F_a, F_b, F_c – are invariant with respect to the Weyl transformations $g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x)$

Logarithmic term in EE in d=4

$$s_{\log} = aF_a + cF_c + bF_b$$ (no boundaries)

- Ryu,Takayanagi, JHEP 0608, 045 (2006),

- Solodukhin, PLB 665, 305 (2008)
- Fursaev, Patrushev, Solodukhin, PRD 88, 044054 (2013)

$$c = b$$ for CFT's

conformal charges in the trace anomaly of a CFT uniquely fix the logarithmic term in EE (no boundaries) !

Holographic entanglement entropy (Ryu-Takayanagi formula)

volume of a holographic surface $\tilde{B}$ in AdS

$$A(\tilde{B}) = \frac{1}{2\varepsilon^2} A(B) + \frac{\pi}{2}(F_a + F_c + F_b)\ln\frac{\mu}{\varepsilon} + \ldots$$

$z = \varepsilon -$ position of the boundary (a UV cutoff in CFT)

(expansion for $A(\tilde{B})$ first found by A.Schwimmer and S.Theisen, arXiv:0802.1017)

$$S(B) = \frac{A(\tilde{B})}{4G_5} \sim \frac{N^2\Lambda^2}{4\pi} A(B) + \frac{1}{4} N^2 (F_a + F_c + F_b)\ln \mu\Lambda + \ldots$$

use AdS/CFT dictionary: $\dfrac{1}{G_5} = \dfrac{2N^2}{\pi}, \quad \varepsilon = 1/\Lambda$

one reproduces correctly the structutre of the leading divergences and exact value

of the logarithmic part of the entropy

the rest of the talk:

We study effects of boundaries in the conformal anomaly and in the entropy of entanglement, when the entangling surface crosses the boundary

- a first systematic classification of the “boundary charges” in the integrated conformal anomaly CFT in d=3,4;

- relation between bulk and boundary charges in d=4;

- calculation of the logarithmic terms in EE in d=3,4;

- new features of distributional (extrinsic and intrinsic) geometry when conical singularities cross boundaries;

- search for the relation between “boundary charges” in the conformal anomaly and in EE;

Local and integrated conformal anomaly

If a classical theory is scale invariant :

$$g'_{\mu\nu}(x) = e^{2\sigma(x)} g_{\mu\nu}(x),$$

the trace of the stress - energy tensor is zero, $T_{\mu}^{\mu} = 0;$ classical property is

broken for quantum everages of the corresponding (renormalized) operators

$\left\langle \hat{T}_{\mu}^{\mu} \right\rangle \neq 0 -$ local (trace) anomaly

the property is known as the conformal or scale anomaly;

we also use the integrated anomaly

$$\mathrm{A} = \partial_{\sigma} W[e^{2\sigma} g_{\mu\nu}]_{\sigma=0} = \int_{M} \left\langle \hat{T}_{\mu}^{\mu} \right\rangle \sqrt{g}\, d^{n}x + \mathrm{b.t.}$$

of the effective action W

Boundary terms in d=4:

a general structure of the integrated anomaly in the presence of boundaries

$$\mathrm{A} = -2a\chi_4 - ci_4 + q_1 j_1 + q_2 j_2 \quad , \; i_4 = \int_M I$$

$$\chi_4 = \int_M E + \frac{1}{32\pi^2}\int_{\partial M} Q$$ – Euler characteristic of M;

$$Q = -8\left[\det K_{ab} + (\hat{R}_{ab} - \frac{1}{2} g_{ab}\hat{R})K^{ab}\right]$$

$$j_1 = \frac{1}{16\pi^2}\int_{\partial M} C_{\mu\nu\lambda\rho} n^\nu n^\rho \hat{K}^{\mu\lambda} \quad , \quad j_2 = \frac{1}{16\pi^2}\int_{\partial M} \mathrm{Tr}(\hat{K}^3)$$

$\hat{K}^{\mu\lambda}$ – traceless part of the extrinsic curvature of the boundary ∂M,

conformal structure of A has been studied first for a scalar field

with the Dirichlet boundary condition (Dowker & Schofield, 1990)

Results for boundary charges in d=4

(DF, JHEP 1512, 112 (2015))

- boundary "charges" q_k are calculated for CFT's, spins 0, 1/2, 1

- a relation between boundary q_k and bulk "charges" a, c is established

Results for d=4

CFT	a	c	q1	q2	b.cond.
Scalar	1 / 360	1 / 120	1 / 15	2 / 35	Dirichlet
Scalar	1 / 360	1 / 120	1 / 15	2 / 45	Robin
Spinor	11 / 360	1 / 20	2 / 5	2 / 7	Mixed
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Absolute
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Relative

- For an Abelian gauge field "charges" do not depend on the boundary conditions:

$$\vec{E}_{\|} = \vec{B}_{\perp} = 0 \quad \text{or} \quad \vec{E}_{\perp} = \vec{B}_{\|} = 0$$

Properties of boundary chargers in d=4

- $q_1 = 8c,$
- as consequence, integrated anomaly has a correct Gibbons-Hawking type

boundary term: the functional

$$c \int_M C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho} + q_1 \int_{\partial M} C_{\mu\nu\lambda\rho} n^\nu n^\rho \hat{K}^{\mu\lambda},$$

under variations has no normal derivatives of the bulk metric on the boundary

(Solodukhin, PLB 752, 131 (2016))

- Boundaries yield a single independent boundary charge q_2 (at $\int \mathrm{Tr}\, \hat{K}^3$)
- q_2 is sensitive to boundary conditions
- q_2 appears in RG equation for 3-point correlation function of the stress-energy

tensor near the boundary (Kuo-Wei Huang (2016), 1604.02138[hep-th])

Some details

Effective action and spectral geometry

$$W = \frac{1}{2}\eta \ \ln\det L \ , \quad \eta = \pm 1$$

$L = -\nabla^2 + X$ – Laplace operators for different spins on M

Asymptotic expansions

$$\mathrm{Tr}\, e^{-tL} \sim \sum_{n=0}^{\infty} t^{\frac{n-d}{2}} A_n \quad t \to 0;$$

$$\mathrm{A} = \partial_\sigma W[e^{2\sigma} g_{\mu\nu}]_{\sigma=0} = \eta A_{n=d}$$ (in d dimensions)

$\eta = +1$ (for Bosons) -1 (for Fermions)

Boundary terms (d=4)

(Branson and Gilkey, Comm. Part. D.E. 15, 245 (1990))

$$L = -\nabla^2 + X, \quad (\nabla_N - S)\Pi_+\phi = 0, \quad \Pi_-\phi = 0, \quad \Pi_+ + \Pi_- = \mathrm{I}$$

$$A_4^{bdr} = \frac{1}{4\pi^2} \int_{\partial M} \mathrm{Tr}\left[\Pi_+ C_4^+ + \Pi_- C_4^- + C_4^{+-}\right],$$

$$C_4^+ = -\frac{1}{360} Q + \frac{1}{15} G_1 + \frac{2}{45} G_1 - \frac{1}{3}\left(X - \frac{1}{6}R\right)K + \frac{1}{2}\nabla_N\left(X - \frac{1}{6}R\right)$$

$$+\frac{4}{3}\left(S\Pi_+ + \frac{1}{3}K\right)^3 - 2\left(X - \frac{1}{6}R\right)S + \left(S + \frac{1}{3}K\right)\left(\frac{2}{15}\mathrm{Tr}K^2 - \frac{2}{45}K^2\right),$$

$$C_4^+ = -\frac{1}{360} Q + \frac{1}{15} G_1 + \frac{2}{35} G_1 - \frac{1}{3}\left(X - \frac{1}{6}R\right)K - \frac{1}{2}\nabla_N\left(X - \frac{1}{6}R\right),$$

$$C_4^{+-} = -\frac{1}{3}(\Pi_+ - \Pi_-)\Pi_{+:a}\Omega_{a\mu}N^\mu - \frac{2}{15}\Pi_{+:a}\Pi_{+:a}K - \frac{4}{15}\Pi_{+:a}\Pi_{+:b}K^{ab} - \frac{4}{3}\Pi_{+:a}\Pi_{+:a}\Pi_+ S$$

Boundary terms (continued)

$$Q = -8\left[\det K + \left(\hat{R}_{ab} - \frac{1}{2} H_{ab}\hat{R}\right) K^{ab}\right]$$

$$G_1 = R_{\mu\nu\lambda\rho} K^{\mu\lambda} N^{\nu} N^{\rho} - \frac{1}{2} R_{\mu\nu}\left(N^{\mu} N^{\nu} K + K^{\mu\nu}\right) + \frac{1}{6} KR$$

$$G_2 = \mathrm{Tr}K^3 - K\,\mathrm{Tr}K^2 + \frac{2}{9} K^3$$

This complicated structure of the heat coefficients drastically simplifies in CFT's as a result of the conformal invariance of the coefficient

Conformal invariance of the heat coefficient

- Let the classical action be invariant

$$I[\phi, g] = \int d^d x \sqrt{g}\, \phi(x) L \phi(x)$$

under conformal transformations:

$$g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x), \quad \phi'(x) = e^{k\omega(x)} \phi(x),$$

$$I[\phi, g] = I[\phi', g']$$

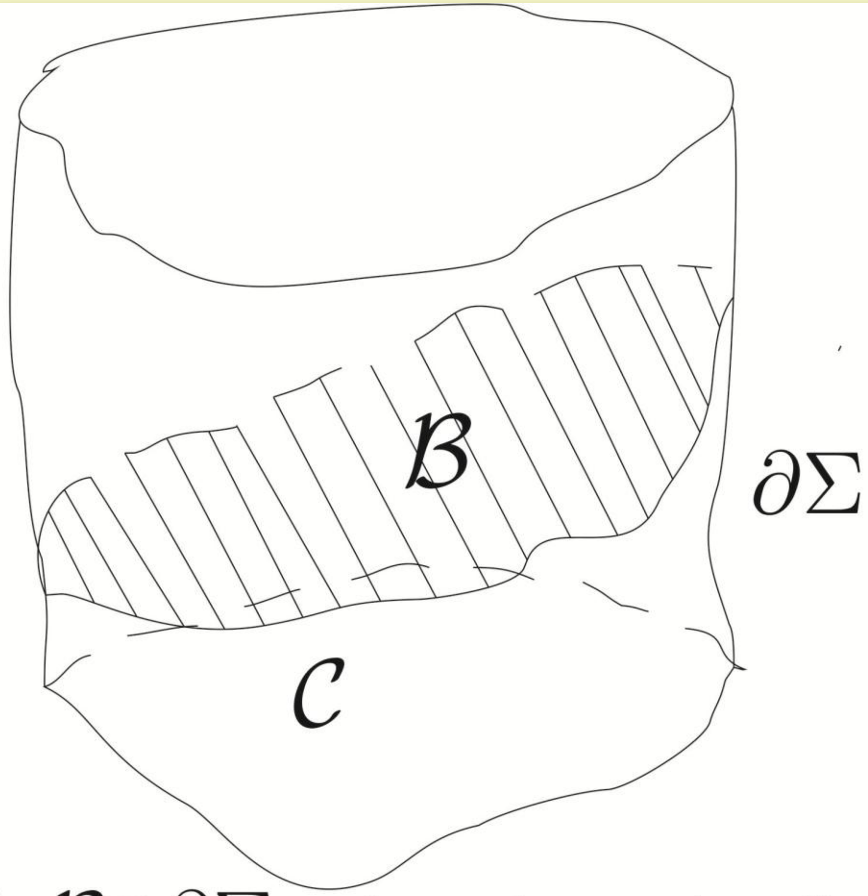
- Let boundary conditions respect the conformal invariance,

for an example: $\phi\big|_{\partial\Sigma} = 0$ (the Dirichlet condition)

Then the heat coefficient $A_{p=d}$ is a conformal invariant:

$$\boxed{A_{p=d}[g] = A_{p=d}[g']}$$

EE for entangling surface crossing the boundary



$\mathcal{C}=\mathcal{B}\cap\partial\Sigma$ - boundary entangling curve

Logarithmic terms in EE in CFT's (d=4)

$$s_{\log}(B) = aF_a + cF_c + bF_b + dF_d + eF_e$$

terms on $C = B \bigcap \partial M$

$$F_a = -\frac{1}{2\pi}\left(\int_B \sqrt{\sigma} d^2x\, R(B) + \int_C ds\, k\right) = -2\chi_2(B) \quad ,$$

$\chi_2(B) -$ Euler characteristics of B

$F_c, F_b -$ are not modified in the presence of boundaries

$F_d = F_d(C), \quad F_e = F_e(C) -$ terms of a new type (pure boundary effects)

$F_d, F_e -$ are dimensionless Weyl invariant (for CFT's) integrals on C

$d, e -$ are boundary coefficients in the entropy

Do d, e are related to charges in the integrated conformal anomaly?

Invariants and coefficients

$$F_d = \frac{3}{2\pi}\int_C ds\, \psi_1\, \hat{K}_{\mu\nu} u^\mu u^\nu\ , \quad u^\nu - \text{tangent vector to } C$$

$$F_e = \frac{1}{\pi}\int_C ds\, \psi_2\, (N\cdot p_i)(\hat{k}_i)_{\mu\nu} u^\mu u^\nu \quad ,$$

$(\hat{k}_i)_{\mu\nu} -$ traceless part of extrinsic curvature of B,

$\psi_1(\alpha), \psi_2(\alpha) -$ are unknown functions of α - a tilt angle of B and ∂M (between normal vector to ∂M and a normal vector to ∂M in B)

coefficient d at F_d can be calculated by assumiming that $\psi_1(0)=1$

$$d = \frac{1}{30}, \frac{7}{60}, \frac{8}{45} -$$ for (Dirichlet) scalars, Dirac and gauge fields

d depends on boundary conditions (Fursaev, JHEP 1307, 119 (2013))

Boundary terms in EE from the integrated anomaly?

explicit derivation of F_d, F_e and d, e by methods of spectral geometry is a technically involved problem,

one can try to derive the boundary terms in $s_{\log}$ by using the integrated anomaly:

$$s_{\log} = \lim_{n\to 1} \frac{n\mathrm{A} - \mathrm{A}(n)}{n-1} \quad ;$$

$\mathrm{A}(n)$ is the integrated anomaly of a CFT on n-'replicated' manifold M_n;

one should use distributional properties of curvatures on conical singularities (c.s.)

this method has proved to be successful for manifolds with squashed c.s.

(Fursaev, Patrushev, Solodukhin (2013))

The difficulty in 4D is that the structure of conical singularities on the boundaries is complicated.

One can get some insights by studying CFT’s in three dimensions (a much simplier case).

see, Fursaev, Solodukhin, PRD 93 (2016) 084021

Why 3D case is interesting:

- there is no local conformal anomaly;

- no analogs of C-theorem and a-theorem;

- the F-theorem and a relation to EE are established, but only for closed 3D manifolds;

- there is an integrated anomaly as a pure boundary effect;

Integrated anomaly of 3D CFT's:

a general structure of 3D integrated anomaly in the presence of boundaries

$$\mathrm{A} = -a\chi_2 + qj \quad ,$$

χ_2 - 2D Euler characteristics of ∂M;

$$j = \frac{1}{4\pi}\int_{\partial M}\sqrt{H}\,d^2x\,\mathrm{Tr}(\hat{K}^2) \quad ,$$

the corresponding heat coefficient

$$A_3 = \frac{1}{384\cdot 4\pi}\int_{\partial M}\mathrm{Tr}\,[-96XR + 16\chi R - 8\chi R_{\mu\nu}N^\mu N^\nu +$$

$$(13\Pi_+ - 7\Pi_-)K^2 + (2\Pi_+ + 10\Pi_-)\mathrm{Tr}K^2 + 96SK + 192S^2 - 12\chi_{:a}\chi_{:a}],$$

for boundary conditions $(\nabla_N - S)\Pi_+\phi = 0, \quad \Pi_-\phi = 0, \; \chi = \Pi_+ - \Pi_-,$

Properties of boundary chargers in d=3

CFT	a	q	b.cond.
Scalar	1 / 96	1 / 64	Dirichlet
Scalar	-1 / 96	1 / 64	Robin
Spinor	0	1 / 32	Mixed

- the anomaly is purely boundary effect
- a depends on boundary conditions
- q depends on the theory, does not depend on b.c.
- q appears in RG equation for 2 - point correlation function

of the stress - energy tensor near the boundary (Kuo-Wei Huang (2016), 1604.02138[hep-th])

- q as a possible candidate for a C - function analogue?

log terms in 3D EE when the entangling line is orthogonal to the boundary

Explicit derivation by using anomaly of the partition function on a replicated manifold M_n with conical singularities

$$s_{\log} = \eta \lim_{n\to 1} \frac{nA_3 - A_3(n)}{n-1} \quad ,$$

$A_3(M_n)$ is the heat coefficient for the heat kernel of $L = -\nabla^2 + X$

for a manifold with conical singularities;

for a plane bounadary and straight entangling line A_3 is known explicitly

$$A_3(M_n) = \eta \frac{1}{48n}(1-n^2)\mathrm{Tr}\ \chi$$

$$s_{\log} = -8a$$

no dependence on q charge for orthogonal entanling line!

log terms in 3D EE from the integrated anomaly

an alternative definition:

$$s^a_{\log} \equiv \lim_{n\to 1} \frac{n\mathrm{A} - \mathrm{A}(\tilde{M}_n)}{n-1} \quad ,$$

where at $n \to 1$ conical singularities are regularized:

$$\mathrm{A}(\tilde{M}_n) = -a\chi_2[\tilde{M}_n] + qj[\tilde{M}_n]$$

the result is non-trivial, since each conical singularity on the boundary contributes

$a(n-1)$ to $\mathrm{A}(\tilde{M}_n)$, therefore

$$s^a_{\log} = -2a$$

$s^a_{\log}$ and $s_{\log}$ coincide in $d=4$ but they don't in $d=3$:

$$s^a_{\log} = s_{\log} + s_{nc} \quad , \quad s_{nc} = 6a$$

non-minimal couplings

contribution of non-minimal couplings of the operator $L = -\nabla^2 + R/8$

to the heat coefficient A_3 is

$$A_3{}^X = -\frac{1}{128\pi}\int_{\partial M} \mathrm{Tr}\,[\chi R], \quad R = 4\pi(1-n)$$

this yields

$$s_{nc} = \lim_{n\to 1}\frac{nA_3{}^X - A_3{}^X(n)}{n-1} = 6a$$

and explains the discrepancy!

Distributional geometry on the boundary:

to carry out calculations of the entropy (when the entangling line is tilted to the boundary) one needs to know distributional properties of intrinsic and extrinsic geometries when conical singularities from the bulk cross the boundary

Previous results:

Fursaev, Solodukhin (1994) - distributional properties of curvature polynomails ('symmetric' conical singularities)

Fursaev, Patrushev, Solodukhin (2013) - generalization to 'squashed' conical singularities

Internal geometry

Simple configuration: let M_β be locally flat (R^3), conical singularities lie on a line C in M_β, the line crossing the plane boundary ∂M_β under a tilt angle α, relation betwee angle deficits $2\pi(1-\beta)$ in M_β and $2\pi(1-\beta_b)$ in ∂M_β is

$$\tan(2\pi(1-\beta_b)) = \cos\alpha \ \tan(2\pi(1-\beta))$$

scalar curavtures are :

$$R = 4\pi(1-\beta)\delta(x) \quad \text{for} \quad M_\beta;$$

$$\hat{R} \simeq 4\pi\cos\alpha\,(1-\beta)\hat{\delta}(x) \quad \text{for} \quad \partial M_\beta \quad \text{at} \quad \beta \to 1;$$

Regularization of conical singularities

Metric $ds^2 = (dx^0)^2 + (dx^1)^2 + (dx^2)^2$, boundary eq.: $x^2 = \tan\alpha \; x^1$

'replicated' manifold M_n can be defined in cylindrical coordinates :

$$x^0 = \rho\cos n\varphi \quad , \quad x^1 = \rho\sin n\varphi \, , \quad 0 \le \varphi < 2\pi$$

equation for the boundary : $x^2 = a\rho\cos n\varphi$, $a = \tan\alpha$

Regularized metric is standard:

$$ds^2 = f_n(\rho) d\rho^2 + n^2\rho^2 d\varphi^2 + (dx^2)^2 \, , \quad f_n = \frac{\rho^2 + n^2 b^2}{\rho^2 + b^2}$$

One also needs to 'regularize' equation of the boundary

$x^2 = ac^{1-n}\rho^n \cos n\varphi$, $n > 1$

External geometry has distributional properties!

Integrals of quadratic invariants of extrinsic curvatures in the limit $n \to 1$:

$$\int_{\partial M_n} K^2 \simeq \int_{\partial M_n} \mathrm{Tr}\, K^2 \simeq 8\pi(1-n)\, f(\alpha) + O((n-1)^2)$$

$$f(\alpha) = -\frac{\sin^2\alpha}{32\cos\alpha}(1 + 2\cos^2\alpha + 5\cos^4\alpha)$$

$$K_{\mu\nu}K_{\lambda\rho} \simeq \pi(1-\beta)\, f(\alpha)\left(H_{\mu\nu}H_{\lambda\rho} + H_{\mu\lambda}H_{\nu\rho} + H_{\mu\rho}H_{\lambda\nu}\right)\hat{\delta}(x)$$

It is consistent with the Gauss-Codazzi equations

$H_{\mu\nu}$ is a metric on the boundary

Logarithmic part of EE for tilted boundary:

Each point, where entangling line meets the boundary, yields the contribution:

$$s_{\log}(\alpha) = -a\,\psi(\alpha) + q\,f(\alpha)$$

$$\psi(\alpha) = 1 + \frac{6}{\cos\alpha},$$

- the log term in the entropy is determined by the boundary charges in the integrated anomaly!
- the q - chrage enters the entropy

Summary and future work:

- boundary terms in integrated conformal anomalies of CFT’s are specified by sets of boundary invariants and charges;

- some boundary charges are connected with the bulk charges (for yet unknown reasons), some don’t; some boundary charges depend on the boundary conditions;

- boundary charges may appear in log part of EE (3D CFT);

- in 3D CFT’s boundary the q-charge is related to RG equations for correlators of the stress-energy tensor (however, its behavior under RG flow is to be understood);

- when conical singularities cross the boundaries new distributional properties of internal and external geometries are revealed (more studies are needed to extend to qubic invariants);

- calculation of boundary terms in EE in d=4, their relation to boundary charges are for the future work

Thank you for attention