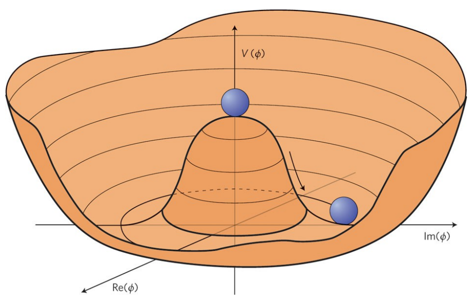
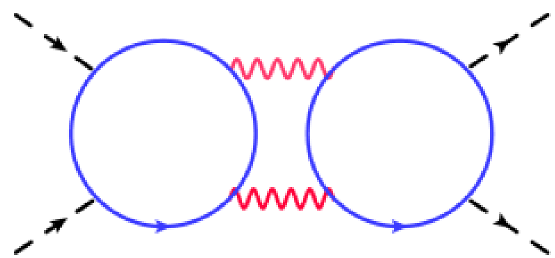
Vacuum stability problem: Multi-loop analysis



A.V. Bednyakov,
BLTP JINR



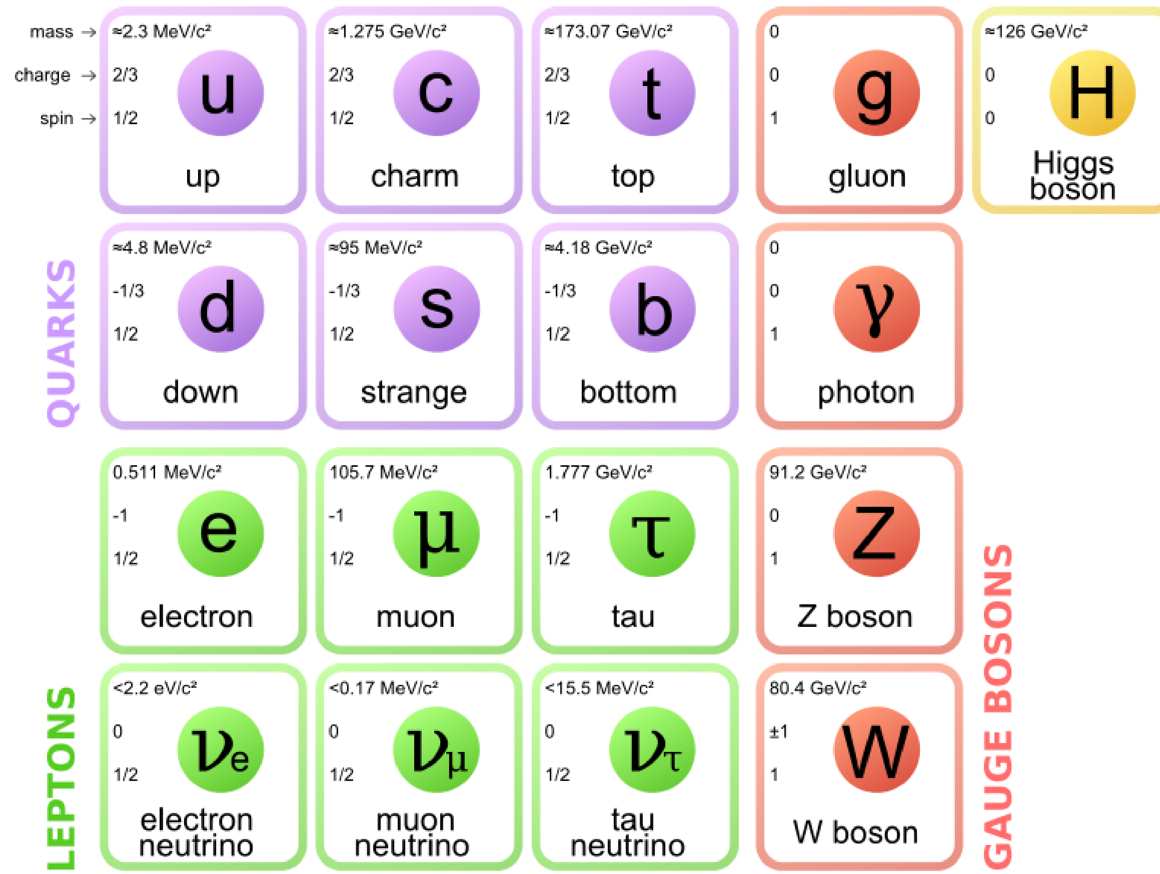
in collaboration with B.A. Kniehl, A.F. Pikelner, O.L. Veretin and V.N. Velizhanin

Outline

- The Standard Model and the Higgs boson
- Effective potential and Vacuum Stability
- Radiative corrections and RG Analysis
- Stability or *Metastability?*
- Critical parameters and scales
- Open issues and Outlook
    - *Gauge-dependence issue*

The Standard Model

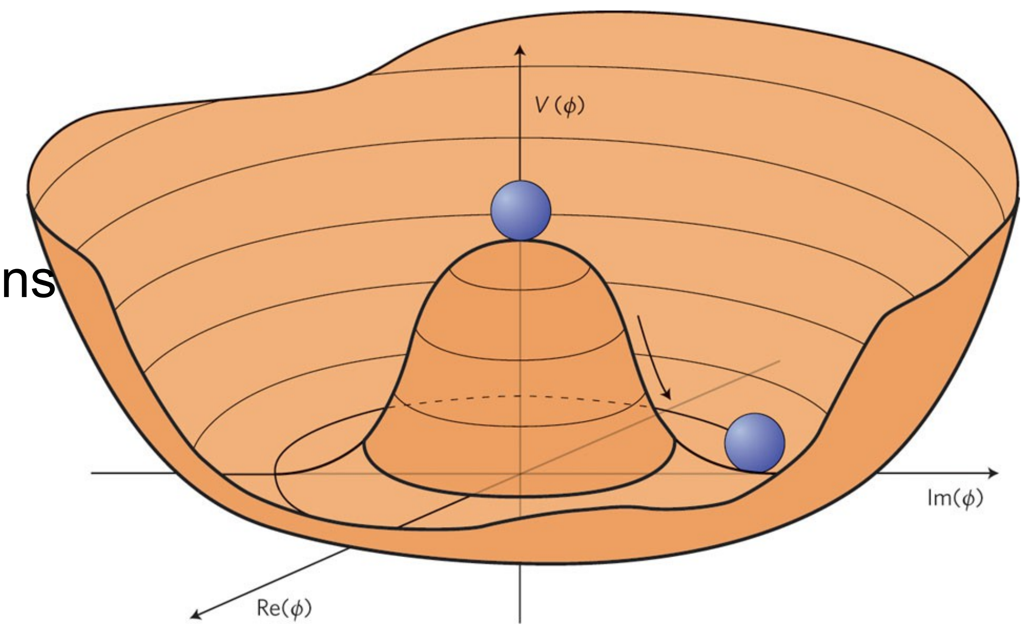
$$
\begin{aligned}
\mathcal{L}_{\text{SM}} = \;& \\
& \mathcal{L}_{\text{Gauge}}(g_1, g_2, g_S) \\
& + \mathcal{L}_{\text{Yukawa}}(Y_u, Y_d, Y_l) \\
& + \mathcal{L}_{\text{Higgs}}(\lambda, m^2) \\
& + \mathcal{L}_{\text{Gauge-fixing}}(\xi) \\
& + \mathcal{L}_{\text{Ghost}}
\end{aligned}
$$



Spontaneous symmetry breaking in the SM

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

$$V(\Phi) = m^2 |\Phi|^2 + \lambda |\Phi|^4$$



Would-be goldstone
"eaten" by W-bosons

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(\phi + i\chi) \end{pmatrix}$$

Neutral higgs field

"eaten" by Z-boson
Would-be goldstone

Spontaneous symmetry breaking in the SM

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

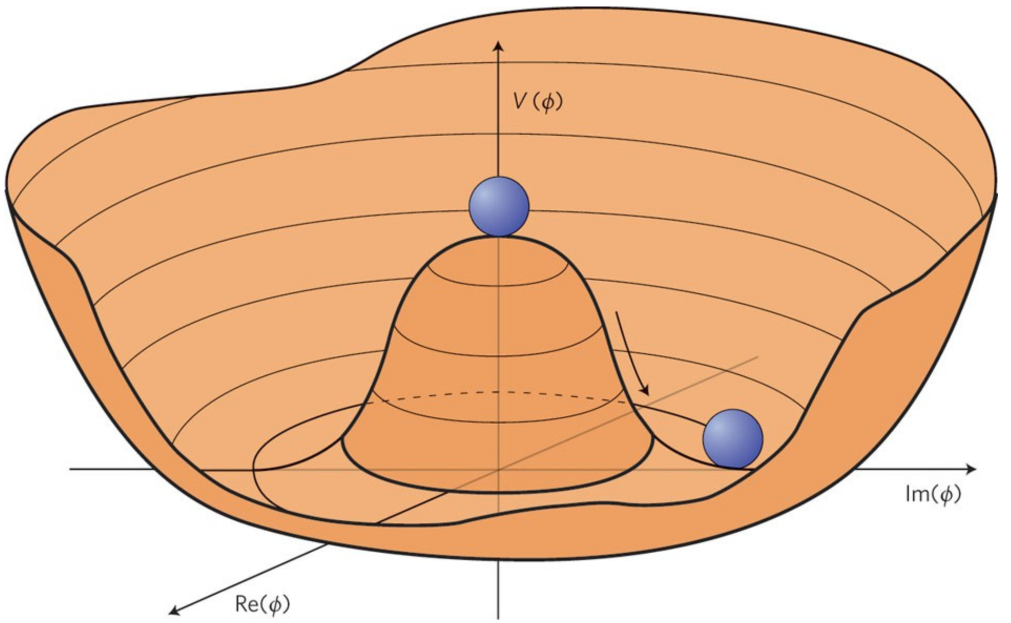
The electroweak vacuum state is characterized by vacuum expectation value

$$\langle\phi\rangle = v \neq 0$$

with $v \simeq 246\ \text{GeV}$

at tree level $$v = \sqrt{\frac{-m^2}{\lambda}}$$

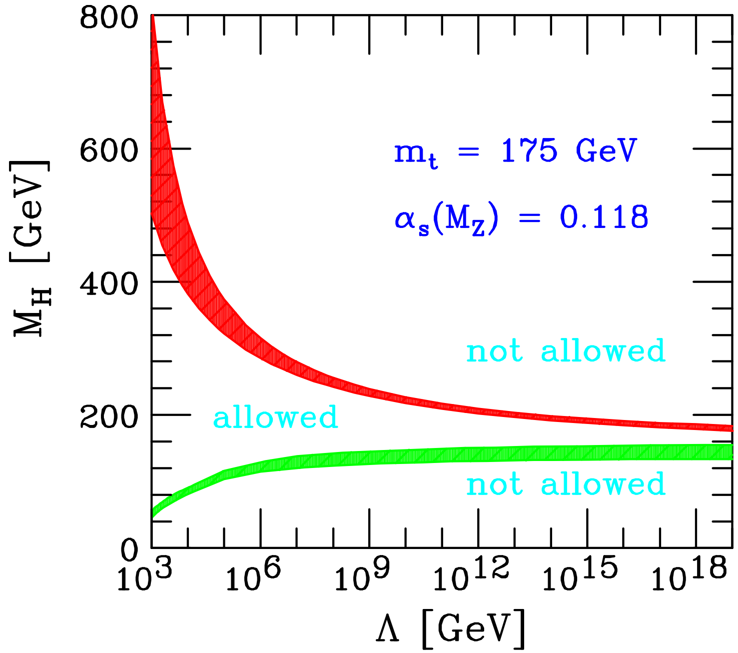
$$M_h^2 = 2\lambda v^2$$

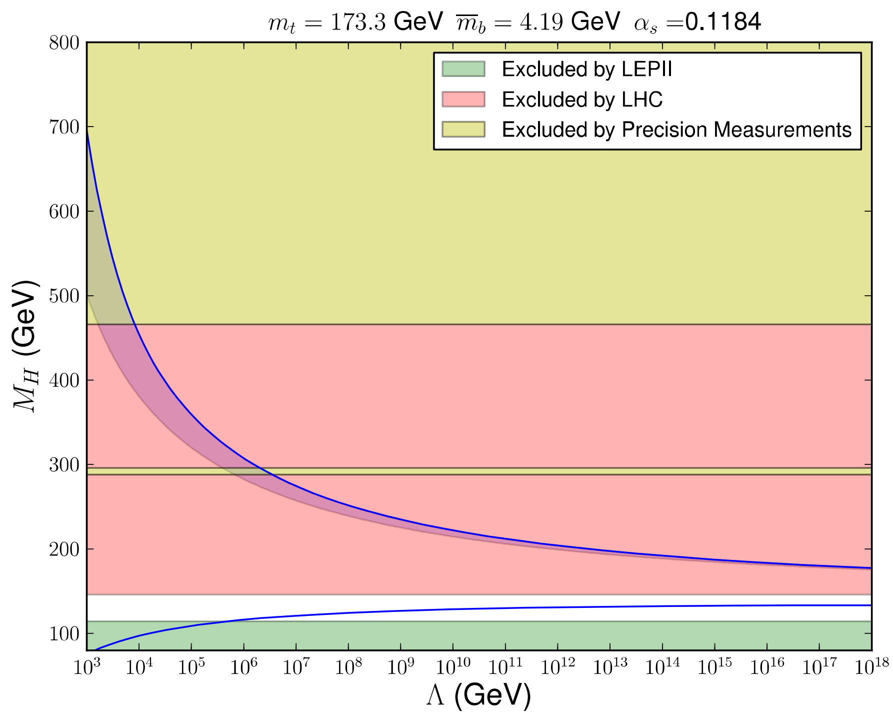


$$\left.\frac{\partial V(\phi)}{\partial \phi}\right|_{\phi=v} = 0$$

Motivation for precision study..

A quest for the Higgs mass *Mh* and/or New Physics scale *Λ* from potential inconsistency of the SM – triviality or vacuum instability





[Hambye, Riesselmann, '97]

[Wingerter,2011]

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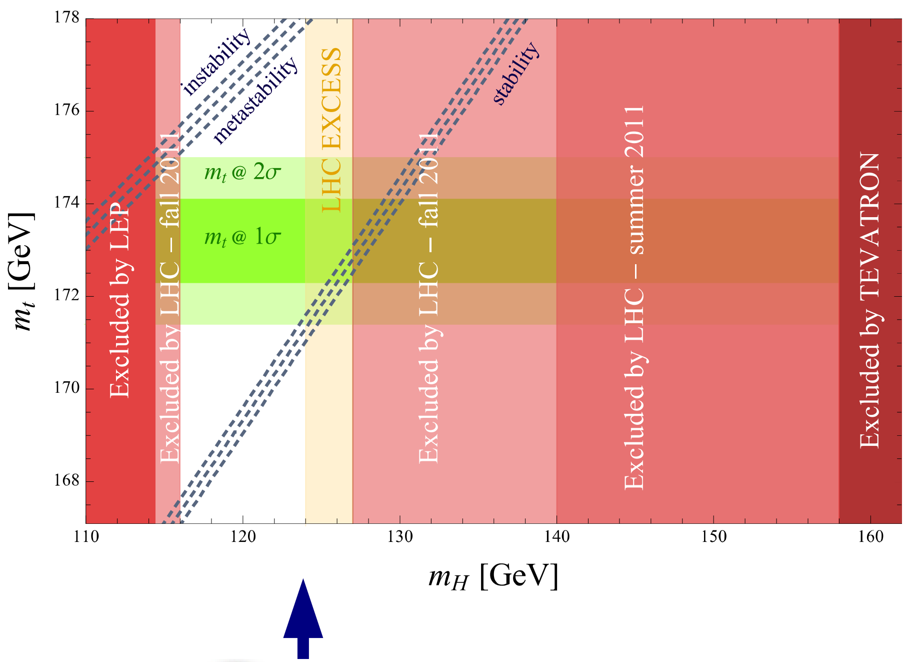
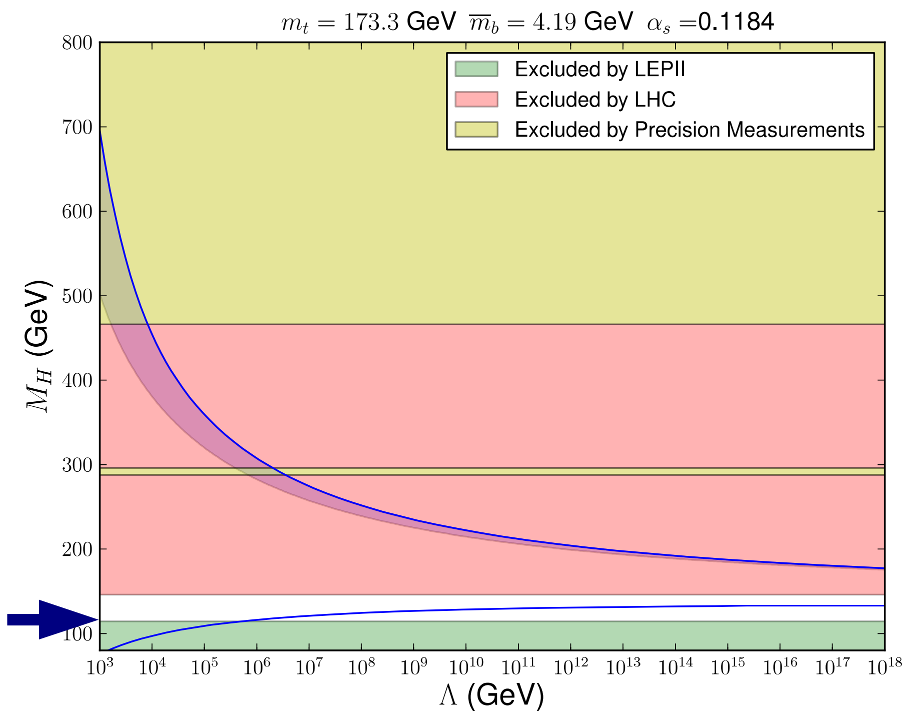
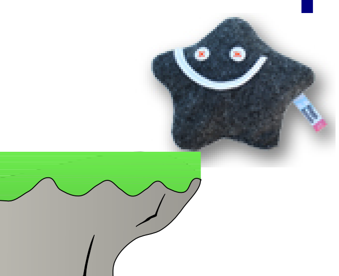






$$M_h = 125.09(24)\ \text{GeV}$$

[ATLAS & CMS '15]

[Wingerter,2011]

Spontaneous symmetry breaking and the Higgs effective potential

A proper way to study the symmetry breaking in the SM is to consider the effective potential for the background Higgs field which takes into account vacuum fluctuations

$$V(\phi) \to V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi)$$

We should consider the solutions of

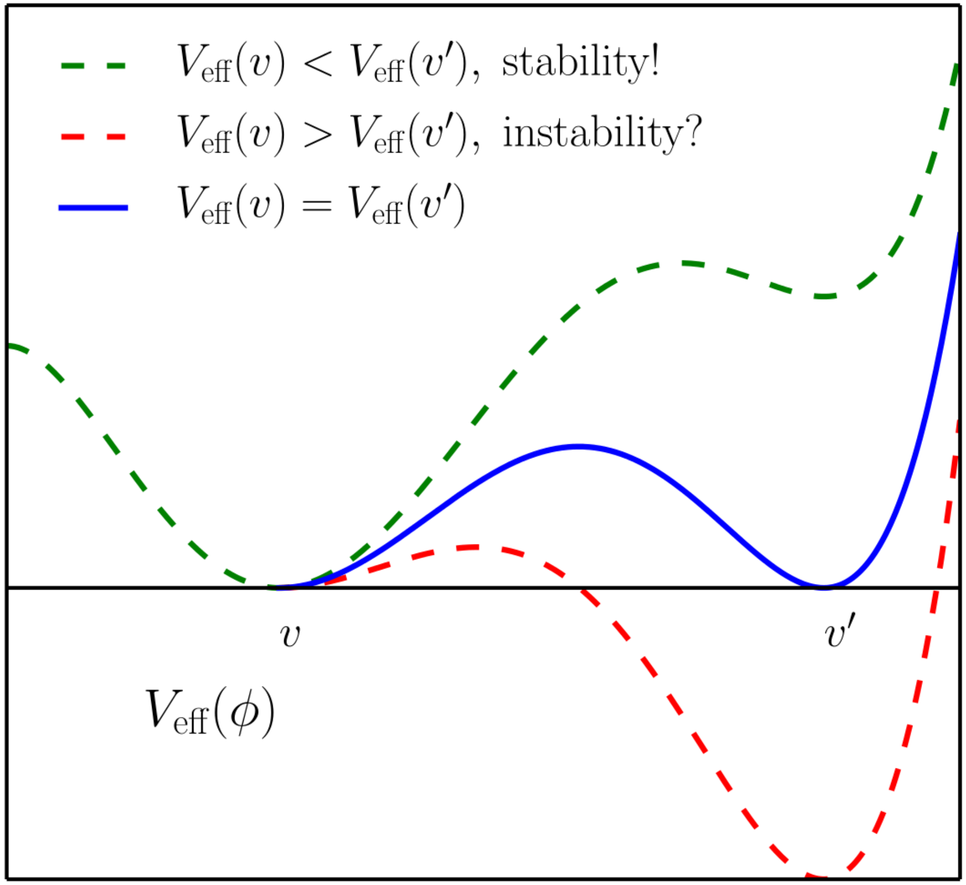
$$\frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} = 0$$

Given the parameters of the SM we should be able to calculate the effective potential order by order!

Questions:

1. Is the SM effective potential **bounded from below**?
2. Does the electroweak vacuum correspond to **the global minimum** of the effective potential or we are living in a false vacuum?

The Higgs field effective potential (schematic view)



$$V_{\rm eff}(v) = V_{\rm eff}(v')$$

Critical situation!

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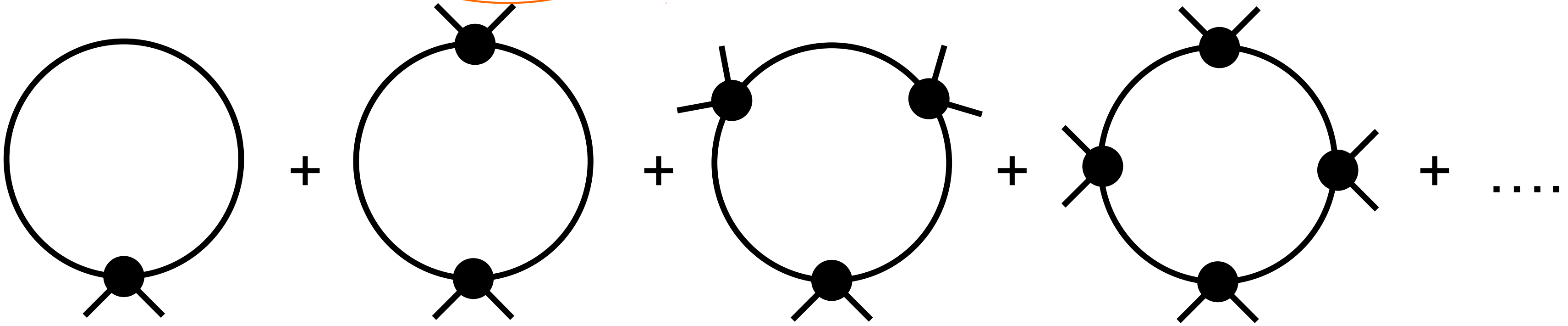
[Coleman, E.Weinberg, '73]
[Jackiw, '74]

See also, [M.Sher' 89]

Loop expansion:

$$\Delta V(\phi) = \Delta^{(1)}V(\phi) + \Delta^{(2)}V(\phi) + \Delta^{(3)}V(\phi) + \dots$$



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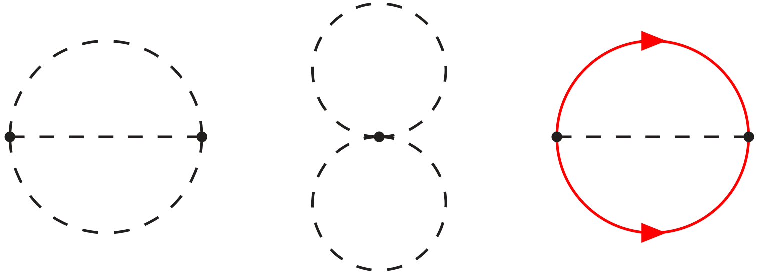
[Ford, Jack, Jones, '92,'97]
[S. Martin, 2002]



Example two-loop diagrams

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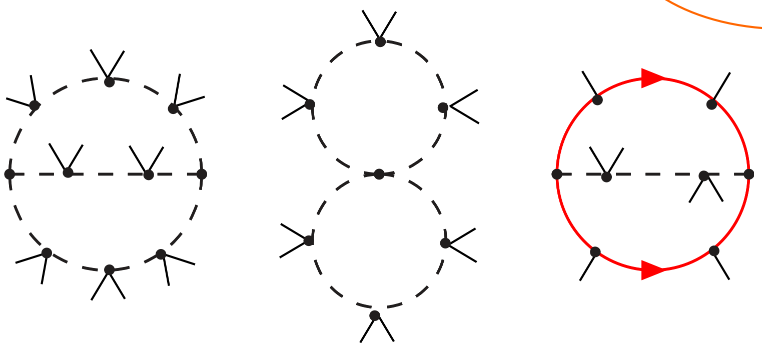

Example two-loop diagrams with field-dependent masses $M_t(\phi) = y_t \phi/\sqrt{2}$

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[S. Martin,'13] only g_s and y_t 3-loop contributions

4-loop g_s contribution [S. Martin,'15]

NB: Zero temperature! For finite T one needs to include $\Delta V(\phi, T)$

[Krasnikov'79]

Why instability?

Counts fermions with (–1)
and bosons with (+1)

$$\Delta^{(1)}V(\phi) = \int \frac{d^4\,k}{2\,(2\pi)^4} \mathrm{STr}\ln\left(k^2 + M^2(\phi)\right)$$

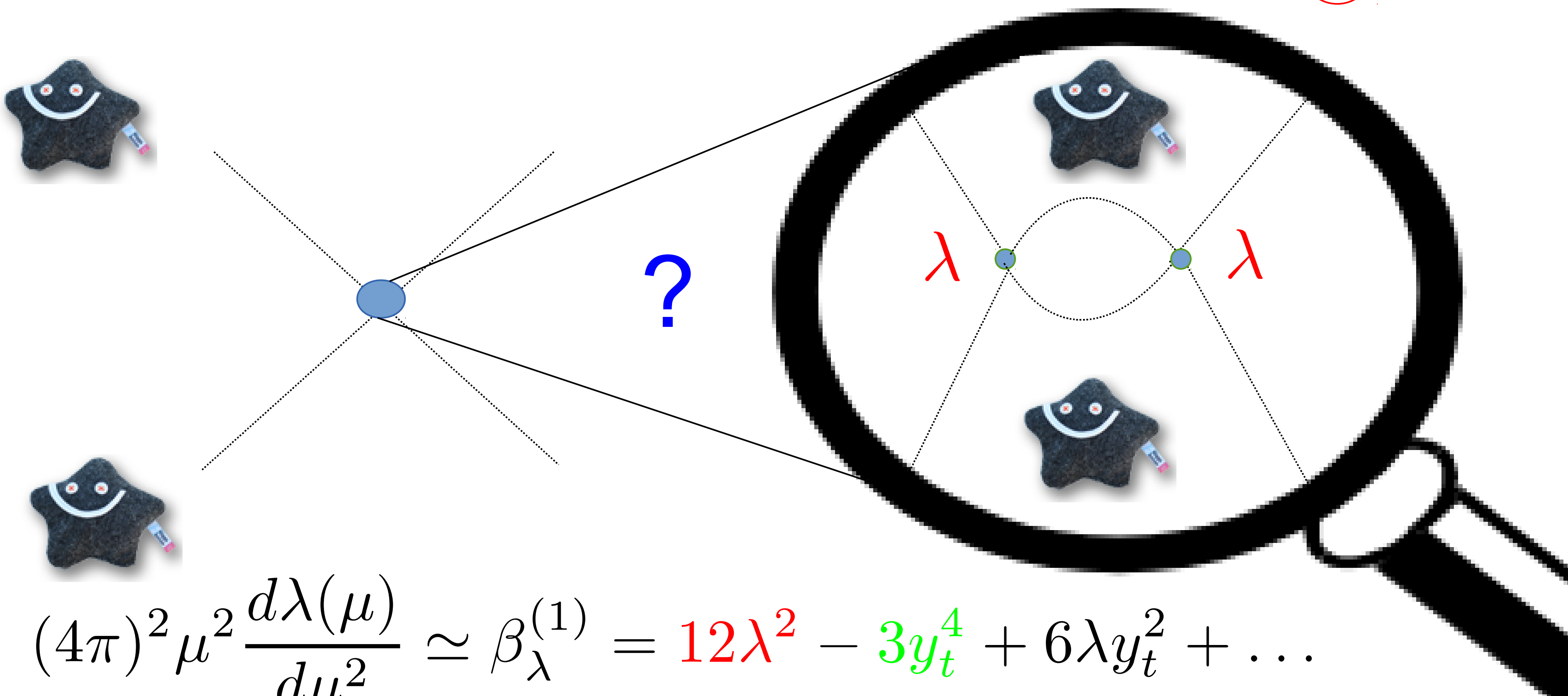
Particle	κ	κ'	n
$W^{\pm}$	$g_2^2/4$	0	2×3
Z	$(g_2^2+g_1^2)/4$	0	3
t	$y_t{}^2/2$	0	4×3
h	3λ	m^2	1
G	λ	m^2	3×1

NB: Landau gauge!

$$M^2(\phi) = \kappa\phi^2 + \kappa'$$

Higgs self-interaction: scale (RG) dependence

$$V(\Phi) = m^2|\Phi|^2 + \lambda|\Phi|^4$$



$$(4\pi)^2\mu^2\frac{d\lambda(\mu)}{d\mu^2} \simeq \beta_\lambda^{(1)} = 12\lambda^2 - 3y_t^4 + 6\lambda y_t^2 + \ldots$$

"Charge screening" at large distances

Higgs self-interaction: scale (RG) dependence

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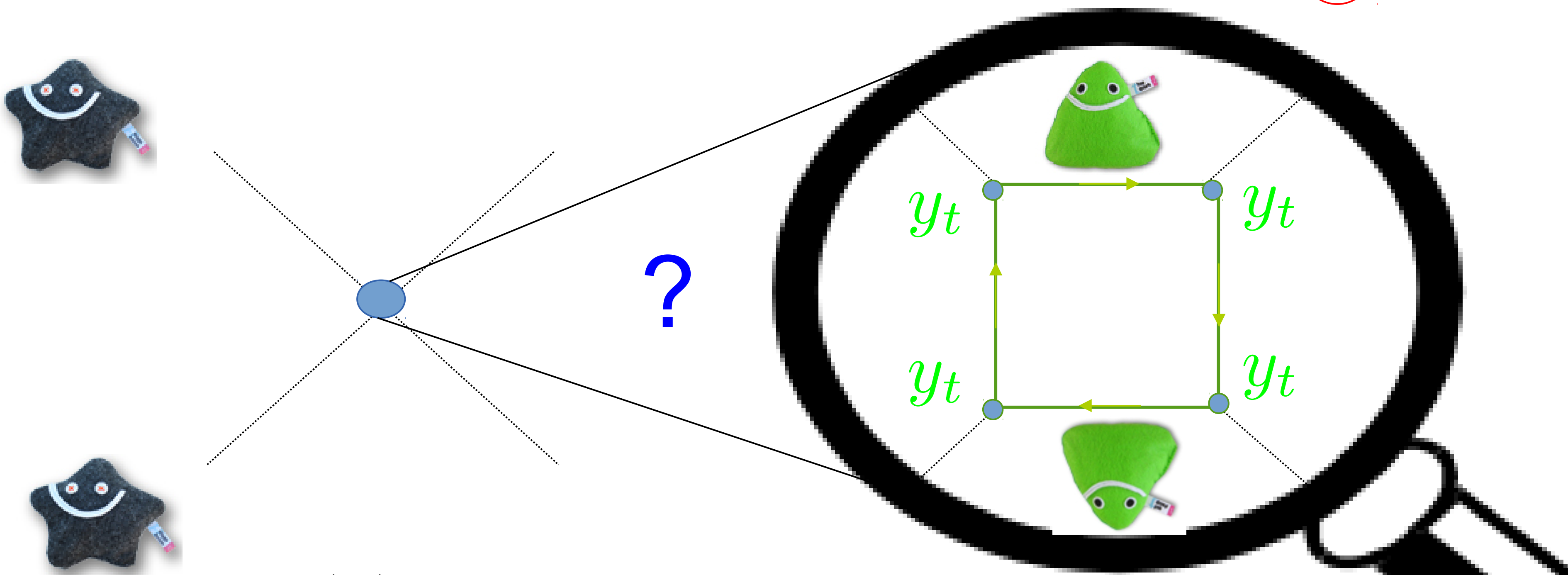

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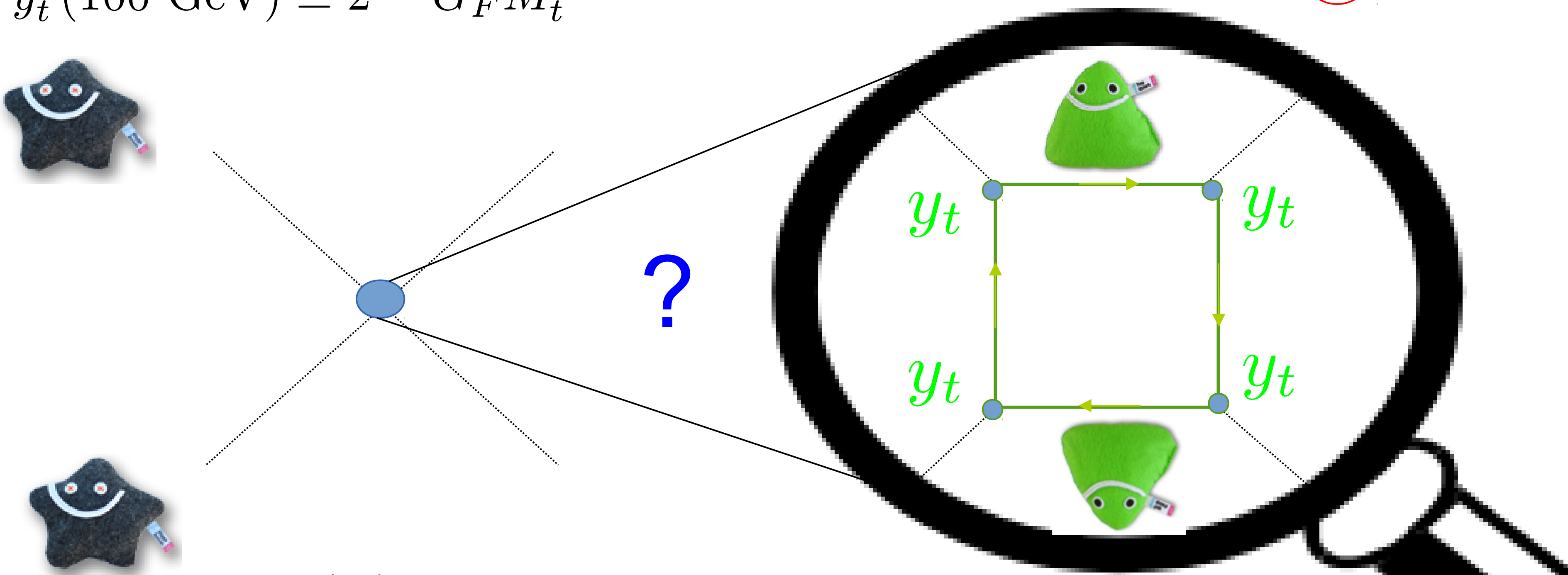
"Charge antiscreening" at large distance

Higgs self-interaction: scale (RG) dependence

$$\lambda(100\ \mathrm{GeV}) \simeq G_F M_h^2/\sqrt{2}$$
$$y_t^2(100\ \mathrm{GeV}) \simeq 2^{3/2} G_F M_t^2$$

$$V(\Phi) = m^2|\Phi|^2 + \lambda|\Phi|^4$$



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"Charge antiscreening" at large distance

The evolution of self-coupling

$$(4\pi)^2\mu^2\frac{d\lambda(\mu)}{d\mu^2} \simeq \beta_\lambda^{(1)} = 12\lambda^2 - 3y_t^4 + 6\lambda y_t^2 + \ldots$$

screening

antiscreening

$$(4\pi)^2\mu^2\frac{dy_t(\mu)}{d\mu^2} \simeq \beta_{y_t}^{(1)} = \frac{9}{4}y_t^3 - 4g_s^2 y_t + \ldots$$

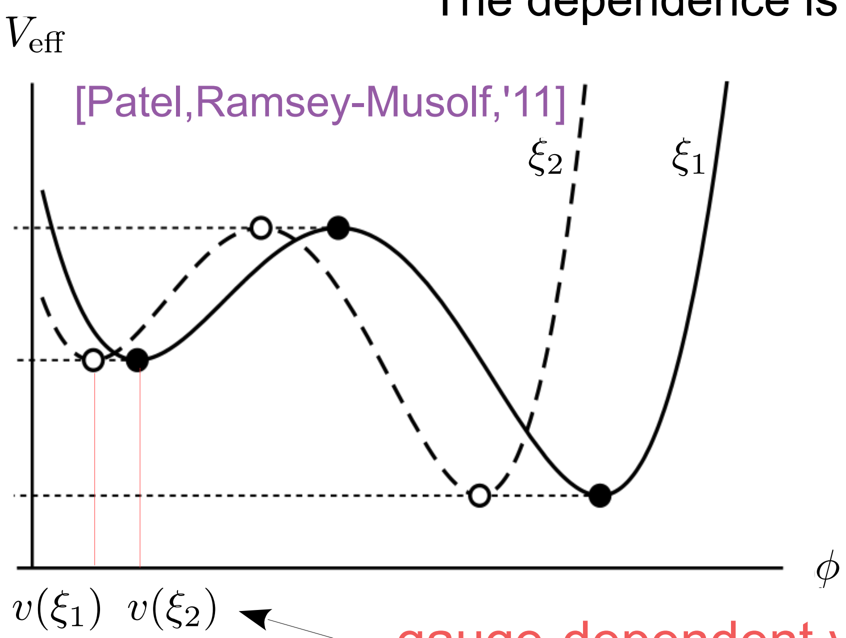
Importance of strong coupling!

The Higgs field effective potential. Gauge-dependence issue (I)

In order to quantize the SM we introduce gauge-fixing terms in the SM Lagrangian parametrized by auxiliary ξ_i for each gauge field of the model.

[Jackiw,'74]

At general field values the effective potential is gauge-dependent. The dependence is governed by Nielsen Identities:



gauge-dependent v.e.v

[Nielsen,'75,'14]

$$\xi \frac{\partial V_{eff}}{\partial \xi} = -C(\phi,\xi)\frac{\partial V_{eff}}{\partial \phi}$$

Which tell us that only at extrema the effective potential is gauge-independent

[Di Luzio,Mihaila'14]

Fermi gauge

$$\Delta^{(1)}V(\phi) = \int \frac{d^4\, k}{2\,(2\pi)^4} \mathrm{STr} \ln \left(k^2 + M^2(\phi)\right)$$

$$M^2(G) \equiv M_G^2 = \lambda\phi^2 + m^2$$

2 more “degrees of freedom”

G_0

$$M_{A\pm}^2 = \frac{1}{2}\left[M_G^2 \pm \sqrt{M_G^4 - 4\xi_W M_W^2 M_G^2}\right]$$

$G_\pm$

$$M_{B\pm}^2 = \frac{1}{2}\left[M_G^2 \pm \sqrt{M_G^4 - 4[(\xi_W - \xi_B)M_W^2 + \xi_B M_Z^2]M_G^2}\right]$$

$$\mathcal{L}_{gf} = -\frac{1}{2\xi_B}(\partial^\mu B_\mu)^2 - \frac{1}{2\xi_W}(\partial^\mu W_\mu^i)^2$$

The Higgs field effective potential

It is known that n-loop corrections to the tree-level potential involve logarithms of the form

$$\alpha^{n+1}(\mu)\left[\ln\frac{\phi^2}{\mu^2}\right]^n$$

with $\alpha(\mu)$ being some SM coupling constant defined at the normalization scale μ. The latter inevitably appears in perturbative calculations beyond the leading order.

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$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta_i\frac{\partial}{\partial a_i}-\gamma\phi\frac{\partial}{\partial\phi}\right)V_{\text{eff}}=0$$

This issue can be addressed by means of renormalization group (RG) improvement which basically corresponds to the choice $\mu^2 \sim \phi^2$

The Higgs field effective potential

At large values of the Higgs field the full effective potential can be approximated by the following expression:

$$V_{\text{eff}}(\phi \gg v) \simeq \lambda_{\text{eff}}(\phi)\frac{\phi^4}{4} \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

[Ford,Jack,Jones'92,'97]

with "running" self-coupling $\lambda(\mu)$ evaluated at the scale $\mu = \phi$ .This effectively re-sums dangerous contributions.

As a consequence, the stability of the electroweak vacuum is related to the behavior of the running Higgs self-coupling constant at large values of the renormalization scale.

If at some point $\lambda(\phi) < 0$, there can be a minimum, which is much deeper than our vacuum, so stability of the latter should be questioned..

The SM parameters in the broken and unbroken phases

In order to calculate the Higgs field effective potential **reliably** one needs to know the values of the **running** SM parameters at different scales!
(in $\overline{\text{MS}}$ scheme)

Gauge and Yukawa couplings are connected to (observed) particle masses:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{g_Z v}{2}, \qquad g_Z = \sqrt{{g_1}^2 + g_2^2}$$

$$M_f = \frac{y_f v}{\sqrt{2}}, \qquad M_h^2 = 2\lambda v^2$$

Given v.e.v, couplings can be extracted from these relations

Observed (Pole) mass – real part of the complex pole of the propagator.

The SM parameters in the broken and unbroken phases

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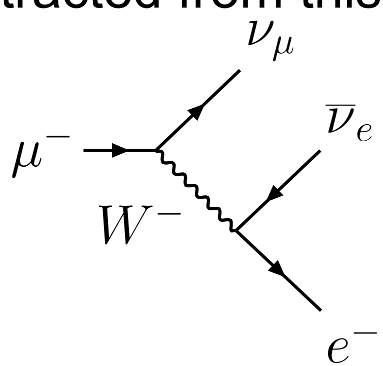
And v.e.v. can be related to the Fermi constant by considering Fermi Theory as a low-energy effective approximation of the SM valid for energies $E \ll M_W$

$$G_f = \frac{1}{\sqrt{2} v^2}$$

NB: VALID AT THE LEADING ORDER!



$$\frac{-g_2^2}{q^2 - M_W^2} \xrightarrow{q \to 0} \frac{g_2^2}{M_W^2} \propto \frac{1}{v^2}$$

Matching relations: high orders

Beyond the tree-level the (matching) relations become non-trivial

"Running" parameters should be expressed in terms of the "physical" ones.
So we need to invert the relations

$$2^{1/2} M_f = y_f v (1 + \bar{\delta}_f), \quad 4 M_W^2 = g_2^2 v^2 (1 + \bar{\delta}_W), \quad 4 M_Z^2 = \left(g_1^2 + g_2^2\right) v (1 + \bar{\delta}_Z),$$
$$M_h^2 = 2\lambda v^2 (1 + \bar{\delta}_h), \quad 2^{1/2} G_f = v^{-2} (1 + \bar{\delta} r), \quad (4\pi)^2 \alpha_s^{(5)}(\mu) = g_s^2 (1 + \bar{\delta}\alpha_s)$$

RHSs depend on "running" parameters and the renormalization scale μ

In principle, from these relations one can (try to) find the values of "running" parameters at any scale.

But, again, large logs can render the corresponding theoretical uncertainty enormous...

Optimal strategy:

Match at the EW scale, run to the scale of interest via RGE!

Matching relations: high orders

Beyond the tree-level the (matching) relations become non-trivial

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1) Real part of the complex pole of the corresponding propagator (SM)

full 2-loop EW corrections are collected in [Kniehl,Veretin,Pikelner'15-16]
recent 4-loop pure QCD result for quark masses [Marquard et al'15-16]

2) Matching to effective non-renormalizable four-fermion theory

2-loop bosonic part [Awramik,Czakon,Veretin,Onischenko,03]
full 2-loop EW corrections [Kniehl,Veretin,Pikelner'15-16]

3) Matching to effective renormalizable QCD(xQED)

leading top-Yukawa 2-loop corrections [Chetyrkin,Kniehl,Steinhauser'97]
full 2-loop EW result [Bednyakov'14]
pure 4-loop QCD corrections are known since 2006

See also, [Actis, Passarino,2007]

Matching relations: Gauge-dependence issue (II)

Physical masses are gauge-independent!

$$2^{1/2} M_f = y_f v (1+\bar{\delta}_f), \quad 4 M_W^2 = g_2^2 v^2 (1+\bar{\delta}_W), \quad 4 M_Z^2 = \left(g_1^2 + g_2^2\right) v^2 (1+\bar{\delta}_Z),$$

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What is v.e.v. here?

1) Minimizes tree-level potential: $v \stackrel{?}{\equiv} v(\mu) \equiv \sqrt{\dfrac{-m^2(\mu)}{\lambda(\mu)}}$

Gauge-**in**dependent together with δ's !

[Fleischer,Jegerlehner,'81,Jegrlehner,Kalmykov, Kniehl'12], [Kniehl,Veretin,Pikelner'15]

+ References therein!

2) Minimizes effective potential: $v \stackrel{?}{\equiv} (\mu) : \left.\dfrac{\partial V_{\text{eff}}(\phi,\mu)}{\partial\phi}\right|_{\phi=\tilde{v}} = 0$

Gauge-dependent together with δ's !

[Degrassi,...'12],[Buttazzo,...'13],[Martin,'14-16]
+ References therein!

Landau gauge!
(no explicit control)

Matching relations: our results

$$x = x_0 + \Delta x_{\alpha_s} \frac{\alpha_s^{(5)}(M_Z) - \alpha_s^{(5),\text{exp}}(M_Z)}{\Delta\alpha_s^{(5),\text{exp}}(M_Z)} + \Delta x_{M_H} \frac{M_H - M_H^{\text{exp}}}{\Delta M_H^{\text{exp}}} + \Delta x_{M_t} \frac{M_t - M_t^{\text{exp}}}{\Delta M_t^{\text{exp}}} \pm \delta x_\mu$$

At the Top mass scale

x	x_0	Δx_{α_s}	Δx_{M_H}	Δx_{M_t}	δx_μ
g_1	0.35838	-3.8×10^{-6}	-2.5×10^{-6}	$+7.1 \times 10^{-5}$	8.5×10^{-5}
g_2	0.64812	$+8.5 \times 10^{-7}$	-6.6×10^{-7}	-9.8×10^{-6}	5.8×10^{-5}
g_s	1.16540	$+2.7 \times 10^{-3}$	$+7.8 \times 10^{-8}$	-4.0×10^{-5}	5.6×10^{-5}
y_t	0.93517	-3.6×10^{-4}	-8.6×10^{-6}	$+5.1 \times 10^{-3}$	8.0×10^{-4}
y_b	0.01706	-5.7×10^{-5}	$+1.3 \times 10^{-7}$	-2.4×10^{-7}	2.5×10^{-4}
λ	0.12714	-6.2×10^{-6}	$+8.2 \times 10^{-4}$	$+6.4 \times 10^{-5}$	5.8×10^{-4}
m	131.86	-2.6×10^{-3}	$+3.8 \times 10^{-1}$	$+1.2 \times 10^{-1}$	7.3×10^{-1}

Obtained from PDG'14 input by means of MR c++ [Kniehl, Veretin, Pikelner,'16]

$$G_F = 1.1663787(6) \times 10^{-5}\ \text{GeV}^{-2},\ \alpha_s^{(5)}(M_Z) = 0.1185(6),$$

$$M_W = 80.385(15)\ \text{GeV},\ M_Z = 91.1876(21)\ \text{GeV},\ M_H = 125.7(4)\ \text{GeV},$$

$$M_t^{\text{MC}} = 173.21(87)\ \text{GeV},\ M_b = 4.78(6)\ \text{GeV}$$

Similar results in [Buttazio,...'12]

but it looks like they underestimate top-yukawa uncertainty by a factor or 2!

Renormalization group equations (RGE) in the SM

[Bogoliubov & Shirkov, 55]

The running of the SM coupling constants is given by the system of coupled Renormalization Group Equations, which basically describe how different SM charges are screened (or anti-screened) with scale variation.

The (anti)screening is due to emission and absorption of virtual particles

$$\mu^2 \frac{d a_i}{d\mu^2} = \beta_{a_i}(a_j)$$

Initial conditions are due to matching!

$$(4\pi)^2 a_i = \{g_1^2, g_2^2, g_s^2, y_b^2, y_t^2, y_\tau^2, \lambda\}$$

The beta-functions are calculated in perturbation theory

$$\beta_{a_i} = \beta_{a_i}^{(1)} + \beta_{a_i}^{(2)} + \beta_{a_i}^{(3)} + \dots$$

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The running of the SM coupling constants is given by the system of coupled Renormalization Group Equations, which basically describe how different SM charges are screened (or anti-screened) with scale variation.

The (anti)screening is due to emission and absorption of virtual particles

$$\mu^2 \frac{d a_i}{d\mu^2} = \beta_{a_i}(a_j)$$

Initial conditions are due to matching!

$$(4\pi)^2 a_i = \{g_1^2, g_2^2, g_s^2, y_b^2, y_t^2, y_\tau^2, \lambda\}$$

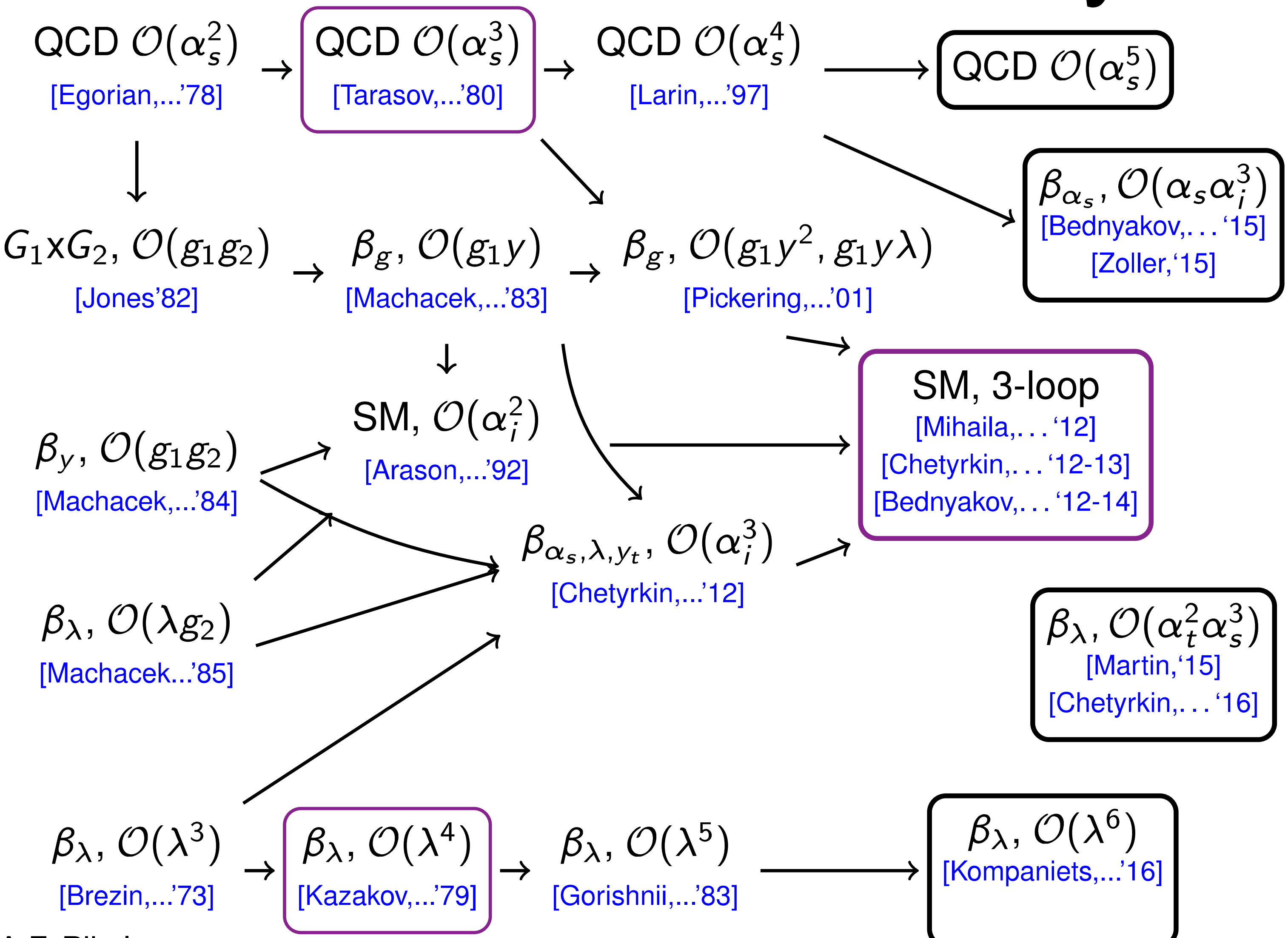
The beta-functions are calculated in perturbation theory

$$\beta_{a_i} = \beta_{a_i}^{(1)} + \beta_{a_i}^{(2)} + \beta_{a_i}^{(3)} + \ldots$$

$\overline{MS}$ renormalization scheme

Our group contributed to the calculation of three-loop RGEs for all SM couplings

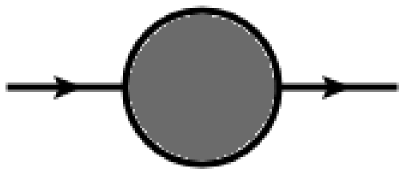
RG functions: some history



Thanks to A.F. Pikelner

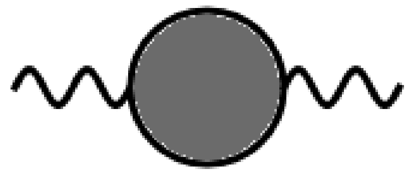
Some details of the calculation

[Bednyakov, Pikelner, **Velizhanin**,12-14]



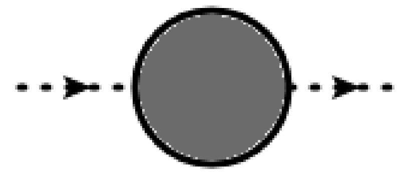
Fermions

$\mathcal{O}\left(10^4\right)$



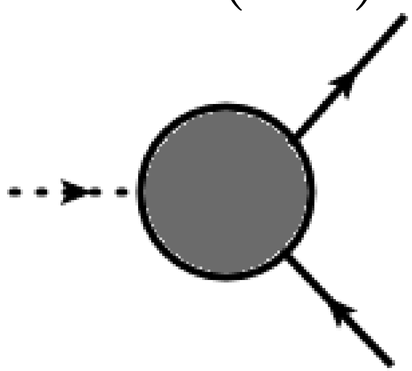
Gauge bosons

$\mathcal{O}\left(10^4\right)$

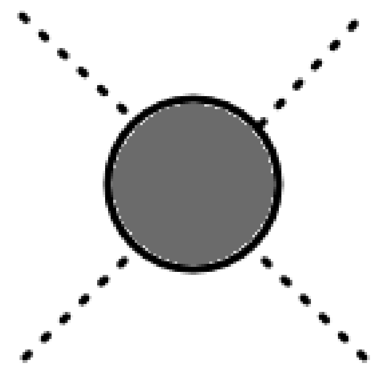


Higgs boson

$\mathcal{O}\left(10^4\right)$



Approximate number
of three-loop diagrams



Yukawa vertices

$\mathcal{O}\left(10^5\right)$

Impossible to evaluate by hand

Higgs vertices

$\mathcal{O}\left(10^6\right)$

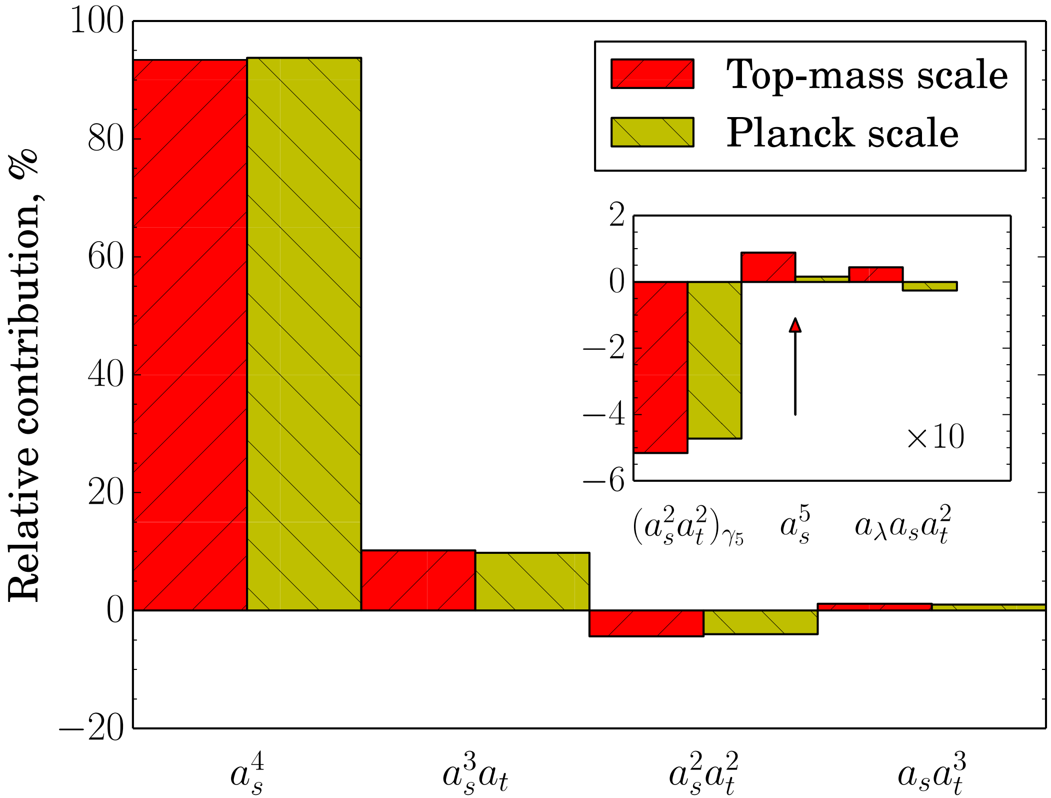
[Bednyakov,Pikelner'15-16]

Our recent result on RGE

Leading four-loop EW contribution to the beta-function of the SM strong coupling from **propagators**.

$$\frac{d\, a_s}{d \ln \mu^2} = \beta_{\alpha_s} a_s$$

$$(16\pi^2) a_i = \{g_s^2, y_t^2, \lambda\}$$

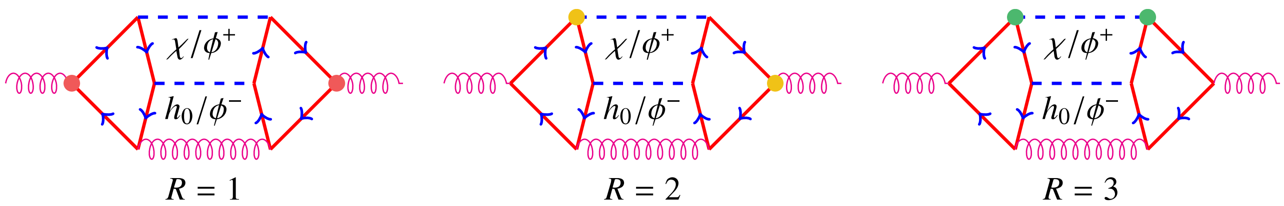


5-loop QCD

[Baikov, Chetyrkin,Kuehn'16]

γ_5 problem

$$\frac{a_s^2 a_t^2 T_F^2}{\epsilon} \cdot R \cdot X$$



Evolution of the SM couplings

The initial conditions at the electroweak scale are obtained by means of relations presented in

[Kniehl,Pikelner,Veretin,2015]

Theoretical uncertainties (due to neglected high-order terms and/or various re-expansions) are studied in

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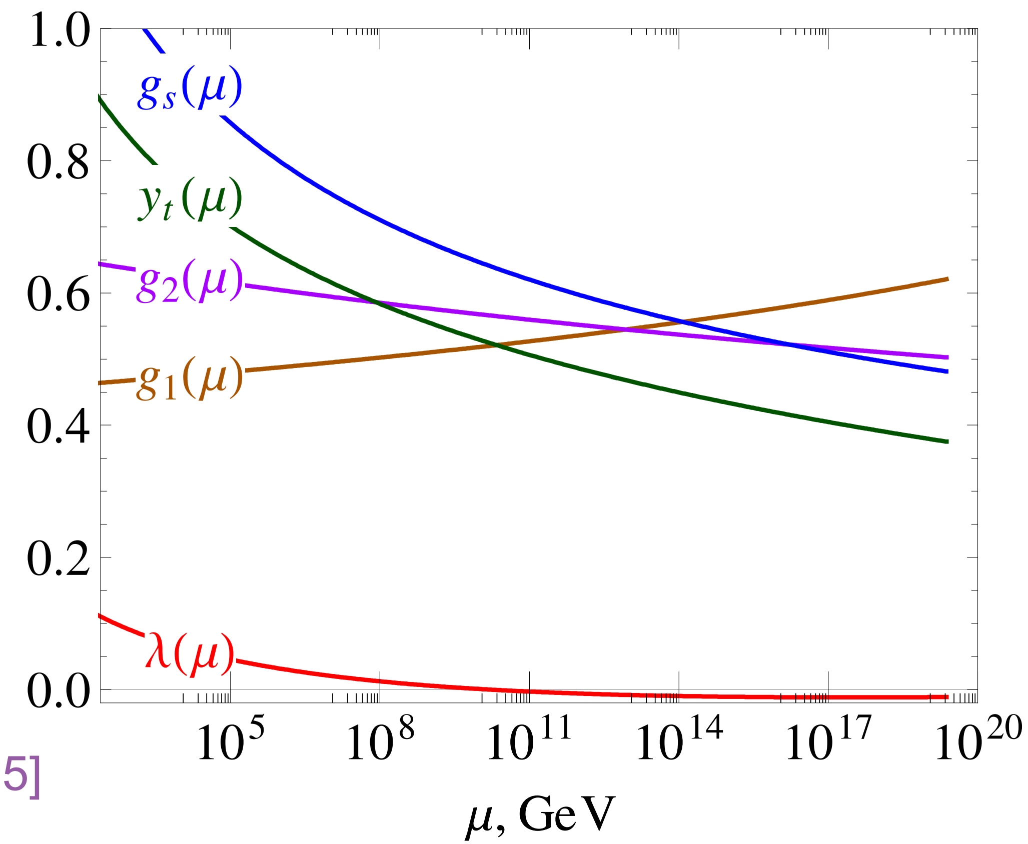

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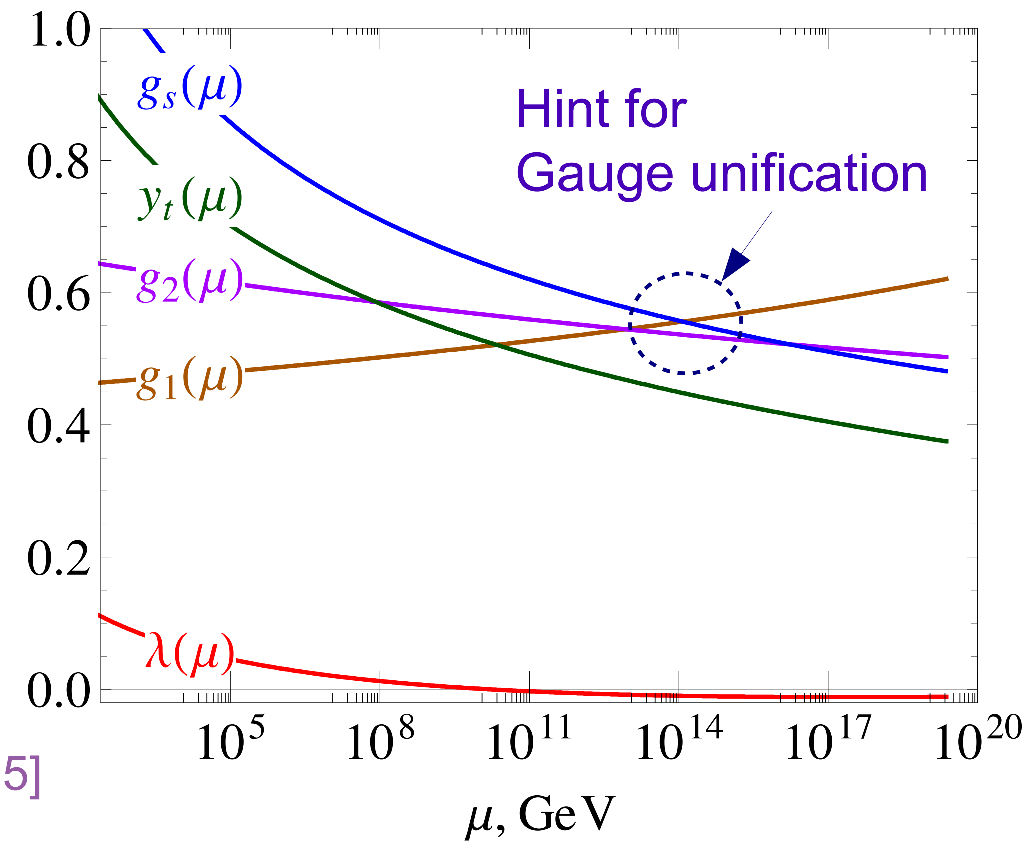

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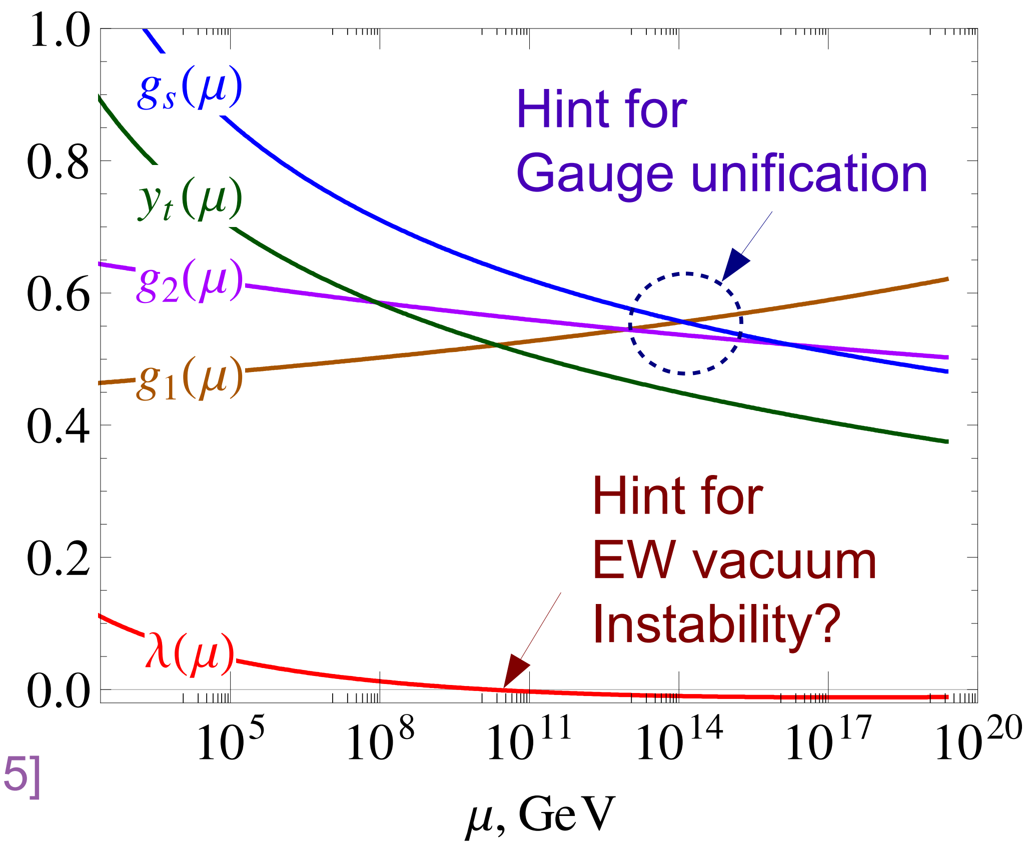


We are interested in the self-coupling evolution

Critical parameters and instability scales: different approaches

- Choose (=fix) "instability" scale Λ and define a critical parameter by implicit equation $\lambda(\mu = \Lambda) = 0$

"stability up to ..."

- Find both the "instability" scale $\mu^{\rm cri}$ and a critical parameter from two implicit equations

$$\lambda(\mu^{cri}) = \beta_\lambda(\mu^{\rm cri}) = 0$$

$\Rightarrow M_h^{\rm cri}$ (M_t fixed)

$\Rightarrow M_t^{\rm cri}$ (M_h fixed)

[Froggat,Nielsen'75],[Bezrukov,...,2012]

Both based on **simple** approximation:

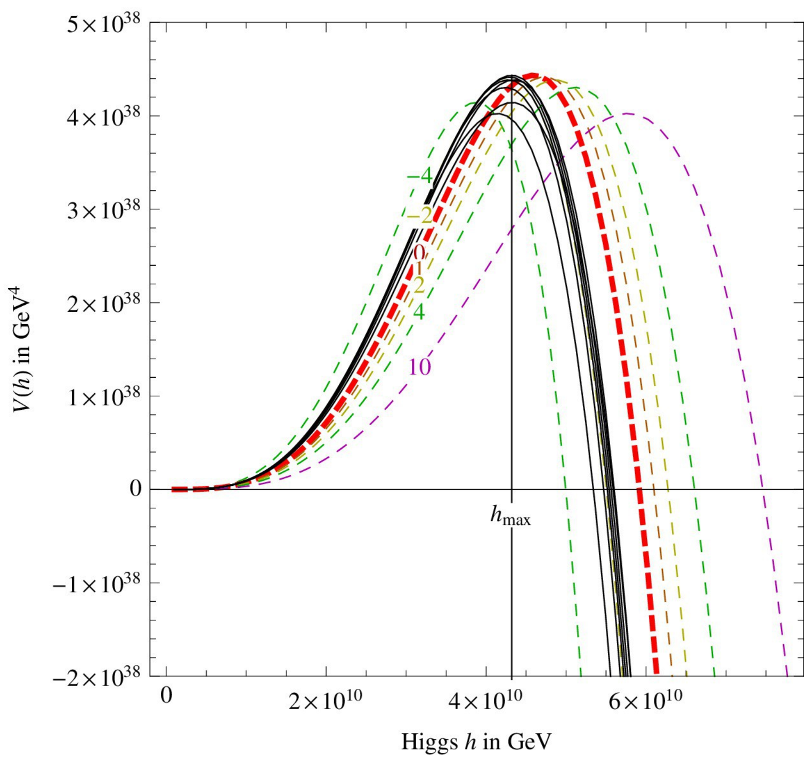
And RGEs!

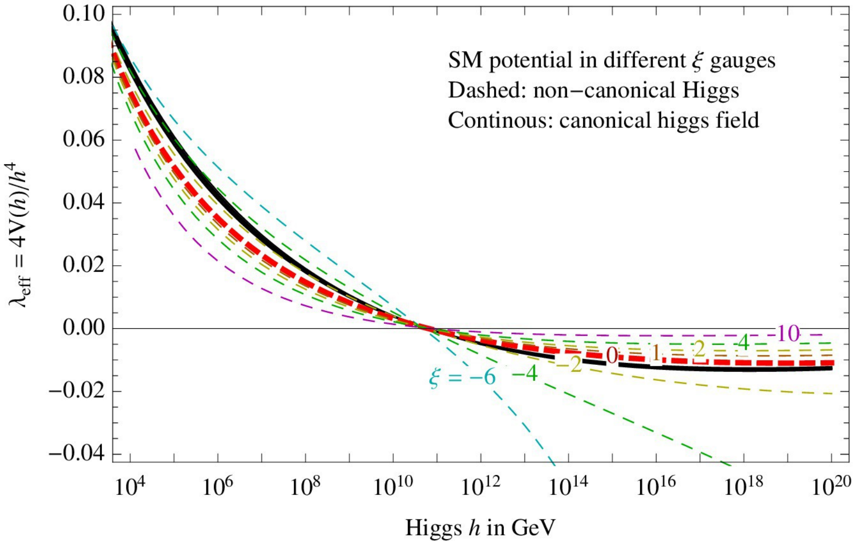
$$V_{\rm eff}(\phi \gg v) \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

Critical parameters and instability scales: different approaches

- More elaborated approaches are based on “full” effective potential, improved via RG

$$V_{\text{eff}}(\phi \gg v) \simeq \frac{\lambda_{\text{eff}}(\phi, \mu = \phi)}{4}\phi^4$$





[Espinosa,..'15]

NB:Gauge-dependent at general values of the field

Critical parameters and instability scales: different approaches

- A consistent approach due to [Andreassen,..'14]. Reorganize loop expansion, which render expansion coefficients at extrema explicitly gauge-independent

$$\lambda \sim \hbar g^4$$

$$V_{eff}^{NLO}(\phi = \mu_X, \mu_X) = \mu_X^4 \left(\hbar \cdot v_1 + \hbar^2 \cdot v_2\right),$$

$$\lambda = \frac{1}{256\pi^2}\left[(g^2+g'^2)^2\left(1 - 3\ln\frac{g^2+g'^2}{4}\right)\right.$$
$$\left. + 2g'^4\left(1 - 3\ln\frac{g'^2}{4}\right) - 48y_t^4\left(1 - \ln\frac{y_t^2}{4}\right)\right]$$

scale, at which is satisfied

$$V_{\text{eff}}^{\text{NLO}}(\mu_{\text{min}}) \geq 0 \quad \text{for} \quad M_h \geq \tilde{M}_h^{\text{cri}} (\text{or}\, M_t \leq \tilde{M}_t^{\text{cri}})$$

Critical parameters and instability scales: our results

$$X = X_0 + \Delta X_{\alpha_s} \frac{\alpha_s^{(5)}(M_Z) - \alpha_s^{(5),\mathrm{exp}}(M_Z)}{\Delta\alpha_s^{(5),\mathrm{exp}}(M_Z)} + \Delta X_M \frac{M - M^{\mathrm{exp}}}{\Delta M^{\mathrm{exp}}} \pm \delta X_{\mathrm{par}} + \delta X_\mu^{\pm} \pm \delta X_{\mathrm{tru}},$$

X	X_0	ΔX_{α_s}	ΔX_M	δX_{par}	δX_μ^+	δX_μ^-	δX_{tru}
M_t^{cri}	171.44	0.23	0.20	0.0007	−0.36	0.17	−0.02
$\log_{10}\mu_t^{\mathrm{cri}}$	17.752	−0.051	0.083	0.007	0.007	−0.006	−0.002
M_H^{cri}	129.30	−0.49	1.79	0.002	0.72	−0.33	0.04
$\log_{10}\mu_H^{\mathrm{cri}}$	18.51	−0.16	0.38	0.008	0.17	−0.08	0.01
$\widetilde{M}_t^{\mathrm{cri}}$	171.64	0.23	0.20	0.001	−0.36	0.17	−0.02
$\log_{10}\tilde{\mu}_t^{\mathrm{cri}}$	21.44	−0.059	0.094	0.005	−0.07	0.02	0.002
$\widetilde{M}_H^{\mathrm{cri}}$	128.90	−0.49	1.79	0.003	0.73	−0.34	0.04
$\log_{10}\tilde{\mu}_H^{\mathrm{cri}}$	22.21	−0.18	0.43	0.007	0.09	−0.06	0.01

Our estimates of theoretical uncertainties

[AVB,Kniehl,Pikelner,Veretin'15]

$M_t = 173.21(87)$ GeV, $M_h = 125.7(4)$ GeV

Critical parameters: Combined results for critical M_t

Absolute stability bound on the least precise parameter..

$$M_t^{\rm cri} = 171.29 \pm 0.30^{-0.36}_{+0.17}\ {\rm GeV} \quad \text{from } \lambda = \beta_\lambda = 0$$

$$\tilde{M}_t^{\rm cri} = 171.49 \pm 0.30^{-0.36}_{+0.17}\ {\rm GeV} \quad \text{from } V_{\rm eff}^{\rm NLO}(\mu_{min}) = 0$$

("scheme-dependence" theoretical uncertainty estimate)

for fixed $M_h = 125.09 \pm 0.24\ {\rm GeV}$

[AVB,Kniehl,Pikelner,Veretin'15]

Up to now we only consider absolute stability..

On metastability of the SM

[Kobzarev,Okun,Voloshin'74]

[Coleman,..'77]

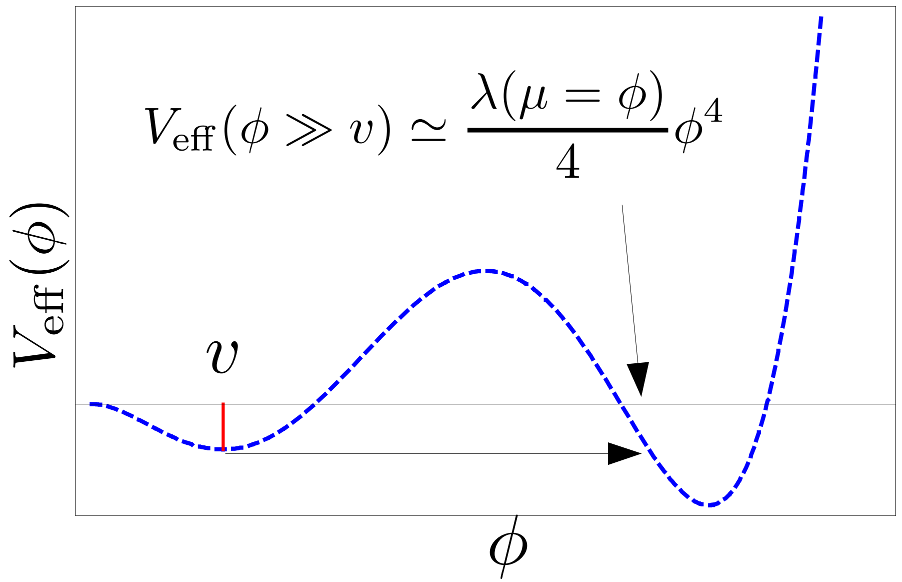
Probability for the EW vacuum to decay during the age of the Universe

$$\tau_U \simeq 13.8 \times 10^9 \text{ years}$$

$$P \simeq \tau_U \underbrace{\left[\frac{\tau_U^3}{R^4} e^{-\frac{8\pi^2}{3|\lambda(1/R)|}} \right]}$$

$$\tau_{EW} \simeq 10^{655} \tau_U$$

[Branchina,...'14]



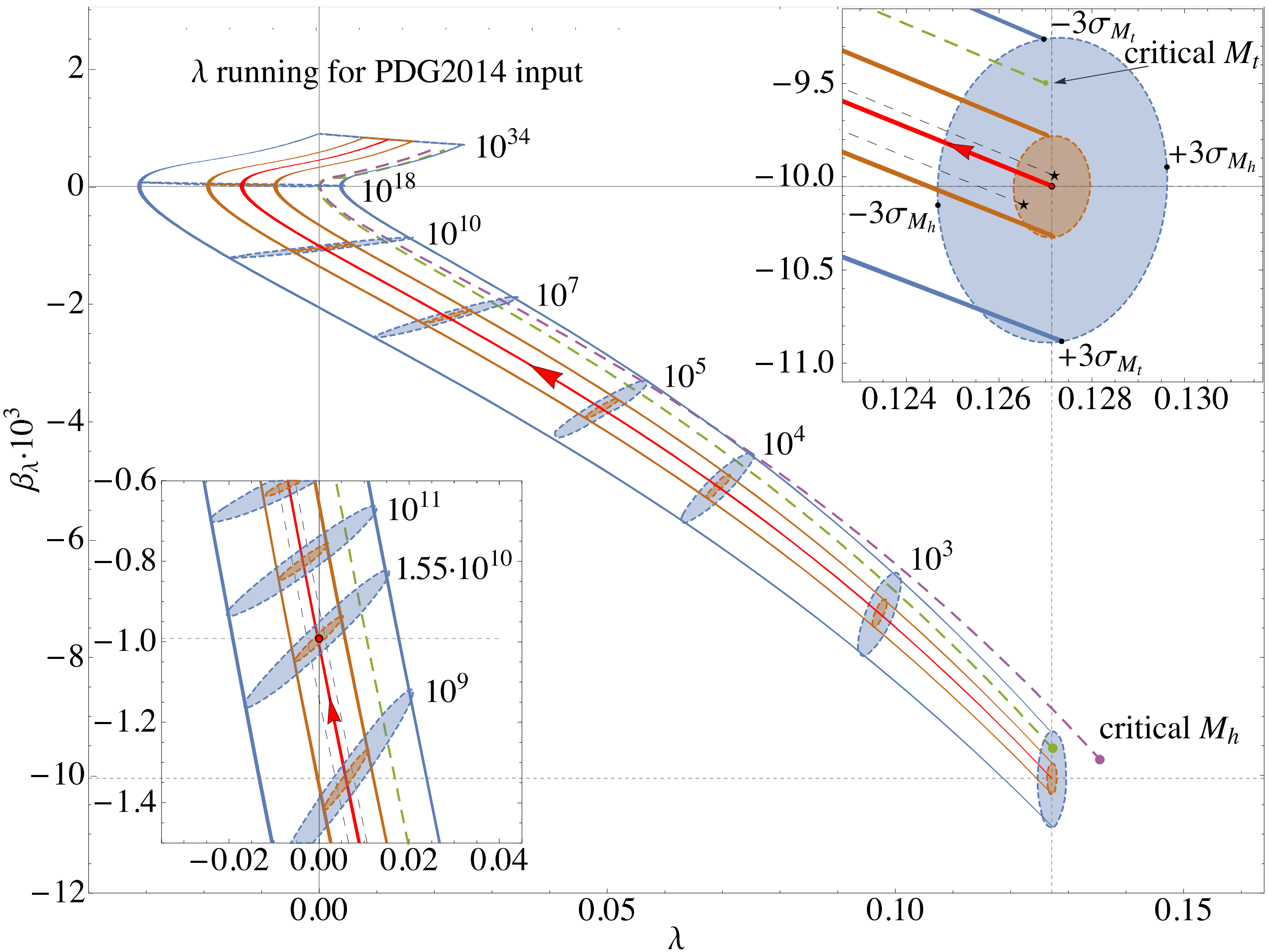
If the true minimum is lower than the electroweak one quantum tunneling is possible.

Critical bubble size R is determined from the **running coupling** for which we require that

$$[-\lambda(\mu)]_{\mu=1/R} \quad \text{has maximum value} \quad \beta_\lambda(\mu = 1/R) = 0$$

improved semiclassical "gauge-independent" approximation

RG flow for Higgs self-coupling



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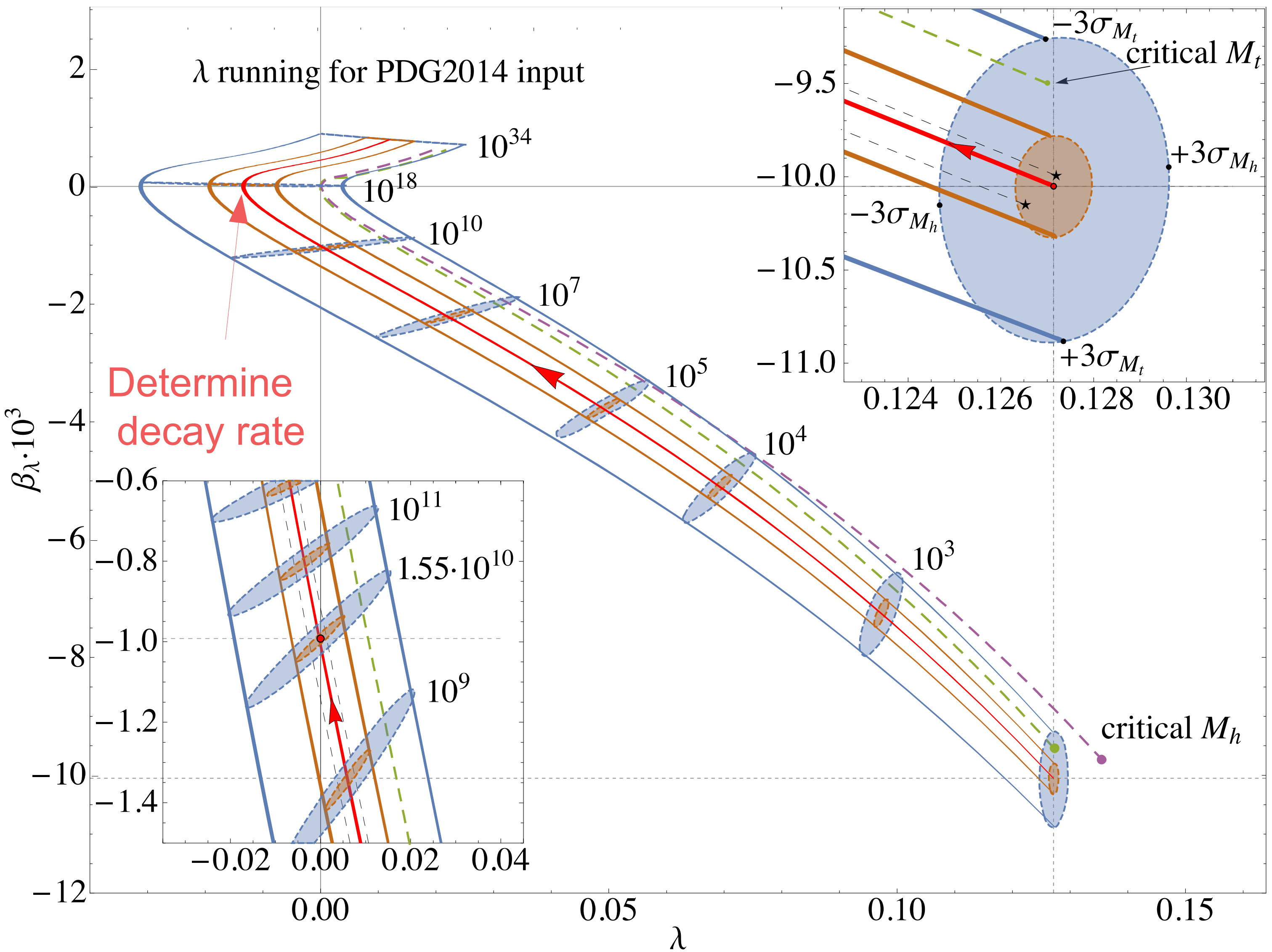
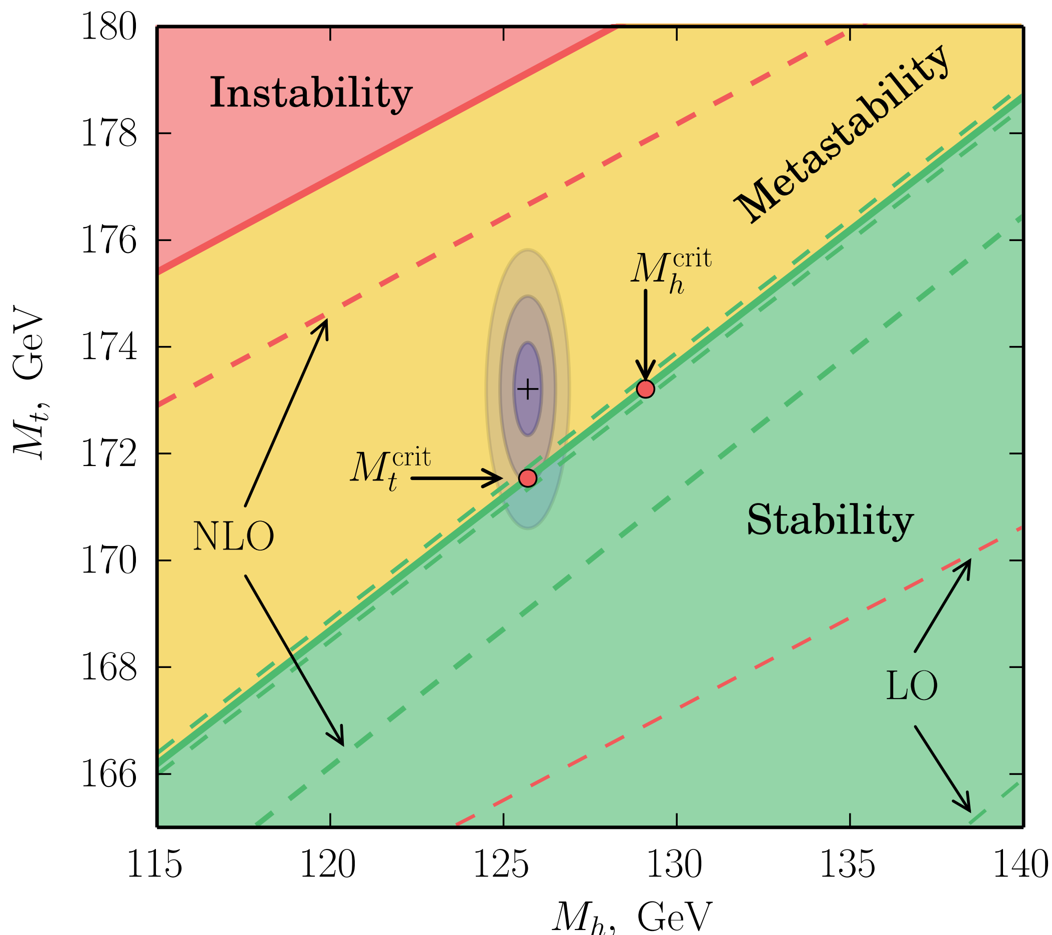


Importance of high-order RGEs

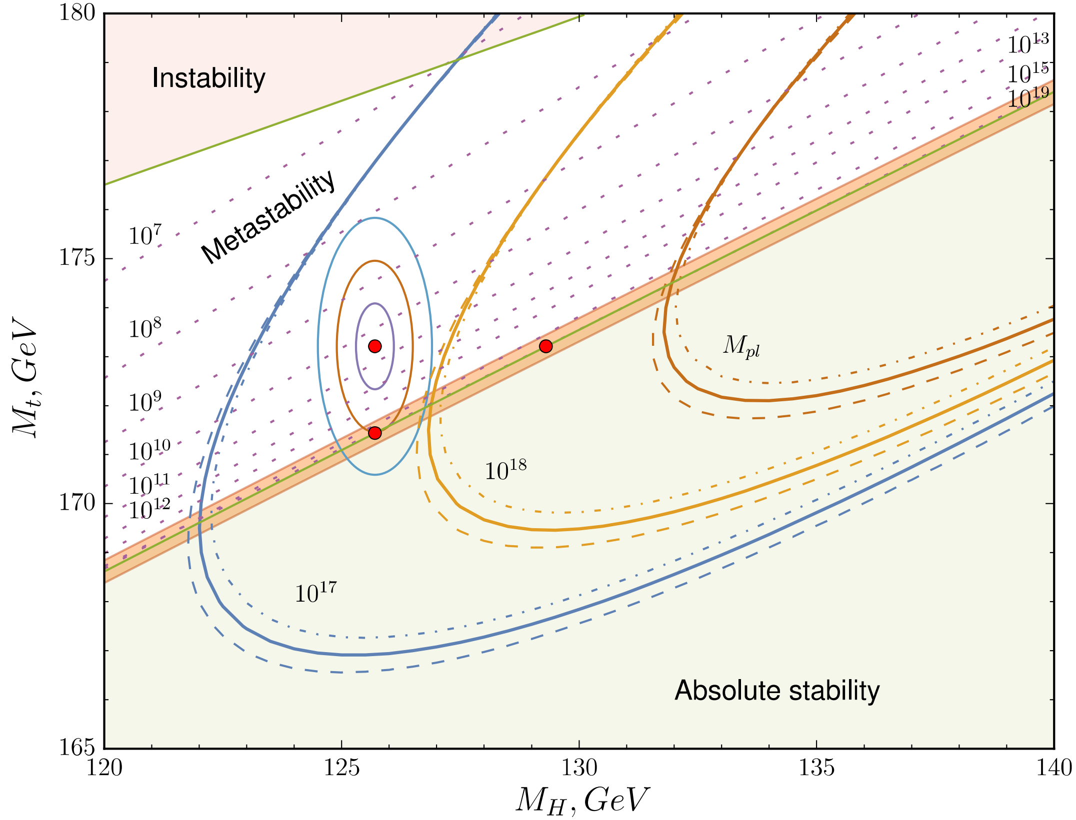
$$M_t^{crit} = 171.19^{-0.36}_{+0.17} + 0.12\left(\frac{M_h - 125.09}{0.24}\right) + 0.49\left(\frac{\alpha_s^{(5)} - 0.1181}{0.0011}\right)$$

$$M_h^{crit} = 129.62^{+0.72}_{-0.34} + 1.79\left(\frac{M_t - 173.21}{0.87}\right) - 1.04\left(\frac{\alpha_s^{(5)} - 0.1181}{0.0011}\right)$$



Instability region: $\tau_{EW} < \tau_U$

Stable, unstable or metastable?

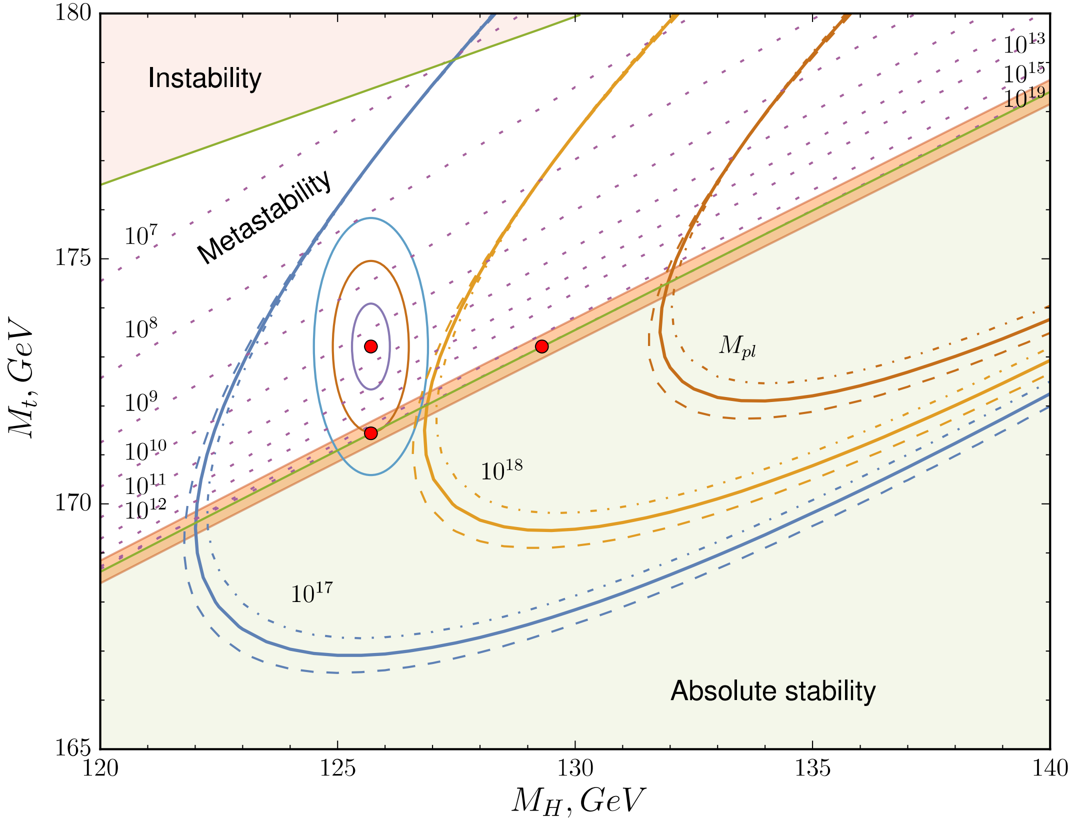


$M_t = 172.38 \pm 0.66$ GeV LHCP2015 [1512.02244]

[AVB, Kniehl, Pikelner, Veretin], Phys. Rev. Lett. **115**, 201802 (2015)

Stable, unstable or metastable?

Taking into account the estimated uncertainties the SM turns out to be compatible with absolute stability at 1.3 sigma level



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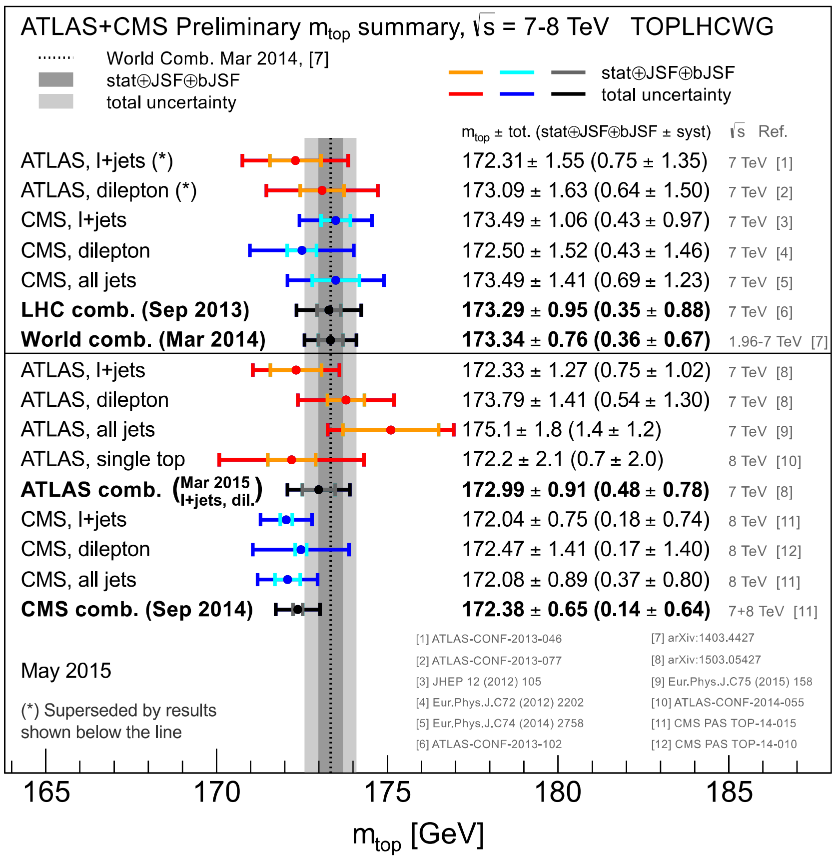
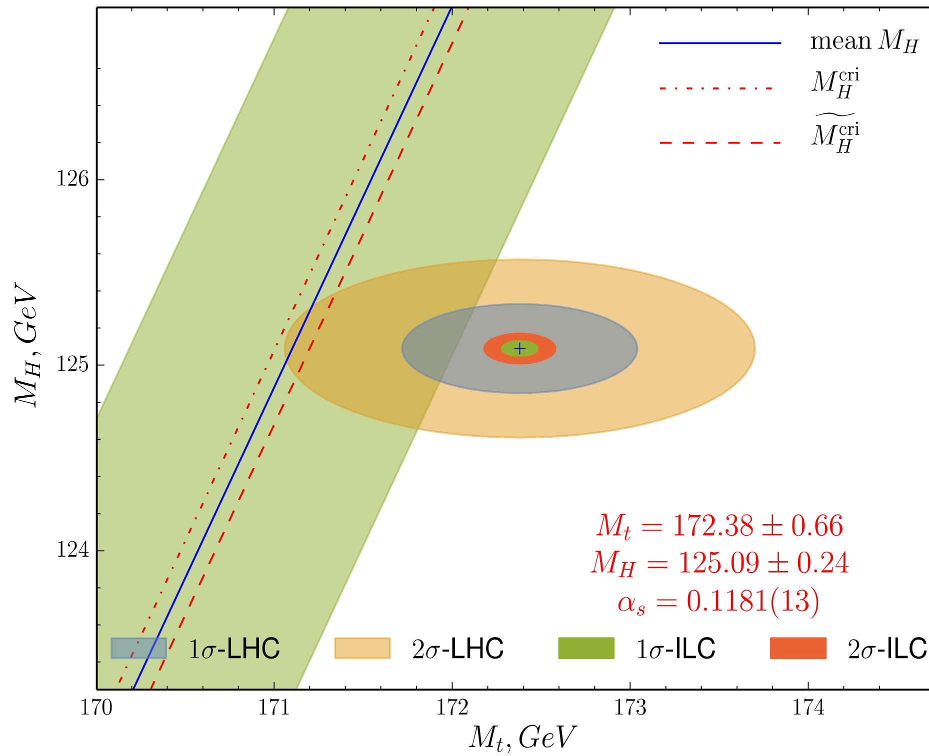
Open Issues

The top-quark mass

- The dominant uncertainty is due to the top-quark mass...





Present...

LHCP2015

Future?

[Moortgat-Pick,...'15]

$$\delta M_t^{ILC} = 0.1\ GeV$$

$$\delta M_h^{ILC} = 0.04\ GeV$$

The top-quark mass

- The dominant uncertainty is due to the top-quark mass.Strictly speaking, this quantity is not a well-defined one (no free quarks)
- The value quoted in PDG is not the pole mass but a parameter in Monte-Carlo code used to generate hadronic events involving jets from top quarks.
- Better understanding of theoretical error in the top mass determination would be desirable in addition to a more precise experimental measurement

See, summary talk on **TOP2015** [Corcella,'15]

Or, maybe, reformulate the bound on running top-Yukawa?

[Bezrukov,Shaposhnikov'15]

$$y_t^{\mathrm{crit}} = 0.9244 + 0.0012 \times \frac{M_h/\mathrm{GeV} - 125.7}{0.4} + 0.0012 \times \frac{\alpha_s(M_Z) - 0.1184}{0.0007}$$

On (meta)stability of the SM

There are technical issues related in extending decay rate calculation procedure beyond the leading order. The main obstacle is the necessity to deal with effective action instead of effective potential

$$S[\phi] = \int d^4 \left(-V_{\text{eff}}(\phi) + \frac{1}{2} Z(\phi)(\partial\phi)^2 + \dots \right)$$

For “bounce” background derivative terms also contribute.

[Espinosa et al’15-16],[Andreassen et al, 15-16]

Derivative expansion may be not justified for tunneling rate calculation!

Again, gauge-dependence and re-summation issues..

On (meta)stability of the SM

The Higgs potential and EW vacuum lifetime can be modified by several factors. Among of them are...

1. (<u>Quantum</u>) Gravity influence... [Espinosa,...'07]

[Coleman,De Luccia'80] may increase the lifetime.. [Abe,...`16]

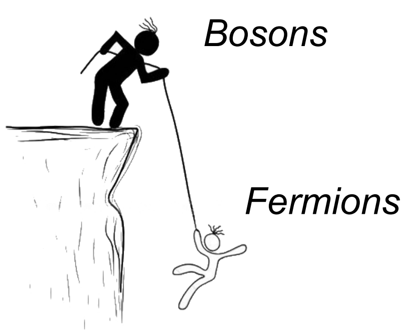
2. Finite-temperature effects...

[Linde'82] Thermal fluctuations of the Higgs field vs thermal corrections to the potential...

[Delle Rossa,...'15]

3. **New Physics....**

SUSY, 2HDM, exotics,...?



4. "Old" Physics...

Scalar 6 $t\bar{t}$ bound-state? [Das,...'16]

Cosmological implications!

See "Cosmological Higgstory ..." [Espinosa,...'15]

Higgs inflation?

[Bezrukov,...'12-15]

Conclusions and Outlook

- Under assumption that there is no New Physics up to the Planck scale the stability of the EW vacuum is studied with the help of the "state of the art" 3-loop RGE (easily reproducible by MR code developed by A.F. Pikelner, https://github.com/apik/mr.)
- The central values of the top-quark and higgs masses prefer metastable scenario but it is still possible to have absolute stability within the SM.
- Theoretical uncertainties due to missing corrections in the critical parameters are comparable to current parametric uncertainties due to known experimental input. New measurements and new calculations (in progress) can improve the situation.
- The EW stability constraint is important when considered in the cosmological and/or New Physics context.

Thank you for your attention!