Analysis of Factorization Scale Dependence in Drell-Yan-like Processes in QED

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Seminar "Quantum Field Theory"at BLTP, JINR

based on arXiv:2511.00437 [hep-ph]

18th Febr. 2026

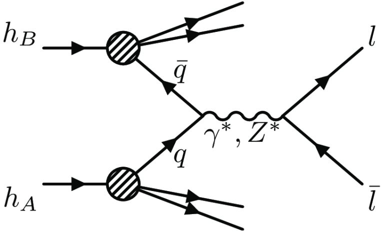
Motivation

- Results of perturbative calculations often depend on factorization and renormalization scales and scheme choices as in QCD as well as in QED (and in EW)
- Existing analytic results in QED provide a unique possibility to check analytically and numerically those dependencies
- QED Drell-Yan-like process $e^+e^- \to \gamma^*, Z \to \mu^+\mu^-$ is the best case for the tests
- Different prescriptions for the factorization scale choice can be compared
- Several other issues can also be addressed: dependence on renormalization scale, scheme choices, exponentiation, effect of scale variation by factor 2, etc.
- Optimization of the scale and scheme choices is required both in QCD and QED/EW

Drell-Yan process in QCD (NC case)

Schematically

$$d\sigma(h_A h_B \to \gamma^*, Z^* \to \mu^+\mu^-) = \sum_{a,b=q,\bar{q},g} D_{ah_A} \otimes D_{bh_B} \otimes d\hat{\sigma}(ab \to \gamma^*, Z^* \to \mu^+\mu^-)$$



[1] V.A. Matveev, R.M. Muradyan, A.N. Tavkhelidze, Production of Muon Pairs in Strong Interactions. . . , JINR Preprint P2-4578 ’1969

[2] S.D. Drell, T.-M. Yan, Massive Lepton-Pair Production in Hadron-Hadron Collisions at High Energies, PRL ’1970

Drell-Yan process in QED: e^+e^- annihilation

The factorization structure is exactly the same as in QCD:

$$\frac{d\sigma_{e^+e^-\to\gamma^*,Z^*\to\mu^+\mu^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes D_{be^+} \otimes \tilde{\sigma}_{ab\to\gamma^*,Z^*\to\mu^+\mu^-}$$

Complete analytic results as function of the final state invariant mass are known in $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$, and in the leading and next-to-leading logarithmic approximations in several higher orders (terms $\sim m_e^2/s$ are neglected). Note, that only initial state radiation is discussed here.

See [J. Ablinger, J. Blümlein, A. De Freitas and K. Schönwald, "Subleading Logarithmic QED Initial State Corrections to $e^+e^- \to \gamma^*/Z^{0^*}$ to $O(\alpha^6 L^5)$," NPB 955 (2020) 115045] and [A.A., U.Voznaya, PRD 2024]

Drell-Yan process in QED: ingredients

$$\frac{d\sigma_{e^+e^-\to\gamma^*,Z^*\to\mu^+\mu^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s')\sum_{a,b=e^-,\gamma,e^+} D_{ae^-}\otimes D_{be^+}\otimes\tilde{\sigma}_{ab\to\gamma^*,Z^*\to\mu^+\mu^-}$$

- $\sigma^{(0)}(s')$ is the Born cross-section, $\tilde{\sigma}$ is the partonic one (normalized to Born)

$$\tilde{\sigma} = \delta(1-z) + \alpha\bar{\sigma}^{(1)}(z) + \alpha^2\bar{\sigma}^{(2)}(z) + \ldots$$

- D_{ae} are electron PDFs, they are solutions of DGLAP evolution eqs. (see the next slide)
- $\alpha \equiv \alpha_{\text{QED}}(\mu_R)$
- $s = (p_{e^+} + p_{e^-})^2, \qquad s' = (p_{\mu^+} + p_{\mu^-})^2 \equiv zs$

The factorization scale and scheme choices are addressed below in detail

QED DGLAP evolution equations

$$\mathcal{D}_{ba}\left(x, \frac{\mu_R^2}{\mu_F^2}\right) = \delta_{ab}\delta(1-x) + \frac{\alpha}{2\pi} d_{ba}^{(1)}(x, \mu_R^2/m_e^2) + \ldots$$
$$+ \sum_{c=e,\gamma,\bar{e}} \int_{\mu_R^2}^{\mu_F^2} \frac{dt}{t} \int_x^1 \frac{dy}{y} P_{bc}(y,t) \mathcal{D}_{ca}\left(\frac{x}{y}, \frac{\mu_R^2}{t}\right)$$

a, b, c are massless partons ($\sim e^{\pm}, \gamma$)

μ_F is a factorization (energy) scale

μ_R is a renormalization (energy) scale

D_{ba} is a parton density function (PDF)

$d_{ba}^{(m)}$ is an initial condition

P_{bc} is a splitting function or kernel of the DGLAP equation

Analytic QED PDFs in the leading and next-to-leading log approximations are known [A.A., U.Voznaya, JPG '2023] see also [S.Frixione et al.]

News: NNLO QED PDFs

There are two recent works

[1] M. Stahlhofen
NNLO electron structure functions (PDFs) from SCET
arXiv:2508.16964 [hep-ph]

[2] M. Schnubel and R. Szafron
Electron and Photon Structure Functions at Two Loops
arXiv:2509.09618 [hep-ph]

where electron and photon QED PDFs were computed in $\mathcal{O}\left(\alpha^2 L^0\right)$

Comparisons with QCD to be still performed

QED splitting functions

The perturbative splitting functions are

$$P_{ba}(x, \bar{\alpha}(t)) = \frac{\bar{\alpha}(t)}{2\pi} P_{ba}^{(0)}(x) + \left(\frac{\bar{\alpha}(t)}{2\pi}\right)^2 P_{ba}^{(1)}(x) + \mathcal{O}(\alpha^3)$$

$$\text{e.g.} \quad P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

They come from loop calculations, e.g., $P_{ba}^{(1)}(x)$ comes from 2-loops

The splitting functions can be obtained by reduction of the ones known in QCD to the abelian case of QED

$\bar{\alpha}(t)$ is the QED running coupling constant in the $\overline{\text{MS}}$ scheme

N.B. Factorization in $\bar{\alpha}(t) \times P_{ba}(x)$ is not unique

Drell-Yan process in QED: sum over partons

$$\frac{d\sigma_{e^+e^-\to\gamma^*,Z\to\mu^+\mu^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes D_{be^+} \otimes \tilde{\sigma}_{ab\to\gamma^*,Z\to\mu^+\mu^-}$$

$a\backslash b$	e^+	γ	e^-
e^-	$D_{e^-e^-}D_{e^+e^+}\sigma_{e^-e^+}$ LO (1)	$D_{\gamma e^-}D_{e^-e^-}\sigma_{e^-\gamma}$ NLO $(\alpha^2 L)$	$D_{e^-e^-}D_{e^-e^+}\sigma_{e^-e^-}$ NNLO $(\alpha^4 L^2)$
γ	$D_{\gamma e^-}D_{e^+\,e^+}\sigma_{e^+\gamma}$ NLO $(\alpha^2 L)$	$D_{\gamma e^-}D_{\gamma e^+}\sigma_{\gamma\gamma}$ NNLO $(\alpha^4 L^2)$	$D_{\gamma e^-}D_{e^-e^+}\sigma_{e^-\gamma}$ NLO $(\alpha^4 L^3)$
e^+	$D_{e^+e^-}D_{e^+e^+}\sigma_{e^+e^+}$ NNLO $(\alpha^4 L^2)$	$D_{e^+e^-}D_{\gamma e^+}\sigma_{e^+\gamma}$ NLO $(\alpha^4 L^3)$	$D_{e^+e^-}D_{e^-e^+}\sigma_{e^+e^-}$ LO $(\alpha^4 L^4)$

Transitions $e^- \leftrightarrow e^+$ were missed in [J. Ablinger, J. Blümlein et al. '2020] presumably because they followed [F. Berends et al. '1988]

Drell-Yan process in QED: perturbative expansion

$$\frac{d\sigma_{e^+e^-\to\gamma^*,Z\to\mu^+\mu^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes D_{be^+} \otimes \tilde{\sigma}_{ab\to\gamma^*,Z\to\mu^+\mu^-}$$

- Large logs $L \equiv \ln(\mu_F^2/\mu_R^2)$ come from running α and anomalous dimensions

$$\frac{d\sigma^{\mathrm{NNLL}}_{\bar{e}e}(s')}{ds'} = \frac{\sigma^{(0)}(s')}{s}\Bigg\{\delta(1-z) + \sum_{\substack{k=1\\l=k}}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k c_{kl}(z)L^l$$

$$+ \sum_{\substack{k=1\\l=k-1}}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k c_{kl}(z)L^l + \sum_{\substack{k=1\\l=k-2}}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k c_{kl}(z)L^l + \mathcal{O}(\alpha^k L^{k-3})\Bigg\} + \mathcal{O}(\frac{m_e^2}{s})$$

where $c_{kl}(z)$ are functions of $z = s'/s$ and they contain

$$\delta(1-z), \quad \zeta(m), \quad N_f, \quad \left[\frac{\ln^m(1-z)}{1-z}\right]_+, \quad \frac{\ln^m(z)}{z}, \quad \mathit{HPOLs}, \; \mathit{elliptic\ int}(?) \; \mathit{etc.}$$

Drell-Yan process in QED: the general structure

$$\frac{d\sigma_{e^+e^-\to\gamma^*,Z\to\mu^+\mu^-}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes D_{be^+} \otimes \tilde{\sigma}_{ab\to\gamma^*,Z\to\mu^+\mu^-}$$

Schematically

$$\begin{array}{llll} c_{00}\alpha^0 \text{ (Born)} & & & \\ c_{11}\alpha^1 L^1 & c_{10}\alpha^1 L^0 & & \\ c_{22}\alpha^2 L^2 & c_{21}\alpha^2 L^1 & c_{20}\alpha^2 L^0 & \\ c_{33}\alpha^3 L^3 & c_{32}\alpha^3 L^2 & c_{31}\alpha^3 L^1 & c_{30}\alpha^3 L^0 \\ \quad\ldots & & & \end{array}$$

where $\alpha = \alpha(\mu_R)$ and $L = \ln(\mu_F^2/\mu_R^2)$ and coefficients c_{kl} in different orders are functions of kinematical variables and N_f

Columns correspond to LO, NLO, NNLO and so on, they are subject of resummation

Drell-Yan process in QED: factorization scale dependence

$$
\begin{array}{llll}
c_{00}\alpha^0 \text{ (Born)} & & & \\
c_{11}\alpha^1 L^1 & c_{10}\alpha^1 L^0 & & \\
c_{22}\alpha^2 L^2 & c_{21}\alpha^2 L^1 & c_{20}\alpha^2 L^0 & \\
c_{33}\alpha^3 L^3 & c_{32}\alpha^3 L^2 & c_{31}\alpha^3 L^1 & c_{30}\alpha^3 L^0 \\
\dots & & &
\end{array}
$$

For a given scheme and μ_R the sum of terms in each line is μ_R-independent, e.g.

$$
\left(\frac{\alpha}{2\pi}\right)^2 L^2 c_{22} + \left(\frac{\alpha}{2\pi}\right)^2 L c_{21} + \left(\frac{\alpha}{2\pi}\right)^2 c_{20} = \left(\frac{\alpha}{2\pi}\right)^2 \hat{L}^2 \hat{c}_{22} + \left(\frac{\alpha}{2\pi}\right)^2 \hat{L} \hat{c}_{21} + \left(\frac{\alpha}{2\pi}\right)^2 \hat{c}_{20}
$$

where

$$
\hat{L} = L + \Delta L, \quad \Delta L = \ln \frac{\hat{\mu}_F^2}{\mu_F^2}
$$

Factorization scale dependence appear if lines (calculations) are incomplete

Factorization scale choices

The final result of calculation in all orders in α and L would not depend on μ_F

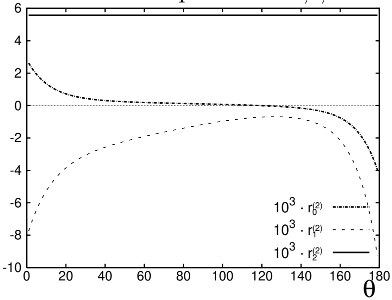
But a fixed-order result for an observable does depend on μ_F (and μ_R)

Many different methods for choosing μ_F were proposed:

- CSS — Conventional Scale Setting ($\mu_F =$ hard momentum transfer)
- FAC — Fastest Apparent Convergence [G. Grunberg; N. Krasnikov]
- PMS — Principle of Minimal Sensitivity [P.M. Stevenson]
- BLM — Brodsky-Lepage-Mackenzie (absorb β_0-dependent terms)
- PMC — Principle of Maximal Conformality [S.Brodsky et al.]
- ...

Factorization scale choice in Bhabha scattering

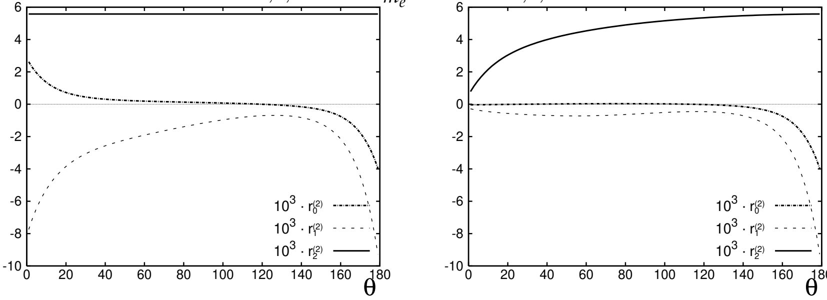
Let's look at soft + virtual $\mathcal{O}\left(\alpha^2\right)$ RC [A. Penin, PRL'2005, NPB'2006]: $\Delta_{2-\text{loop}} = \sum_{n=0,1,2} C_n \ln^n \frac{\mu_F^2}{m_e^2} = \sum_{n=0,1,2} r_n$



Soft and virtual second order photonic relative radiative corrections in permil versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $\mu_F = \sqrt{s}$

Factorization scale choice in Bhabha scattering

Let's look at soft + virtual $\mathcal{O}\left(\alpha^2\right)$ RC [A. Penin, PRL'2005, NPB'2006]: $\delta_2 = \alpha^2 \sum_{n=0,1,2} c_n \ln^n \frac{\mu_F^2}{m_e^2} = \sum_{n=0,1,2} r_n$



Soft and virtual second order photonic relative radiative corrections in permil versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV;
$\mu_F = \sqrt{s}$ on the left side and $\mu_F = \sqrt{-t}$ on the right side
CSS choice works here quite well [A.A., E.S.Scherbakova, JETP Lett. 2006]

Factorization scale choice in $e^+e^- \to \gamma^*, Z \to \mu^+\mu^-$

We propose the FAC-like prescription, i.e., hide the bulk of one-loop corrections into the scale choice

For e^+e^- annihilation in $\mathcal{O}(\alpha^1)$ we have

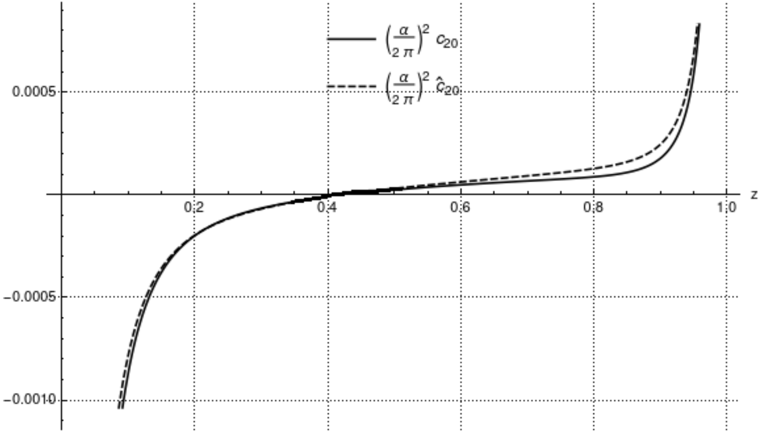
$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \left(\ln\frac{s}{m_e^2} - 1\right) + \delta(1-z)\left(2\zeta(2) - \frac{1}{2}\right) \Rightarrow \mu_F^2 = s \text{ or } \frac{s}{e}$$

Remind QCD Drell-Yan processes where we usually take $\mu_F^2 = s' \equiv zs \sim M_Z$, i.e., the energy scale of the hard subprocess (CSS choice)

F.Berends at al. and J. Blümlein et al. used $\mu_F = zs$ without any justification

Numerical results: example $\mathcal{O}(\alpha^2 L^0)$

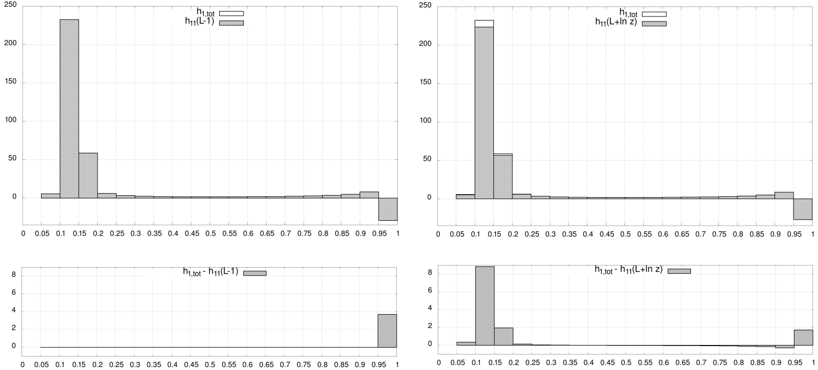
Let's look at $\mathcal{O}(\alpha^2 L^0)$ contribution. We tried to absorb it into $\mathcal{O}(\alpha^2 L^2)$ and $\mathcal{O}(\alpha^2 L^1)$ terms by adjusting μ_F



$\mathcal{O}(\alpha^2)$ contribution as function of z for $\mu_F = \sqrt{s}$ and $\mu_F = \sqrt{zs}$

Numerical results for differential distribution: $\mathcal{O}(\alpha^1)$

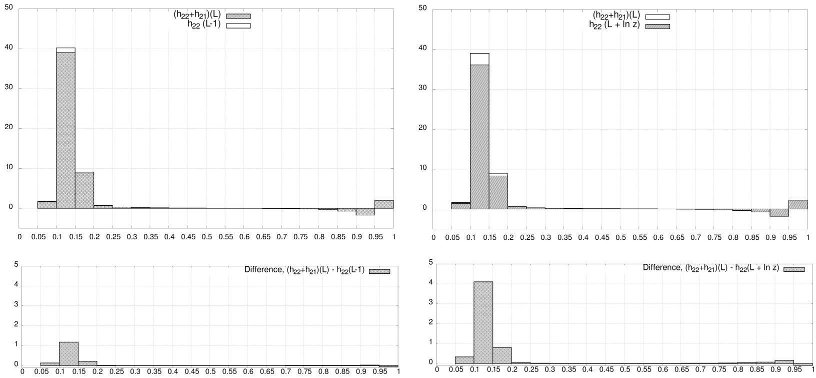
Try to reproduce NLO by adjusting μ_F in LO



$\mathcal{O}(\alpha^1)$ corrections and differences for three μ_F: $\sqrt{s}$, $\sqrt{s/e}$ (left), $\sqrt{sz}$ (right), at $\sqrt{s} = 240$ GeV in %

Numerical results for differential distribution: $\mathcal{O}(\alpha^2)$ (I)

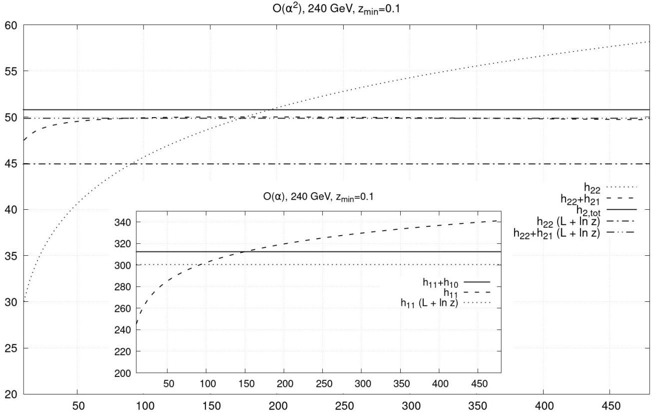
Try to reproduce NLO by adjusting μ_F in LO



$\mathcal{O}(\alpha^2)$ corrections and differences for three μ_F: $\sqrt{s}$, $\sqrt{s/e}$ (left), $\sqrt{sz}$ (right), at $\sqrt{s} = 240$ GeV in %

Numerical results for total cross section at 240 GeV

Compare LO, NLO, and NNLO results as functions of μ_F

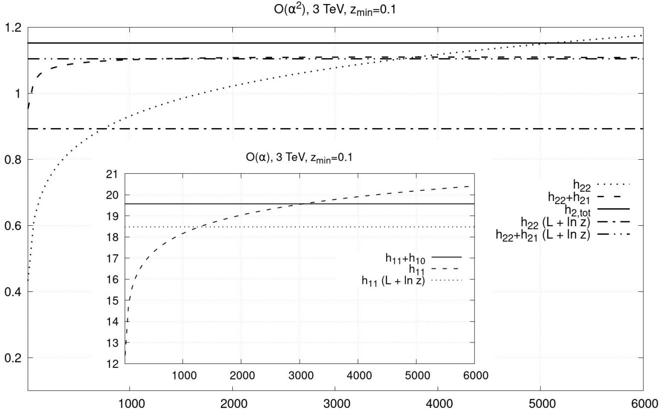


Corrections in % vs. factorization scale for $\sqrt{s} = 240$ GeV, $z_{min} = 0.1$

N.B. Comment on π^2-terms

Numerical results for total cross section at 3 TeV

Compare LO, NLO, and NNLO results as functions of μ_F



Corrections in % vs. factorization scale for $\sqrt{s} = 3$ TeV, $z_{min} = 0.1$

Factorization scale choice: preliminary conclusions

Sensitivity to factorization scale choice is relevant numerically even in QED

Higher-order calculations are required to reduce the uncertainty

The comparison of several concrete schemes shows:

- CSS — Conventional Scale Setting (μ_F = hard momentum transfer) fails
- FAC — Fastest Apparent Convergence looks good
- PMS — Principle of Minimal Sensitivity looks reasonable
- BLM — Brodsky-Lepage-Mackenzie not applicable
- PMC — Principle of Maximal Conformality not applicable

Factorization (subtraction) scheme choice

NLO exponentiation in the MSbar scheme is ambiguous:

explicit solution for $D_{ee}(x)$ in the $\overline{\mathrm{MS}}$ scheme in the limit $x \to 1$ doesn't match the (pure photonic) exact solution by Gribov and Lipatov '1972

$$\mathcal{D}_{ee}^{(\gamma)}(x)\bigg|_{x\to 1} = \frac{\beta}{2}\,\frac{(1-x)^{\beta/2-1}}{\Gamma(1+\beta/2)}\exp\left\{\frac{\beta}{2}\left(\frac{3}{4}-C\right)\right\}$$

where $\beta = 2\alpha/\pi(L-1)$ and C is the Euler constant

See also [A.V. Kotikov et al., "α_s from DIS data with large x resummation," JPG '2025]

We suggest a DIS-like scheme with the following modification of the NLO initial condition

$$d_{ee}^{(1)}\Big|_{\overline{\mathrm{MS}}} = \left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_R^2}{m_e^2} - 1 - \ln(1-x)\right)\right]_+ \to \tilde{d}_{ee}^{(1)} = \left[\frac{1+x^2}{1-x}\ln\frac{\mu_R^2}{m_e^2}\right]_+ = 0$$

for $m_R = m_e$ with subsequent modification of $\sigma^{(1)}$ to preserve the NLO matching. Fixed-order results for cross-sections remain unchanged.

Scale variation test: $\mu_F \to \mu_F/2, \quad \mu_F \times 2$

True (Δ) shifts and the ones estimated (δ) by factorization scale variation by factor 2 in $\mathcal{O}(\alpha^2)$ for $\mu_F = \sqrt{s}$ in the total cross section ($s' \geq s z_{\mathrm{min}}$)

	LO		NLO	
	Δ	δ	Δ	δ
$\sqrt{s} = M_z, \quad z_{min} = 0.1$	0.436	0.524	0.0064	0.0250
$\sqrt{s} = M_z, \quad z_{min} = 0.5$	0.436	0.525	0.0063	0.0250
$\sqrt{s} = 240, \quad z_{min} = 0.1$ GeV	2.468	5.569	0.518	0.148
$\sqrt{s} = 240, \quad z_{min} = 0.5$ GeV	0.114	0.106	0.0088	0.0061

$$\Delta^{\mathrm{LO}} = h_{21}, \qquad \Delta^{\mathrm{NLO}} = h_{20}$$

$$\delta^{\mathrm{LO}} = \frac{|h_{22} - h_{22}(1/2)| + |h_{22} - h_{22}(2)|}{2}$$

$$\delta^{\mathrm{NLO}} = \frac{|h_{22} + h_{21} - (h_{22} + h_{21})(1/2)| + |h_{22} + h_{21} - (h_{22} + h_{21})(2)|}{2}$$

Outlook

- Current and future high-precision HEP experiments challenge theory. New calculations of two-loop and higher-order corrections within QED and full SM are required
- There is progress in NLO and NNLO QED PDFs and fragmentation functions
- QED provides explicit results and serves for various tests
- Optimization of factorization scale and scheme choices is important as in QCD as well as in QED (and EW!)
- There is no perfect choice, compromises are inevitable

Next steps:

- NLO exponentiation in the proposed DIS-like scheme (in progress)
- Implementation into ZFITTER (in progress)
- Muon and (light) hadron contributions (in progress)
- BFKL effects in QED are large (might be interesting)
- k_T dependence in QED PDFs (there are some developments)
- Monte Carlo applications (within SANC)