Semiclassics for composite operators

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Conformal field theories (CFT)

Extrema of the RG flow

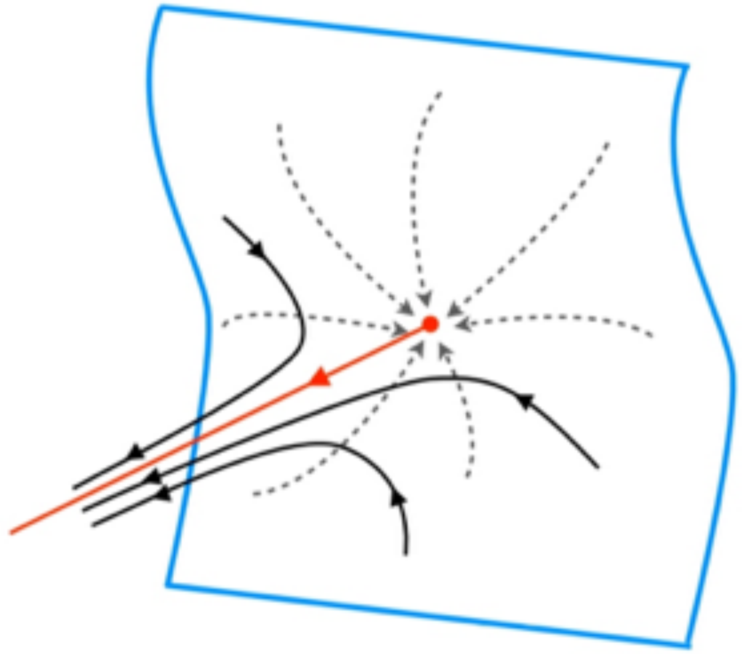
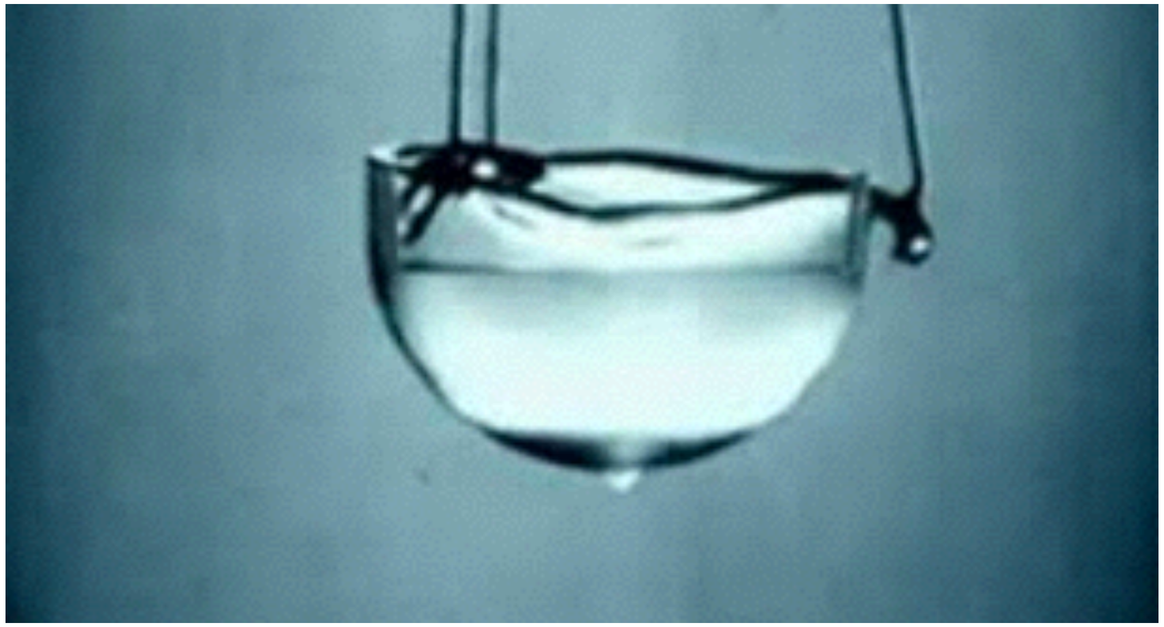
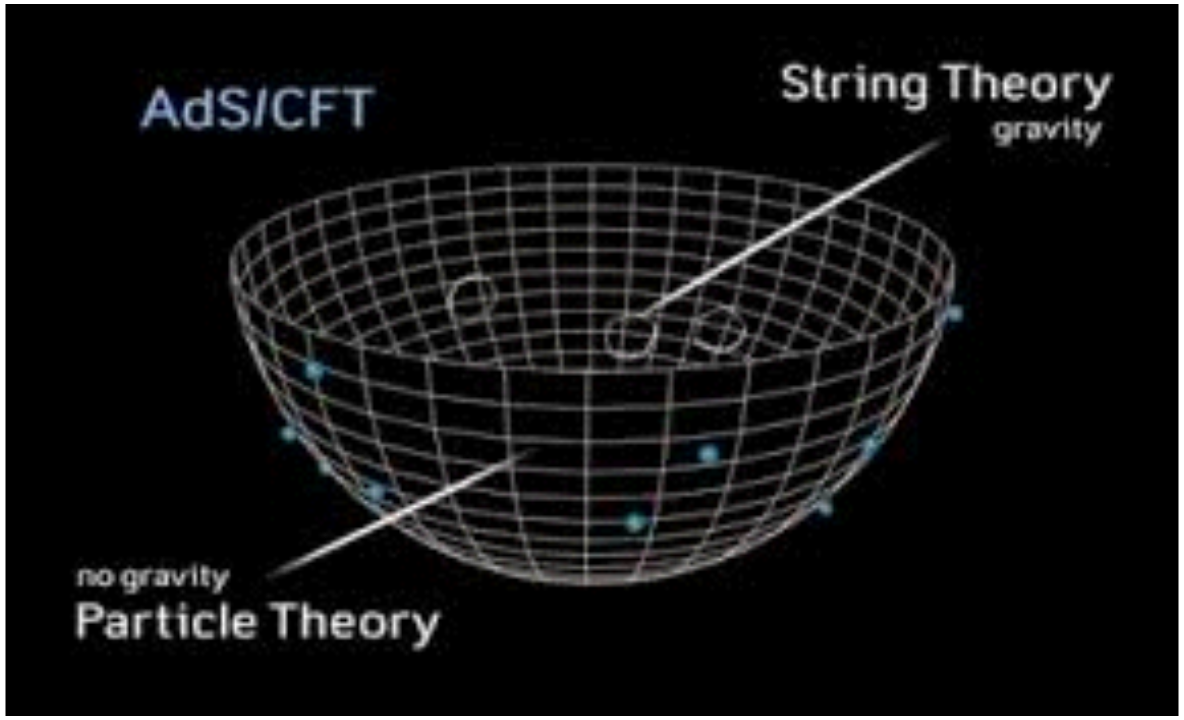


Figure 5: *Theories on the critical surface flow (dashed lines) to a critical point in the IR. Turning on relevant operators drives the theory away from the critical surface (solid lines), with flow lines focussing on the (red) trajectory emanating from the critical point.*

AdS/CFT
correspondence

Critical phenomena





CFT data

To define a CFT we need to specify:

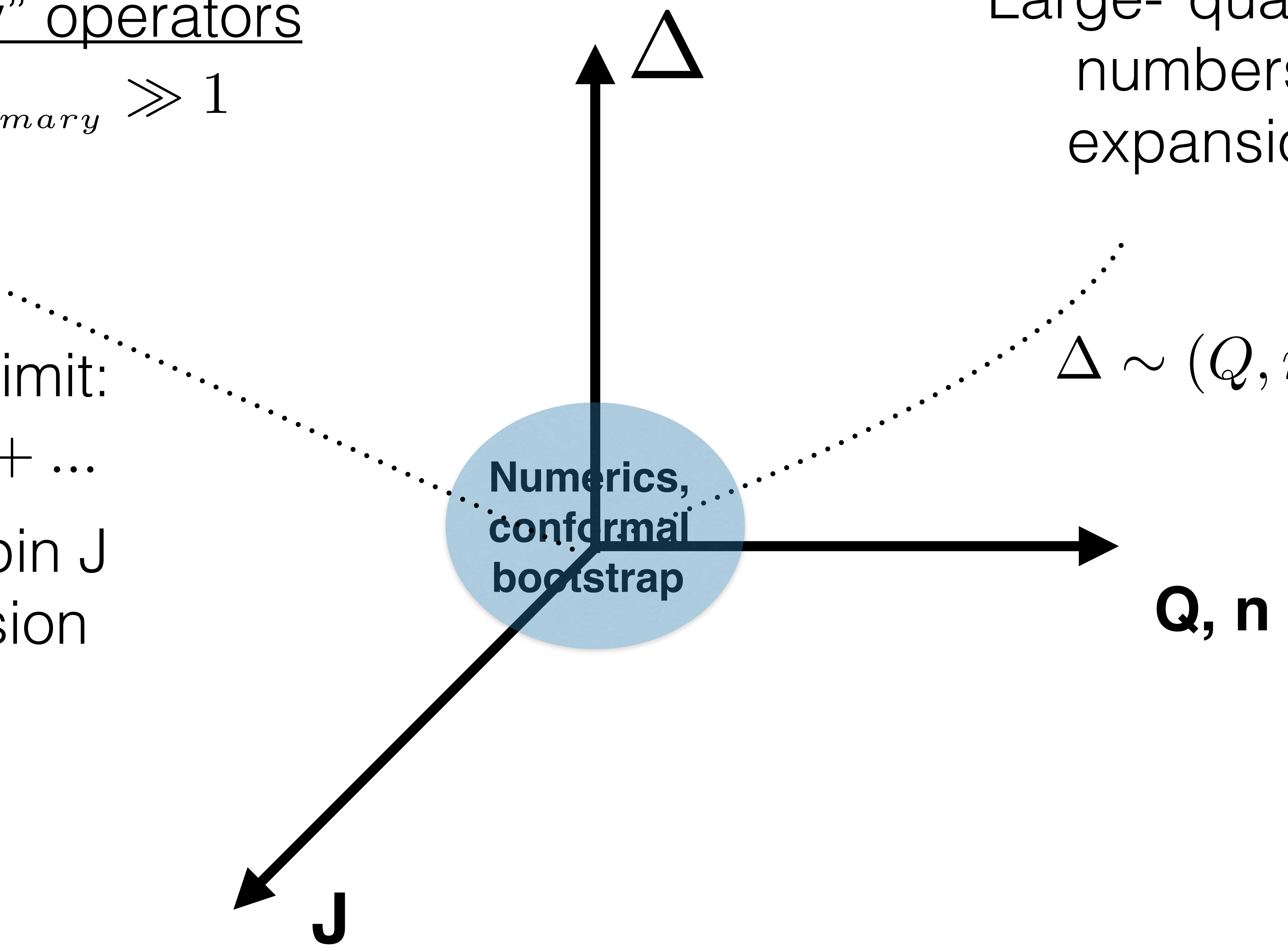
1. Spectrum of primary operators: $\left(\underset{\substack{\uparrow \\ \text{Conformal} \\ \text{dimension}}}{\Delta_{O_{primary}}}, \underset{\substack{\uparrow \\ \text{Representation}}}{\rho_{primary}}\right)$

2. OPE coefficients (structure constants)

“Heavy” operators

$\Delta_{O_{primary}} \gg 1$

Large-“quantum numbers“ expansion



$\Delta \sim (Q,n)^{\frac{d}{d-1}}$

Regge limit:

$\Delta = J + ...$

large-spin J expansion

This talk is about 1/Q and 1/n semiclassical expansions

Why semiclassical?

Bohr's correspondence principle:

Behaviour of systems described by quantum theory reproduces classical physics in the limit of large quantum numbers (Q,n)

Classical theory is easier to solve

Example: scalar primaries $\phi^Q(x)$

1) Global U(1) symmetry with (electric) charge Q

$$L = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\lambda}{4} \left(\bar{\phi}\phi\right)^2$$

The operators $\phi^Q(x)$ and $\bar{\phi}^Q(x)$ carry U(1) charge $+Q(-Q)$

Composite operators **charged** under the symmetry

$$\phi^Q(x) = \underbrace{\phi(x) \times \phi(x) \times \phi(x) \times \ldots.}_{Q \text{ times}}$$

2) In general, you can also construct **uncharged** (singlet) operators

$$\phi^n(x) = [\phi(x)\bar{\phi}(x)]^{n/2}$$

Accessed via 1/Q and 1/n expansions respectively

Outside of the CFT limit:

EFT: Higgs SMEFT

$$L_H = \partial_\mu H^\dagger \partial^\mu H - \lambda_H (H H^\dagger)^2 + \sum_i c_i O_i$$

$$\mu \frac{d}{d\mu} c_i \quad \sim \gamma_{c_i}$$

Requires knowledge of anomalous dimensions for the operators.
Our results apply outside of CFT

uncharged operators

$$\phi^n(x) = [\phi(x)\bar{\phi}(x)]^{n/2}$$

(**charged** operators do not enter in the Lagrangian)

for n=2: Higgs mass $H H^\dagger$

for n=4: Higgs quartic $(H H^\dagger)^2$

for n=6: Higgs sextic $(H H^\dagger)^3$

……

For a given model with action: $S = \int d^4x \; L(\phi, \partial\phi)$

Semiclassical expansion

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

Solves equation of motion
for a classical system

Template example: O(N) model

$$\mathcal{L} = \frac{1}{2}(\partial\phi_a)^2 - \frac{\lambda}{4}(\phi_a\phi_a)^2 \, , \qquad a = 1, \dots, N$$

In d=4-ε there is an IR WF fixed point at $\lambda_* \sim \dfrac{\epsilon}{N+8} + \cdots$

CFT limit:

$$\langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle_{CFT} = \frac{1}{|x_f - x_i|^{2\Delta_{\phi^Q}}}$$

Goal : compute $\boxed{\Delta_{\phi^Q} \equiv Q\left(\frac{d-2}{2}\right) + \gamma_{\phi^Q}}$

In the double scaling limit $\lambda \to 0$ $Q \to \infty$ $\lambda Q = fixed$

conformal dimensions take the form:

$$\Delta_Q = \sum_{k=-1} \frac{\Delta_k(\lambda Q)}{Q^k}$$

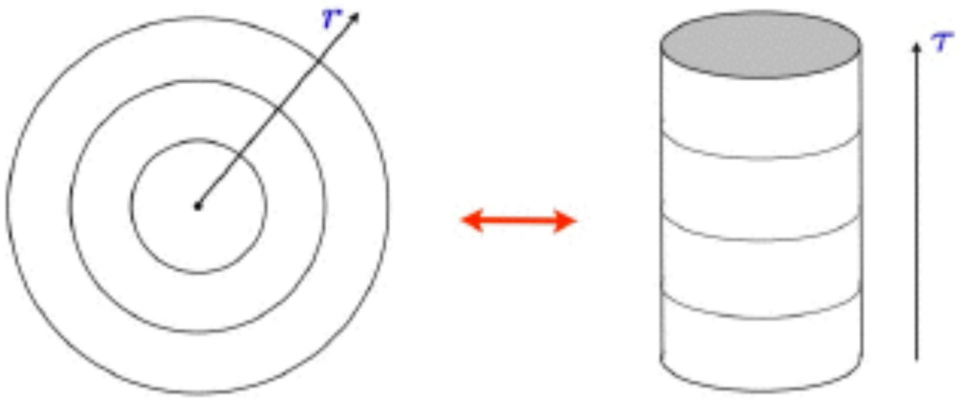
Δ_k is (k+1)-loop correction to the saddle point equation

We will compute Δ_{-1} and Δ_0

Weyl map and operator/state correspondence

Working at the WF fixed point we can map the theory to the cylinder.

$$\mathbb{R}^d \to \mathbb{R} \times S^{d-1}, \qquad r = Re^{\tau/R}$$



The eigenvalues of the dilation charge, i.e. the scaling dimensions, become the energy spectrum on the cylinder.

$$\boxed{E_{\phi^Q} = \Delta_{\phi^Q}/R}$$

State-operator correspondence:
States and operators are in 1-to-1 correspondence.

$$\tau_f - \tau_i \equiv T \qquad \langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle_{cyl} \overset{T\to\infty}{=} Ne^{-E_{\phi^Q}T}$$

Basic picture

Charge density $\rho \sim \frac{Q}{R^{d-1}}$ sets the UV scale $\Lambda_{UV} \sim \rho^{\frac{1}{d-1}} \sim \frac{Q^{\frac{1}{d-1}}}{R}$

For $Q \gg 1 :$ $\quad \Lambda_{UV} \gg \Lambda_{IR} \sim \frac{1}{R}$

0

$\Lambda_{IR} \sim \frac{1}{R}$

$\Lambda_{UV} \sim \rho^{\frac{1}{d-1}}$

Massless Goldstones

Large charge EFT for CFT

Massive modes

Write Weyl-invariant EFT for the Goldstone χ
(with conformal weight=0) :

$$L_{EFT} = c_1 \left(\partial_\mu \chi g^{\mu\nu} \partial_\nu \chi\right)^{d/2} + c_2 \left(\partial_\mu \chi g^{\mu\nu} \partial_\nu \chi\right)^{\frac{d-2}{2}} \mathcal{R} + \cdots$$

$$S_{eff} = c_1 \int d^d x \sqrt{-g} \left[(\partial_\mu \chi g^{\mu\nu} \partial_\nu \chi)^{d/2} + c_2 \, (\partial_\mu \chi g^{\mu\nu} \partial_\nu \chi)^{\frac{d-2}{2}} \, \mathcal{R} + \cdots \right]$$

$$\chi = -i\mu\tau$$

Chemical
potential

→

$$\frac{Q}{R^{d-1}\Omega_{d-1}} = j_0 = \rho = i \frac{\partial L}{\partial \dot{\chi}} = c_1 d \mu^{d-1}$$

→

$$\mu R \sim Q^{\frac{1}{d-1}}$$

$$\Delta_Q = \frac{E_Q}{R} \sim L_{EFT}\big|_{\chi = -i\mu\tau} \sim \mu^d = \alpha_1 Q^{\frac{d}{d-1}} + \cdots$$

$$\alpha_1 = \frac{c_1 (d-1) \Omega_{d-1}}{(c_1 d \Omega_{d-1})^{\frac{d}{d-1}}}$$

Consider model with U(1) global symmetry

$$L = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\lambda}{4} \left(\bar{\phi}\phi\right)^2$$

In d=4-ε there is an IR WF fixed point

$$\lambda^* = \frac{3}{10}\epsilon + \cdots$$

Weyl map the theory to the cylinder:

$$S_{cyl} = \int d^d x \sqrt{-g} \left(g_{\mu\nu} \partial^\mu \bar{\phi} \partial^\nu \phi + m^2 \bar{\phi}\phi + \frac{\lambda}{4} (\bar{\phi}\phi)^2 \right)$$

$$m^2 = \left(\tfrac{d-2}{2R}\right)^2$$

stemming from the coupling to Ricci scalar

Classical solution: $S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \ldots$

$$\phi = \frac{\rho}{\sqrt{2}} e^{i\chi} \qquad m^2 = \left(\tfrac{d-2}{2R}\right)^2$$

$$S_eff = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \left(\frac{1}{2}(d\rho)^2 + \frac{1}{2}\rho^2 (d\chi)^2 + \frac{m^2}{2}\rho^2 + \frac{\lambda}{16}\rho^4 \right)$$

Stationary solution: $\rho = f$ $\qquad \chi = -i\mu\tau$

$$\int d^{d-1}x \; j_0(x) = Q \qquad j_\mu(x) = \bar{\phi}\partial_\mu \phi - \phi \partial_\mu \bar{\phi}$$

$$\mu^2 - m^2 = \frac{\lambda}{4} f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}} \qquad i\partial_\mu(\rho^2 g^{\mu\nu}\partial_\nu \chi) = 0$$

e.o.m for ρ $\qquad$ charge-fixing $\qquad$ e.o.m for χ

EFT regimes: charged operators

$$\mu^2 - m^2 = \frac{\lambda}{4} f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

Solve:

$$\boxed{\mu(\mu^2 - m^2) = \frac{\lambda Q}{4R^{d-1}\Omega_{d-1}}}$$

$$m^2|_{d=4} = \frac{1}{R^2}$$

$$\lambda Q \ll 1 \qquad \mu R = 1 + \frac{\lambda Q}{16\pi^2} + \cdots$$

$\lambda \sim \epsilon \ll 1$

violating $\mu \sim Q^{\frac{1}{d-1}}$

$$\lambda Q \gg 1 \qquad \mu R = \frac{(\lambda Q)^{1/3}}{2\pi^{2/3}} + \cdots$$

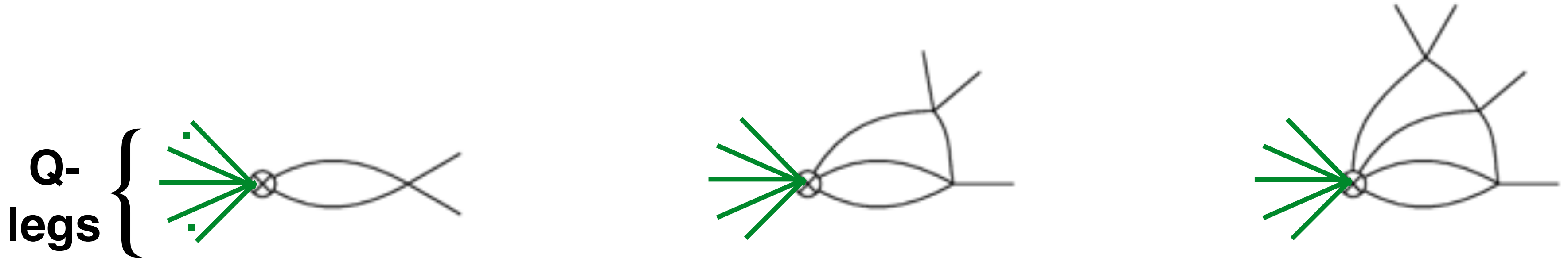
$S_{eff}R = E_{-1}R = \Delta_{-1}$

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}}$$

$x \equiv 6\lambda Q$

$$\frac{\Delta_{-1}}{\lambda_*} \underset{\lambda Q \ll 1}{=} Q\left[1+\frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right)-\frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right)^2+\cdots\right]$$

Resums infinite number of Feynman diagrams



$$\frac{\Delta_{-1}}{\lambda_*} \overset{\lambda Q \gg 1}{=} \frac{8\pi^2}{\lambda^*}\left[\frac{3}{4}\left(\frac{\lambda_* Q}{8\pi^2}\right)^{4/3}+\frac{1}{2}\left(\frac{\lambda_* Q}{8\pi^2}\right)^{2/3}+\cdots\right]$$

Leading quantum correction:

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$$\rho = f + r(x) \qquad \chi = -i\mu\tau + \frac{\pi(x)}{\sqrt{2}f}$$

$$S^{(2)} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \Big(\frac{1}{2}(\partial r)^2 + \frac{1}{2}(\partial\pi)^2 - 2i\mu r \partial_\tau \pi + (\mu^2 - m^2)^2 \Big)$$

One relativistic (Type I) Goldstone boson (the conformal mode=phonon) and one massive state and their excitations

$$\omega_\pm^2(\ell) = J_l^2 + 3\mu^2 - m^2 \pm \sqrt{4J_l^2\mu^2 + (3\mu^2 - m^2)^2}$$

$$J_l^2 = \ell(\ell + d - 2)/R^2$$

Energy= sum of zero point energies

$$\Delta_0 = \frac{R}{2}\sum_{\ell=0}^{\infty} n_\ell \left[\omega_+(\ell) + \omega_-(\ell)\right]$$

The MSbar renormalized result in the limiting cases reads:

$$\lambda Q \ll 1 \quad : \qquad \Delta_0 = -\frac{3\lambda_* Q}{(4\pi)^2} + \frac{\lambda_*^2 Q^2}{2(4\pi)^4} + \cdots$$

$$\lambda Q \gg 1 \ : \ \Delta_0 = [\alpha + \frac{5}{24}\log\left(\frac{\lambda_* Q}{8\pi^2}\right)^{4/3}] + [\beta - \frac{5}{36}\log\left(\frac{\lambda_* Q}{8\pi^2}\right)^{2/3}] + \cdots$$

EFT regimes

$$\omega_{\pm}^2(\ell) = J_l^2 + 3\mu^2 - m^2 \pm \sqrt{4J_l^2\mu^2 + (3\mu^2 - m^2)^2}$$

$\lambda Q \ll 1$

$$\mu R = 1 + \frac{\lambda Q}{16\pi^2} + \cdots$$

0

$\mu \sim \Lambda_{IR} \sim \frac{1}{R}$

$\mu \sim \Lambda_{UV} \sim \rho^{\frac{1}{d-1}}$

ω_-

$(\omega_+)_{\lambda Q \ll 1}$

μ controls the gap of the massive radial mode

Massless phonon

0

$\mu \sim \Lambda_{IR} \sim \frac{1}{R}$

$\mu \sim \Lambda_{UV} \sim \rho^{\frac{1}{d-1}}$

ω_-

$\lambda Q \gg 1$

$$\mu R = \frac{(\lambda Q)^{1/3}}{2\pi^{2/3}} + \cdots$$

$(\omega_+)_{\lambda Q \gg 1}$

EFT regimes: charged operators

$$\lambda Q \ll 1 \qquad \mu R = 1 + \frac{\lambda Q}{16\pi^2} + \cdots$$

$$\lambda \sim \epsilon \ll 1$$

so violates $\mu \sim Q^{\frac{1}{d-1}}$

$$\lambda Q \gg 1 \qquad \mu R = \frac{(\lambda Q)^{1/3}}{2\pi^{2/3}} + \cdots$$

$$\lambda \to 0 \qquad Q \to \infty \qquad \lambda Q = fixed$$

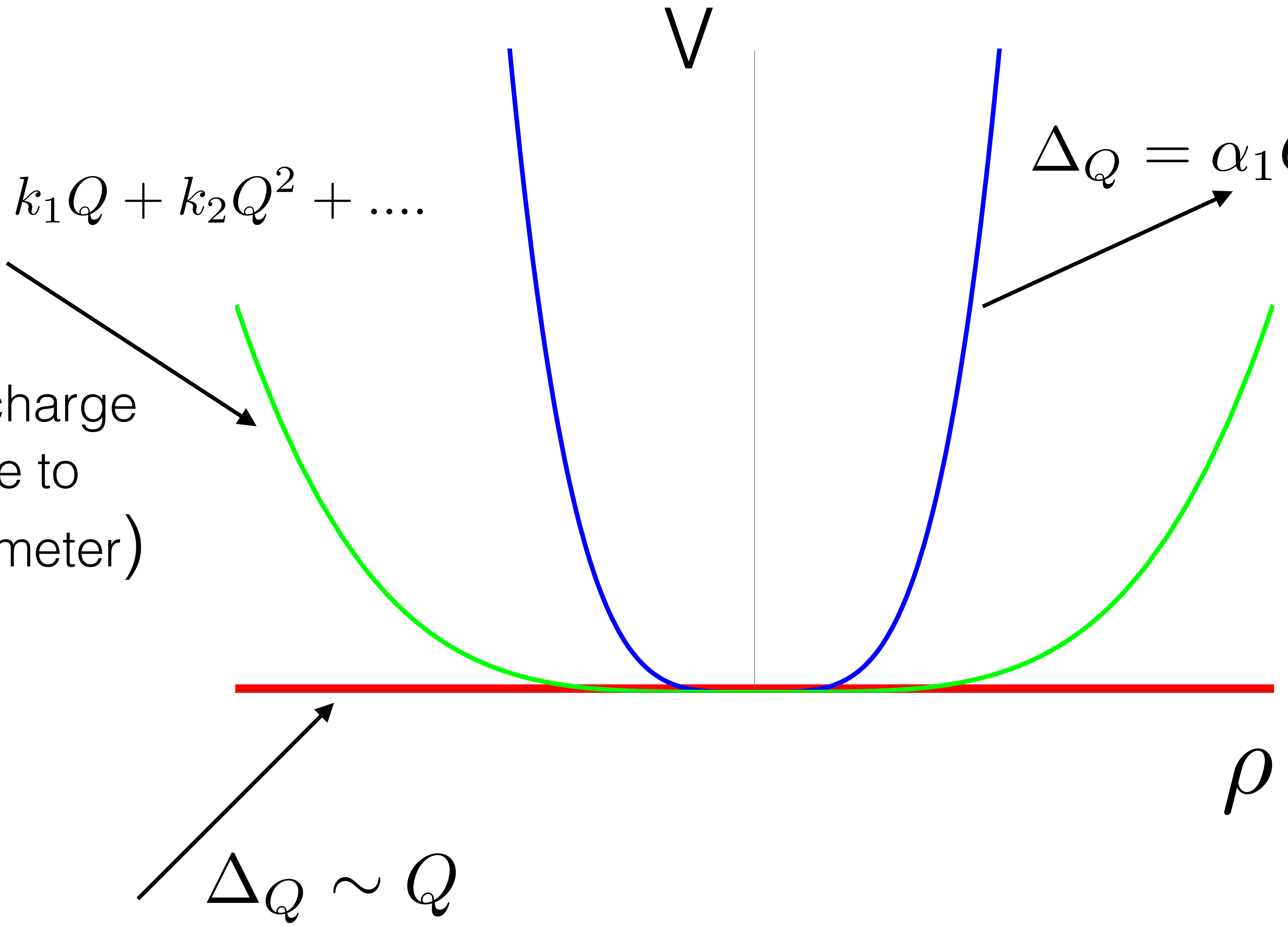
<<1 Superfluid interacts with light radial mode

>>1 Radial mode decouples

Free theory: $\lambda Q = 0$ $\quad \Delta_Q = \frac{d-2}{2} Q$

Also valid for CFT with moduli space: $\Delta_Q \sim Q$ (in SUSY BPS states)

CFT with moduli space + **potential** $V \sim \lambda\phi^4$ for the radial mode ρ



$$\Delta_Q \sim \frac{d-2}{2}Q + k_1 Q + k_2 Q^2 + ...$$

(violates large charge prediction due to extra small parameter)

$$\Delta_Q = \alpha_1 Q^{\frac{d}{d-1}} + \cdots$$

$$\Delta_Q \sim Q$$

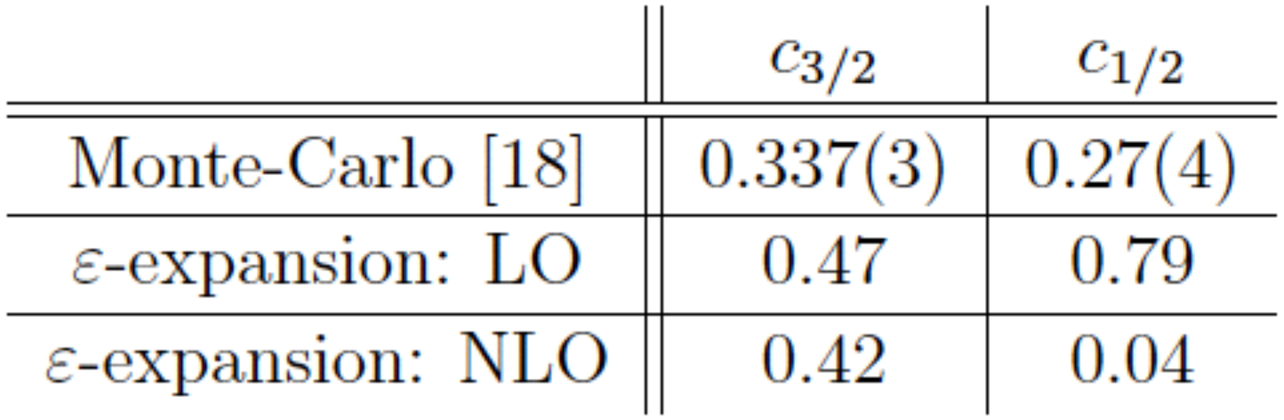
(exact flat direction)

Large charge limit in d=3: LO+NLO

$$\Delta_Q = Q^{\frac{d}{d-1}}\left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \dots\right] + Q^0\left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \dots\right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}}\right)$$

$$\Delta_Q \simeq c_{3/2} Q^{3/2} + c_{1/2} Q^{1/2} - 0.0937 + c_{-1/2} Q^{-1/2} + \mathcal{O}\left(Q^{-1}\right)$$

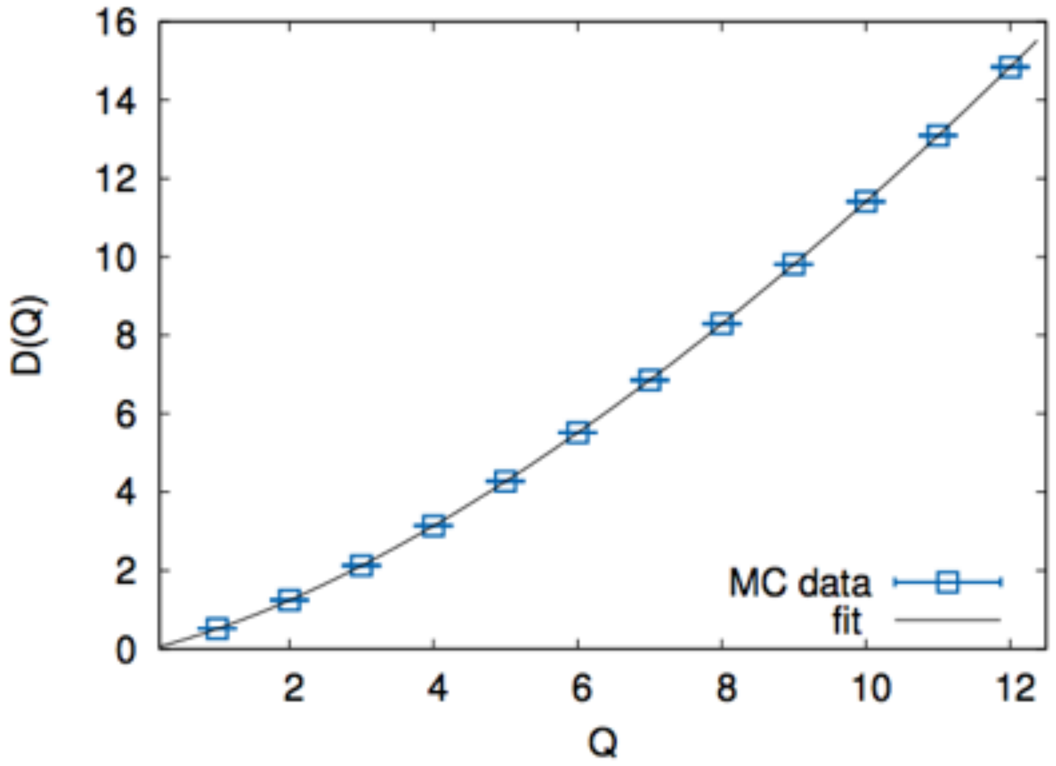
We can fix Wilson coefficients by fitting to the lattice computation.
Then we compare to the large charge prediction



	$c_{3/2}$	$c_{1/2}$
Monte-Carlo [18]	0.337(3)	0.27(4)
ε-expansion: LO	0.47	0.79
ε-expansion: NLO	0.42	0.04

NLO result is closer to the Monte-Carlo

Orlando et al. 1707.00711



(Un)charged $\phi^n(x)$ **operators:** (Real) complex scalar

$$\phi = \frac{\rho}{\sqrt{2}}\cancel{e^{i\chi}} \qquad\qquad m^2 = \left(\tfrac{d-2}{2R}\right)^2$$

$$S_eff = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1}\left(\frac{1}{2}(d\rho)^2 + \frac{1}{2}\rho^2(\cancel{d\chi})^2 + \frac{m^2}{2}\rho^2 + \frac{\lambda}{16}\rho^4\right)$$

Classical solutions:

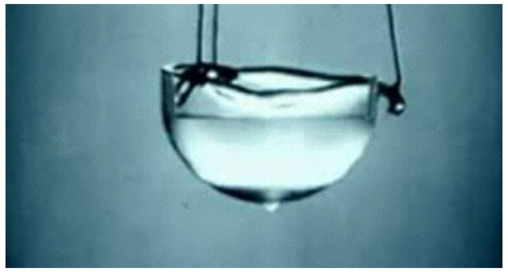
Charged | **Uncharged**

E.O.M:

$$-\partial^2\rho + (m^2 + (\partial\chi)^2)\rho + \frac{\lambda\rho^3}{4} = 0 \qquad\qquad \frac{d^2\rho}{dt^2} + m^2\rho + \lambda\rho^3 = 0$$

$$\rho = f \qquad \chi = -i\mu\tau \qquad\qquad \phi_0 \qquad\qquad \rho(t) = \sqrt{n}\,x_0\,\mathrm{cn}(\omega t|\kappa)$$

time-independent
superfluid solution



time-dependent
<u>periodic</u> solution

Charged

$$-\partial^2\rho + (m^2 + (\partial\chi)^2)\rho + \frac{\lambda\rho^3}{4} = 0$$

$$\rho = f \qquad\qquad \chi = -i\mu\tau$$

Uncharged

$$\rho(t) = \sqrt{n}\, x_0\, \mathrm{cn}(\omega t|\kappa)$$

Quantization conditions

$$\int d^{d-1}x\ j_0(x) = Q \qquad j_\mu(x) = \bar{\phi}\partial_\mu\phi - \phi\partial_\mu\bar{\phi}$$

$$\mu^2 - m^2 = \frac{\lambda}{4}f^2 \qquad \mu f^2 = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

$$\boxed{\mu(\mu^2 - m^2) = \frac{\lambda Q}{4R^{d-1}\Omega_{d-1}}}$$

$$\underbrace{2\pi^2}_{V_{S^3}} \int_0^{\mathcal{T}} \left(\frac{d\rho}{dt}\right)^2 dt = 2\pi n \qquad \mathcal{T} = 4\mathcal{K}(\kappa)/\omega$$

$$\Big\downarrow$$

$$\boxed{\lambda n = \frac{8\pi}{3(1-2\kappa)^{3/2}}\left[(2\kappa - 1)\mathcal{E}(\kappa) + (1-\kappa)\mathcal{K}(\kappa)\right]}$$

$\mathcal{K}(\kappa), \mathcal{E}(\kappa)$ elliptic integrals of 1st and 2nd kind

Plug the solution into the action:

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x + \sqrt{-3 + x^2}\right)^{1/3}}{3^{1/3} + \left(x + \sqrt{-3 + x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3} + \left(x + \sqrt{-3 + x^2}\right)^{2/3}\right)}{\left(x + \sqrt{-3 + x^2}\right)^{1/3}}$$

$$x = 6\lambda_* Q$$

$$\Delta_{-1} = \frac{2\pi^2\kappa\,(1-\kappa)}{\lambda n\,(1-2\kappa)^2}$$

Weak coupling limit

$$x = 6\lambda_* Q$$

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}}$$

$$\frac{\Delta_{-1}}{\lambda_*} = Q\left[1+\frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right)-\frac{1}{2}\left(\frac{\lambda_* Q}{16\pi^2}\right)^2+\cdots\right]$$

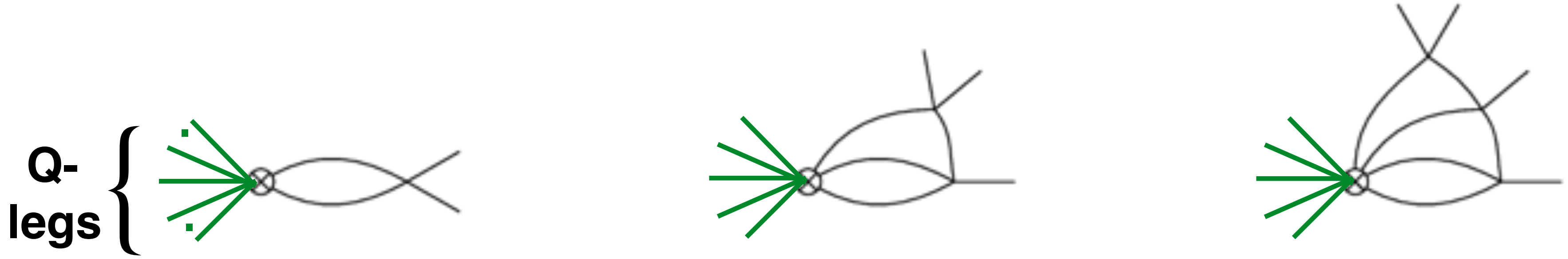
$\lambda Q \ll 1$

$$\Delta_{-1} = \frac{2\pi^2\kappa\left(1-\kappa\right)}{\lambda n\left(1-2\kappa\right)^2}$$

$$\Delta_{-1} = n\left[1+\frac{3\lambda n}{16\pi^2}-\frac{17\lambda^2 n^2}{256\pi^4}+\mathcal{O}\left(\lambda^3 n^3\right)\right]$$

$\lambda n \ll 1$

Classical computation
resums infinite
number of leading
Feynman diagrams



Strong coupling limit

$$\frac{\Delta_{-1}}{\lambda_*} = \frac{8\pi^2}{\lambda^*}\left[\frac{3}{4}\left(\frac{\lambda_* Q}{8\pi^2}\right)^{4/3}+\frac{1}{2}\left(\frac{\lambda_* Q}{8\pi^2}\right)^{2/3}+\cdots\right]$$

$\lambda Q \gg 1$

$$\Delta_{-1} = \left(\frac{3\Gamma\left(\frac{3}{4}\right)}{2^{5/4}\Gamma\left(\frac{1}{4}\right)}\right)^{4/3}\lambda^{1/3}n^{4/3}+\mathcal{O}\left(n^{2/3}\lambda^{-1/3}\right)$$

$\lambda n \gg 1$

EFT matching: Large quantum number expansion

$$\Delta \sim (Q,n)^{\frac{d}{d-1}}$$

Leading quantum correction:

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \ldots$$

$$\rho = f + r(x) \qquad \chi = -i\mu\tau + \frac{\pi(x)}{\sqrt{2}f}$$

$$S^{(2)} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \left[\frac{1}{2}(\partial r)^2 + \frac{1}{2}(\partial \pi)^2 - 2i\mu\, r\partial_\tau \pi + (\mu^2 - m^2) r^2\right]$$

$$\omega_{\pm}^2(\ell) = J_l^2 + 3\mu^2 - m^2 \pm \sqrt{4J_l^2\mu^2 + (3\mu^2 - m^2)^2}$$

$$J_l^2 = \ell(\ell + d - 2)/R^2$$

One relativistic Goldstone boson (the conformal mode=phonon) and one massive state

$$\rho(t) = \sqrt{n}\, x_0\, \mathrm{cn}(\omega t|\kappa) \qquad \phi = \rho(t) + \eta(\vec{x}, t)$$

$$L_2 = \frac{1}{2}\eta O_2 \eta$$

Lame operator:

$$O_2 = -\partial_t^2 + \Delta_{S^{d-1}} - \mu^2 - 3\lambda\, \rho^2(t)$$

Solution is periodic. During the period there instances when v(t) is small/large so radial mode becomes light/heavy

Energy= (divergent) sum of zero point energies

$$\omega_\ell T = \nu_\ell$$

$$\Delta_0 = \frac{R}{2} \sum_{\ell=0}^{\infty} n_\ell \left[\omega_+(\ell) + \omega_-(\ell)\right]$$

$$\Delta_0 = \frac{R}{2T} \sum_{\ell=0}^{\infty} n_l (q_l + \frac{1}{2}) \nu_l$$

Renormalized result: weak coupling limit

$\lambda Q \ll 1$

$$\Delta_0 = -\frac{3\lambda_* Q}{(4\pi)^2} + \frac{\lambda_*^2 Q^2}{2(4\pi)^4} + \cdots$$

$\lambda n \ll 1$

$$\Delta_0^{q_\ell} = \sum_{\ell=1}^{\infty} q_\ell \ell - 2\left(2 + \sum_{\ell=1}^{\infty} \frac{(\ell-1)q_\ell}{\ell+1}\right)\frac{n\lambda_*}{(4\pi)^2}$$
$$+\ldots$$

Classical computation resums infinite number of subleading Feynman diagrams

Strong coupling limit

$$\Delta_Q = Q^{\frac{d}{d-1}}\left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \ldots\right] + Q^0\left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \ldots\right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}}\right)$$

EFT matching: Large quantum number expansion

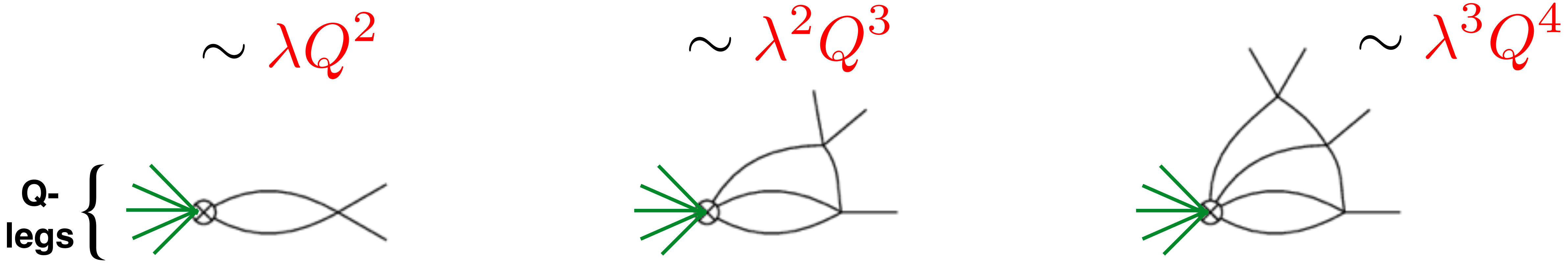
Weakly coupled CFT
(Perturbative asymptotic)

Charged operators **perturbative LO+NLO** result

$$\Delta_{\phi^Q} \equiv Q\left(\frac{d-2}{2}\right) + \gamma_{\phi^Q} \quad = \Delta_{-1} + \Delta_0 + \cdots$$

$$\overset{\text{2-loops}}{\gamma_{\phi^Q}} = Q\left[\frac{\lambda_*}{16\pi^2}\,\frac{(Q-1)}{2} - \left(\frac{\lambda_*}{16\pi^2}\right)^2 \frac{2Q^2-2Q-1}{4}\right] + \cdots$$

Perfect agreement for coloured terms with diagrammatics



Our results are valid for any λ just as Feynman diagrams do not know about fixed point

Overlap of two expansions

	1-loop	**2-loop**	**3-loop**	
Δ_{-1}	$Q^2\lambda_0$	$Q^3\lambda_0^2$	$Q^4\lambda_0^3$	$\cdots$
Δ_0	$Q\lambda_0$	$Q^2\lambda_0^2$	$Q^3\lambda_0^3$	$\cdots$
Δ_1		$Q\lambda_0^2$	$Q^2\lambda_0^3$	$\cdots$
Δ_2			$Q\lambda_0^3$	$\cdots$
$\vdots$				

Neutral operators **perturbative LO+NLO result**

$$\Delta_{n,\{q_\ell\}} = n + \sum_{\ell=1}^{\infty} q_\ell \ell + \frac{1}{6}\left[n^2 - 2\left(2 + \sum_{\ell=1}^{\infty}\frac{(\ell-1)q_\ell}{\ell+1}\right)n + O(n^0)\right]\epsilon$$

$$-\frac{1}{324}\left[17n^3 - \left(67 + 3\sum_{\ell=1}^{\infty}\frac{(\ell-1)\left(17\ell^4+78\ell^3+135\ell^2+98\ell+12\right)q_\ell}{\ell(\ell+1)^3(\ell+2)}\right)n^2 + O(n,n^0)\right]\epsilon^2 + O(\epsilon^3)$$

q_l

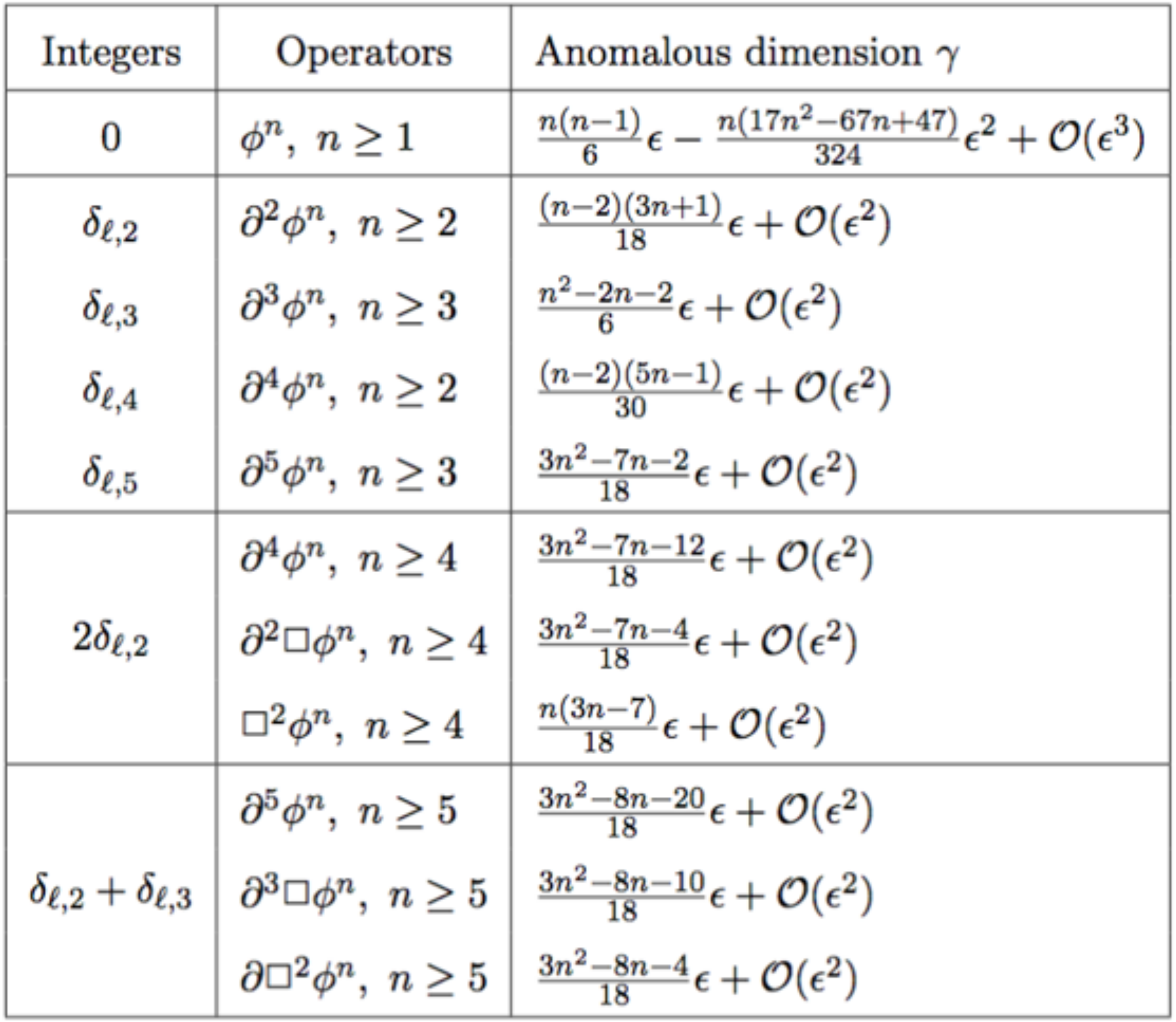
Exciting mode-L adds L derivatives to the operator

Exciting l=2 mode twice

$4 \oplus 2 \oplus 0$

Exciting l=2 and l=3 mode once

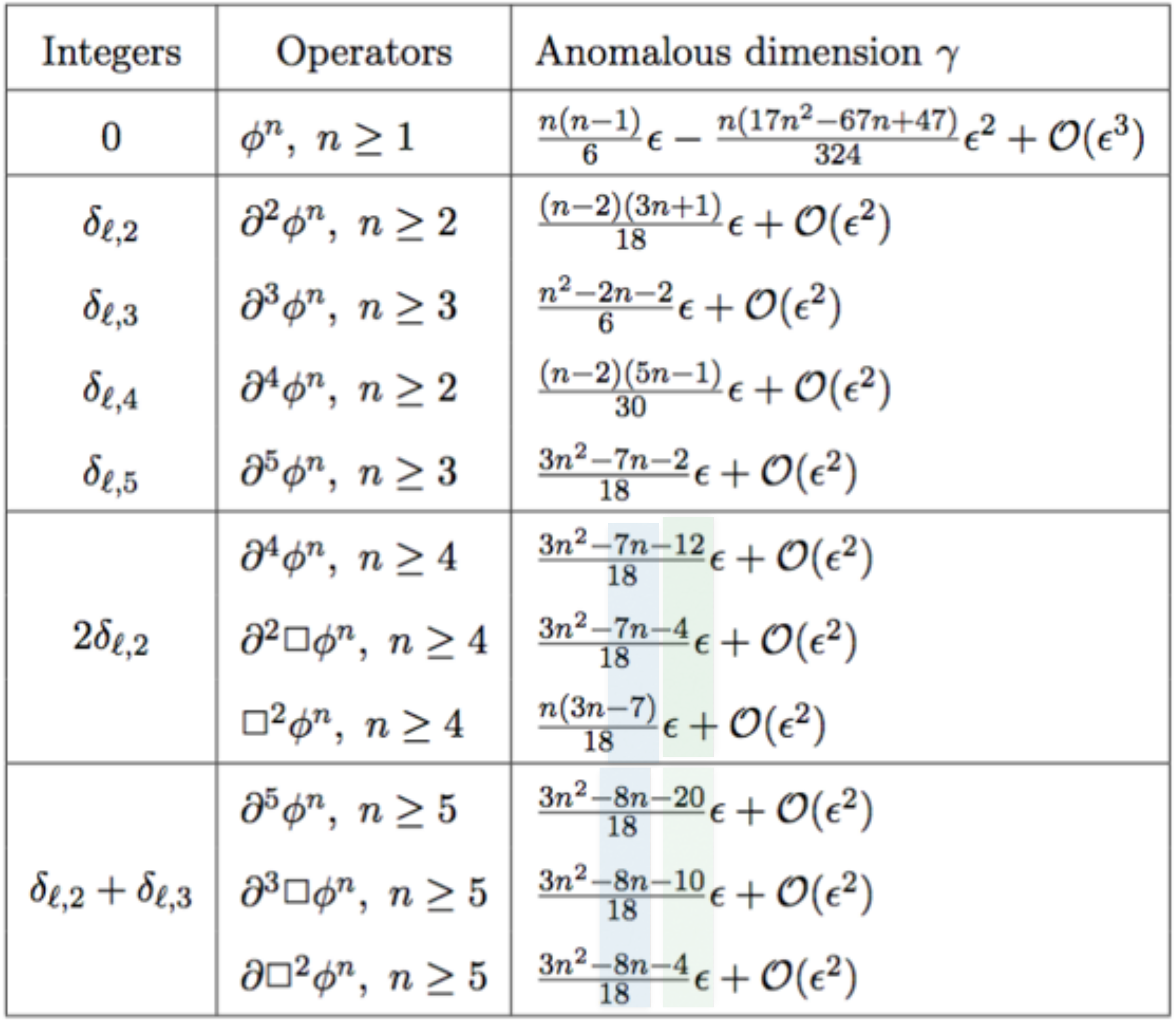
$5 \oplus 3 \oplus 1$



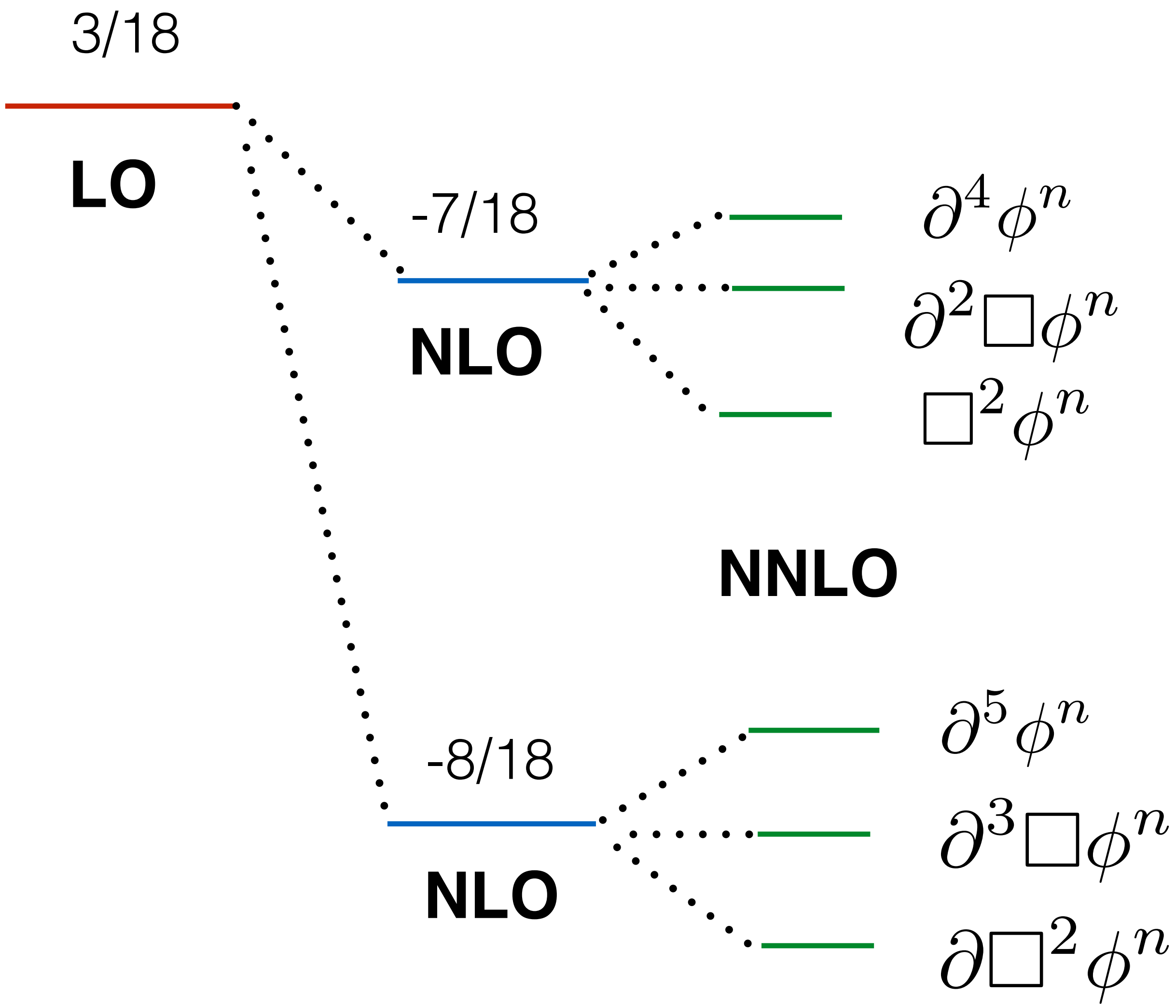
Integers	Operators	Anomalous dimension γ
0	$\phi^n,\ n\geq 1$	$\frac{n(n-1)}{6}\epsilon - \frac{n(17n^2-67n+47)}{324}\epsilon^2 + \mathcal{O}(\epsilon^3)$
$\delta_{\ell,2}$	$\partial^2\phi^n,\ n\geq 2$	$\frac{(n-2)(3n+1)}{18}\epsilon + \mathcal{O}(\epsilon^2)$
$\delta_{\ell,3}$	$\partial^3\phi^n,\ n\geq 3$	$\frac{n^2-2n-2}{6}\epsilon + \mathcal{O}(\epsilon^2)$
$\delta_{\ell,4}$	$\partial^4\phi^n,\ n\geq 2$	$\frac{(n-2)(5n-1)}{30}\epsilon + \mathcal{O}(\epsilon^2)$
$\delta_{\ell,5}$	$\partial^5\phi^n,\ n\geq 3$	$\frac{3n^2-7n-2}{18}\epsilon + \mathcal{O}(\epsilon^2)$
	$\partial^4\phi^n,\ n\geq 4$	$\frac{3n^2-7n-12}{18}\epsilon + \mathcal{O}(\epsilon^2)$
$2\delta_{\ell,2}$	$\partial^2\Box\phi^n,\ n\geq 4$	$\frac{3n^2-7n-4}{18}\epsilon + \mathcal{O}(\epsilon^2)$
	$\Box^2\phi^n,\ n\geq 4$	$\frac{n(3n-7)}{18}\epsilon + \mathcal{O}(\epsilon^2)$
	$\partial^5\phi^n,\ n\geq 5$	$\frac{3n^2-8n-20}{18}\epsilon + \mathcal{O}(\epsilon^2)$
$\delta_{\ell,2}+\delta_{\ell,3}$	$\partial^3\Box\phi^n,\ n\geq 5$	$\frac{3n^2-8n-10}{18}\epsilon + \mathcal{O}(\epsilon^2)$
	$\partial\Box^2\phi^n,\ n\geq 5$	$\frac{3n^2-8n-4}{18}\epsilon + \mathcal{O}(\epsilon^2)$

one-loop perturbative results

Spectroscopy of composite operators



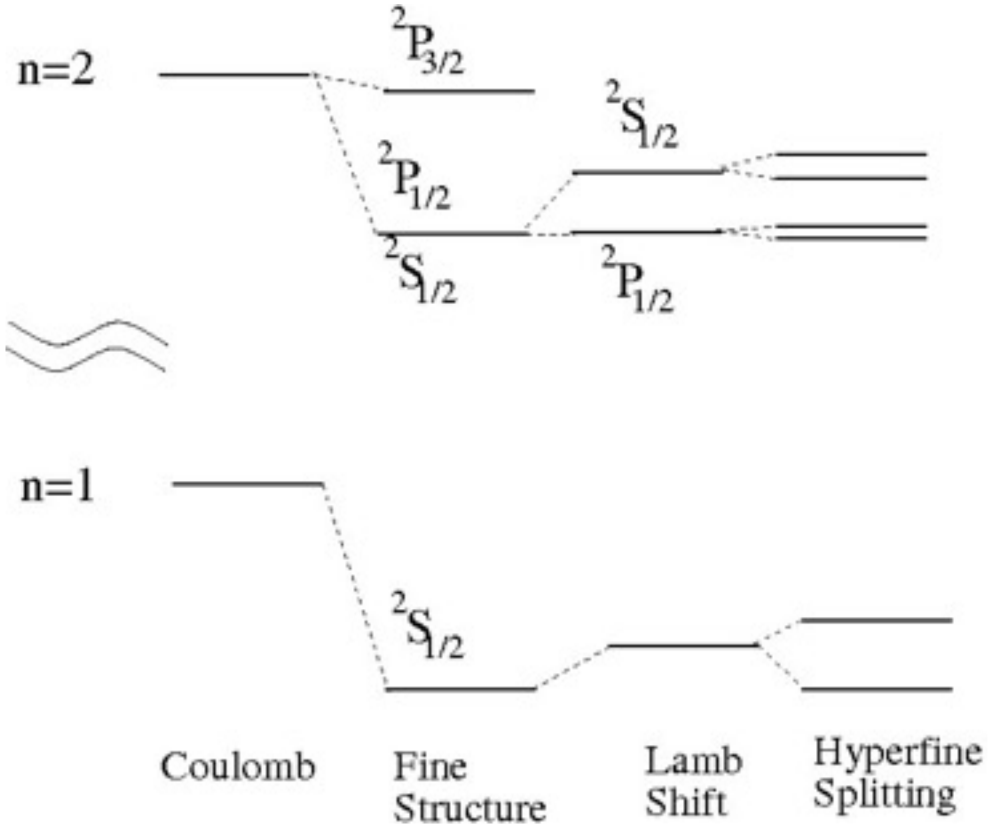
Integers	Operators	Anomalous dimension γ
0	$\phi^n,\ n\geq 1$	$\frac{n(n-1)}{6}\epsilon-\frac{n(17n^2-67n+47)}{324}\epsilon^2+\mathcal{O}(\epsilon^3)$
$\delta_{\ell,2}$	$\partial^2\phi^n,\ n\geq 2$	$\frac{(n-2)(3n+1)}{18}\epsilon+\mathcal{O}(\epsilon^2)$
$\delta_{\ell,3}$	$\partial^3\phi^n,\ n\geq 3$	$\frac{n^2-2n-2}{6}\epsilon+\mathcal{O}(\epsilon^2)$
$\delta_{\ell,4}$	$\partial^4\phi^n,\ n\geq 2$	$\frac{(n-2)(5n-1)}{30}\epsilon+\mathcal{O}(\epsilon^2)$
$\delta_{\ell,5}$	$\partial^5\phi^n,\ n\geq 3$	$\frac{3n^2-7n-2}{18}\epsilon+\mathcal{O}(\epsilon^2)$
$2\delta_{\ell,2}$	$\partial^4\phi^n,\ n\geq 4$	$\frac{3n^2-7n-12}{18}\epsilon+\mathcal{O}(\epsilon^2)$
	$\partial^2\Box\phi^n,\ n\geq 4$	$\frac{3n^2-7n-4}{18}\epsilon+\mathcal{O}(\epsilon^2)$
	$\Box^2\phi^n,\ n\geq 4$	$\frac{n(3n-7)}{18}\epsilon+\mathcal{O}(\epsilon^2)$
$\delta_{\ell,2}+\delta_{\ell,3}$	$\partial^5\phi^n,\ n\geq 5$	$\frac{3n^2-8n-20}{18}\epsilon+\mathcal{O}(\epsilon^2)$
	$\partial^3\Box\phi^n,\ n\geq 5$	$\frac{3n^2-8n-10}{18}\epsilon+\mathcal{O}(\epsilon^2)$
	$\partial\Box^2\phi^n,\ n\geq 5$	$\frac{3n^2-8n-4}{18}\epsilon+\mathcal{O}(\epsilon^2)$



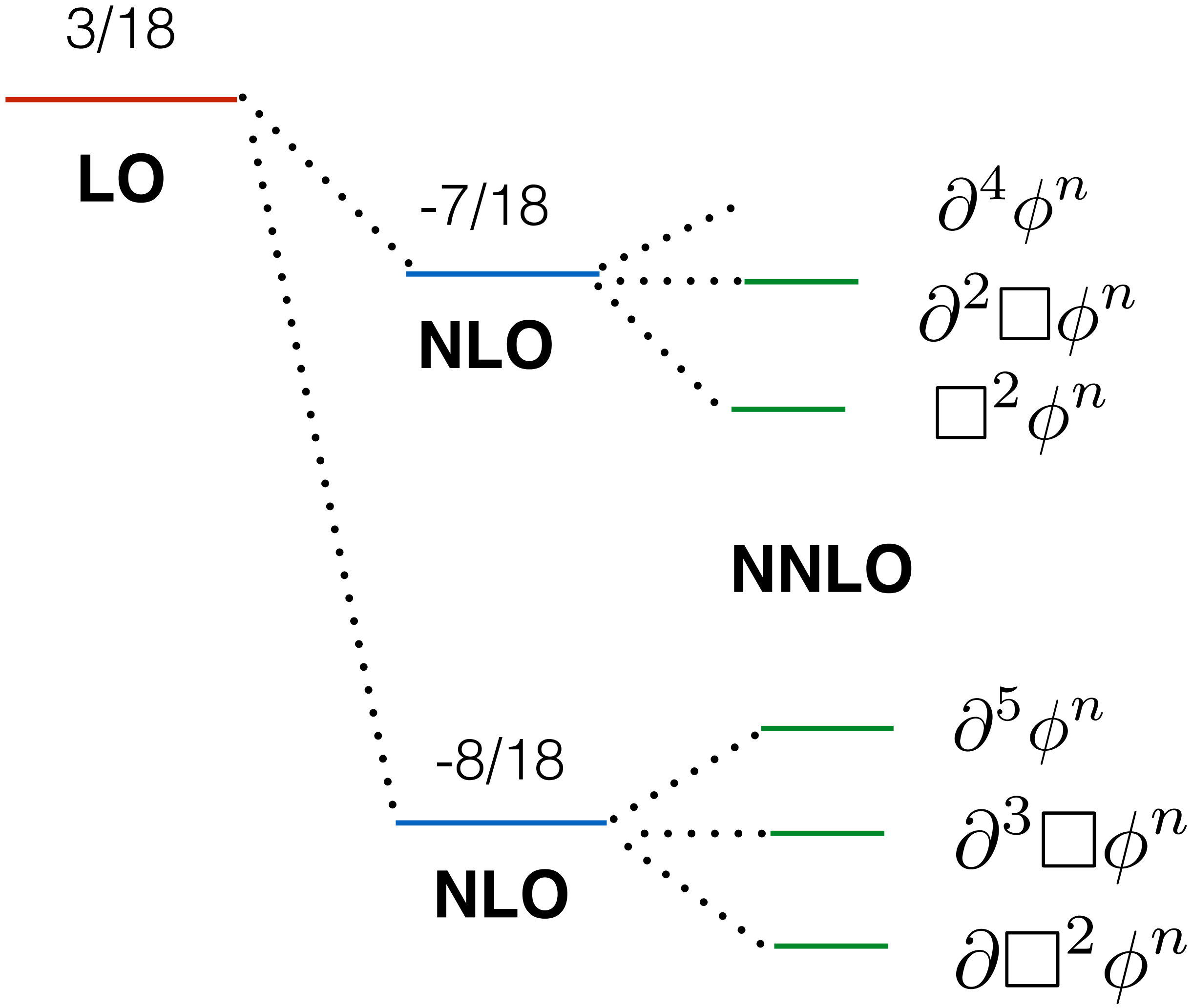
Neutral operators to NLO

$$\Delta_{n,\{q_\ell\}} = n + \sum_{\ell=1}^{\infty} q_\ell \ell + \frac{1}{6}\left[n^2 - 2\left(2+\sum_{\ell=1}^{\infty}\frac{(\ell-1)q_\ell}{\ell+1}\right)n + O(n^0)\right]\epsilon$$

$$-\frac{1}{324}\left[17n^3 - \left(67+3\sum_{\ell=1}^{\infty} \frac{(\ell-1)\left(17\ell^4+78\ell^3+135\ell^2+98\ell+12\right)q_\ell}{\ell(\ell+1)^3(\ell+2)}\right)n^2 + O(n,n^0)\right]\epsilon^2 + O(\epsilon^3)$$



hydrogen atom



3/18

LO

-7/18

NLO

$\partial^4\phi^n$

$\partial^2\Box\phi^n$

$\Box^2\phi^n$

NNLO

-8/18

NLO

$\partial^5\phi^n$

$\partial^3\Box\phi^n$

$\partial\Box^2\phi^n$

Other directions/aspects

- In a generic scalar QFT, I showed how to semiclassically compute anomalous dimensions for neutral and charged operators

- Large order behaviour of the series (resurgence)
- Higher correlation functions
- Condensed matter applications (3d Ising, for example)
- Test dualities between different CFTs in their charged sectors
- Phenomenology: Multi-particle (a.k.a. Higgsplosion)
- Add Yukawa and gauge interactions towards full Higgs SMEFT
- …………

Thank you!

(Some) references

1) Hellerman et al	*JHEP 12 (2015) 071*	Original paper for large charge EFT
2) Rattazzi et al	*JHEP 11 (2019) 110*	Semiclassical method
3) Orlando et al	Phys.Rept. 933 (2021)	Mini-review
4) Jack and Jones	*Phys.Rev.D* 103 (2021) 8, 085013	Higher loops checks
5) Antipin et al	*Phys.Rev.D* 102 (2020) 4, 045011	O(N) model charged
6) Antipin et al	Phys.Rev.D 111 (2025) 4, L041701	Neutral operators to LO
7) Antipin et al	hep-th: **2511.08276**	Neutral operators to NLO
8) Bednyakov et al	hep-th: **2512.05059**	phi6 model to NLO diagrammatically

EFT : Large quantum number expansion

Double scaling limit….

$\lambda \to 0 \qquad Q \to \infty \qquad \lambda Q = fixed$

<<1

Superfluid interacts with
light radial mode

>>1

Radial mode decouples
and we write EFT for phonon

$$\Delta_Q = Q^{\frac{d}{d-1}} \left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \ldots\right] + Q^0 \left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \ldots\right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}}\right)$$

Now, let’s discuss deeper full LO+NLO perturbative results…