Method of Hyperlogarithms: a non-trivial application to the theory of turbulence

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- Motivation for hyperlogarithm approach
- Integration algorithm
- Connection to MZV
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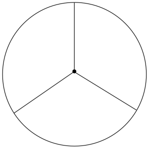
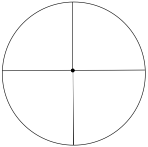
MZVs in Feynman diagrams

A wide class of Feynman diagrams are expressed in terms of the multiple zeta functions (MZV).

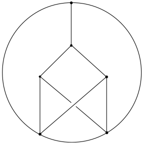
$$\zeta(n_1, n_2, ..., n_m) = \sum_{0<k_1<k_2<...<k_m} \frac{1}{k_1^{n_1} k_2^{n_2} ... k_m^{n_m}}$$



$6\zeta(3)$



$20\zeta(5)$



$36\zeta(3)^2$

- What diagrams are expressed in terms of MZV?
- What is more general structure for their values?
- **How to analytically compute diagrams evaluated to MZVs?**

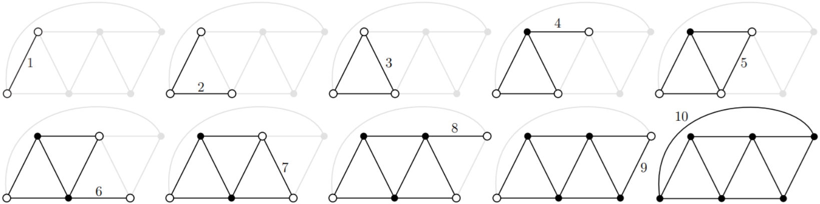
MZVs in Feynman diagrams

The structure of MZV expressions depends on the topology of a diagram.

Theorem

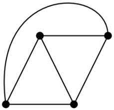
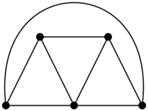
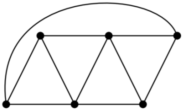
If a diagram has vertex-width ≤ 3, then all coefficients of the its ε-expansion are rational linear combinations of MZVs. (F. Brown, 2009)

Example of a diagram with vertex-width $= 3$:



MZVs in Feynman diagrams

Example of such diagrams is a class of zig-zag diagrams:

$ZZ_3 =$  $ZZ_4 =$  $ZZ_5 =$  $\cdots$

Theorem

$$I_{ZZ_n} = 4\frac{(2n-2)!}{n!(n-1)!}\left(1 - \frac{1-(-1)^n}{2^{2n-1}}\right)\zeta(2n-3)$$

(F. Brown, O. Schnetz, 2012)

Feynman parametrization

Consider a massless diagram G with l loops in $d = 4 - 2\varepsilon$. Parameters u_e correspond to each edge e with propagator q_e.

$$I_G = \int_{\mathbb{R}^d} dk_1^d ... dk_l^d \prod_{e \in E(G)} \left(\frac{1}{q_e} \right)$$

$$\frac{1}{q_1 ... q_n} = \Gamma(n) \int_0^1 du_1 ... du_n \frac{\delta(\sum_{i=1}^n u_i - 1)}{\left[\sum_{i=1}^n q_i u_i \right]^n}$$

$$\frac{1}{(2\pi)^{dl}} \int dk_1^d ... dk_l^d \frac{1}{(v_{ij} k_i k_j + a_i k_i + c)^\alpha} = \frac{(4\pi)^{-dn/2} \Gamma(\alpha - dn/2)}{\Gamma(\alpha)(det\, v)^{d/2} \left[c - (v^{-1})_{ij} a_i a_j \right]^{\alpha - dn/2}}$$

Let G be primitive and log-divergent. Hence, the pole residue is proportional to the following convergent integral

$$I_G(\varepsilon = 0) \propto \int_0^1 du_1 ... du_n \frac{\delta(\sum_{i=1}^n u_i - 1)}{(det\, v)^2} = \int_0^\infty du_1 ... du_{n-1} \frac{1}{(det\, v)^2} \Big|_{u_n = 1}$$

det $v \equiv \Psi_G = \sum_T \prod_{e \notin T} u_e$ (Symanzik polynomial) is **linear** in each integration parameter.

Motivation for hyperlogarithms

Since Ψ_G is linear in each parameter u_i, we can integrate over u_1 (with $u_n = 1$):

$$\int_0^\infty du_1 ... du_{n-1} \frac{1}{\Psi_G^2} = \int_0^\infty du_1 ... du_{n-1} \frac{1}{(V_1 + U^{(1)} u_1)^2} = \int_0^\infty du_2 ... du_{n-1} \frac{1}{V_1 U^{(1)}}$$

With V_1 and $U^{(1)}$ being linear in all remaining parameters, the integration over u_2 is also straightforward:

$$\int_0^\infty du_2 \dots du_{n-1} \frac{1}{V_1 U^{(1)}} = \int_0^\infty du_2 \dots du_{n-1} \frac{1}{(U_2 + U^{(2)} u_2)(V_2 + V^{(2)} u_2)} =$$

$$\int_0^\infty du_2 \dots du_{n-1} \frac{1}{U^{(2)} V_2 - U_1 V^{(2)}} \left(\frac{U^{(2)}}{U_2 + U^{(2)} u_2} - \frac{V^{(2)}}{V_2 + V^{(2)} u_2} \right) =$$

$$\int_0^\infty du_3 \dots du_{n-1} \frac{\ln U^{(2)} - \ln U_2 - \ln V^{(2)} + \ln V_2}{U^{(2)} V_2 - U_2 V^{(2)}}$$

Due to *Dodgson identity* the denominator factorizes as the square of a polynomial which is linear in each variable, so we can integrate over u_3 as well. As long as there exists a variable with respect to which all polynomials which occur in the integrand are linear, this can be repeated. On the 4th integration step, we has to introduce dilogarithm and etc.

Hyperlogarithms

Hyperlogarithm(HL) is defined as iterated integral:

$$L_{\omega_{\sigma_1}\omega_{\sigma_2}...\omega_{\sigma_n}}(z) := \int_0^z \frac{dz_1}{z_1-\sigma_1}\int_0^{z_1}\frac{dz_2}{z_2-\sigma_2}...\int_0^{z_{n-1}}\frac{dz_n}{z_n-\sigma_n}$$

The letter ω_{σ_i} is a symbol for $dz_i/(z_i-\sigma_i)$, where $\sigma_i \in \mathbb{C}$. The combination of letters forms a word $w = \omega_{\sigma_1}...\omega_{\sigma_n}$.

$$L_{\omega_\sigma}(z) = \int_0^z \frac{dz_1}{z_1-\sigma} = \ln(z-\sigma) - \ln(-\sigma)$$

The derivative of HL follows from its definition:

$$\partial_z L_{\omega_\sigma w}(z) = \frac{1}{z-\sigma} L_w(z)$$

Hyperlogarithms $L_w(z)$ satisfy the *shuffle relations*:

$$L_{w_1}(z)\cdot L_{w_2}(z) = L_{w_1 ш w_2}(z)$$

This allows to rewrite the product of HLs in terms of combination of HLs with higher weight:

$$L_{\omega_{\sigma_1}}(z)\, L_{\omega_{\sigma_2}}(z) = L_{\omega_{\sigma_1}\omega_{\sigma_2}}(z) + L_{\omega_{\sigma_2}\omega_{\sigma_1}}(z)\ .$$

Hyperlogarithms

It turns out that on the k^{th} integration step the integrand can be expressed as the sum of products of rational functions $g_i(z)$ and HLs with some words w_i.

$$\int_0^\infty dz_k f_{k-1}(z_k) = \int_0^\infty dz_k \sum_i g_i(z_k) L_{w_i}(z_k)$$

If polynomials in $g_i(z_k)$ are linear in z_k, by decomposing them into partial fractions the evaluation of the integral is reduced to finding primitives of functions of the form

$$(z_k - \sigma_i)^n L_w(z_k)\ , \quad n \in \mathbf{Z}\,.$$

In case of $n = -1$, integration result is obvious:

$$\int_0^\infty dz_k \frac{L_w(z_k)}{z_k - \sigma} = L_{\omega_\sigma w}(z_k)\Big|_0^\infty$$

.

For other values of n, one needs to apply partial integration formula

$$\int_0^\infty dz \frac{L_w(z)}{(z-\sigma_i)^{n+1}} = -\frac{L_w(z)}{n(z-\sigma_i)^n}\Big|_0^\infty + \int_0^\infty dz \frac{\partial_z L_w(z)}{n(z-\sigma_i)^n}\ .$$

Regularization

The next step is to evaluate the limits

$$\int_0^\infty f(z)dz = \lim_{z\longrightarrow\infty} F(z) - \lim_{z\longrightarrow 0} F(z) = \underset{z=\infty}{\mathrm{Reg}} F(z) - \underset{z=0}{\mathrm{Reg}} F(z) \ .$$

There is a potential logarithmic divergence at ∞ which demands proper regularization. Even in case of convergent overall integral, during integration process divergent HLs occurs. Regularization at 0 is trivial

$$\underset{z=0}{\mathrm{Reg}}\, L_w(z) = 0 \ .$$

There are also some obvious examples of regularization at ∞:

$$\underset{z=\infty}{\mathrm{Reg}} L_{\omega_\sigma}(z) = \underset{z=\infty}{\mathrm{Reg}} \int_0^z \frac{dz}{z-\sigma} = \underset{z=\infty}{\mathrm{Reg}} \left[\ln(z-\sigma) - \ln(\sigma)\right] = -\ln(\sigma) \ ,$$

$$\underset{z=\infty}{\mathrm{Reg}} L_{\omega_0}(z) = \underset{z=\infty}{\mathrm{Reg}} \int_0^z \frac{dz}{z} = 0 \ .$$

It turns out that the result of integration is expressed in term of HL with letters $0, -1$ that provides connection to MZV.

Regularization

Integration process is reduced to regularization of the HL $L_{\omega_{\sigma_1}\ldots\omega_{\sigma_r}}(z)$, where $\sigma_k \in \{0, -1\}$. Formally, the regularization is defined as some manipulation with HL word, which cancels divergencies:

$$\operatorname*{Reg}_{z=\infty} L_w(z) = L_{\mathsf{reg}^\infty(w)}(\infty) \ ,$$

The origin of this can be seen from the fact, that if we subtract from a word itself with the first letter changed to ω_{-1}, it would produce a HL finite at ∞:

$$L_{(\omega_\sigma - \omega_{-1})w}(\infty) = \int_0^\infty dz' \, \frac{(1+\sigma)L_w(z')}{(z'-\sigma)(z'+1)} \neq \infty \ , \text{where} \quad L_{(\omega_\sigma - \omega_{-1})w} = L_{\omega_\sigma w} - L_{\omega_{-1} w}$$

Even though this expression is finite, this procedure is not correct regularization because besides subtraction of divergence it subtracts some finite part as well. The general regularization operation is provided by

$$\mathsf{reg}^\infty(\omega_{\sigma_1}\ldots\omega_{\sigma_r}) := \sum_{k=1}^{r} (\omega_{\sigma_k} - \omega_{-1}) \left[(-\omega_{-1})^{k-1} ш \ \omega_{\sigma_{k+1}}\ldots\omega_{\sigma_r}\right] \ ,$$

Regularization

Some examples of words regularization:

$$\text{reg}^{\infty}(\omega_0\omega_{-1}) = (\omega_0 - \omega_{-1})\left[e \text{ ш } \omega_{-1}\right] + (\omega_{-1} - \omega_{-1})\left[(-\omega_{-1}) \text{ ш } e\right] = \omega_0\omega_{-1} - \omega_{-1}^2$$

$$\begin{aligned}
&\text{reg}^{\infty}(\omega_0\omega_0\omega_{-1}\omega_{-1}) = \\
&= (\omega_0 - \omega_{-1})\left[e \,\text{ш}\, \omega_0\omega_{-1}\omega_{-1}\right] + (\omega_0 - \omega_{-1})\left[(-\omega_{-1}) \,\text{ш}\, \omega_{-1}\omega_{-1}\right] + 0 + 0 \\
&= (\omega_0 - \omega_{-1})\omega_0\omega_{-1}\omega_{-1} - 3(\omega_0 - \omega_{-1})\omega_{-1}^3
\end{aligned}$$

Connection to MZVs

Regularized HL with letters $\{0, -1\}$ can be expressed in terms of MZVs. Let us recall the integral representation for Riemann ZV:

$$\zeta(n) = -\int_0^1 \frac{dt_1}{t_1} \int_0^{t_1} \frac{dt_2}{t_2} ... \int_0^{t_{n-2}} \frac{dt_{n-1}}{t_{n-1}} \ln(1 - t_{n-1}) = L_{\omega_0^{n-1}\omega_1}(1) \ .$$

MZV integral representation can be obtained by inserting certain amount of letters ω_1, namely,

$$\zeta(n_1, n_2, ..., n_m) = (-1)^m L_{\omega_0^{n_m-1}\omega_1 ... \omega_0^{n_2-1}\omega_1\omega_0^{n_1-1}\omega_1}(1) \ .$$

Such HL can be constructed from the regularized limits $\operatorname*{Reg}_{z=\infty} L_{\omega_{\sigma_1}\omega_{\sigma_2}...\omega_{\sigma_n}}(z)$ with letters ω_0 and ω_{-1} with the following change of variables

$$z_i = \frac{t_i}{1 - t_i}, \quad \frac{dz_i}{z_i} = \frac{dt_i}{t_i} + \frac{dt_i}{1 - t_i}, \quad \frac{dz_i}{z_i + 1} = \frac{dt_i}{1 - t_i} \ ,$$

which corresponds to the change of letters

$$\omega_0 \ \longrightarrow \ \omega_0 - \omega_1 \ , \qquad \omega_{-1} \longrightarrow \ -\omega_1 \ .$$

Connection to MZVs

It is known that MZVs with weights (sum of the arguments) up to and including 7 can be expressed as the sum of products of ZVs $\zeta(n)$. For example,

$$\operatorname*{Reg}_{z=\infty} L_{\omega_0\omega_{-1}^n}(z) = L_{(\omega_0-\omega_{-1})\omega_{-1}^n}(\infty) = (-1)^n L_{\omega_0\omega_1^n}(1) = (-1)^n \zeta(\underbrace{1,1,...,1}_{n},2) = \zeta(n+1)$$

$$\begin{aligned}\operatorname*{Reg}_{z=\infty} L_{\omega_0\omega_0\omega_{-1}\omega_{-1}}(z) =& L_{(\omega_0-\omega_{-1})\omega_0\omega_{-1}^2}(\infty) - 3L_{(\omega_0-\omega_{-1})\omega_{-1}^3}(\infty) = \\ & L_{\omega_0(\omega_0-\omega_1)\omega_1^2}(1) + 3L_{\omega_0\omega_1^3}(1) = L_{\omega_0^2\omega_1^2}(1) + 2L_{\omega_0\omega_1^3}(1) = \\ & \zeta(1,3) - 2\zeta(1,1,2) = \frac{1}{10}\zeta^2(2) - \frac{4}{5}\zeta^2(2) = -\frac{7}{10}\zeta^2(2)\end{aligned}$$

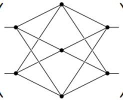
Requirements

Requirements of the hyperlogarithm method:

- Even critical dimension
- Diagrams need to be written in Feynman representation
- Not complex kinematics
- Finiteness of calculated expression
- **Linear reducibility**

Linear reducibility is a property of multiple integral that ensures that there is an integration order for which on each integration step rational functions in the integrand are linear in the next integration parameter. If it is, one always can express the next integral in terms of HLs. Reducibility of the integrand can be checked before integration.

6-loop diagram which is **not** linear reducible, but still evaluates to MZVs:

$\Phi\Bigg($  $\Bigg) = \frac{288}{30\varepsilon}\left(58\zeta_8 - 45\zeta_3\zeta_5 - 24\zeta_{3,5}\right) + \mathcal{O}\left(\varepsilon^0\right)$

HyperInt - Maple implementation of the HL integration algorithm

E. Panzer, Algorithms for the symbolic integration of hyperlogarithms with applications to Feynman integrals, Comput. Phys. Commun. 188 (2015)

Achievments:

- 6- and 7-loop calculations in ϕ^4 theory

  M. V. Kompaniets and E. Panzer, Minimally subtracted six-loop renormalization of $O(n)$-symmetric ϕ^4 theory and critical exponents, Phys.Rev.D 96 (2017) p

  O. Schnetz, ϕ^4 theory at seven loops, Phys.Rev.D 107 (2023)

- Multiloop calculations in QCD

  A. von Manteuffel, E. Panzer and R. M. Schabinger, Computation of form factors in massless QCD with finite master integrals, Phys.Rev.D 93 (2016)

  B. Agarwal, A. von Manteuffel, E. Panzer and R. M. Schabinger, Four-loop collinear anomalous dimensions in QCD and N=4 super Yang-Mills, Phys.Lett.B 820 (2021)

QFT approach in the theory of turbulence

The fully developed turbulence is described by the stochastic Navier-Stokes equation with the random stirring force

$$\partial_t v_i = -\partial_i P - (v_k \partial_k) v_i + \nu_0 \partial^2 v_i + f_i \ , \qquad \partial_i = \partial_{x_i}, \qquad \partial_i v_i = 0 \ .$$

The random force f_i is defined by the following statistics

$$\langle f_i(\mathbf{x}_1, t_1) f_j(\mathbf{x}_2, t_2) \rangle \equiv D^f_{ij}(\mathbf{x}_1 - \mathbf{x}_2, t_1 - t_2)$$

$$D^f_{ij}(\mathbf{k}, t) = \delta(t) P_{ij}(\mathbf{k}) d_f(\mathbf{k}) \ , \qquad P_{ij}(\mathbf{k}) = \delta_{ij} - \frac{k_i k_j}{k^2} \ .$$

The pumping function $d_f(\mathbf{k})$ describes injection of energy into the system. In the inertial interval of the wavenumbers $m \ll k \ll k_{diss}$

$$d_f(\mathbf{k}) = D_0 k^{4-d-2\varepsilon}, \quad \varepsilon_{phys} = 2.$$

The parameter ε is **not** related to the dimension of the space.

QFT approach in the theory of turbulence

The Martin-Siggia-Rose formalism relates SDE to QFT with a given action with a doubled set of fields

$$S_0 = \frac{1}{2} v' D^f v' + v' \left[-\partial_t v - (v\partial) v + \nu_0 \partial^2 v \right] .$$

The diagrams in the perturbation theory with action S_0 contain UV divergences in the limit $\varepsilon \to +0$, which take place only in the 1-irreducible Green function $\Gamma_{ij} = \left\langle v_i v'_j \right\rangle_{1-irr}$. The renormalized action is given by

$$S = \frac{1}{2} v' D^f v' + v' \left[-\partial_t v - (v\partial) v + \nu Z_\nu \partial^2 v \right] ,$$

$$D_0 = g_0 \nu_0^3 = g \mu^{2\varepsilon} \nu^3 \ , \quad \nu_0 = \nu Z_\nu \ , \quad g_0 = g \mu^{2\varepsilon} Z_g \ , \quad Z_g = Z_\nu^{-3} \ .$$

QFT approach in the theory of turbulence

The free propagators in the (k, t) representation take the form

$$\langle v_i(t_1)v_j(t_2)\rangle(\mathbf{k}) = \frac{g\mu^{2\varepsilon}\nu^2}{2}k^{2-d-2\varepsilon}e^{-\nu k^2\cdot|t_1-t_2|}P_{ij}(\mathbf{k})$$

$$\langle v_i(t_1)v'_j(t_2)\rangle(\mathbf{k}) = \theta(t_1-t_2)e^{-\nu k^2(t_1-t_2)}P_{ij}(\mathbf{k})$$
$$\langle v'_i(t_1)v'_j(t_2)\rangle(\mathbf{k}) = 0 \qquad .$$

The interaction in (1) is represented by a triple vertex $-v'(v\partial)v = v'_m V_{mnp} v_n v_p$ with a vertex factor

$$V_{mnp} = ik_n\delta_{mp}$$

where k_n is the momentum argument of the field v'. The crossed line corresponds to the field v', the line with the dot corresponds to the field v_n contracted with ik_n, and the plain line represents the field v_p.

Consideration of $d \to \infty$ asymptotics

It is possible to construct perturbation theory with expansion in two parameters (1/d,ε), where the limit $d \to \infty$ would determine the first term. There are few arguments indicating that $d_c = \infty$ is a critical dimension for theory of stochastic turbulence in which K41 theory becomes valid.

In the $d \to \infty$ limit all internal momenta are orthogonal, so the dependence of angles factorizes as S_d (unity d-sphere surface). This leads to two major simplifications:

- Significant reduction of a number of nonzero diagrams - from 417872 to 1693 in 4 loop
- Diagonalization of the graph polynomial $\Psi_G = det\, v$ which guarantees linear reducibility at least in the 4-loop order.

Renormalization scheme

Aim: express RG function directly in terms of finite renormalized diagrams without calculating Z-factors to apply HL algorithm.

$$(\mu\partial_\mu + \beta\partial_g - \gamma_\nu \nu\partial_\nu)\Gamma^R = 0 \tag{1}$$

$$\overline{\Gamma}(k,\omega) \equiv \frac{\Gamma(k,\omega)}{-\nu k^2} \tag{2}$$

$$(\mu\partial_\mu + \beta\partial_g - \gamma_\nu \nu\partial_\nu)\overline{\Gamma}^R = \gamma_\nu \overline{\Gamma}^R \tag{3}$$

Considering this equation in the normalization point $(k=\mu, \omega=0)$

$$\overline{\Gamma}^R|_{k=\mu,\omega=0} = 1, \quad \partial_g \Gamma^R|_{k=\mu,\omega=0} = 0, \quad \partial_\nu \Gamma^R|_{k=\mu,\omega=0} = 0, \tag{4}$$

$$\gamma_\nu = (\mu\partial_\mu \overline{\Gamma}^R)|_{k=\mu,\omega=0} \; . \tag{5}$$

Using the dependence of the dimensionless function $\overline{\Gamma}^R$ on the ratio k/μ and its independence of ν,

$$\gamma_\nu = -(k\partial_k \overline{\Gamma}^R)|_{k=1,\omega=0} \; . \tag{6}$$

Renormalization scheme

Anomalous dimension γ_ν is expressed in terms of renormalized n-loop diagrams χ_n:

$$\gamma_\nu = -\sum_{n\geq 1} h^n \sum_i k\partial_k \left(R'\chi_n^{(i)}(k, \omega = 0)\right)\Big|_{k=1}, \qquad h \equiv \frac{S_d g}{(2\pi)^d}$$

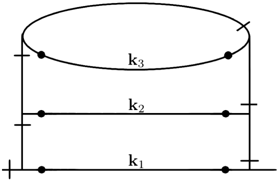
$$\beta = -h(2\varepsilon - 3\gamma_\nu)$$

$$\omega = \partial_h \beta(h)|_{h=h_*}$$

- There are no overlapping subgraphs due to the absence of vertex divergences which leads to the significant simplifications.
- In the static massless theories ϕ^3 and ϕ^4, trivial external momentum dependence allows to factorize 2-point subgraphs from a diagram and calculate them separately. In our case, it is not possible due to the presence of not only external momentum but also external frequency.

3 loop example

Hyperlogarithm method allows for separate integration of divergent subtractive terms of R'-operation.



$$\chi^R = \left(-k\partial_k R'\chi(k)\right)\Big|_{k=1} = -\partial_k\left[\chi_1(k) - \chi_2(k) - \chi_3(k) + \chi_4(k)\right]\Big|_{k=1}$$

$$\begin{aligned}
\partial_k(\chi_1(k))\Big|_{k=1} &= \int_0^1 du_1 \ldots du_5 \frac{(u_1+3u_2+3u_3+3u_4+u_5)\,\delta(u_1+u_2+u_3+u_4+u_5-1)}{u_3\,(u_2+u_3+u_4)^2\,(u_1+u_2+u_3+u_4+u_5)^3} \\
&= \int_0^\infty du_2 du_4 du_5 du_1 \frac{(u_1+3u_2+3u_4+u_5+3)}{(u_2+u_4+1)^2\,(u_1+u_2+u_4+u_5+1)^3} \\
&= \int_0^\infty du_1 \frac{\ln(u_1+1)}{u_1} = \int_0^\infty du_1 \frac{L_{\omega_{-1}}(u_1)}{u_1} = \\
&= \operatorname*{Reg}_{u_1=\infty} L_{\omega_0\omega_{-1}}(u_1) - \operatorname*{Reg}_{u_1=0} L_{\omega_0\omega_{-1}}(u_1) = \zeta(2) - 0
\end{aligned}$$

Results

$$H := 2^{\varepsilon-2}\frac{S_d}{(2\pi)^d}g$$

$$\begin{aligned}\gamma_\nu &= H\left(1+\frac{\pi^2\varepsilon^2}{6}+0\cdot\varepsilon^3\right)+H^2\left(\frac{1}{2}-\frac{(\pi^2+9)\varepsilon}{6}+\frac{(\pi^2+8\zeta(3))\varepsilon^2}{4}\right)+\\ &+ H^3\left(2+\frac{\pi^2}{8}-\frac{(43+4\pi^2+32\zeta(3))\varepsilon}{8}\right)+H^4\left(\frac{15}{2}+\frac{7\pi^2}{16}+\frac{5\zeta(3)}{2}\right)+\dots\end{aligned}$$

$$H_* = \frac{2\varepsilon}{3}-\frac{2\varepsilon^2}{9}+\left(\frac{2}{9}-\frac{2\pi^2}{27}\right)\varepsilon^3+\left(\frac{7}{81}+\frac{2\pi^2}{81}-\frac{16\zeta(3)}{81}\right)\varepsilon^4+\dots$$

Obtained 4 loop expression for the correction exponent ω, responsible for IR stability of the fixed point:

$$\omega = \partial_H\beta(H_*) = 2\varepsilon+\frac{2}{3}\varepsilon^2+\frac{10}{9}\varepsilon^3+\frac{56}{27}\varepsilon^4+\dots$$

Prospects

- **5-loop calculation of exponent ω in the considered model.**
  Very likely, the majority (or even all of them) of 5-loop diagrams will turn out to be linear reducible which make them suitable for applying the hyperlogarithm method.
- *A* model of critical dynamics (dynamic analogue of ϕ^4 theory)
- *A* model based on ϕ^3 theory

Thank you!