Loop radiative corrections and the fully non-perturbative regime in strong-field QED

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Credits

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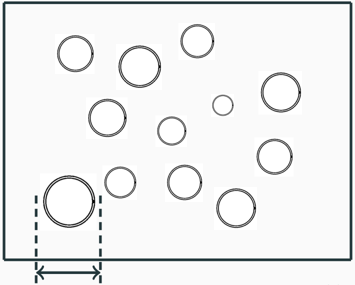
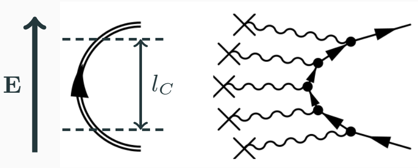
Outline

Strong field in QED



$l_C = \hbar/mc \approx 3.9 \times 10^{-11}$ cm



$\mathbf{E} = const$, $\mathbf{H} = 0$, pair is created if $eEl_C = mc^2$

$$E_S = \frac{m^2c^3}{e\hbar}$$ — critical field (F. Sauter 1931)

$$E_S = 1.32 \times 10^{16} \text{ V/cm} = 4.4 \times 10^{13} \text{ G}, \qquad I_S = \frac{c}{4\pi}E_S^2 \sim 10^{29} \text{ W/cm}^2$$

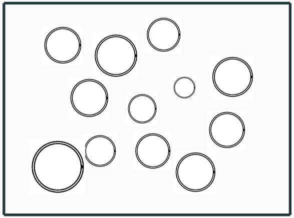
For other particles, e.g. π, W, H: $E_{cr} = \dfrac{m^2_{\pi,W,H}c^3}{e\hbar}$

Strong-field effects

Electric

$$E^2 - H^2 > 0$$

Vacuum polarization



- Birefringence
- Pair creation

Analogous effect was observed in graphene [A. Schmitt et al, Nature Physics 19, 6, 830 (2023)]

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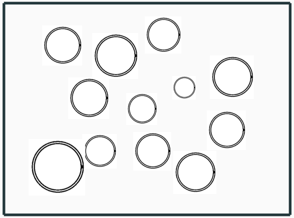

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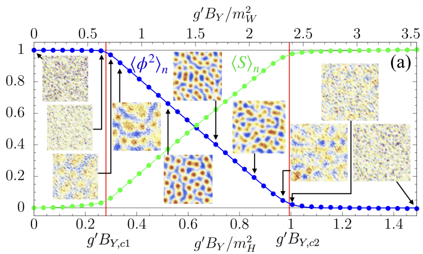
Magnetic

$$E^2 - H^2 < 0$$

No vacuum polarization

- Dynamical chiral symmetry breaking and mass generation [Gusynin et al Nucl. Phys. B563, 361 (1999); Kogut & Sinclair PRD 109, 034511 (2024)]
- Field-induced phase transitions (in electroweak + Higgs sectors)



[Chernodub et al PRL 130, 111802 (2023)]

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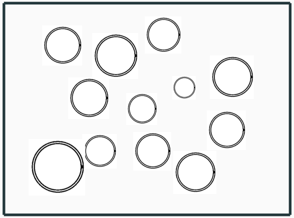

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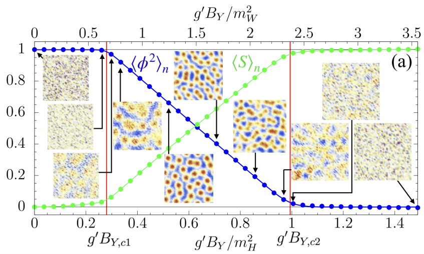


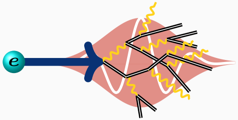
[Chernodub et al PRL 130, 111802 (2023)]

Crossed

$$E^2 = H^2,\ \mathbf{E} \perp \mathbf{H}$$

No vacuum polarization

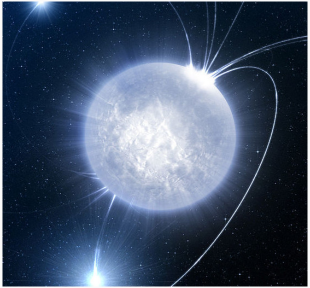
Effects with ultra-relativistic particles propagating transversely in the field

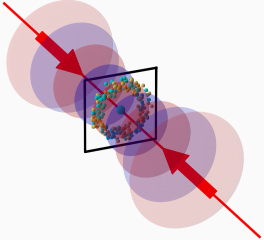


- Field-induced scattering processes, cascades
- Dynamical mass generation and spontaneous symmetry breaking???

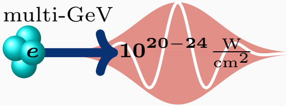
Reaching strong EM fields

- Compact astrophysical objects: magnetars, pulsars, black holes
- Strongly focused multi-petawatt laser beams
- Lorentz-boosted field in collision of high-energy particle bunches with targets





SLAC e^--beam + PW laser

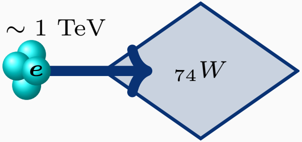


SLAC

~ 100 GeV dense e-bunches

CERN

~ 1 TeV



RHIC, LHC peripheral collisions

$\sqrt{s} \sim 100$ GeV heavy ions

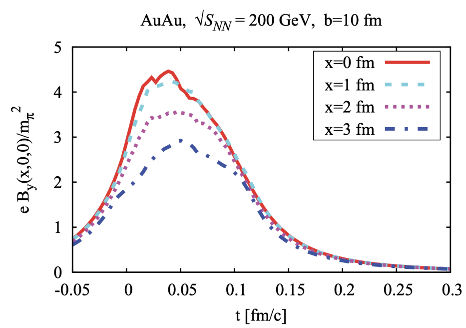
External (classical) field

The number of absorbed/emitted photons in a mode (~ 1) $\ll$ their total number N_γ:

$$N_\gamma \sim \frac{(E^2/4\pi)L^3}{\hbar\omega} \gg 1 \iff E \gg \sqrt{\hbar\omega/L^3} = \sqrt{\alpha\left(\frac{l_C}{L}\right)^3 \frac{\hbar\omega}{mc^2}} \times E_S$$

Focused optical laser: $L \sim \lambda \sim 10^{-6}$ m, $\hbar\omega \sim 1$ eV $\Longrightarrow E \gg 10^{-14} E_S$ ($l_C \approx 3.86 \times 10^{-13}$ m)

Ultra peripheral heavy ion collisions:



[V. Voronyuk et al, PRC 83, 054911 (2011)]

$L \sim c\Delta t \sim 0.1$ fm, $\omega \sim 1/\Delta t$

$$\Longrightarrow E \gg \sqrt{\alpha}\left(\frac{l_C}{c\Delta t}\right)^2 E_S \sim 10^6 E_S$$

Actual field: $E \approx 5\dfrac{m_\pi^2 c^3}{e\hbar} \approx 3 \times 10^5 E_S$

Field = photons

Field range can be extended by using crystals

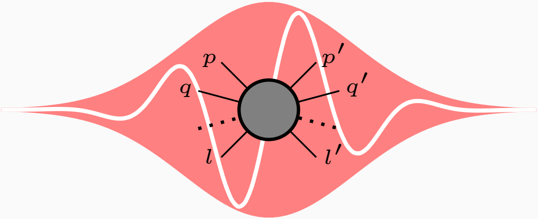
[Di Piazza, Wistisen, Tamburini, Uggerhøj, PRL 124, 044801 (2020)]

Some reviews on SFQED

- A. Fedotov, A. Ilderton, F. Karbstein, B. King, D. Seipt, H. Taya, and G. Torgrimsson, Advances in QED with intense background fields, Phys. Rep. 1010, 1 (2023).
- S. V. Popruzhenko and A. M. Fedotov, Dynamics and radiation of charged particles in ultra-intense laser fields, Phys. Usp. 66, 460 (2023).
- A. Gonoskov, T. G. Blackburn, M. Marklund, and S. S. Bulanov, Charged particle motion and radiation in strong electromagnetic fields, Rev. Mod. Phys. 94, 045001 (2022).

- A. Di Piazza, C. Müller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. 84, 1177 (2012)
- N. B. Narozhny and A. M. Fedotov, Contemp. Phys. 56, 249 (2015)
- F. Gelis and N. Tanji, Prog. Part. Nucl. Phys. 87 (2016)

Invariant parameters



Field:

$A^{\mu}, \quad F^{\mu\nu}$

Electron:

p^{μ} — generalized momentum

ψ_p — mode

Photon: l^{μ}

$\Longrightarrow$

$$\mathscr{F} = E^2 - H^2, \quad \mathscr{G} = (\mathbf{E}, \mathbf{H})$$

Classical non-linearity parameter*

$$a_0 = \frac{e\sqrt{-\langle A^{\mu} A_{\mu} \rangle}}{mc} \sim \frac{eE}{m\omega c}$$

Quantum dynamical parameter

$$\chi = \frac{e\hbar\sqrt{-(F^{\mu\nu} p_{\nu})^2}}{m^3 c^4} \sim \frac{\mathscr{E}}{mc^2} \frac{E}{E_S}$$

* For a discussion of gauge-invariance see [T. Heinzl, A. Ilderton, Opt. Commun. (2009)]

Strong-field QED

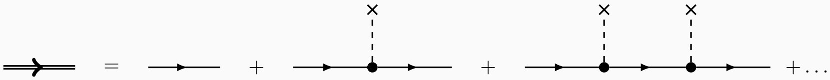
- $A^\mu(x) = \underbrace{A^\mu_{\text{ext}}(x)}_{\text{classical field, non-perturbative}} + \underbrace{A^\mu_{\text{rad}}(x)}_{\text{quantized radiation, perturbative}}$

$$\mathscr{L} = \mathscr{L}_{e^-e^+} + \mathscr{L}^{\text{rad}}_{\text{Maxwell}} + \mathscr{L}^{\text{ext}}_{\text{int}} + \boxed{\mathscr{L}^{\text{rad}}_{\text{int}}} \longleftarrow \text{perturbation,}$$

$$\mathscr{L}_{e^-e^+} + \mathscr{L}^{\text{ext}}_{\text{int}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - e\gamma^\mu A^{\text{ext}}_\mu - m\right)\psi,$$

$$\mathscr{L}^{(\bullet)}_{\text{Maxwell}} = -\frac{1}{4}F^{(\bullet)}_{\mu\nu}F^{\mu\nu}_{(\bullet)}, \quad \mathscr{L}^{\text{rad}}_{\text{int}} = -J^\mu A^{\text{rad}}_\mu = -e\bar{\psi}\gamma^\mu A^{\text{rad}}_\mu\psi$$

- The Furry picture



$$\boxed{\left(\not{p} - e\not{A}_{\text{ext}} - m\right)\psi_p = 0}$$

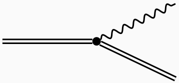
$\{\psi_p\} \Longrightarrow$ scattering theory $\Longrightarrow$ cross sections of various processes in a SF

Conventions: $\hbar = c = 1$, $e > 0$, metric$=\text{diag}(+---)$, $\not{p} = \gamma^\mu p_\mu$

$$\left(\not{p} - e\not{A} - m\right)\psi_p = 0$$

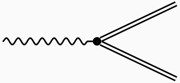
$\{\psi_p\}$ is known in limited types of field: constant, plane wave, Coulomb...

Examples of calculated processes cross sections (probability rates) in a plane wave:



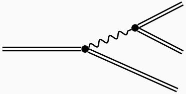
Nonlinear Compton

Nikishov-Ritus 1964



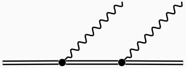
Nonlin. Breit-Wheeler

Nikishov-Ritus 1964



Trident pair production

King-Ruhl 2013, Dinu-Torgrimsson 2020



...

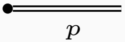
Double Compton

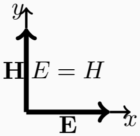
Mackenroth-Di Piazza 2013, King 2015, Torgrimsson 2020

See recent reviews [A. Gonoskov, T. G. Blackburn, and M. Marklund, Rev. Mod. Phys. 94 (2022);

A. Fedotov, A. Ilderton, F. Karbstein, B. King, D. Seipt, H. Taya, G. Torgrimsson , https://arxiv.org/abs/2203.00019 (2021)]

$$(\not{\hat{p}} + e\not{A} - m)\,\psi_p = 0$$





- CCF: $A^\mu = a^\mu \varphi, \quad \varphi = kx, \quad k^2 = ka = 0$

- Volkov solutions: $\boxed{\psi_{p,\sigma}(x) = E_p(x) u_{p,\sigma}}, \qquad u_{p,\sigma} = (\not{p} + m)(1 \pm \gamma_5 \not{n})w$

$$E_p(x) = \left[1 + \frac{e}{2(kp)}\not{k}\not{A}\right] e^{iS_p}$$ — Ritus E_p-function (4x4 matrix)

$$S_p(x) = -px + \frac{e(ap)}{2(kp)}\varphi^2 + \frac{e^2a^2}{6(kp)}\varphi^3$$ — classical action

- Properties of the E_p-functions:

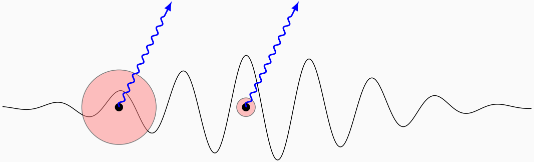
$$\int d^4x \overline{E}_p(x) E_q(x) = (2\pi)^4 \delta^{(4)}(p - q),$$

$$\int \frac{d^4p}{(2\pi)^4} E_p(x) \overline{E}_p(y) = \delta^{(4)}(x - y), \quad \overline{E}_p = \gamma^0 E_p^\dagger \gamma^0,$$

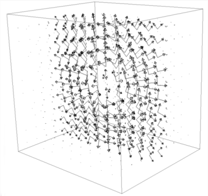
$$\hat{\not{P}} E_p = E_p \not{p}, \quad \hat{\not{P}} = \hat{\not{p}} + e\not{A}$$

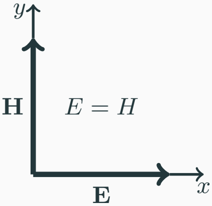
Locally constant field approximation (LCFA)

1. Formation scale $\ll$ field scale: locally *constant*; $\tau_F = \frac{m}{eE} \ll \lambda \iff \omega \to 0$, $\boxed{a_0 = \frac{eE}{m\omega} \to \infty}$



2. e^- is ultrarelativistic: locally *crossed*

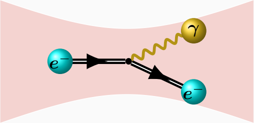




Total probability rate: calculate $W(\chi)$ locally and then integrate over slowly a varying field

$$\chi_{e,\gamma} \sim \frac{\varepsilon_{e,\gamma}}{m}\frac{E_\perp}{E_S}$$ — quantum dynamical parameter

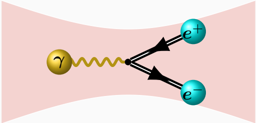
Non-linear Compton emission



$$W_{\text{rad}} \approx \begin{cases} 1.44\dfrac{\alpha}{\tau_C}\dfrac{m}{\varepsilon_e}\chi_e, & \chi_e \ll 1 \\ 1.46\dfrac{\alpha}{\tau_C}\dfrac{m}{\varepsilon_e}\chi_e^{2/3}, & \chi_e \gg 1 \end{cases}$$

Mean free path time $t_{\text{em}} \sim W_{\text{rad}}^{-1}$

Non-linear Breit-Wheeler e^-e^+ production



$$W_{\text{cr}} \approx \begin{cases} 0.23\dfrac{\alpha}{\tau_C}\dfrac{m}{\varepsilon_\gamma}\chi_\gamma e^{-8/3\chi_\gamma}, & \chi_\gamma \ll 1 \\ 0.38\dfrac{\alpha}{\tau_C}\dfrac{m}{\varepsilon_\gamma}\chi_\gamma^{2/3}, & \chi_\gamma \gg 1 \end{cases}$$

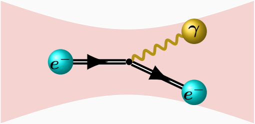
Pair creation rate becomes significant at $\chi_\gamma \gtrsim 1$

[AI Nikishov, VI Ritus JETP 19(5) (1964); A Gonoskov et al Rev. Mod. Phys. 94 (2022); A Fedotov et al Phys. Rep. 1010(1) (2023)]

Leading order strong-field QED effects

$$\chi_{e,\gamma} \sim \frac{\varepsilon_{e,\gamma}}{m}\frac{E_\perp}{E_S}$$ — quantum dynamical parameter

Non-linear Compton emission



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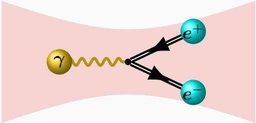
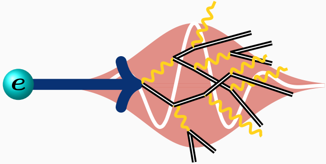
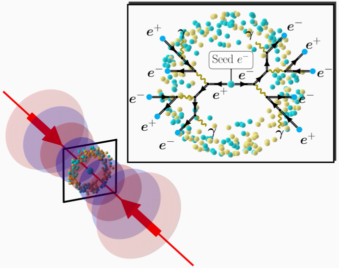


QED cascades:

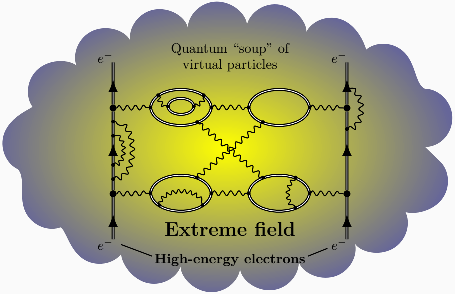




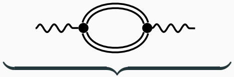
[AAM, E. G. Gelfer, A. M. Fedotov PRA 104, 012221 (2021); Mercuri-Baron, AAM, Riconda, et al, arXiv:2402.04225 (2024)]

Loop corrections to QED processes in a CCF

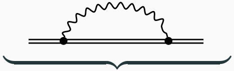
Nikishov, Ritus 1964



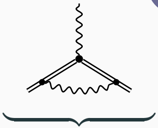
At $\chi \gg 1$ $\boxed{W_{\text{rad,cr}} \propto \alpha\chi^{2/3}}$



Narozhny 1969



Ritus 1970



Morozov 1981, Di Piazza 2020

At $\chi \gg 1$ also scale as $\boxed{g = \alpha\chi^{2/3}}$! $\alpha\chi^{2/3} \sim 1$ (or $\chi \approx 1600$) signifies a new regime of interaction

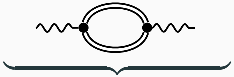
$$m_\gamma^2 \simeq \alpha m^2 \chi^{2/3} \simeq m^2, \quad \Delta m_e \simeq \alpha m \chi^{2/3} \simeq m$$

meaning that **radiative corrections cease being small**.

Loop corrections to QED processes in a CCF

Nikishov, Ritus 1964

At $\chi \gg 1$ $\boxed{W_{\text{rad,cr}} \propto \alpha\chi^{2/3}}$



Narozhny 1969

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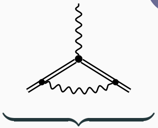
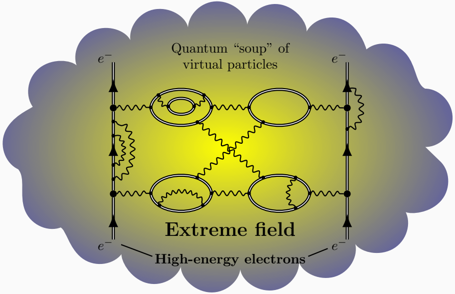


Morozov 1981, Di Piazza 2020



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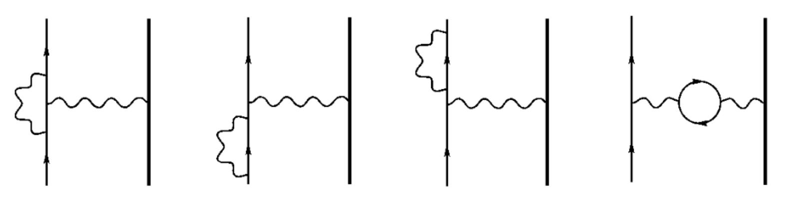
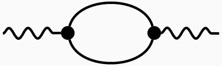
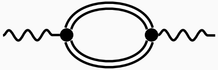
$\varepsilon_{\text{in}} = m\gamma_{\text{in}}$ [GeV]	800	80	8	0.8
E/E_S	10^{-3}	10^{-2}	0.1	1
I_L [W/cm^2]	5×10^{23}	5×10^{25}	5×10^{27}	5×10^{29}



- l^μ — virtual photon momentum, $l^2 \neq 0$
- Loops contribute only to virtual lines: $\int dl^2 \ldots$
- $\Pi_{\mu\nu}(l) =$  $= \widehat{\Pi}(l^2)(l^2 g_{\mu\nu} - l_\mu l_\nu)$
- UV divergent: $\widehat{\Pi}(l^2) \simeq \frac{\alpha}{3\pi} l^2 \log \frac{|l^2|}{m^2}, \quad l^2 \gg m^2$
- Main effect: renormalization of e and m, running coupling $\alpha_{\text{eff}} = \dfrac{\alpha}{1 - \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{m^2}}$

Radiative corrections in a CCF

N.B. Narozhny Sov. Phys. JETP 28 (2) (1969); Ritus V.I. Annals of Physics 69.2 (1972)

$\Pi_{\mu\nu}(l) =$  $= \underbrace{\widehat{\Pi}(l^2,\chi_l)(l^2 g_{\mu\nu} - l_\mu l_\nu)}_{\text{Modified standard QED}} + \underbrace{\sum_{i=1}^{2} \Pi_i(l^2,\chi_l)\epsilon_\mu^{(i)}(l)\epsilon_\nu^{(i)}(l)}_{\text{Field-induced}}$

$$\widehat{\Pi}(0,\chi_l \gg 1) \simeq \frac{\alpha m^2}{3\pi} \log \chi_l^{2/3}$$

$$\boxed{\Pi_i(l^2,\chi_l \gg 1) \simeq \alpha m^2 \chi_l^{2/3}}$$

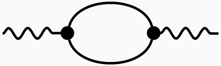
$$\epsilon_\mu^{(1,2)}(l) = \frac{e(F,F^\star)_{\mu\nu} l^\nu}{m^3 \chi_l}, \qquad l^\mu \epsilon_\mu^{(1,2)} = 0$$

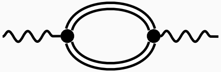
- Contribution to both virtual and real lines
- Photon acquires effective mass, vacuum has a refractive index
- Field-induced part is finite. Renormalization as in QED, $\alpha_{\text{eff}} = \dfrac{\alpha}{1 - \frac{2\alpha}{9\pi}\log\chi_l} \sim \alpha$

Relation to fundamental UV behavior of QED

T. Podszus and A. Di Piazza, Phys. Rev. D 99, 076004 (2019).

A. Ilderton, Phys. Rev. D 99, 085002 (2019).

QED:  $\simeq \dfrac{\alpha}{3\pi} l^2 \log \dfrac{|l^2|}{m^2}$

SFQED:  $\simeq m^2 \alpha \chi_l^{2/3}$

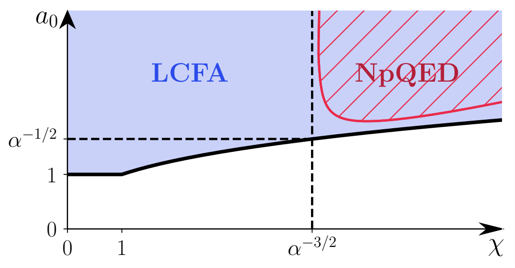
Result depends on the limit order: $a_0 \to \infty$, $\chi \to \infty$!

- $\chi_l \ll a_0^3$, $a_0 \to \infty$, $\chi_l \to \infty$

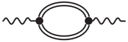
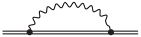
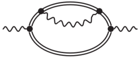
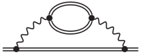
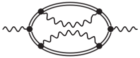
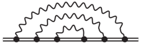
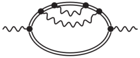
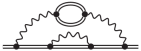
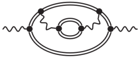
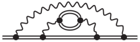
  SFQED, $\Pi^{(2)} \sim \alpha \chi_l^{2/3}$

- $\chi_l \to \infty$, then $a_0 \to \infty$

  NO LCFA, $\Pi^{(2)} \sim \log \dfrac{|l^2|}{m^2}$

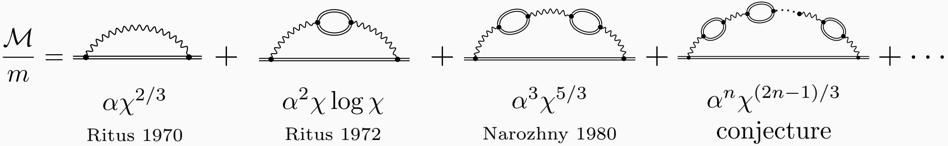


Summary of known radiative corrections in a CCF

				1 loop			
(1a)		$\alpha\chi^{2/3}$	[12]	(1b)		$\alpha\chi^{2/3}$	[13]
				2 loops			
(2a)		$\alpha^2\chi^{2/3}\log\chi$	[16]	(2b)		$\alpha^2\chi\log\chi$	[14,21]
				(2c)		$\alpha^2\chi^{2/3}\log\chi$	[15]
				3 loops			
(3a)		$\alpha^3\chi^{2/3}\log\chi$	[17]	(3d)		$\alpha^3\chi^{2/3}\log^2\chi$	[17]
(3b)		$\alpha^3\chi^{2/3}\log\chi$	[17]	(3e)		$\alpha^3\chi^{4/3}$	[17]
(3c)		$\alpha^3\chi\log^2\chi$	[18]	(3f)		$\alpha^3\chi\log^2\chi$	[18]
				(3g)		$\alpha^3\chi^{5/3}$	[18]

Bubble-chain mass operator to all orders

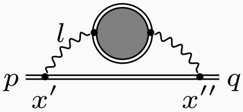
[AAM, S. Meuren and A. M. Fedotov PRD 102 (2020)]



For the electron elastic scattering amplitude to all orders:

1. Main contribution to $\mathscr{M}^{(n-loop)}$ at $\chi \gg 1$ — from polarization loop insertions
2. $\dfrac{\mathscr{M}^{(n+1)}}{\mathscr{M}^{(n)}} \sim \alpha\chi^{2/3}$ (except $n = 1$) for the bubble-type corrections
3. $\boxed{g = \alpha\chi^{2/3}}$ is the effective PT parameter, at least for the bubble-type corrections
4. $g \gtrsim 1$ the one-loop bubble-type corrections are resummed

Bubble-chain mass operator



$$-i\Sigma(q,p) = \Lambda^{2(D-4)} \int d^D x'\, d^D x''\, \bar{E}_q(x'')(ie\gamma^\mu) S_0^c(x'',x')(ie\gamma^\nu) E_p(x') D^c_{\mu\nu}(x'',x')$$

- $S_0^c(x'',x')$ — LO electron propagator

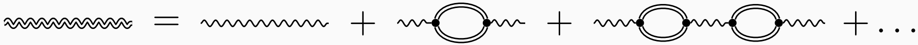
$$S_0^c(x'',x') = e^{i(ax)\Phi} e^{-i\frac{\pi}{2}\frac{D-2}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2}} \int_0^\infty \frac{ds}{s^{D/2}} \exp\left\{-im^2 s - i\frac{x^2}{4s} + i\frac{s}{12}e^2 (Fx)^2\right\}$$
$$\times\left[m + \frac{(\gamma x)}{2s} - \frac{s}{3}e^2(\gamma F^2 x) + \frac{i}{2}mse(\sigma F) + \frac{i}{2}e(\gamma F^\star x)\gamma^5\right],$$

where $x = x'' - x'$, $X = (x' + x'')/2$, $\Phi = (kX)$, s — proper time

E_p-representation: $S_0^c(p) = \Lambda^{D-4}\int d^D x \bar{E}_p(x'') S_0^c(x'',x') E_p(x') = i\dfrac{(\gamma p)+m}{p^2 - m^2 + i0}$

- $D^c_{\mu\nu}(x'',x')$ — photon propagator with loops inserted

$$\left[l^2 g^{\mu\nu} - l^\mu l^\nu - \Pi^{\mu\nu}(l^2,\chi_l)\right] D_{\nu\lambda} = -i\delta^\mu_\lambda$$



$$D^c_{\mu\nu}(l) = D_0(l^2,\chi_l) g_{\mu\nu} + \sum_{i=1}^{2} D_i(l^2,\chi_l)\epsilon^{(i)}_\mu(l)\epsilon^{(i)}_\nu(l),$$

$$D_0(l^2,\chi_l) = \frac{-i}{l^2+i0}, \quad D_{1,2}(l^2,\chi_l) = \frac{i\Pi_{1,2}}{(l^2+i0)\,(l^2-\Pi_{1,2})} = \frac{-i}{l^2+i0} - \frac{-i}{l^2-\Pi_{1,2}(l^2,\chi_l)}$$

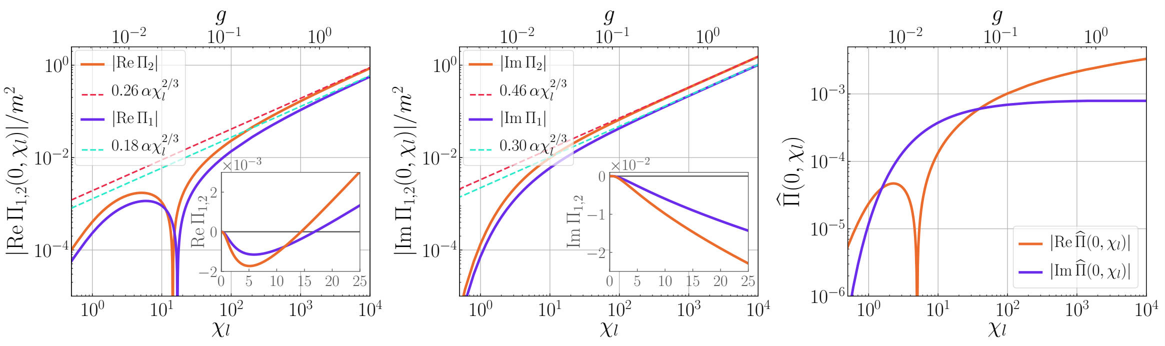
Polarization operator eigenfunctions:

$$\Pi_{1,2}(l^2,\chi_l) = m^2\frac{4\alpha\chi_l^{2/3}}{3\pi}\int_4^\infty \frac{dv}{v^{13/6}}\,\frac{v+0.5\mp1.5}{\sqrt{v-4}}f'(\zeta), \quad \zeta = \left(\frac{v}{\chi_l}\right)^{2/3}\left(1-\frac{l^2}{vm^2}\right)$$

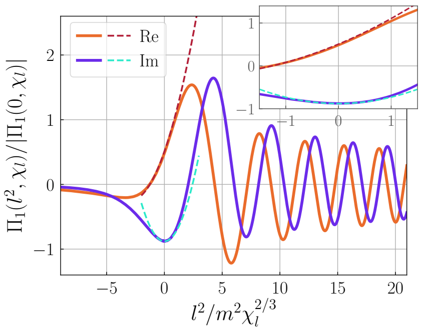
$$f(\zeta) = i\int_0^\infty d\sigma\, e^{-i(\zeta\sigma+\sigma^3/3)}$$

Graphic representation of pol. op. eigenfunctions

$$g = \alpha \chi_l^{2/3}$$



Dependence on l^2



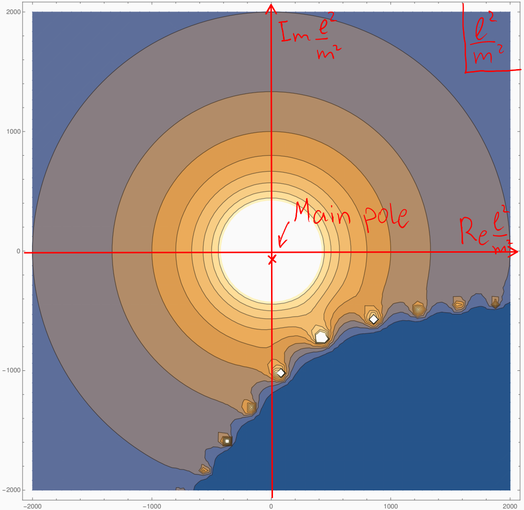
If $l^2 \lesssim m^2\chi_l^{2/3}$:

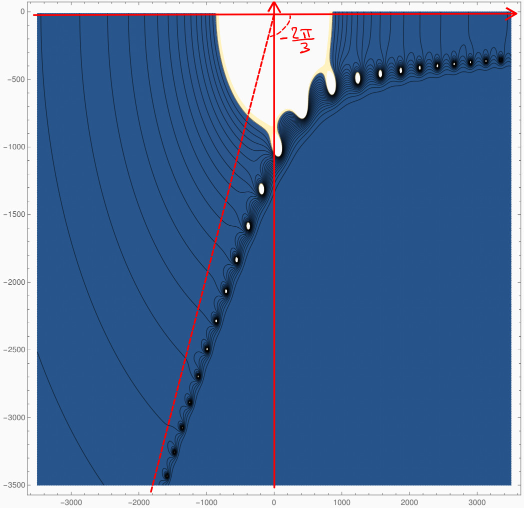
$$\Pi_i(l^2, \chi_l) \approx m^2\alpha\chi_l^{2/3}\left[K_i + K_i^{(1)}\frac{l^2}{m^2\chi_l^{2/3}} + K_i^{(2)}\left(\frac{l^2}{m^2\chi_l^{2/3}}\right)^2\right]$$

where K_i, $K_i^{(1,2)}$ are constants

Analytic properties

- $\Pi_{1,2}$ are a whole transcendent functions of l^2
- Poles: $\dfrac{1}{|l^2 - \Pi_{1,2}|}$ in complex plane of l^2





Plan of further calculation [V.I. Ritus]

$$-i\Sigma(q,p) = \Lambda^{2(D-4)} \int d^D x'\, d^D x''\, \bar{E}_q(x'')(ie\gamma^\mu) S_0^c(x'',x')(ie\gamma^\nu) E_p(x') D^c_{\mu\nu}(x'',x')$$

1. Tedious Dirac matrices algebra (using FeynCalc)...
2. Integrals are sequentially carried out, so that $\int d^D x' d^D x'' \longrightarrow \int dl^2 dq^2 d\chi_l$
3. Scattering amplitude $\bar{u}_{p',s}\Sigma(p',p)u_{p,s} = -(2\pi)^4\delta^{(4)}(p-p')\cdot 2p^0 T_s(p)$:

$$\boxed{\mathcal{M}(\chi) \equiv \bar{u}_{p,s}\Sigma(p,F)|_{p^2=m^2} u_{p,s} = \mathcal{M}_0(\chi) + \delta\mathcal{M}(\chi), \quad \delta\mathcal{M} = \sum_{i=1}^{2} \delta\mathcal{M}_i}$$

4. Residual renormalization reduces to the subtraction $\mathcal{M} \mapsto \mathcal{M}^{(\mathrm{ren})} = \mathcal{M} - \mathcal{M}|_{F=0}$
   OR can be carried out in a standard way: $D = 4 - \varepsilon$, $\varepsilon \to 0$

Scattering amplitude: $\mathscr{M}_0(\chi)$

$$\mathscr{M}_0(\chi) = \frac{\alpha m^4}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{du}{(1+u)^2} \int_{-\infty}^{+\infty} d\lambda \int_{-\infty}^{+\infty} \frac{d\mu}{\mu + i0} D_0(m^2\lambda, \chi_l)$$
$$\times \left\{ (2+\lambda)\,\mathrm{Ai}_1(t) + 2\frac{u^2+2u+2}{1+u} \left(\frac{\chi}{u}\right)^{2/3} \mathrm{Ai}'(t) \right\}$$

- Notations: $\lambda = l^2/m^2$, $\mu = (q^2 - m^2)/m^2$, $u = \chi_l/\chi_q$
- $\mathscr{M}_0$ is divergent and renormalized via the replacement

$$\mathrm{Ai}_1(t) \mapsto \mathrm{Ai}_1^{(\mathrm{ren})}(t) = -i \int_{-\infty}^{\infty} \frac{d\sigma}{2\pi\sigma} e^{-it\sigma} \left(e^{-i\sigma^3/3} - 1 \right)$$

- $\mathscr{M}_0^{(\mathrm{ren})}(\chi) \leftrightarrow D_0$ is the 2nd order contribution with no vacuum polarization loops [cf. Eq. (23) in Ritus 1972] with an asymptotic behavior [*ibid.*, Eq. (72)]:

$$\mathscr{M}_0^{(\mathrm{ren})}(\chi \gg 1) \simeq 0.843(1 - i\sqrt{3})\alpha\chi^{2/3} m^2$$

Scattering amplitude: $\delta\mathcal{M}(\chi)$

$$\delta\mathcal{M}_{1,2}(\chi) = -\frac{\alpha m^4}{(2\pi)^2}\int_{-\infty}^{+\infty}\frac{du}{(1+u)^2}\int_{-\infty}^{+\infty}d\lambda\int_{-\infty}^{+\infty}\frac{d\mu}{\mu+i0}\,D_{1,2}(m^2\lambda,\chi_l)$$
$$\times\left\{\left[1+\lambda\frac{u^2+2u+2}{2u^2}\right]\mathrm{Ai}_1(t)+\left(\frac{u^2+2u+2}{1+u}\pm 1\right)\left(\frac{\chi}{u}\right)^{2/3}\mathrm{Ai}'(t)\right\}$$

$$t=\left(\frac{u}{\chi}\right)^{2/3}\left(1+\frac{1+u}{u^2}\lambda+\frac{1+u}{u}\mu\right),\quad \chi_l=\frac{\chi u}{1+u},$$

$$\mathrm{Ai}_1(t)=-i\int_{-\infty}^{\infty}\frac{d\sigma}{2\pi}\,\frac{1}{(\sigma-i0)}e^{-i\sigma^3/3-it\sigma},\quad \mathrm{Ai}'(t)=-i\int_{-\infty}^{\infty}\frac{d\sigma}{2\pi}\,\sigma e^{-i\sigma^3/3-it\sigma}$$

- $\delta\mathcal{M}_{1,2}$ are finite and vanish on switching the field off.
- Reproduces [Eq. (42) in Narozhny 1980], except minor differences;
- As Narozhny 1980, from now on we also assume $Z\approx 1$;
- As Narozhny 1980, we drop the subleading spin-dependent terms;
- In contrast to Narozhny 1980, we have also dropped the terms $\propto \mu$ in $\{\dots\}$, as they eventually vanish after $\int d\mu$;
- In contrast to Narozhny 1980, no perturbative expansion in powers of $\Pi_{1,2}$ is assumed here and below!

Photon propagator in the proper time representation

Propagator in x-space $\mapsto$ probability to travel between two space-time points $x = x'' - x'$

$$D^c_{\mu\nu}(x) = \frac{\Lambda^{4-D}}{(2\pi)^D} \int d^D l \, D^c_{\mu\nu}(l) e^{-ilx}$$

$$D^c_{0\mu\nu}(x) = e^{-i\frac{\pi}{2}\frac{D-4}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2+1}} \int \frac{d\tau}{\tau^{D/2}} \exp\left(-i\frac{x^2}{4\tau}\right) \boxed{2\pi i\theta(\tau)} g_{\mu\nu}$$

τ — proper time of a virtual photon

$\mathcal{J}_0(\tau) = \int dl^2 \frac{e^{-il^2\tau}}{l^2 + i0} \equiv 2\pi i\theta(\tau) \Longrightarrow \tau > 0$, photon propagation respects causality

Photon propagator in the proper time representation

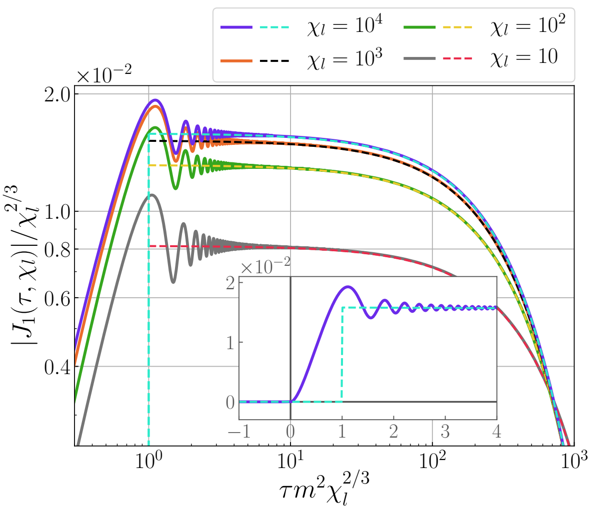
Propagator in x-space $\mapsto$ probability to travel between two space-time points $x = x'' - x'$

$$D^c_{\mu\nu}(x) = \frac{\Lambda^{4-D}}{(2\pi)^D} \int d^D l \, D^c_{\mu\nu}(l) e^{-ilx}$$

$$\begin{aligned} D^c_{\mu\nu}(x) = e^{-i\frac{\pi}{2}\frac{D-4}{2}} \frac{\Lambda^{4-D}}{(4\pi)^{D/2+1}} \int_0^\infty \frac{d\tau}{\tau^{D/2}} \exp\left(-i\frac{x^2}{4\tau}\right) \Big\{ & \mathcal{J}_0(\tau, \chi_l) g_{\mu\nu} \\ & + \frac{\mathcal{J}_1(\tau, \chi_l)}{m^2 \xi^2 \varphi^2} e^2 \left[(Fx)_\mu (Fx)_\nu - 2it(F^2)_{\mu\nu} \right] \\ & + \frac{\mathcal{J}_2(\tau, \chi_l)}{m^2 \xi^2 \varphi^2} e^2 \left[(F^\star x)_\mu (F^\star x)_\nu - 2it(F^2)_{\mu\nu} \right] \Big\} \end{aligned}$$

where now $\mathcal{J}_n(\tau, \chi_l) = -i \int_{-\infty}^{\infty} dl^2 \, D_n(l^2, \chi_l) e^{-il^2\tau}, \quad n = 0, 1, 2, \qquad \varphi = (kx), \; \chi_l = \dfrac{\xi\varphi}{2m^2 t}$

- $\mathcal{J}_n(\tau, \chi_l)$ smear causal $\theta(\tau)$-functions: $\mathcal{J}_n(\tau < 0, \chi_l) = 0$
- $\mathcal{J}_n(\tau, \chi_l)$ contain all information about the pole structure of $D^c_{\mu\nu}$



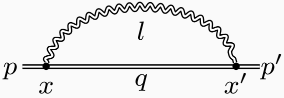
$$J_1(\tau,\chi_l) = -i\int_{-\infty}^{\infty} dl^2\, \frac{i\Pi(l^2,\chi_l)}{(l^2+i0)\,[l^2-\Pi(l^2,\chi_l)]} e^{-il^2\tau}$$

Key approximation for studying $\chi \gg 1$ limit

$$J_1(\tau,\chi_l) \approx -2\pi i\,\theta\left(\operatorname{Re}\tau - \tau_{\rm eff}^{(1)}\right)\frac{\Pi(0,\chi_l)}{m^2} e^{-i\Pi(0,\chi_l)\tau}$$

Note that it is already good at $\chi_l \gtrsim 1$!

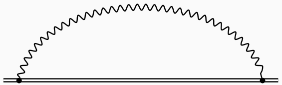
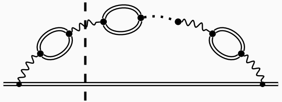
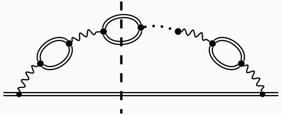
Next steps:



$\mathscr{M}(\chi) = \bar{u}_{p,s}\Sigma(p,p')|_{p^2=m^2}u_{p,s}$ at $\chi \gg 1$

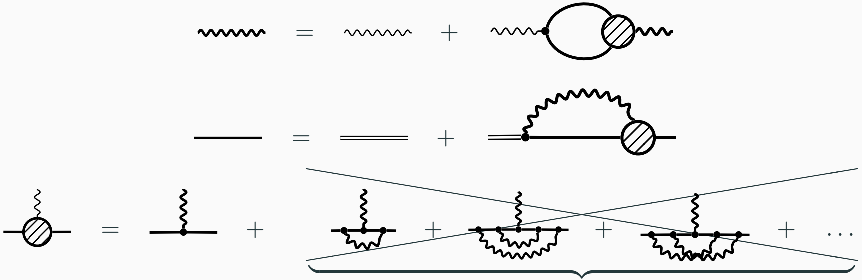
Calculation of $\delta\mathscr{M}$: Summary

Mass radiative correction: $\mathscr{M}(\chi) = \mathscr{M}_0^{(\mathrm{ren})} + \delta\mathscr{M}, \quad \delta\mathscr{M} = \delta\mathscr{M}^{(\mathrm{II})} + \delta\mathscr{M}^{(\mathrm{III})}$

Lowest-order PQED correction $\mathscr{M}_0^{(\mathrm{ren})}$	$0.843(1 - i\sqrt{3})\alpha\chi^{2/3}m^2$	
NPQED correction due to photon emission $\delta\mathscr{M}^{(\mathrm{II})}$	$(-0.995 + 1.72i)\alpha^{3/2}\chi^{2/3}m^2$	
NPQED correction due to trident pair production* $\delta\mathscr{M}^{(\mathrm{III})}$	$-(0.103 + 1.18i)\alpha^2\chi m^2$	

* Cf. 2-loop PQED result [Eq.(76) in Ritus 1972]:

$$\delta\mathscr{M}^{(2-\mathrm{loop})} = -[0.208 + (0.133\ln\chi - 0.725)i]\alpha^2\chi m^2$$



Not proven, though some evedence presented in Di Piazza & Lopez-Lopez, PRD (2020)

RN conjecture $\Longrightarrow$ DS equations become closed!

In order to proceed, we need to:

(i) Define structures of the exact propagators
(ii) Find a gauge where the proper vertex $\Gamma^\mu \longrightarrow ie\gamma^\mu$
(iii) Calculate exact mass and polarization operators
(iv) Plug everything to the DS equations and try solving them

Structure of radiative corrections

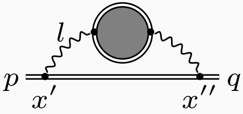
Renormalized exact photon propagator:

$$D^c_{\mu\nu}(l) = \frac{-i}{l^2 + i0}\left[g_{\mu\nu} - \xcancel{(1-d_l)\frac{l_\mu l_\nu}{l^2}}\right] + \sum_{i=1}^{2} \frac{i\Pi_i(l^2, \chi_l)}{(l^2 + i0)\,(l^2 - \Pi_i(l^2, \chi_l))}\epsilon^{(i)}_\mu(l)\epsilon^{(i)}_\nu(l),$$

$$\epsilon^{(1)}_\mu(l) = \frac{eF_{\mu\nu}l^\nu}{m^3\chi_l}, \qquad \epsilon^{(2)}_\mu(l) = \frac{eF^\star_{\mu\nu}l^\nu}{m^3\chi_l}, \qquad \chi_l = \frac{\xi(kl)}{m^2}, \qquad \xi^2 = -e^2a^2/m^2$$

Structure of radiative corrections

Electron mass operator with accounting for the exact photon propagator:

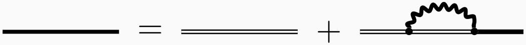


$$\Sigma(p,F)=\sum_{i=0}^{2}\Big[mS_i(p^2,\chi)+(\gamma p)V_i^{(1)}(p^2,\chi)+\frac{e^2(\gamma F^2 p)}{m^4\chi^2}V_i^{(2)}(p^2,\chi)$$
$$+\frac{e(\sigma F)}{m\chi}T_i(p^2,\chi)+\frac{e(\gamma F^{\star}p)\gamma^5}{m^2\chi}A_i(p^2,\chi)\Big]$$

- Scalar factors $S_i(p^2,\chi),\ldots$ can be expressed explicitly as multi-dim integrals with Π_i-s
- $n=0$ — 1-loop mass operator [Ritus 1970]; $n=1,2$ — nontrivial contribution

Structure of radiative corrections

Exact electron propagator:



$$S^c(p,F) = i\left[mS - (\gamma p)V^{(1)} - \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} - \frac{e(\sigma F)}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]\sum_{\pm}\frac{1\pm(\gamma n_D)\gamma^5}{2D_\pm},$$

$$D_\pm = m^2S^2 - p^2V^{(1)\,2} + m^2\left(A^2 - 2V^{(1)}V^{(2)}\right) \pm 2m^2\left(SA - 2TV^{(1)}\right)$$

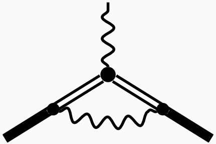
$$S = -1 - \sum_{i=0}^{2} S_i,\ V^{(1)} = 1 - \sum_{i=0}^{2} V_i^{(1)},\ V^{(2)} = -\sum_{i=0}^{2} V_i^{(2)},\ T = -\sum_{i=0}^{2} T_i,\ A = -\sum_{i=0}^{2} A_i.$$

At $\chi \gg 1$ the $V^{(2)}$-term dominates in the adopted approximation

$$\Sigma(p,F) \propto \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)}, \quad S^c(p,F) \propto \frac{e^2(\gamma F^2 p)}{m^4\chi^2 D_\pm}V^{(2)}$$

Vertex correction at $\chi \gg 1$

Suppose we insert a 1-loop correction into a vertex connecting two exact S^c

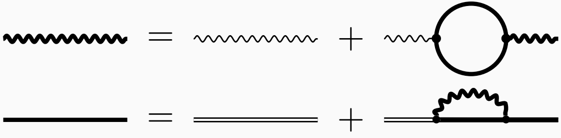


- Leading contribution $=$ LO term in Γ^μ $\times$ LO term in S^c

  If this is true, vertex insertion will enhance the total amplitude by $g = \alpha\chi^{2/3}$

- Γ^μ: dominant $\mathcal{O}(g)$ contribution is $\propto (\gamma k)k^\mu$ [Morozov 1981, Di Piazza PRD 2020]
- S^c: dominant contribution is $\propto (\gamma F^2 p)V^{(2)} = -a^2(\gamma k)(kp)V^{(2)}$
- HOWEVER: $(\gamma k)k^\mu \times (-a^2)(\gamma k)(kp)V^{(2)} \propto (\gamma k)^2 = 0$
- Therefore, the LO nonvanishing contribution should be enhanced by a factor weaker than $\alpha\chi^{2/3}$!

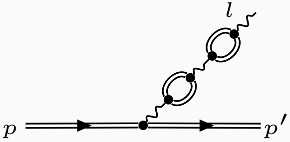
Consistent resummation: Dyson-Schwinger equations

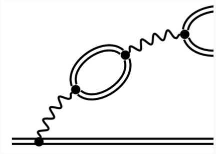


RN conjecture $\Longrightarrow$ DS equations become closed!

- **(i)** $+$ Define structures of the exact propagators
- **(ii)** ? Find a gauge where the proper vertex $\Gamma^\mu \longrightarrow ie\gamma^\mu$
- **(iii)** $\pm$ Calculate exact mass and polarization operators
- **(iv)** ? Plug everything to the DS equations and try solving them

For details see [AAM, A. M. Fedotov, PRD 105, 033005 (2022)]

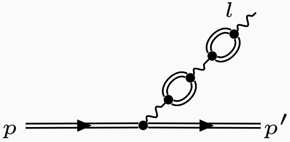
Modified photon emission: 

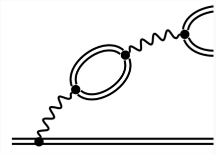
, trident 

Can we plug modified particle modes into tree-level amplitudes and calculate them?

$$A_{i\mu}(x) = \frac{Z_A^{1/2}}{\sqrt{2\omega_i}}\varepsilon_{i\mu}e^{-ilx}\Big|_{l^0=\omega_i}, \; l^2 - \Pi_i(l^2,\chi_l) = 0\Big|_{l^0=\omega_i} \; ; \qquad \psi(x) = E_p(x)\overline{D}(1 \pm \gamma_5 \not{n}_D)w\Big|_{p^0=\sqrt{m_{\uparrow\downarrow}^2+\mathbf{p}^2}}$$

Corrections to tree-level processes

Modified photon emission: 

, trident 

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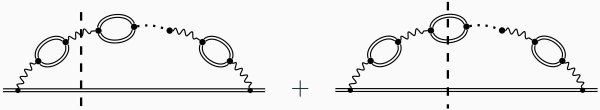
Unitarity (conservation of probability):

$$\langle f|\, T\, |i\rangle - \langle f|\, T^\dagger\, |i\rangle = i \sum_j \langle f|\, T^\dagger\, |j\rangle \langle j|\, T\, |f\rangle$$

[Veltman (1963)]: unitarity is satisfied by the inclusion of *only* the asymptotically stable states

Cutting rules for unstable particles

Optical theorem: $2\mathrm{Im}\Sigma =$ 

Cut through a stable photon state:

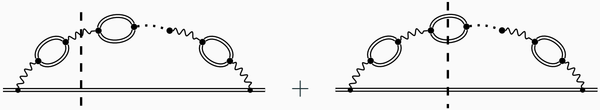
$$\frac{i}{l^2 + i0} \longrightarrow 2\pi\theta(l_0)\delta(l^2)$$

... and replace all $D \longrightarrow D^*$ on the r.h.s. of the cut

$$-2\mathrm{Im}\,\Sigma = \begin{cases} |\mathcal{M}|^2, & l^2 = 0, \\ 0, & l^2 \neq 0 \end{cases}$$

[Donoghue, Menezes PRD 100, 105006 (2019)]

Cutting rules for unstable particles

Optical theorem: $2\mathrm{Im}\Sigma =$ 

Cut through a **un**stable photon state:

$$\frac{i}{l^2 - \Pi} \longrightarrow 2\pi\theta(l^0) \int_0^\infty ds\, \delta(l^2 - s) \frac{\rho(s)}{\pi}$$

$$-2\mathrm{Im}\,\Sigma \propto \rho(s) \quad \forall s$$

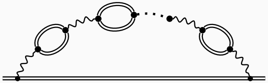
Instability doesn't break unitarity, but we can't associate $2\mathrm{Im}\,\Sigma$ to $|\mathcal{M}|^2$, no asymptotic states

Narrow-width approximation: $\rho(s) \approx \frac{\mathrm{Im}\,\Pi}{(s - \mathrm{Re}\,\Pi)^2 + \mathrm{Im}\,\Pi^2} \xrightarrow{\mathrm{Im}\,\Pi \to 0} \pi\delta(s - \mathrm{Re}\,\Pi)$

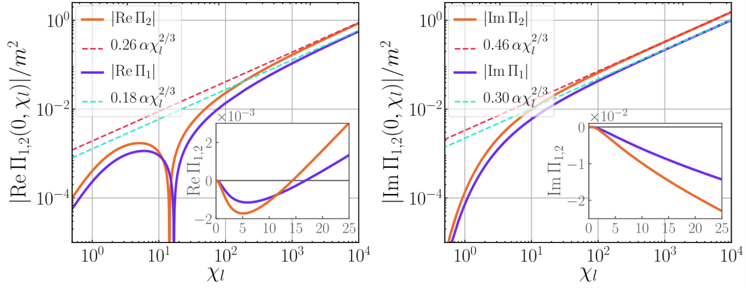
Cutting through unstable photon states is valid for $\chi_l \ll 1$

[Donoghue, Menezes PRD 100, 105006 (2019)]

Cutting the bubble-chain mass operator (scalar case)

What can we extract out of the bubble-chain Σ =  ?

$$\frac{d\Sigma_i}{du} = \frac{\alpha m^2}{2\pi}\frac{1}{(1+u)^2}\int_{\sigma_0(u)}^{\infty} d\sigma \left[\frac{\tilde{z}_i}{z}\frac{1}{\sigma} + \frac{2\mp 1}{z}\sigma\right]\exp\left(-i\tilde{z}_i\sigma - i\frac{\sigma^3}{3}\right), \qquad \tilde{z}_i = z\left(1 + \frac{\chi_{p'}\chi_p}{\chi_l^2}\boxed{\frac{\Pi_i(0,\chi_l)}{m^2}}\right)$$



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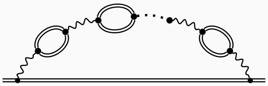

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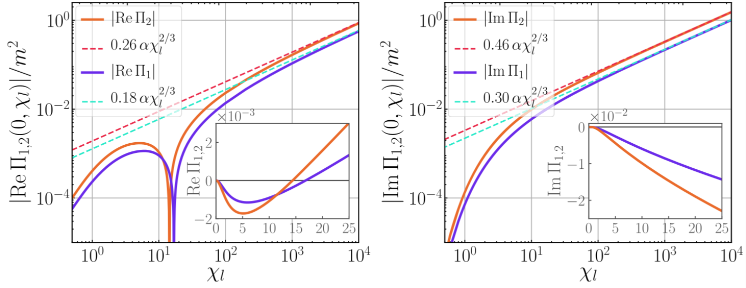
- Set $\Pi_i = 0$

  bare γ emission

  $\mathrm{Im}\Sigma = -W_{rad}/2p^0$



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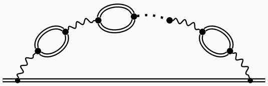

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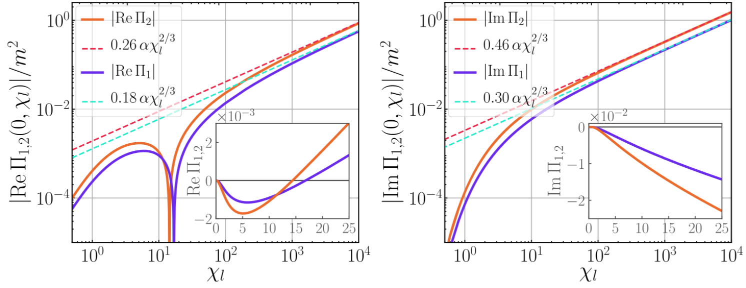
- $\mathrm{Im}\,\Pi_i(\chi_l \ll 1) \ll \mathrm{Re}\,\Pi_i$
  quasi-stable dressed γ emission
  $\mathrm{Im}\Sigma \sim W^{e\to e\gamma} + W^{e\to eee}$



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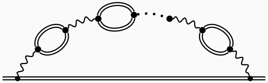
What can we extract out of the bubble-chain Σ =  ?

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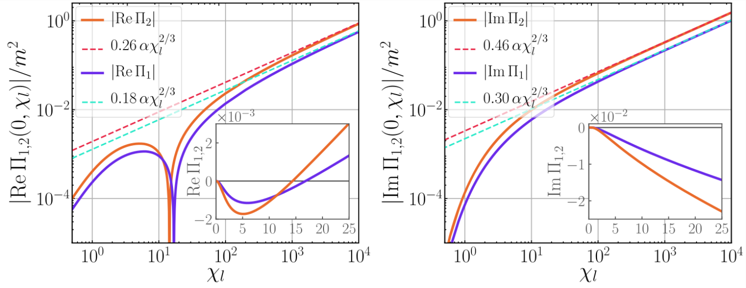
- $\mathrm{Im}\,\Pi_i(\chi_l > 1) \sim \mathrm{Re}\,\Pi_i$
  no stable final γ states!
  $\mathrm{Im}\Sigma \sim \tilde{W}^{e\to eee}$



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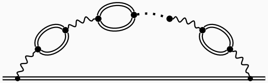
What can we extract out of the bubble-chain Σ =  ?

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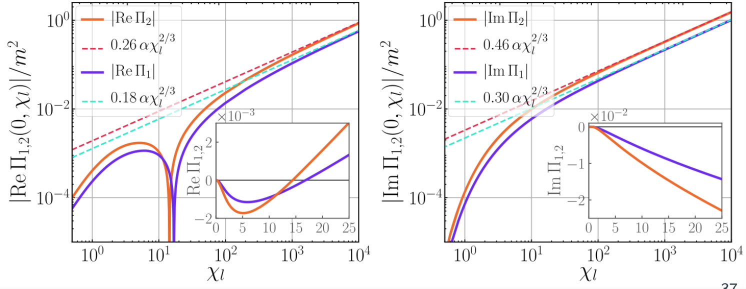
  no stable final γ states!
  $\tau_{\text{form}} \sim \tau_{\text{decay}} \sim \dfrac{m}{eE}\dfrac{\chi^{1/3}}{\sqrt{\alpha}}$



https://github.com/ArsenyMironov/SFQED-Loops

How to cite: https://doi.org/10.5281/zenodo.5866682

README.md

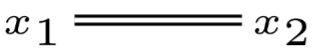
SFQED-Loops

This is a collection of Mathematica scripts developed for calculating loop processes in Strong-Field QED specifically in a constant crossed external electromagnetic field.

DOI 10.5281/zenodo.5866682

The list of diagrams

The tree-level electron propagator in a CCF in the proper time representation



Filename: *tree-level e propagator.nb*

The script shows how to transform the electron propagator in the Ritus E-p representation into the proper time representation (in accordance with J. Schwinger's result)

The bubble-chain photon propagator in a CCF in the proper time representation

Filename: *bubble-chain photon propagator.nb*

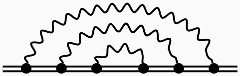
Transformation of the bubble-chain photon propagator in the momentum representation (obtained by N.B. Narozhny)

Non-perturbative regime: outlook

- Strong-field effects in a CCF differ from that of in E- and H-type fields. A supercritical CCF induces a new regime of interaction (e.g. on the impact of high-energy particles with slowly varying fields).

- We made only the first steps towards the formulation of the consistent theory in the nonperturbative regime. E.g. vertex corrections are yet to be rigorously studied.

- At $\chi \gg 1$ bubble-chain diagrams are dominant at each level of PT. This does not automatically provide that the **sum** of them is also dominant over a **sum over sub-dominant** types of diagrams



- Defining single-particle states is an intricate problem: NpSFQED is a theory with unstable particle states. We still cannot calculate cross sections, but within the RN conjecture it seems to be viable

Open questions

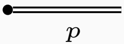
- Can we tune heavy ion colliders to study SFQED effects?
- Borrowing tools from QCD: lattice simulations (TODO list: formulation, spinor vs scalar QED, the sign problem, defining the operators to calculate)
- Can an extreme CCF induce phase transitions (line in supercritical magnetic fields)?
- Electroweak theory and QCD in a strong CCF?

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

Electron propagation (at $\alpha\chi^{2/3} < 1$) [Kostyukov et al, PRD 108, 093007 (2023)]

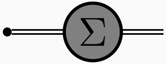
$$\left(\hat{\slashed{p}} + e\slashed{A} - m\right)\psi_p(x) = 0$$



- CCF: $A^\mu = a^\mu \varphi, \quad \varphi = kx, \quad k^2 = ka = 0$
- Volkov solutions: $\psi_{p,\sigma}(x) = E_p(x) u_{p,\sigma} \longrightarrow E_p$-representation; $\qquad u_{p,\sigma} = (\slashed{p} + m)(1 \pm \gamma_5 \slashed{n})w$

Electron propagation (at $\alpha\chi^{2/3} < 1$) [Kostyukov et al, PRD 108, 093007 (2023)]

$$D(p,F)\psi_p = \left[\not{p} - m - \Sigma(p,F)\right]\psi_p = 0$$



$$D(p,F) = \left[mS + \not{p}V^{(1)} + \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} + \frac{e\sigma_{\mu\nu}F^{\mu\nu}}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]$$

$[S,\, V^{(i)},\, T,\, A] = [S,\, V^{(i)},\, T,\, A](p^2,\chi)$ — known explicitly at 1-loop

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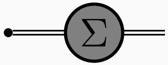

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Solution exists if $\det D = 0$:

$$\psi_p = \underbrace{\left[mS - \not{p}V^{(1)} - \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} - \frac{e\sigma_{\mu\nu}F^{\mu\nu}}{m\chi}T + \frac{e(\gamma F^{\star} p)\gamma^5}{m^2\chi}A\right]}_{\overline{D}=D^{-1}}(1 \pm \gamma_5 \not{n}_D)w$$

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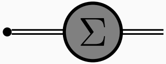

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- Mass shift: $p^2 = m^2_{\uparrow\downarrow} = \frac{m^2}{V^{(1)\,2}}\left[S^2 + \left(A^2 - 2V^{(1)}V^{(2)}\right) \pm 2\left(SA - 2TV^{(1)}\right)\right]$
- Polarization axis is now fixed: $n^\mu \longrightarrow n^\mu_D = \frac{e(F^\star p)^\mu}{m^3\chi}$

[Ritus 1970], see also [Podszus, Di Piazza PRD 104 016014 (2021) and Podszus, Dinu, Di Piazza arXiv:2206.10345 (2022)]

Short time evolution of a wave packet ($t < W_{rad}^{-1} \sim [\operatorname{Im}\Sigma_{on-shell}]^{-1}$)

$$w = C(\mathbf{p})\{0,1,0,0\}, \qquad C(\mathbf{p}) = N \exp\left[-\frac{(p_z+\varepsilon)^2}{2\Delta_z^2} - \frac{p_\perp^2}{2\Delta_\perp^2}\right]$$

$$\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} E_p(x) C(\mathbf{p}) U_{\uparrow\downarrow}, \qquad U_{\uparrow\downarrow} = (\not{p} + m)(1 \pm \gamma_5 \not{n}) \times \{0,1,0,0\}$$

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$$\rho(x) = \psi^\dagger(x)\psi(x)$$

$$\boxed{\rho(x) = C\,(\rho_0 + \alpha\rho_\alpha)\exp\left[-Q(D_\perp x, D_\perp y, D_z z) - W_{rad}\left(t + \frac{(eE)^2\varphi^3}{6m^3\varepsilon^2}\right)\right]}$$

$$\rho_0 = \frac{1}{2\varepsilon^2}\left\{(\varepsilon+m)^2 + D_z^4(t+z)^2 + \ldots\right\}, \qquad \rho_\alpha = \left[2 + \frac{(eE)^2\varphi^2}{2m^2\varepsilon^2}\right]\mathrm{Re}\left[\frac{V^{(1)}-1}{\alpha} \mp \chi\frac{m}{2\varepsilon}\frac{A}{\alpha}\right].$$

Short time evolution of a wave packet ($t < W_{rad}^{-1} \sim [\mathrm{Im}\,\Sigma_{on-shell}]^{-1}$)

$$w = C(\mathbf{p})\{0,1,0,0\}, \qquad C(\mathbf{p}) = N\exp\left[-\frac{(p_z+\varepsilon)^2}{2\Delta_z^2} - \frac{p_\perp^2}{2\Delta_\perp^2}\right]$$

$$\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} E_p(x) C(\mathbf{p}) U_{\uparrow\downarrow}, \qquad U_{\uparrow\downarrow} = (\not{p} + m)(1 \pm \gamma_5 \not{n}) \times \{0,1,0,0\}$$

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- The main effect of radiative corrections is:
  1. Dampening by $e^{-W_r T}$, where $W_r = -\mathrm{Im}\, m_{\uparrow\downarrow}^2/2\varepsilon$ is the emission probability rate
     This dampening is enhanced by additional term compared to the expected $e^{-W_r t}$
  2. $\alpha\rho_\alpha$ — 1-loop correction to the pre-exponent
- Trajectory of the wave packet and its width are not modified by the radiative corrections

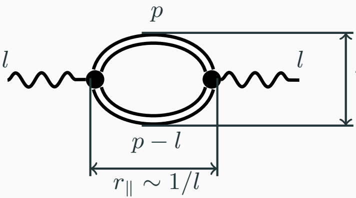
In the same fashion, we can calculate spin expectation value $\langle \mathbf{S} \rangle = \int_{-\infty}^{+\infty} d^3\mathbf{r}\, \psi^\dagger(x)\mathbf{\Sigma}\psi(x)$, etc

Intuitive insight into a loop scale formation

- Electron gains $p \sim eEt$ during loop life-time t
- **Energy mismatch** in a virtual state is estimated as

$$\Delta\varepsilon \approx \sqrt{p_{\|}^2 + p_{\perp}^2} + \sqrt{(l - p_{\|})^2 + p_{\perp}^2} - l \approx \ /p_{\|} \sim l,\, p \gg m/ \ \approx \frac{e^2E^2t^2}{l}$$

- Uncertainty principle: $\Delta\varepsilon \times t \sim 1 \Longrightarrow t \sim (l/e^2E^2)^{1/3}$

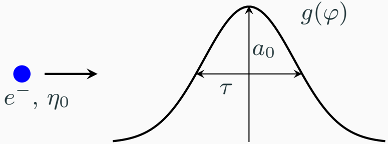


$$r_{\perp} \sim \frac{1}{eEt} \sim \frac{1}{(eEl)^{1/3}} \sim \boxed{\frac{1}{m\chi_l^{1/3}}}$$

- Photon dynamical mass $\sim$ plasma frequency of a 'relativistic plasma of virtual pairs':

$$\boxed{m_\gamma^2 \simeq \omega_p^2 \equiv \frac{8\pi e^2}{m\gamma} n_{e^+e^-} \simeq \frac{\alpha}{k}\frac{1}{V_{loop}} \simeq \alpha m^2 \chi_l^{2/3}}$$

Reaching the non-perturbative regime



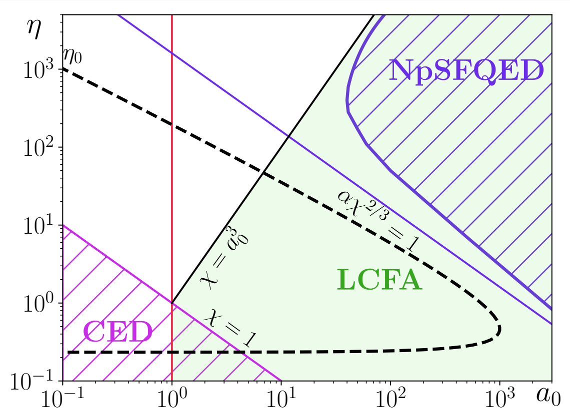
Let η_0, a_0: $\alpha\chi_0^{2/3} \gg 1$

e^- radiates upon entering the field

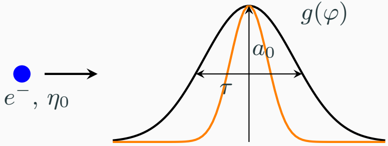
Energy losses:

$$\frac{1}{\eta_f^{1/3}} = \frac{1}{\eta_0^{1/3}} + \int_{-\infty}^{\varphi} d\varphi\, \alpha(a_0 g(\varphi))^{2/3}$$

At max field e^- lost it's energy!



Reaching the non-perturbative regime



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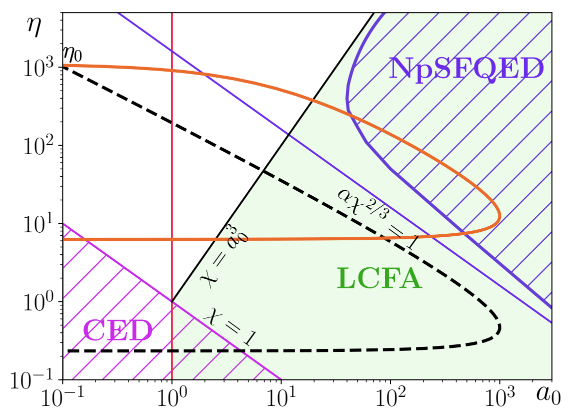


In a short pulse e^- may reach NpQED regime

Yakimenko et al., PRL 122 (2019); Di Piazza, Wistisen, Tamburini, and Uggerhoj, PRL 124 (2020); Blackburn, Ilderton, Marklund, and Ridgers NJP 21 (2019); Baumann and Pukhov PPCF 61 (2019); Baumann, Nerush, Pukhov, and Kostyukov Sci. Rep. 9 (2019)

Plane wave case: Volkov solution $(\not{\hat{p}} - e\not{A} - m)\,\psi_p = 0$

Plane wave: $A^\mu = A^\mu(\varphi), \quad \varphi = kx, \quad k^2 = kA = 0$

Solution:

$$\boxed{\psi_{p,\sigma}(x) = e^{iS_p}\Sigma_p u_{p,\sigma}}, \ (\not{p} - m)u_{p,\sigma} = 0$$

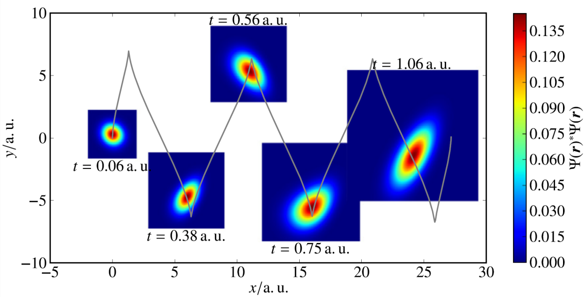
Spin factor:

$$\Sigma_p = 1 + \frac{e}{2(kp)}\not{k}\not{A}$$

Classical action:

$$S_p = -px - \frac{e}{kp}\int_0^\varphi \left(pA(\varphi) - \frac{e}{2}A^2(\varphi)\right)\,d\varphi$$

$\{\psi_p\}$ — full orthonormal set of solutions



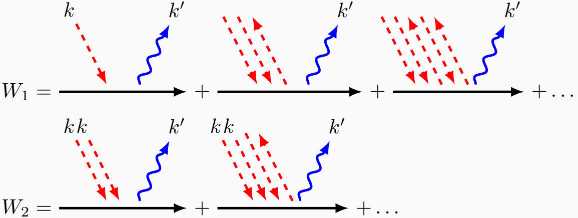
Free wave packet evolution in a plane wave field. The solid gray — classical trajectory.

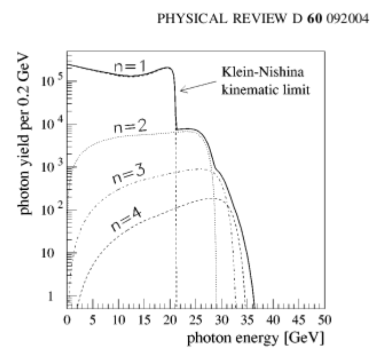
Bauke and Keitel, Computer Physics Communications 182, 12 (2011);

A. Di Piazza, Rev. Mod. Phys. 84 (2012)

Compton scattering in a circularly polarized plane wave $A^\mu(\varphi) = a_1^\mu \cos\varphi + a_2^\mu \sin\varphi$

$$S_{i\to f} = -ie\int d^4x\,\bar\psi_{p'}(x)\not\epsilon_l^{*} e^{ilx}\psi_p(x) = (2\pi)^4\sum_{s\geq 1} M^{(s)}\delta^{(4)}(q'+k'-q-sk)$$





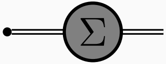
$$W_{i\to f}^{(s)}(\xi,\chi) = \frac{\alpha m^2}{4q_0}\int_0^{u_s}\frac{du}{(1+u)^2}\left\{-4J_s^2(z) + a_0^2\left(2+\frac{u^2}{1+u}\right)\times\left[J_{s+1}^2(z)+J_{s-1}^2(z)-2J_s^2(z)\right]\right\},$$

$$z = \frac{a_0^2\sqrt{1+a_0^2}}{\chi}\sqrt{u\,(u_s-u)},\qquad u_s = \frac{2s\chi}{a_0(1+a_0^2)}$$

Narozhny, Nikishov, Ritus, Sov. Phys. JETP 20, 622–629 (1965)

Accounting for radiative corrections in sub-NpQED regime ($\alpha\chi^{2/3} < 1$)

$$D(p,F)\psi_p = \left[\not{p} - m - \Sigma(p,F)\right]\psi_p = 0$$



$$D(p,F) = \left[mS + \not{p}V^{(1)} + \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} + \frac{e\sigma_{\mu\nu}F^{\mu\nu}}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]$$

$$[S,\, V^{(i)},\, T,\, A] = [S,\, V^{(i)},\, T,\, A](p^2,\chi)$$

- Solution exists if $\det D = 0$:

$$\psi_p = \underbrace{\left[mS - \not{p}V^{(1)} - \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} - \frac{e\sigma_{\mu\nu}F^{\mu\nu}}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]}_{\overline{D}}(1 \pm \gamma_5 \not{n}_D)w$$

- Polarization axis is now fixed: $n^\mu \longrightarrow n^\mu_D = \dfrac{e(F^\star p)^\mu}{m^3\chi}$
- Mass shift: $p^2 = m^2_{\uparrow\downarrow} = \dfrac{m^2}{V^{(1)\,2}}\left[S^2 + \left(A^2 - 2V^{(1)}V^{(2)}\right) \pm 2\left(SA - 2TV^{(1)}\right)\right]$

[Ritus 1970], see also [Podszus, Di Piazza PRD 104 016014 (2021) and Podszus, Dinu, Di Piazza arXiv:2206.10345 (2022)]

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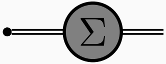

$$D(p,F) = \left[mS + \not{p}V^{(1)} + \frac{e^2(\gamma F^2 p)}{m^4\chi^2}V^{(2)} + \frac{e\sigma_{\mu\nu}F^{\mu\nu}}{m\chi}T + \frac{e(\gamma F^\star p)\gamma^5}{m^2\chi}A\right]$$

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- At 1-loop

$$S = -1 - \frac{\alpha}{\pi}\int_0^\infty \frac{du}{(1+u)^2} f_1(\lambda), \qquad f_1(\lambda) = \int_0^\infty \frac{d\sigma}{\sigma} e^{-i\lambda\sigma}\left[e^{-i\frac{\sigma^3}{3}} - 1\right], \quad \lambda = \left(\frac{u}{\chi}\right)^{2/3}$$

$$V^{(i)},\, T,\, A = \ldots$$

Polarization operator in a CCF

$$\Pi_{\mu\nu}(l) = l^2\widehat{\Pi}(l^2,\chi_l)g_{\mu\nu} + \sum_{i=1}^{2}\Pi_i(l^2,\chi_l)\epsilon_\mu^{(i)}(l)\epsilon_\nu^{(i)}(l),$$

$$\epsilon_\mu^{(1)}(l) = \frac{eF_{\mu\nu}l^\nu}{m^3\chi_l}, \quad \epsilon_\mu^{(2)}(l) = \frac{eF^\star_{\mu\nu}l^\nu}{m^3\chi_l}$$

$$l^2\widehat{\Pi}(l^2,\chi_l) = -l^2\frac{4\alpha}{\pi}\int_4^\infty \frac{dv}{v^{5/2}\sqrt{v-4}}\left[f_1(\zeta) - \log\left(1-\frac{l^2}{vm^2}\right)\right] \simeq \frac{2\alpha}{9\pi}\log\chi_l \ll 1$$

$$\Pi_{1,2}(l^2,\chi_l) = m^2\frac{4\alpha\chi_l^{2/3}}{3\pi}\int_4^\infty \frac{dv}{v^{13/6}}\,\frac{v+0.5\mp 1.5}{\sqrt{v-4}}f'(\zeta), \quad \zeta = \left(\frac{v}{\chi_l}\right)^{2/3}\left(1-\frac{l^2}{vm^2}\right)$$

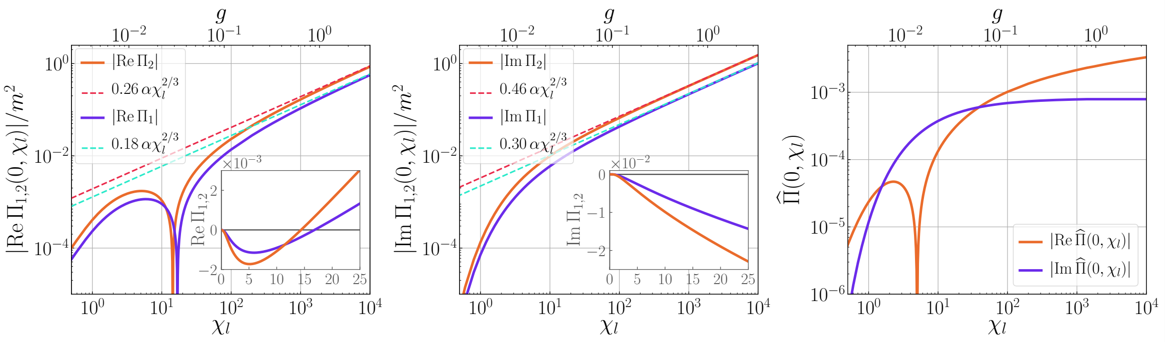
$$f(\zeta) = i\int_0^\infty d\sigma\, e^{-i(\zeta\sigma+\sigma^3/3)},$$

$$f_1(\zeta) = \int_\zeta^\infty dz\left[f(z) - \frac{1}{z}\right]$$

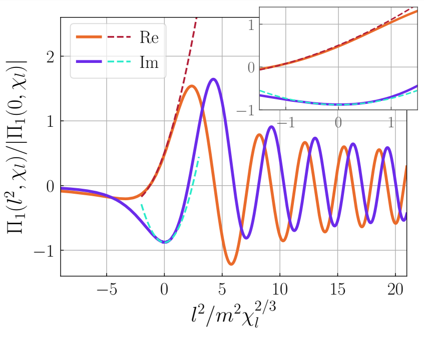
$$\Pi_1(0,\chi_l\gg 1) \approx e^{-i\pi/3}\frac{2}{3\sqrt[3]{6}\sqrt{\pi}}\frac{\Gamma^2(2/3)}{\Gamma(13/6)}\boxed{m^2\alpha\chi_l^{2/3}}, \quad \Pi_2(0,\chi_l\gg 1) = \frac{3}{2}\Pi_1$$

Graphic representation and high-χ asymptotics validity

$$g = \alpha \chi_l^{2/3}$$



Dependence on l^2



If $l^2 \lesssim m^2\chi_l^{2/3}$:

$$\Pi_i(l^2,\chi_l) \approx m^2\alpha\chi_l^{2/3}\left[K_i + K_i^{(1)}\frac{l^2}{m^2\chi_l^{2/3}} + K_i^{(2)}\left(\frac{l^2}{m^2\chi_l^{2/3}}\right)^2\right]$$

where K_i, $K_i^{(1,2)}$ are constants

Integration over $\lambda = l^2$ ($m = 1$ for brevity)

$$\pi(\lambda) = \int_0^\infty d\xi\, \tilde{\pi}(\xi)\, e^{i\xi\lambda}, \qquad \boxed{\xi_{\rm eff} \simeq \chi_l^{-2/3}}$$

Consider the master integral:

$$J_1(z) = \int_{-\infty}^{+\infty} d\lambda\, \frac{\pi(\lambda) e^{-i\lambda z}}{\lambda - \pi(\lambda)} = \int_{-\infty}^{+\infty} d\lambda\, e^{-i\lambda z} \sum_{n=0}^{\infty} \left(\frac{\pi(\lambda)}{\lambda + i0} \right)^{n+1}$$

$$= \sum_{n=0}^{\infty} \left(\prod_{a=1}^{n+1} \int_0^\infty d\xi_a \tilde{\pi}(\xi_a) \right) \int_{-\infty}^{+\infty} \frac{d\lambda}{(\lambda + i0)^{n+1}} \exp\left[i \left(\sum_{a=1}^{n+1} \xi_a - z \right) \lambda \right]$$

$$= -2\pi i \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \left(\prod_{a=1}^{n+1} \int_0^\infty d\xi_a \tilde{\pi}(\xi_a) \right) \left(z - \sum_{a=1}^{n+1} \xi_a \right)^n \theta\left(z - \sum_{a=1}^{n+1} \xi_a \right)$$

Note that $n_{\rm eff} \sim z_{\rm eff} \simeq 1/\pi(0)$, hence

$$\sum_{a=1}^{n+1} \xi_a \sim n_{\rm eff} \cdot \xi_{\rm eff} \simeq \frac{z}{\chi_l^{2/3}} \ll z, \quad \pi^{n+1}(0) \cdot n z^{n-1} \sum_{a=1}^{n+1} \xi_a \sim \chi_l^{-2/3} \ll 1, \quad \text{etc.}$$

We arrive at the approximation: $\boxed{J_1(z) \approx -2\pi i\, \theta(z - \xi_{\rm eff}) \pi(0) e^{-i\pi(0) z}}$