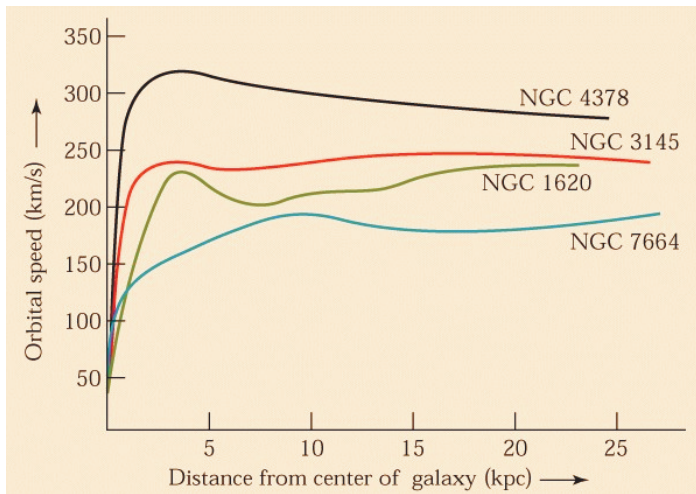


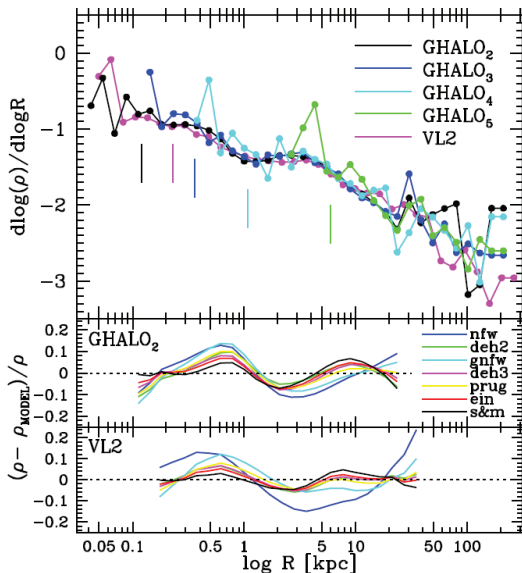
Cusps in the center of galaxies: a real conflict with observations or a numerical artefact of cosmological simulations?

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Dark matter. Discovery.



Density profiles. N-body simulations (Stadel et al. 2009)



Density profiles. Theory.

Isothermal profile

$$\rho \sim r^{-2}$$

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

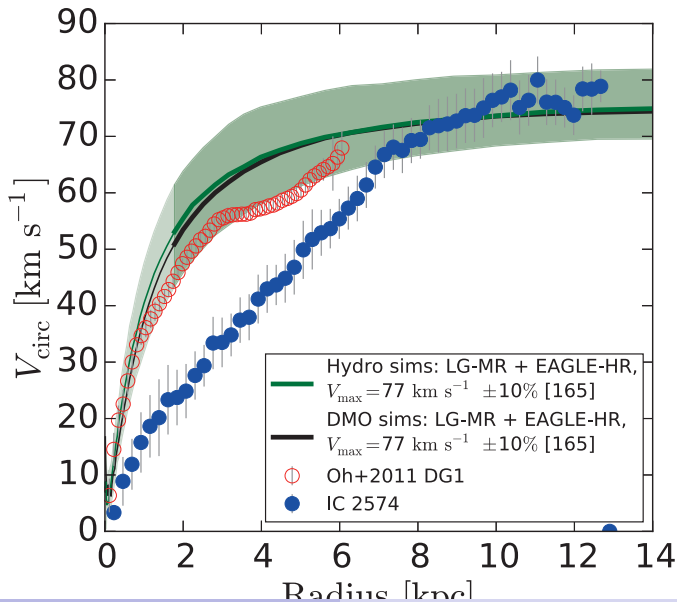
Einasto profile

$$\rho_{Ei} = \rho_s \exp \left\{ -2n \left[\left(\frac{r}{r_s} \right)^{\frac{1}{n}} - 1 \right] \right\}$$

Hernquist profile

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3}$$

Simulations vs. observations (Oman et al. 2015)



Relaxation time

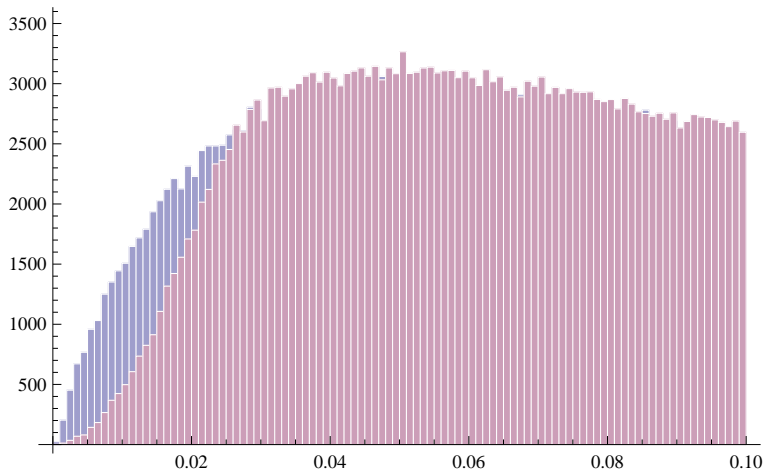
$$\langle \Delta v \rangle \simeq 0 \quad \langle \Delta v^2 \rangle \simeq \frac{8v^2 \ln \Lambda}{N(r)}$$

$$\tau_r(r) = \frac{N(r)}{8 \ln \Lambda} \cdot \tau_d(r) \quad \tau_d(r) \sim \frac{r}{v}$$

(Power et. al. 2003) $t_0 \leq 1.7\tau_r$

(Hayashi et al. 2003; Klypin et al. 2013) $t_0 \leq 30\tau_r$

Core formation



Simulation details. Gadget-3.

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3} \quad \phi(r) = -\frac{GM}{r+a}$$

$M = 10^9 M_\odot$, $a = 100$ pc. We use $N = 10^6$ test bodies.

The relaxation time at $r = a$ is $\simeq 8.8 \cdot 10^{16}$ s $\simeq 2.8 \cdot 10^9$ years.

Therefore, we make 200 snapshots with the time interval

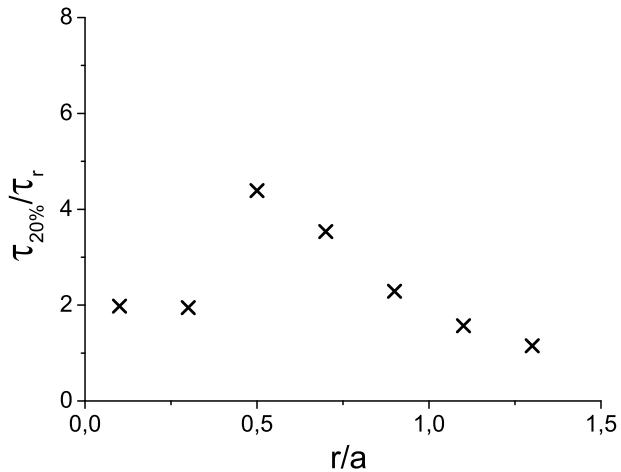
$\Delta t = 10^{15}$ s, covering the time from 0 to

$t_{max} = 2 \cdot 10^{17}$ s $\simeq 6.5 \cdot 10^9$ years.

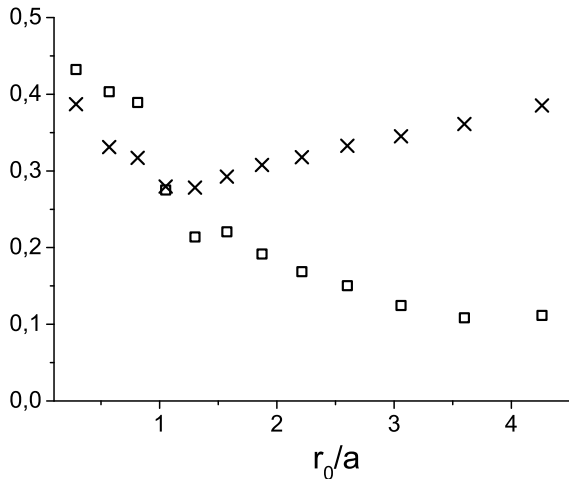
The integrals of motion $\epsilon = \phi(r) + v^2/2$, $\vec{K} = [\vec{v} \times \vec{r}]$, r_0 :

$$\epsilon = \phi(r_0) + K^2/2r_0$$

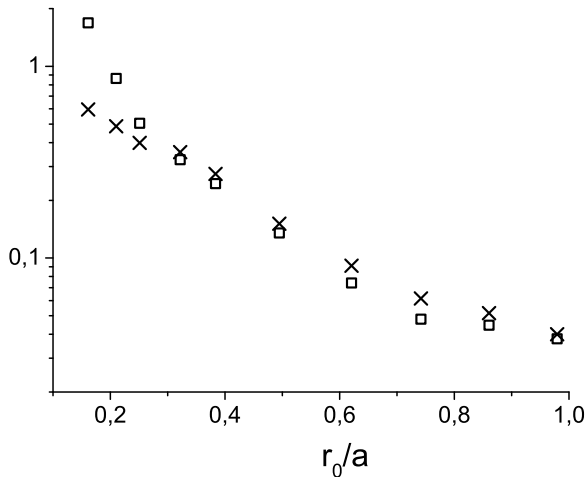
Core formation



$\langle \Delta K / K_{circ} \rangle$ (squares) and $\langle \Delta r_0 / r_0 \rangle$ (crosses)



The ratios $\frac{K_{circ}}{\tau_r} \left\langle \frac{\Delta K}{\Delta t} \right\rangle^{-1}$ (squares) and $\frac{1}{\tau_r} \left\langle \frac{\Delta r_0}{r_0 \Delta t} \right\rangle^{-1}$ (crosses)



Kinetic equations

$$\frac{df}{dt} = \frac{\partial}{\partial p_\alpha} \left\{ \tilde{A}_\alpha f + \frac{\partial}{\partial p_\beta} [B_{\alpha\beta} f] \right\}$$

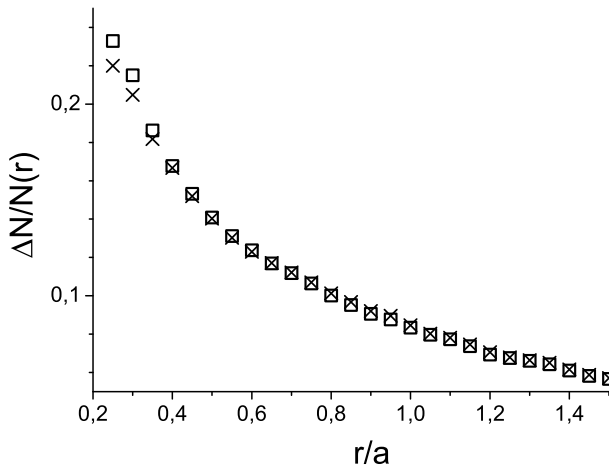
where \vec{q} is the momentum changing $\vec{p} \rightarrow \vec{p} - \vec{q}$ in a unit time.

$$\tilde{A}_\alpha = \frac{\sum q_\alpha}{\delta t} \quad B_{\alpha\beta} = \frac{\sum q_\alpha q_\beta}{2\delta t}$$

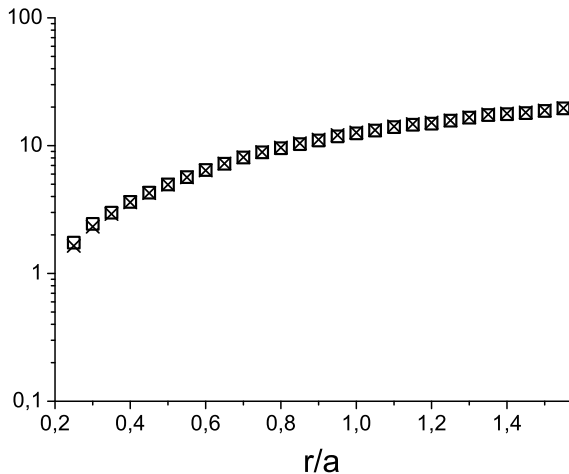
The Fokker-Planck equation has an attractor solution $\rho \propto r^{-\beta}$, where $\beta \approx 1$ (Evans & Collett 1997, Baushev 2015)

$$\frac{df}{dt} = 0 \quad \text{vs} \quad \frac{df}{dt} = \frac{\partial^2 [B_{\alpha\beta} f]}{\partial p_\alpha \partial p_\beta}$$

The upward $\Delta N_+(r)/\Delta t$ (squares) and downward $\Delta N_-(r)/\Delta t$ (crosses) Fokker-Planck streams



$1.7\tau_r \frac{\Delta N_+(r)}{N(r)\Delta t}$ (squares) and $1.7\tau_r \frac{\Delta N_-(r)}{N(r)\Delta t}$ (crosses)



Conclusions

- 1) Though the cuspy profile is stable, all integrals of motion characterizing individual particles suffer strong unphysical variations along the whole halo, revealing an effective interaction between the test bodies.
- 2) This result casts doubts on the reliability of the velocity distribution function obtained in the simulations.
- 3) We find unphysical Fokker-Planck streams of particles in the cusp region. The same streams should appear in cosmological N-body simulations, being strong enough to change the shape of the cusp or even to create it.
- 4) A much better understanding of the N-body simulation convergency is necessary before a 'core-cusp problem' can properly be used to question the validity of the CDM model.