Phase transitions in hexagonal, graphen-like lattice sheets and nanotubes

D. Ebert

Institute of Physics, Humboldt-University Berlin

Collaborators: K.G.Klimenko, P.B.Kolmakov, V.Ch.Zhukovsky
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JINR Dubna
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Introduction

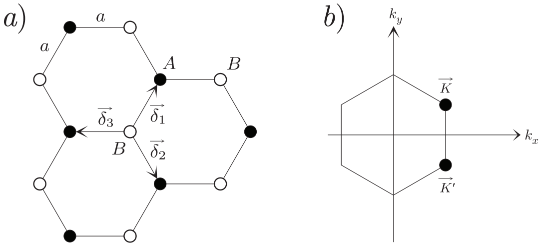


Figure: *a.) Hexagonal honeycomb lattice with two interpenetrating triangular lattices of A and B sites.* $\vec{\delta}_i$, $i = 1, 2, 3$ *are the nearest neighbor vectors*

b.) Corresponding Brillouin zone: the Dirac cones of the fermion spectrum are located at the K and K′ points

- ▷ Two sublattice degrees of freedom (pseudospin)
- ▷ Two valley degrees of freedom (2 Dirac points)

$\Rightarrow$ reducible 4-spinor description in $D = 2 + 1 \;\; \Leftrightarrow \;\;$ chiral γ^5

- Chiral "valley-sublattice" symmetry $U(2)_{\mathrm{vs}}$
- Inclusion of Coulomb interaction and general four-fermion interactions $\Rightarrow$ extended schematic graphen-like model
- Chiral symmetry breaking: fermion mass and exciton spectrum
- Construction of the effective potential. Phase transitions under external conditions: temperature, chemical potential, and Zeeman effect
- Compactification of one dimension: nanotubes, Aharonov–Bohm effect

Effective free low-energy model

Tight-binding Hamiltonian:

$$H_0 = -t \sum_{\vec{r}\in B} \sum_{i=1,2,3} \left[\psi^{+Aa}(\vec{r}+\vec{\delta}_i)\psi^{Ba}(\vec{r}) + h.c. \right]$$

• t — hopping constant; ψ^{+Aa}, ψ^{Ba} — fermion field operators belonging to triangular sublattices with A and B sites; $\vec{\delta}_i$ nearest neighbor vectors.
"Multilayer" case of $N_{\rm f} = 2N$ degenerate fermion species (flavors) of real spin $\uparrow$ and $\downarrow$, living on N hexagonal monolayers; flavor index $a = (1, ..., N_{\rm f} = 2N)$.
• Low energy expansion around 2 Dirac points K, K' and continuous limit $\Rightarrow$ effective free low-energy Lagrangian:

$$\mathcal{L}_0 = \overline{\psi} \left[i\gamma^0\partial_0 + iv_{\rm F}\gamma^1\partial_x + iv_{\rm F}\gamma^2\partial_y \right] \psi = \overline{\psi} i\gamma^\mu \tilde{\partial}_\mu \psi$$

$$\tilde{\partial}_\mu = (\partial_0, v_{\rm F}\vec{\nabla}),\ \ v_{\rm F} = \frac{3}{2}ta$$

$$\psi^t = (\psi_K^{Aa}, \psi_K^{Ba}, -i\psi_{K'}^{Ba}, i\psi_{K'}^{Aa})$$

- Reducible chiral (Weyl) 4×4 representation of Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 0 & \mathrm{I}_2 \\ \mathrm{I}_2 & 0 \end{pmatrix},\ \gamma^1 = \begin{pmatrix} 0 & -\tau^1 \\ \tau^1 & 0 \end{pmatrix},\ \gamma^2 = \begin{pmatrix} 0 & -\tau^2 \\ \tau^2 & 0 \end{pmatrix}$$
$$\gamma^3 = \begin{pmatrix} 0 & -\tau^3 \\ \tau^3 & 0 \end{pmatrix},\ \gamma^5 = \begin{pmatrix} \mathrm{I}_2 & 0 \\ 0 & -\mathrm{I}_2 \end{pmatrix},\ \gamma^{35} = \frac{1}{2}\left[\gamma^3, \gamma^5\right] = \begin{pmatrix} 0 & \tau^3 \\ \tau^3 & 0 \end{pmatrix}.$$

"Right" and "left" spinors:

$$\psi_\pm = \mathcal{P}_\pm \psi,\ \ \mathcal{P}_\pm = \frac{1}{2}(1 \pm \gamma^5)$$
$$\gamma^5 \psi_\pm = \pm \psi_\pm$$

Chirality eigenvalues ± 1 corresponding to valley indices for K, K'.

- Emergent continuous $U(2)_{\mathrm{vs}}$-symmetry:

$$t^1 = \frac{1}{2} i\gamma^3,\ t^2 = \frac{1}{2}\gamma^5,\ t^3 = \frac{1}{2}\gamma^{35}$$
$$\left[t^i, t^j\right] = i\varepsilon_{ijk} t^k$$

- Invariance under larger group $U(2N_{\mathrm{f}})$, Generators $t^i \otimes \frac{\lambda^\alpha}{2} \otimes \sigma^m$.
- Discrete symmetries $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$:

$$\psi(x^0, x, y) \xrightarrow{\mathcal{P}} i\gamma^1\gamma^5\psi(x^0, -x, y),$$

$$\psi(x^0, \vec{r}) \xrightarrow{\mathcal{C}} \gamma^1\overline{\psi}^t(x^0, \vec{r}),$$

$$\psi(x^0, \vec{r}) \xrightarrow{\mathcal{T}} i\sigma^2\gamma^1\gamma^5\psi(-x^0, \vec{r})$$

Four-fermion interactions

• "Reduced" QED scenario with Dirac-Maxwell interaction:

$$S = \int d^3x \overline{\psi} i\gamma^\mu \widetilde{D}_\mu \psi - \frac{\varepsilon_0}{4} \sum_{\mu,\nu=(0,\dots,3)} \int d^4x F_{\mu\nu} F^{\mu\nu}$$

$$\widetilde{D}_\mu = (\partial_0 - ieA_0, v_{\mathrm{F}}(\vec{\nabla} + ie\vec{A}))$$

Fermion quasiparticles run in $(2+1)$-dim. space-time $x^{(3)} = (x^0, x^1, x^2)$ with Fermi velocity v_{F}; $U(1)$ gauge field propagates in $(3+1)$-dim. bulk space-time $x^{(4)} = (x^0, x^1, x^2, x^3)$ with speed of light $c\,(=1)$.

• Partition function

$$Z = \int D\psi D\overline{\psi} D_\mu[A_\mu] \exp[iS],$$

• Integration over gauge field yields Coulomb interaction

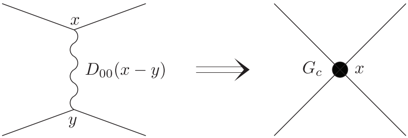
$$S = S_0 - \frac{v_\mathrm{F}}{2c} \int d^{(3)}x' \int d^{(3)}x \left[\overline{\psi}(x^0, \vec{r})\gamma^0\psi(x^0, \vec{r})\right] U_0^C(x^0 - x'^0, |\vec{r} - \vec{r}\,'|)\times$$
$$\times \left[\overline{\psi}(x'^0, \vec{r}\,')\gamma^0\psi(x'^0, \vec{r}\,')\right]$$

Instantaneous Coulomb potential

$$U_0^C(x^0, |\vec{r}|) = \frac{e^2\delta(x^0)}{\varepsilon_0 v_\mathrm{F}} \int \frac{d^2k}{(2\pi)} \exp(i\vec{k}\vec{r})\frac{1}{|\vec{k}|} = \frac{\alpha}{\varepsilon_0}\left(\frac{c}{v_\mathrm{F}}\right)\frac{\delta(x^0)}{|\vec{r}|}$$

with $v_\mathrm{F}/c \sim 1/300$ and $\alpha_\mathrm{eff} = \alpha\frac{c}{v_\mathrm{F}} \sim 2 \Rightarrow$ strong interaction!

- Low-energy contact approximation:



Local $U(2N_{\rm f})$-invariant four-fermion interaction Lagrangian:

$$\mathcal{L}_{\rm int}^{C} = -\frac{G_c v_{\rm F}}{2}\left[\overline{\psi}(x)\gamma^0\psi(x)\right]^2$$

- Coulomb interaction on the lattice contains additionally a small on-site scalar repulsive interaction term:

$$\Delta L_{\rm int} = \frac{G v_{\rm F}}{2}(\overline{\psi}\psi)^2 \quad \Rightarrow \quad \mathcal{X}\mathrm{SB}: \; U(2N_{\rm f}) \longrightarrow U(N_{\rm f})_{t^0} \otimes U(N_{\rm f})_{t^3}$$

Inclusion of phonon-mediated interaction with coupling strength g yields symmetry breaking interaction Lagrangian:

$$\mathcal{L}_{\rm int} = -\frac{1}{2} G_c v_{\rm F} (\overline{\psi}\gamma^0\psi)^2 + \frac{\widetilde{G} v_{\rm F}}{2}(\overline{\psi}\psi)^2, \qquad \widetilde{G} = G + g$$

- Fierz-transformation:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{\rm int} = \overline{\psi} i\widetilde{\partial\!\!\!/}\psi \\ &+ \Big\{ \frac{1}{2N_{\rm f}} G_1 v_{\rm F}(\overline{\psi}\psi)^2 + \frac{1}{2N_{\rm f}} G_2 v_{\rm F}(\overline{\psi}\gamma^{35}\psi)^2 \\ &+ \frac{1}{2N_{\rm f}} H_1 v_{\rm F}(\overline{\psi} i\gamma^5\psi)^2 + \frac{1}{2N_{\rm f}} H_2 v_{\rm F}(\overline{\psi}\gamma^3\psi)^2 \Big\} \end{aligned}$$

Here we omitted any constrains between coupling constants ⇒ extended schematic Gross–Neveu model

Effective potential; gap equation and exciton spectrum

- Introduce (auxiliary) excitonic fields σ_1, σ_2, φ_1, φ_2 via the Hubbard–Stratonovich transformation

$$\mathcal{L}[\overline{\psi}, \psi, \sigma_i, \varphi_i] = \overline{\psi}\Big[i\tilde{\not\partial} - \sigma_1 - \sigma_2\gamma^{35} - \varphi_1 i\gamma^5 - \varphi_2\gamma^3\Big]\psi - N_\mathrm{f}\sum_{k=1}^{2}\left(\frac{\sigma_k^2}{4v_\mathrm{F}G_k} + \frac{\varphi_k^2}{4v_\mathrm{F}H_k}\right) \quad (1)$$

- Field equations for exciton fields

$$\sigma_1 = -2\frac{G_1 v_\mathrm{F}}{N_\mathrm{f}}\overline{\psi}\psi, \qquad \sigma_2 = -2\frac{G_2 v_\mathrm{F}}{N_\mathrm{f}}\overline{\psi}\gamma^{35}\psi,$$

$$\varphi_1 = -2\frac{H_1 v_\mathrm{F}}{N_\mathrm{f}}\overline{\psi}i\gamma^5\psi, \qquad \varphi_2 = -2\frac{H_2 v_\mathrm{F}}{N_\mathrm{f}}\overline{\psi}\gamma^3\psi$$

- Gap equations:

$$\begin{aligned}
\langle\sigma_1\rangle &= -2\frac{G_1 v_\mathrm{F}}{N_\mathrm{f}}\langle\overline{\psi}\psi\rangle = 2\frac{G_1 v_\mathrm{F}}{N_\mathrm{f}}\mathrm{Tr}_\mathrm{sf}\left[iG(x,x)\right],\\
\langle\sigma_2\rangle &= -2\frac{G_2 v_\mathrm{F}}{N_\mathrm{f}}\langle\overline{\psi}\gamma^{35}\psi\rangle = 2\frac{G_2 v_\mathrm{F}}{N_\mathrm{f}}\mathrm{Tr}_\mathrm{sf}\left[\gamma^{35}iG(x,x)\right],\\
\langle\varphi_1\rangle &= -2\frac{H_1 v_\mathrm{F}}{N_\mathrm{f}}\langle\overline{\psi}i\gamma^{5}\psi\rangle = 2\frac{H_1 v_\mathrm{F}}{N_\mathrm{f}}\mathrm{Tr}_\mathrm{sf}\left[i\gamma^{5}iG(x,x)\right],\\
\langle\varphi_2\rangle &= -2\frac{H_2 v_\mathrm{F}}{N_\mathrm{f}}\langle\overline{\psi}\gamma^{3}\psi\rangle = 2\frac{H_2 v_\mathrm{F}}{N_\mathrm{f}}\mathrm{Tr}_\mathrm{sf}\left[\gamma^{3}iG(x,x)\right],
\end{aligned}$$

Inverse fermion propagator

$$\left[G^{-1}(x,x')\right]^{ab}_{\alpha\beta} = \left[i\widetilde{\partial\!\!\!/} - \langle\sigma_1\rangle - \langle\sigma_2\rangle\gamma^{35} - \langle\varphi_1\rangle i\gamma^5 - \langle\varphi_2\rangle\gamma^3\right]_{\alpha\beta}\delta^{ab}\delta^{(3)}(x-x').$$

- Transformation properties of condensates

$\langle\overline{\psi}\Gamma_i\psi\rangle$	$\langle\overline{\psi}\psi\rangle$	$\langle\overline{\psi}\gamma^{35}\psi\rangle$	$\langle\overline{\psi}i\gamma^{5}\psi\rangle$	$\langle\overline{\psi}\gamma^{3}\psi\rangle$
$\mathcal{P}$	1	−1	−1	1
$\mathcal{C}$	1	1	−1	1
$\mathcal{T}$	1	−1	1	1
γ^5	−1	1	−1	1
γ^3	−1	1	1	−1

Table: *Transformation properties of various condensates $\langle\overline{\psi}\Gamma_i\psi\rangle$, where now $\Gamma_i = \{\mathrm{I}_4, \gamma^{35}, i\gamma^5, \gamma^3\}$, under discrete $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$ and γ^5, γ^3 transformations (here we consider $\mathcal{P}: (x^0, x, y) \to (x^0, -x, y)$).*

i $\langle\overline{\psi}\psi\rangle$ breaks $U(2N_\mathrm{f})$ and discrete γ^5, γ^3, but preserves $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$

ii $\langle\overline{\psi}\gamma^{35}\psi\rangle$ preserves $U(2N_\mathrm{f})$, $\mathcal{C}$, γ^5, γ^3, but breaks $\mathcal{P}$, $\mathcal{T}$. ,,Haldane mass" $m_2 = \langle\sigma_2\rangle/{v_\mathrm{F}}^2$ related to parity anomaly in $D = (2+1)$ dimension

iii $\langle\overline{\psi}i\gamma^5\psi\rangle$ breaks $U(2N_\mathrm{f})$ and discrete $\mathcal{P}$, $\mathcal{C}$, γ^5, but preserves $\mathcal{T}$ and γ^3

iv $\langle\overline{\psi}i\gamma^3\psi\rangle$ breaks $U(2N_\mathrm{f})$ and γ^3, but preserves $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$ and γ^5

- Partition function of semi-bosonized Lagrangian:

$$Z = \int D\overline{\psi}D\psi \int D\sigma_1 D\sigma_2 D\varphi_1 D\varphi_2 \exp\left\{ i \int dx^0 d^2x \mathcal{L}[\overline{\psi}, \psi, \sigma_i, \varphi_i] \right\} \qquad (2)$$

Fermion determinant of Dirac operator $\hat{D}(x,y) = D(x,y)I_{N_\mathrm{f}}$ (being the inverse propagator) rewritten by using $\mathrm{Det}(\hat{D}) = (\mathrm{Det}\, D)^{N_f} = \exp(N_f \mathrm{Tr}_{sx} \ln D)$:

$$\begin{aligned} Z &= \int D\sigma_1 D\sigma_2 D\varphi_1 D\varphi_2 \exp\left\{ iN_\mathrm{f} S_\mathrm{eff}(\sigma_i, \varphi_i) \right\}, \\ S_\mathrm{eff}(\sigma_i, \varphi_i) &= -\int dx^0 d^2x \sum_{k=1}^{2} \left(\frac{\sigma_k^2}{4v_\mathrm{F} G_k} + \frac{\varphi_k^2}{4v_\mathrm{F} H_k} \right) \\ &\quad - i\mathrm{Tr}_{sx} \ln(i\widetilde{\partial\!\!\!/} - \sigma_1 - \sigma_2\gamma^{35} - \varphi_1 i\gamma^5 - \varphi_2\gamma^3) \end{aligned}$$

- Effective potential (large-N_f saddle point $\Rightarrow \sigma_i, \varphi_i = \mathrm{const}$)

$$V_{\text{eff}}(\sigma_i,\varphi_i)\int dx^0 d^2x = -S_{\mathrm{eff}}(\sigma_i,\varphi_i)\Big|_{\sigma_i,\varphi_i=\mathrm{const}},$$

$$V_{\text{eff}}(\sigma_i,\varphi_i) = \sum_{k=1}^{2}\left(\frac{\sigma_k^2}{4v_{\mathrm{F}}G_k}+\frac{\varphi_k^2}{4v_{\mathrm{F}}H_k}\right)+i\int\frac{dp_0 d^2\vec{p}}{(2\pi)^3}\mathrm{Tr}_s \ln D(p),$$

$$D(p) = p_0\gamma^0 - v_{\mathrm{F}}\vec{p}\vec{\gamma} - \sigma_1 - \sigma_2\gamma^{35} - \varphi_1 i\gamma^5 - \varphi_2\gamma^3$$

Using $\mathrm{Tr}_s \ln D(p) = \sum_i \ln \epsilon_i$ with ϵ_i the four eigenvalues of the 4×4 matrix $D(p)$, one can calculate the momentum integral and obtain (for $M_k/\Lambda \ll 1$):

$$V_{\text{eff}}(\sigma_i,\varphi_i) \;\; = \;\; \sum_{k=1}^{2}\left\{\frac{g_k\sigma_k^2}{4v_{\mathrm{F}}}+\frac{h_k\varphi_k^2}{4v_{\mathrm{F}}}+\frac{M_k^3}{6\pi v_{\mathrm{F}}^2}\right\},$$

$$M_{1,2} = |\sigma_2 \pm \rho|,\; \rho = \sqrt{\sigma_1^2+\varphi_1^2+\varphi_2^2},$$

where $g_k = \frac{1}{G_k} - \frac{1}{G_{\mathrm{cr}}}$, $h_k = \frac{1}{H_k} - \frac{1}{H_{\mathrm{cr}}}$, $(G_{\mathrm{cr}}^{-1} = H_{\mathrm{cr}}^{-1} = \frac{2\Lambda}{\pi})$

- Gap equations

$$\frac{\partial V_{\text{eff}}(\sigma_i, \varphi_i)}{\partial \sigma_i} = 0, \quad \frac{\partial V_{\text{eff}}(\sigma_i, \varphi_i)}{\partial \varphi_i} = 0, \quad i = 1, 2 \tag{3}$$

Illustration: $g_1 = g_2 = h_1 = h_2 = g$
Solutions

i $\langle\sigma_1\rangle = -\pi g v_{\mathrm{F}}/2$, $\langle\sigma_2\rangle = \langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$

ii $\langle\sigma_2\rangle = -\pi g v_{\mathrm{F}}/2$, $\langle\sigma_1\rangle = \langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$

iii $\langle\varphi_1\rangle = -\pi g v_{\mathrm{F}}/2$, $\langle\sigma_1\rangle = \langle\sigma_2\rangle = \langle\varphi_2\rangle = 0$

iv $\langle\varphi_2\rangle = -\pi g v_{\mathrm{F}}/2$, $\langle\sigma_1\rangle = \langle\sigma_2\rangle = \langle\varphi_1\rangle = 0$

- Exciton Spectrum:

$$\sigma_k(x) \to \langle\sigma_k\rangle + \sigma_k(x),\ \varphi_k(x) \to \langle\varphi_k\rangle + \varphi_k(x)$$

Consider now the phase with $\langle\sigma_1\rangle = m_1 v_{\mathrm{F}}^2$, $\langle\sigma_2\rangle = \langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$.
Two point 1PI Green function (inverse propagators) of fluctuating fields:

$$\Gamma_{\phi_k\phi_k}(x-y) = \frac{\delta^2 S_{\mathrm{eff}}}{\delta\phi_k(x)\delta\phi_k(y)}\Big|_{\sigma_i,\varphi_i=0}\ ,\ \phi_k = \{\sigma_1,\sigma_2,\varphi_1,\varphi_2\},$$

$$\Gamma_{\phi_k\phi_k}(x-y) = -\frac{1}{2v_{\mathrm{F}}G_{\phi_k}}\delta^{(3)}(x-y) + i\mathrm{Tr}_s\left[\hat{t}_k G_0(x-y)\hat{t}_k G_0(y-x)\right].$$

Notations:

$$G_{\phi_k} = \{G_1, G_2, H_1, H_2\}\ ,\qquad \hat{t}_k = \left\{\mathrm{I}_4, \gamma^{35}, i\gamma^5, \gamma^3\right\}\ ,\qquad k = (1, ..., 4),$$

$$G_0(x-y)_{\alpha\beta} = \int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{\tilde{\not{p}} - m_1 v_{\mathrm{F}}^2}\right)_{\alpha\beta} \mathrm{e}^{-ip(x-y)} \qquad (\widetilde{p} = (p^0, v_{\mathrm{F}}\vec{p}))$$

• The straightforward loop calculations yields in momentum space (Minkowski metric):

$$
\begin{aligned}
\Gamma_{\sigma_1\sigma_1}(p) &= \frac{\widetilde{p}^2 - (2m_1 v_{\rm F}^2)^2}{2\pi v_{\rm F}^2\sqrt{-\widetilde{p}^2}}\Gamma(p), \\
\Gamma(p) &= \tan^{-1}\left(\frac{\sqrt{-\widetilde{p}^2}}{2m_1 v_{\rm F}^2}\right), \\
\Gamma_{\sigma_2\sigma_2}(p) &= -\frac{1}{2v_{\rm F}}(g_2 - g_1) + \frac{\widetilde{p}^2 - (2m_1 v_{\rm F}^2)^2}{2\pi v_{\rm F}^2\sqrt{-\widetilde{p}^2}}\Gamma(p), \\
\Gamma_{\varphi_k\varphi_k}(p) &= -\frac{1}{2v_{\rm F}}(h_k - g_1) - \frac{\sqrt{-\widetilde{p}^2}}{2\pi v_{\rm F}^2}\Gamma(p).
\end{aligned}
$$

The inverse expressions are just the exciton propagators, the singularities of which determine their mass spectrum and dispersion laws. Scalar excitation σ_1 corresponds to a stable particle with a mass $m_\sigma = 2m_1$. Quasiparticle σ_2 is scalar resonance

• Under certain restrictions of coupling constants, the model Lagrangian acquires additional continuous symmetry.
Illustration: $g_1 = h_1 = g < 0$, $g_2 = h_2 > g \Rightarrow \langle \sigma_1 \rangle \sim \langle \overline{\psi}\psi \rangle \neq 0$
Lagrangian is invariant under continuous chiral symmetry:

$$U_{\gamma^5}(1) \;\; : \;\; \psi \to \exp(i\alpha\gamma^5)\psi,$$

$\Rightarrow$ massless GB: φ_1.

Nanotubes

- Compactification:

One spatial dimension compactified and lattice sheet is rolled up to a cylinder. Compactification of coordinate $x^2 = R\varphi$ with a length $L = 2\pi R$ (R cylinder radius) and x^1 pointing in z-direction, parallel to cylinder axis.

There exists a constant gauge field $\mathcal{A}_2$ (not to be gauged away) to be included by $\partial_2 \to D_2 = \partial_2 + ie\mathcal{A}_2$. Alternatively, keep ∂_2 and include an effective magnetic phase ϕ into the boundary condition:

$$\phi = \frac{e\mathcal{A}_2 L}{2\pi} = \frac{\Phi_m}{\Phi_m^0}$$

Φ_m — the magnetic flux passing through the tube cross section, $\Phi_m^0 = 2\pi/e$ is magnetic flux quantum.

Boundary condition:

$$\begin{aligned}\psi_K(x^0, \vec{r} + \vec{L}) &= e^{2\pi i(\phi - \frac{1}{3}\nu)}\,\psi_K(x^0, \vec{r}), \quad \nu = (0, \pm 1),\\ \psi_{K'}(x^0, \vec{r} + \vec{L}) &= e^{2\pi i(\phi + \frac{1}{3}\nu)}\,\psi_{K'}(x^0, \vec{r}).\end{aligned}$$

Fourier decomposition of spinors:

$$\psi = \frac{1}{L}\sum_{n=-\infty}^{\infty} e^{i\left[\frac{x^2}{R}(n+\phi) + p_1 x^1 + p_0 x^0\right]} \begin{pmatrix}\psi_{Kn}^{(1)}\\ \psi_{K'n}^{(2)}\end{pmatrix},$$

$$\psi_{Kn}^{(1)} = \begin{pmatrix}\psi_{Kn}^{A}\\ \psi_{Kn}^{B}\end{pmatrix} e^{-i\frac{x^2}{R}\left(\frac{\nu}{3}\right)},$$

$$\psi_{K'n}^{(2)} = \begin{pmatrix}-i\psi_{K'n}^{B}\\ i\psi_{K'n}^{A}\end{pmatrix} e^{i\frac{x^2}{R}\left(\frac{\nu}{3}\right)}.$$

- Azimuthal component of the p_2 momentum:

$$p_{\nu\phi}(n) = \frac{2\pi}{L}(n + \phi - \frac{\nu}{3}),$$

$\nu \neq 0 \Rightarrow$ "semiconductor" energy gap between conduction/valence bands

$$\Delta\mathcal{E}(n = \phi = p_1 = 0) = v_{\mathrm{F}} \frac{4\pi}{L} \frac{|\nu|}{3} \neq 0.$$

$\nu = 0 \Rightarrow$ "metallic" behavior.

Insulator phase for dynamical mass

$$\Delta\mathcal{E}(n = p_1 = \phi = 0) = 2\sqrt{v_{\mathrm{F}}^2 \left(\frac{2\pi}{L}\right)^2 \left(\frac{\nu}{3}\right)^2 + (m v_{\mathrm{F}}^2)^2}.$$

• Thermodynamic potential $\Omega_T(\to V_{\rm eff})$:
Inclusion of temperature T and extended "chemical" potential $\hat{\mu} = \mu - \frac{g}{2} s\mu_{\rm B} B_{\|}$ describing Zeeman interaction.
Replace p_0-integration in effective potential by summation over Matsubara frequencies ω_ℓ using rule:

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} f(p_0) \to \frac{i}{\beta} \sum_{\ell=-\infty}^{\infty} f(i\omega_\ell),$$

$$\omega_\ell = \frac{2\pi}{\beta}\left(\ell + \frac{1}{2}\right), \quad \ell = 0, \pm1, \pm2, ...$$

$$\beta = \frac{1}{T}, \text{ inverse temperature.}$$

Standard shift

$$\omega_\ell \to \omega_\ell - i\hat{\mu}, \quad \hat{\mu} = \mu - \frac{g}{2} s\mu_{\rm B} B_{\|} \tag{4}$$

where $s = \pm1$ for up/down spin, g Landé factor, $\mu_B = e/(2m)$ the Bohr magneton and $B_{\|}$ longitudinal in-plane magnetic field.
Boundary condition for nanotubes gives $p_2 \to p_{\nu\phi}(n) = \frac{2\pi}{L}(n + \phi - \frac{\nu}{3})$;
ϕ expressed by magnetic AB flux.

- Thermodynamic potential

$$
\begin{aligned}
V_{\rm eff}(\sigma_i, \varphi_i, T, \hat{\mu}, \phi) \;\; = \;\; & \sum_{k=1}^{2} \left\{ \left(\frac{\sigma_k^2}{4 v_{\rm F} G_k} + \frac{\varphi_k^2}{4 v_{\rm F} H_k} \right) \right. \\
& - \frac{1}{\beta L} \sum_{s=\pm 1} \sum_{\ell=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int \frac{dp_1}{2\pi} \ln \left[\left(\frac{2\pi}{\beta} \left(\ell + \frac{1}{2} \right) - i\hat{\mu} \right)^2 \right. \\
& \left. \left. + v_{\rm F}^2 \left(\frac{2\pi}{L} \right)^2 \left(n + \phi - \frac{\nu}{3} \right)^2 + v_{\rm F}^2 p_1^2 + M_k^2 \right] \right\} .
\end{aligned}
$$

Phase transitions: Aharonov–Bohm effect

- Numerical investigation of the global minima of the thermodynamic potential $V_{\text{eff}}(\sigma, \phi, T)$.

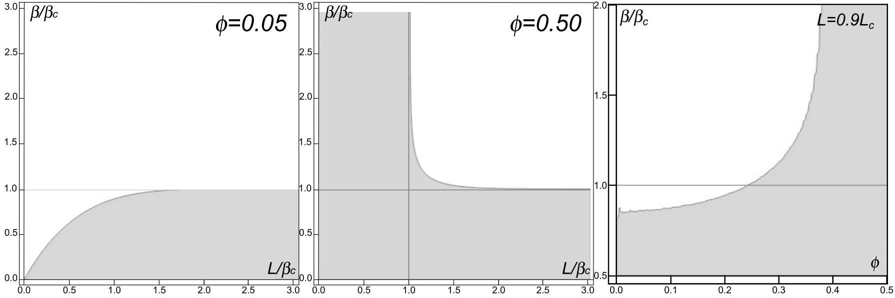


Figure: *Phase diagrams of the model in the plane (L, β) with different values of the magnetic phase ϕ and in the plane (ϕ, β) with fixed $L < L_c$ ($L_c = v_F \beta_c$).*
Painted area: symmetrical phase
Unpainted area: broken symmetry

Phase transitions: Zeeman effect

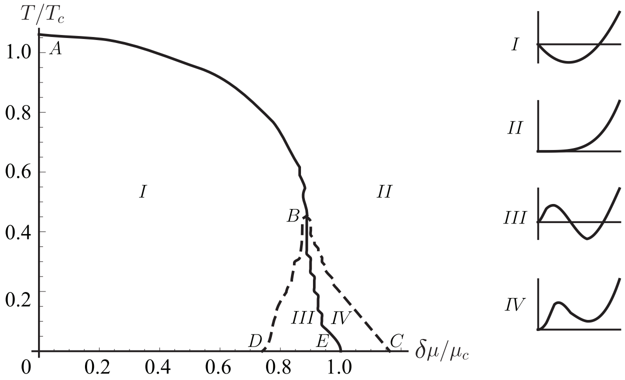


Figure: Phase diagram of the model in the plane $(\delta\mu, T)$
Area I: broken symmetry, only one minimum at $\sigma \neq 0$
Area II: symmetrical phase, only one minimum at $\sigma = 0$
Area III: broken symmetry, global minimum at $\sigma \neq 0$, local minimum at $\sigma = 0$
Area IV: symmetrical phase, global minimum at $\sigma = 0$, local minimum at $\sigma \neq 0$
Line AB: phase transition of second kind
Line BE: phase transition of first kind
Lines BC and BD: no phase transition, local minima appear/vanish

Summary

- Tight binding Hamiltonian $\longrightarrow$ effective low energy Dirac-like model of massless electrons
  - ▷ (reducible) 4-spinors
    - 2 sublattice (A, B — pseudospin)
    - 2 valley (Dirac points) d.o.f.
  - ▷ Chirality operator γ^5 (pseudohelicity)
- $U(2N_{\mathrm{f}})$ chiral symmetry
- Four-fermion contact Coulomb, one-site scalar, and phonon-mediated interactions $\rightarrow$ $\mathcal{X}$SB by condensates
- Fierz-transformation and generalization to extended schematic GN model
- Effective potential: gap eqs. and exciton spectrum
- Nanotubes by compactification and boundary conditions
  - ▷ Phase transitions at L, T, ϕ with AB effect
  - ▷ Phase transitions at $\delta\mu$ and T with Zeeman effect