PHYSICS OF ATMOSPHERIC NEUTRINOS
(introductory overview)

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Abstract

This short introduction into the current status of the atmospheric neutrino problem aims to provide some comments to the set of transparencies which represents the content of the lecture and to list the original and review papers for more profound study.

1 The role of atmospheric neutrinos in astroparticle physics

The mechanism of neutrino production in the atmosphere is well understood [1]. Electron and muon neutrinos and antineutrinos $(\nu_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu)$ come into being from the decay of unstable particles generated in the collisions of primary and secondary cosmic rays with air nuclei. Fraction of tau neutrinos and antineutrinos in the atmospheric neutrino (AN) flux is very small, because ν_τ and $\overline{\nu}_\tau$ arise only from the decay of heavy particles (like D_s or B mesons) whose production cross sections are small (in comparison with the cross sections for light meson production) at all energies of present interest.

The process of neutrino generation is rather intricate seeing that the primaries and secondaries (both stable and unstable) can repeatedly interact in the atmosphere with absorption, regeneration or overcharging, and dissipation of energy through electromagnetic interactions. At low energies ($E_\nu \lesssim 2-3$ GeV), the Earth's magnetic field give rise to the spatial (longitudinal and latitudinal) and angular (zenithal and azimuthal) asymmetries in the AN flux. Complicated structure of the real geomagnetic field, the Earth's penumbra, and re-entrant albedo embarrass the analysis of the geomagnetic effects. Quasi-periodical variations of solar activity modify the low-energy part of the primary cosmic-ray spectrum and therefore affect the neutrino spectrum (below hundreds of MeV), making the AN flux time-dependent. At very low energies ($E_\nu \lesssim 200$ MeV), the 3-dimensionality of nuclear reactions and decays are important. With increasing energy, life-times of light mesons grow and the production and decay chains become branchy: "anything produce everything". Cosmic-ray muons (whose decay is an important source of neutrinos up to the multi-TeV energy range) change their polarization due to energy loss and multiply scattering, affecting the AN spectra. Meteorological effects are also essential at all energies of interest.

Consequently, an accurate calculation of the AN flux presents a hard multi-factor problem complicated by uncertainties in the primary cosmic-ray spectrum and composition, inclusive

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and total inelastic cross sections for particle interactions and by pure computational difficulties. But solution of this problem is a prime necessity for the study of many fundamental issues of particle physics, astrophysics, and cosmology.

The AN flux represents an annoying and unavoidable background for some key low-energy experiments with underground detectors, e. g., search for proton decay and $n \to \overline{n}$ transitions in nuclei [2], and also for most of experiments on high-energy neutrino astrophysics with present-day and future large underwater/ice neutrino telescopes [2, 3]. Among the astrophysical experiments are the detection of neutrinos from the (quasi)diffuse neutrino backgrounds, like pregalactic neutrinos, neutrinos from the bright phase of galaxy evolution, from active galactic nuclei (AGN), and other astrophysical sources, indirect detection of non-relativistic dark matter (presumably composed of neutralinos) through neutrinos produced in the annihilation of the dark-matter particles captured in the Earth and the Sun, or the direct detection of relativistic WIMP (weakly-interacting massive particles) of astrophysical or cosmological origin [4].

At the same time, the AN flux is a natural instrument for studying neutrino oscillations and neutrino interactions with matter at energies beyond the reach of accelerator experiments. Search for neutrino oscillations with underground detectors is the main issue of this lecture (see also ref. [5]). Let us sketch here the problem of neutrino interactions at very high energies.

Measurements of the cross sections for $\nu_\ell N$ and $\overline{\nu}_\ell N$ charged-current interactions at $\sqrt{s} \sim m_W$ ($E_\nu \sim 3.4$ TeV) provide an important test for the standard model of electroweak interactions [6]. With modern accelerators, the interactions of neutrinos are studied at energies up to several hundreds of GeV (besides the single very high energy HERA data point extracted from the $ep \to \nu X$ cross section), whereas deep underwater experiments with AN will enable to enlarge the region of neutrino energies up to a few tens of TeV. Future "KM3" (cubic-kilometer-size) deep-underwater/ice neutrino telescopes will be able to study the production of the standard vector $q\overline{q}$ resonances (ρ, D_s^* and possibly $\overline{t}b$) and the resonant W^- production ($E_\nu^{\rm res} = m_W^2/(2m_e) \simeq 6.3$ PeV) in $\overline{\nu}_e e^-$ annihilation [7] as well as hypothetical non-standard interactions of neutrinos like interactions induced by off-diagonal neutral currents or the charged-current processes with production of supersymmetric particles or with an exchange of light leptoquarks [8] and so forth[1].

Further still the AN flux, along with the atmospheric muon (AM) flux, provides a way of testing the inputs of nuclear-cascade models that is parameters of the primary cosmic-ray flux and cross sections for hadron-nucleus and nucleus-nucleus interactions at very high energies. In particular, the AN flux measurements have much potential for yielding information about the mechanism of charm hadroproduction [10].

In any event, – to correct for the neutrino background and to use the AN flux as the subject of investigations or as a tool for particle physics, – there is a need to employ accurate, detailed, and reliable calculations for the energy spectra, spatial and angular distributions of AN over a wide range of neutrino energies (from ~ 100 MeV up to the multi-PeV energy range) as well as calculations of the transport of neutrinos through the Earth with taking account for their absorption due to charged currents and regeneration via neutral currents.[2] Admittedly, we are as yet far from that goal, despite of a considerable progress made in the past years.

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[1] Last years, considerable attention has been focused on a possible nonperturbative behavior in the electroweak sector of the standard model, at energies above a threshold $\sqrt{\hat{s}_0} \gg m_W$, responsible for multiple production of gauge and Highs bosons in νN interactions with a sizeable cross section. But the AN flux of the appropriate energies (above ~ 10 PeV) proves to be too small. Hopefully, neutrinos from AGN or gamma-ray barsters may provide a possibility for studying this phenomenon with future large-scale neutrino telescopes [9].

[2] The latter effects become essential for $E_\nu \gtrsim 1$ TeV (see, e.g., [11] and references therein).

2 Conventional (π, K) neutrinos

"Low-energy" AN ($E_\nu < 10 - 15$ GeV) are produced mainly in the two-particle leptonic decays of charged pions and kaons and also in the muon decay which is the basic source of ν_e and $\overline{\nu}_e$ in this energy range (table 1); muon polarization is an essential factor affecting the neutrino flavor ratio

$$R = \frac{\nu_\mu + \overline{\nu}_\mu}{\nu_e + \overline{\nu}_e}$$

as well as the neutrino to antineutrino ratios ($\nu_e/\overline{\nu}_e$ and $\nu_\mu/\overline{\nu}_\mu$).

Table 1: The main sources of conventional neutrinos.

Particle	Exclusive decay mode	Branching ratio
$\mu^\pm$	$e^\pm + \nu_e(\overline{\nu}_e) + \overline{\nu}_\mu(\nu_\mu)$	$\simeq 100\%^*$
$\pi^\pm$	$\mu^\pm + \nu_\mu(\overline{\nu}_\mu)$	$\simeq 100\%$
$K^\pm$	$\mu^\pm + \nu_\mu(\overline{\nu}_\mu)$	$(63.51 \pm 0.18)\%$
	$\pi^0 + e^\pm + \nu_e(\overline{\nu}_e)$	$(4.82 \pm 0.06)\%$
	$\pi^0 + \mu^\pm + \nu_\mu(\overline{\nu}_\mu)$	$(3.18 \pm 0.08)\%$
K^0_L	$\pi^\pm + e^\mp + \overline{\nu}_e(\nu_e)$	$(38.78 \pm 0.27)\%$
	$\pi^\pm + \mu^\mp + \overline{\nu}_\mu(\nu_\mu)$	$(27.17 \pm 0.25)\%$
*Including radiative mode $e^\pm + \nu_e(\overline{\nu}_e) + \overline{\nu}_\mu(\nu_\mu) + \gamma$.		

The problem of low-energy AN flux is being discussed intensively at present in the context of the anomalous flavor ratio and zenith-angle distribution observed in Kamiokande (for the sub-GeV [12] and multi-GeV [13] energy ranges), IMB-3 [14], and then in SOUDAN 2 [15]. Recently, these effects were confirmed with highly improved statistics in the Super-Kamiokande experiment for the sub-GeV [16] and multi-GeV[17] energy ranges. Preliminary results from detector MACRO at Gran Sasso on low-energy neutrino events also confirm the AN anomaly [18]. Another confirmation come from the angular distribution of upgoing through-going muons measured MACRO, Kamiokande, and Super-Kamiokande. However, situation here is very uncertain today and the data from IMB and Baksan do not shed light upon the problem.

The observed anomalies give the long-expected hint for "new physics" (neutrino oscillations or proton decay!). Although two other (tracking) experiments, Fréjus [19, 20] and NUSEX [21], have reported no effect, they do leave (because of small statistics) some room for an interpretation of the Kamioka–IMB–SOUDAN–MACRO data in terms of maximal $\nu_\mu - \nu_\tau$ or $\nu_\mu - \nu_s$ mixing[3]. Although the explanation of the observed anomalies with the proton decay hypothesis [22] formally remains to be among the living, after Super-Kamiokande it seems to be highly improbable or, at least, unpopular.

With the hopes for new physics we must be on call to correct the underground data subject to possible neutron background [23] (see also the discussion in ref. [20]). According to Ryazhskaya [23], a significant high-energy neutron flux originating from interactions of cosmic-ray muons in the surrounding rock near a detector could be expected. Some of these neutrons

[3]The number of papers with different scenarios for two-, three-, and even four-neutrino mixing run into the hundreds. Many of these scenarios can explain not only the (positive) results of the underground neutrino experiments under discussion, but also the data from solar neutrino experiments, as well as the combined available data from accelerators and reactors; generally, they take into account cosmological and laboratory constraints on neutrino masses and mixing parameters.

traversed the veto counter designed for charged particles and the neutral pions produced by interactions of the neutrons inside the detector may lead to an excess of electron-like (showering) events. This background is expected to be negligible for the detectors NUSEX and Fréjus located at large depths, but it might be significant for (Super-)Kamiokande, IMB-3, SOUDAN 2, and MACRO due to the shallow depths of these experiments. Careful investigations were performed in all the mentioned experiments in order to estimate the real effect from the neutron, gamma-ray, and cosmic-ray muon backgrounds. The final conclusion is that these backgrounds are too small in order to explain the observed anomalies in the AN flux.

Calculations of the low-energy AN flux were reported in a number of papers [24, 25, 26, 27, 28, 29, 30] (see also refs. [31] for further information and refs. [32, 33, 34, 35, 36, 37] for reviews of the present-day status of the problem). These calculations are complex convolutions of the primary spectrum with the inclusive energy distributions of secondaries: the lack of Feynman scaling in the hadron-nucleus interactions, non-power spectrum of primary cosmic rays (+ geomagnetic effects and solar modulation), ionization energy losses, and the nonisothermality of the atmosphere are included into the nuclear-cascade calculations. Besides, the simplest superposition model for nucleus-nucleus collisions (commonly accepted in cosmic ray physics at high energies) is inoperable here as a good approximation and other, more sophisticated approaches are necessary (see, e. g., refs. [38, 35, 29]). Although the inputs and methods employed in the cited papers are significantly distinct, all calculations agree within a range of about 5% for the flavor ratio. Much larger differences exist among the results for the absolute flux and shape of the energy spectra. These are the central problems in the right interpretation of the R anomaly.

As an example, we show in fig. 1 the $\nu_\mu + \overline{\nu}_\mu$ and $\nu_e + \overline{\nu}_e$ energy spectra averaged over the zenith and azimuth angles for the Kamioka site, as calculated in refs. [24, 25, 27, 29]. Figure. 2 shows some important relative characteristics of the low-energy AN flux for the same site.

As is seen from the fig. 1, the calculation ref. [25] has a harder spectrum than the other calculations[4]. The maximum difference between the calculations for the absolute flux ranges up to almost 100% at $E_\nu = 100$ MeV. In the new, improved calculation by the Bartol group [30], the AN spectra were significantly reduced in the sub-GeV energy range (especially for the Kamioka site), due mainly to the improved treatment of the geomagnetic cutoffs [39]. Now, their spectra are in good agreement with the result of Honda *et al.* [29]. The maximum difference with the result of ref. [25] is about 55% for the Kamioka site but, as before, about 90% for the IMB site. A discussion of the main roots for the disagreements among the calculations at low energies is given in ref. [37]. All low-energy calculations are in close agreement above 2–3 GeV; the multi-GeV energy range of the AN spectrum has been studied in greater detail.

At higher energies (above 10–15 GeV), the semileptonic decays of charged and neutral kaons become important (table 1) and hence the differential cross sections for kaon production in NA, πA and KA interactions are required for the calculations. The high-energy AN flux consists mainly of ν_μ and $\overline{\nu}_\mu$. For instance, within the energy range 1 to 100 TeV, the flavor ratio R for the conventional AN flux is a monotonically increasing function of energy varying from about 28 to 34 at $\vartheta = 0^\circ$ and from about 13 to 34 at $\vartheta = 90^\circ$ (where ϑ is the zenith angle) [10]. However, the contribution from decay of charmed particles results in a decrease of the AN flavor ratio (see below).

[4] A remark is in order. Several bugs in the implementation of the code of Lee and Koh [27] (which is a 3-dimensional version of the model of hadronic interactions used by Bartol group [24]) have been discovered [37]. When these are removed, the results of ref. [27] are essentially the same as those of ref. [24] at $E_\nu \gtrsim 200$ MeV.

Table 2: The most important pionic decays of kaons.

Particle	Exclusive decay mode	Branching ratio
K_S^0	$\pi^+ + \pi^-$	$(68.61 \pm 0.28)\%$
K_L^0	$\pi^+ + \pi^- + \pi^0$	$(12.56 \pm 0.20)\%$
$K^\pm$	$\pi^\pm + \pi^0$	$(21.16 \pm 0.14)\%$
	$\pi^\pm + \pi^\pm + \pi^\mp$	$(5.59 \pm 0.05)\%$
	$\pi^\pm + \pi^0 + \pi^0$	$(1.73 \pm 0.04)\%$

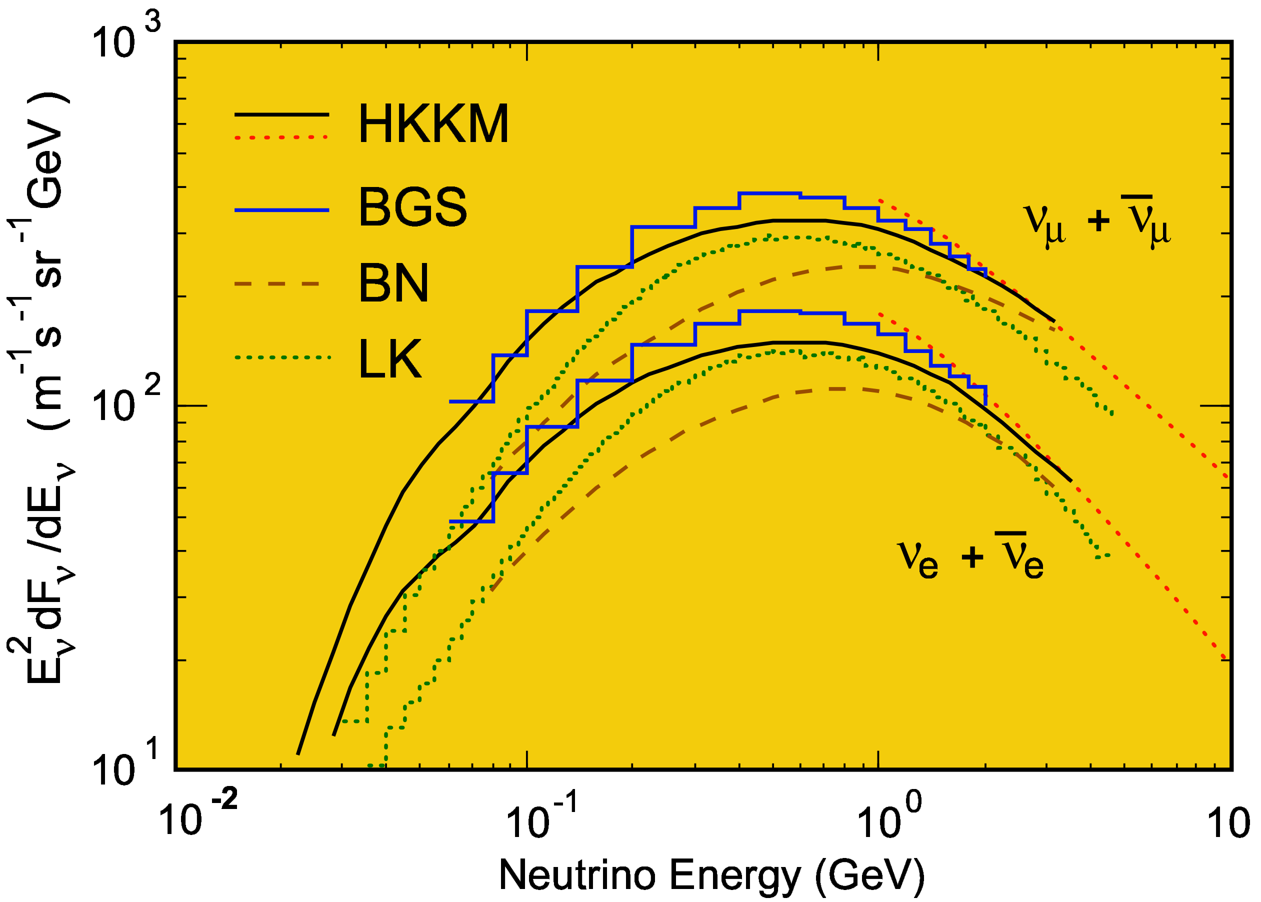

Figure 1: Predicted low-energy AN fluxes (multiplied by E_ν^2) for Kamioka site at medium solar activity. The curves marked by HKKM, BGS, BN, and LK are from refs. [29], [24], [25], and [27], respectively. The dotted lines represent the results of ref. [29] obtained without geomagnetic cutoff. [Figure is taken from ref. [29].]

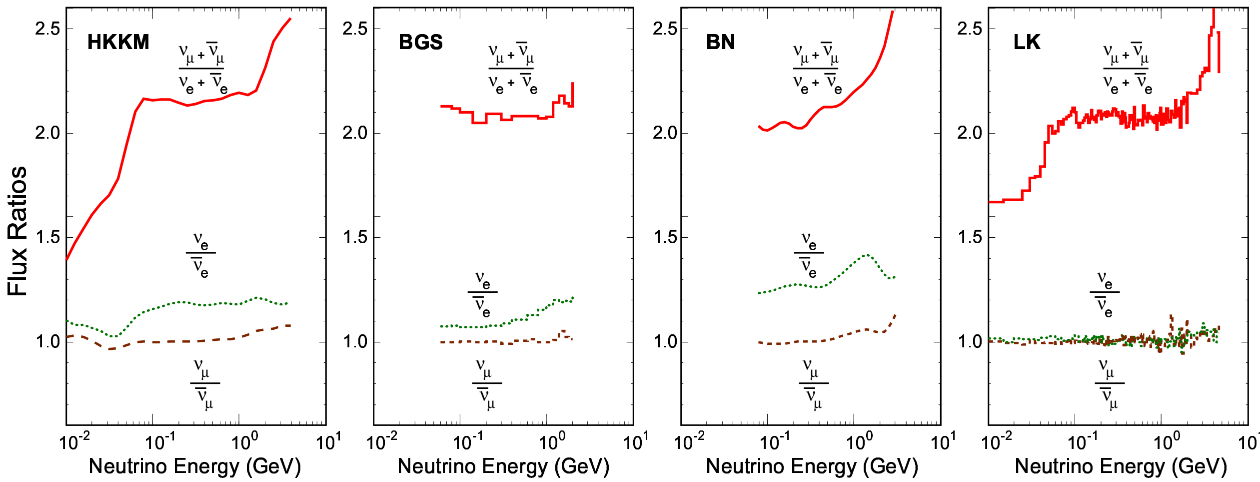

Figure 2: Predicted AN flux ratios for the Kamioka site at medium solar activity. The notation is the same as in fig. 1. [Figure is taken from ref. [29].]

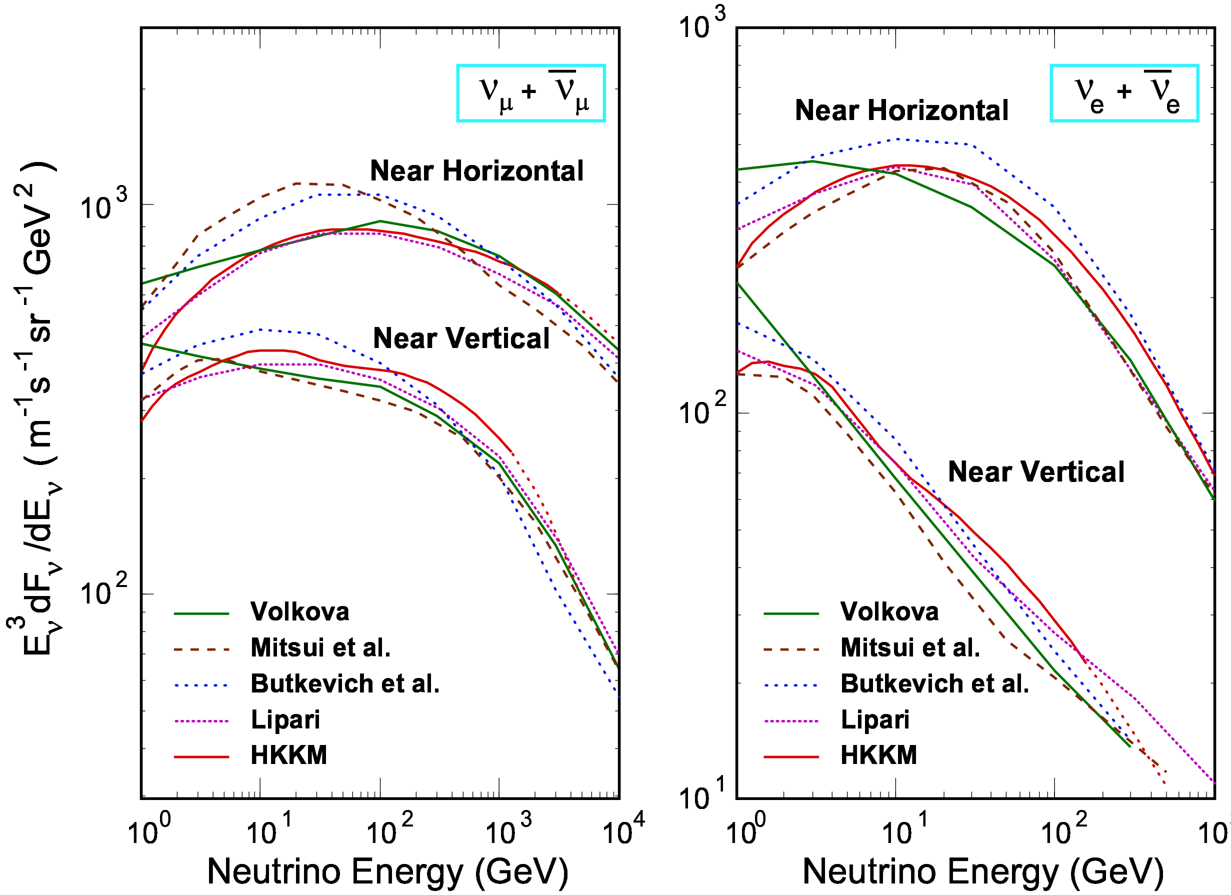

Figure 3: Predicted AN fluxes (multiplied by E_ν^3) at $E_\nu > 1$ GeV for near horizontal and near vertical directions. The curves marked by HKKM, V, MMK, BDZ, and L are from refs. [29] ($\cos\vartheta = 0 - 0.1$ and $0.9 - 1.0$), [47] ($\cos\vartheta = 0.05$ and 1), [48] ($\vartheta = 87°$ and 0°) [41] ($\cos\vartheta = 0$ and 1), and [43] ($\cos\vartheta = 0.05$ and 1), respectively. The dotted curves have the same sense as in fig. 1. [Figure is taken from ref. [29].]

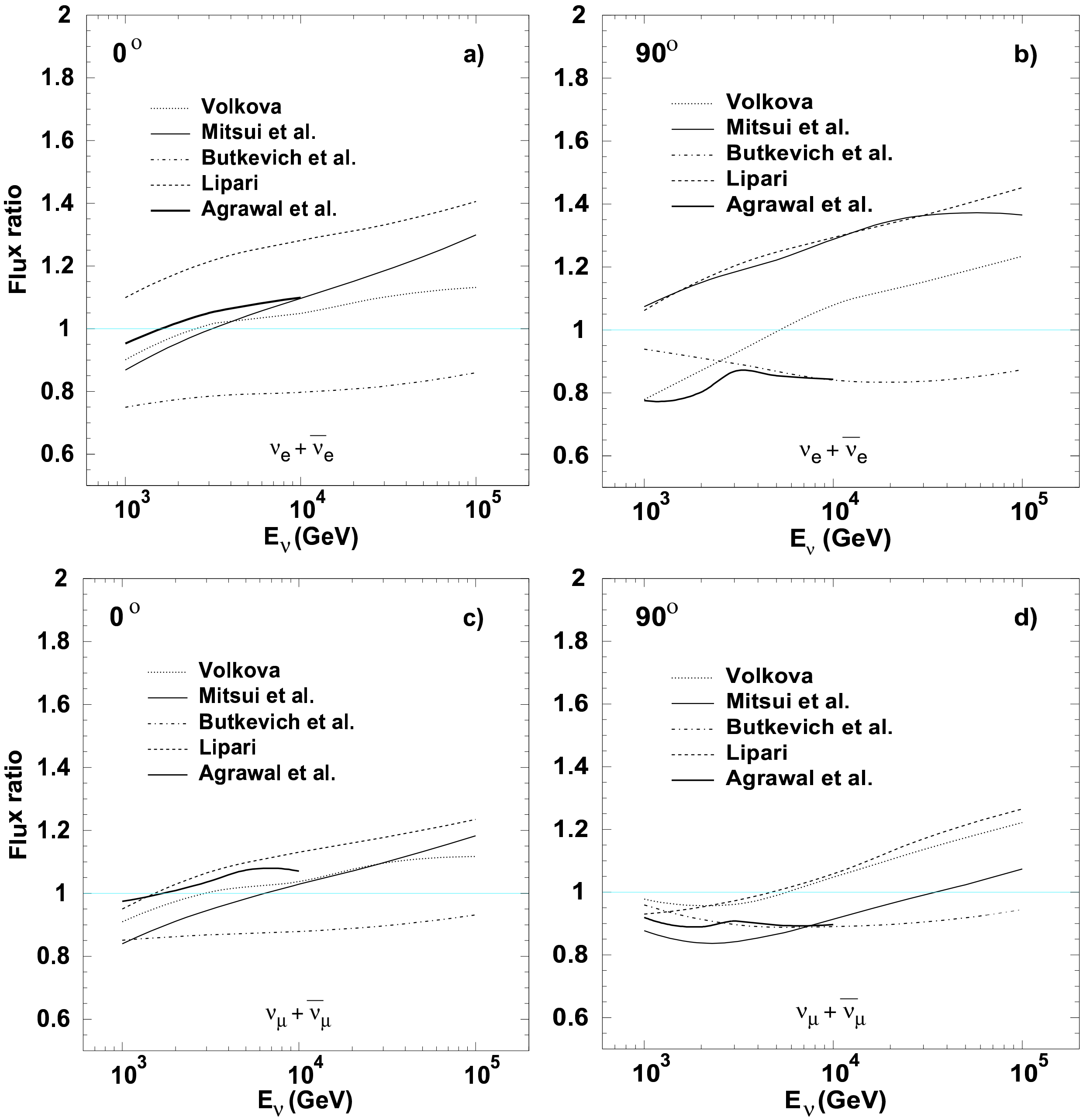

Figure 4: Conventional $\nu_e+\overline{\nu}_e$ and $\nu_\mu+\overline{\nu}_\mu$ fluxes at $\vartheta = 0°$ and 90° from refs. [47, 48, 41, 43, 45], normalized to the fluxes calculated in ref. [10]. [Figure is taken from ref. [10].]

Muon decays give an essential contribution to the electron neutrino flux up to $E_\nu \simeq 1$ TeV but already at $E_\nu \gtrsim 100$ GeV (and up to about 10 TeV), the K_{e3} decays are the dominant channels for ν_e and $\overline{\nu}_e$ production and thus their inclusion is essential for the evaluation of the flavor ratio in the high-energy AN flux. Pion production due to kaon decays (table 2) gives a small but not completely negligible contribution to the AN flux and can affect the flavor ratio [40].

Detailed calculations for the (π, K) neutrino flux at high energies were performed in refs. [29, 41, 42, 43, 44, 45] and, recently, in [10]. For earlier calculations see refs. [38, 46, 47, 48] and references therein. Comparison between five different predictions for the energy spectra of $\nu_\mu+\overline{\nu}_\mu$ (within the energy range $1-10^4$ GeV) and $\nu_e+\overline{\nu}_e$ (within the energy range $1-10^3$ GeV) for near-vertical and near-horizontal directions is shown in fig. 3. Figures 4 (a–d) represent the same spectra, calculated in refs. [41, 43, 45, 47, 48], but within the energy range $1 - 100$ TeV. All these spectra are normalized to the most recent calculation of ref. [10].

3 Prompt neutrinos

The dominant contribution to the AN flux at very high energies[5] is due to semileptonic decays of charmed hadrons (mainly $D^{\pm}$, D^0, $\overline{D}^0$ and Λ_c^+). The AN from this source are called prompt neutrinos. There are numerous exclusive decay modes of charmed particles with a lepton pair and one or more hadrons in the final state [49]; the inclusive semileptonic decays of D, D_s, and Λ_c^+ are shown in table 3. The dashes indicate the absence of direct data but, owing to the $\mu - e$ universality, one can expect that the branching ratios for electron and muon inclusive modes are close to each other. Branching ratios for pure leptonic modes (with $\ell\nu_\ell$ in final state) are very small except the case of $D_s^{\pm} \to \tau^{\pm} + \nu_\tau(\overline{\nu}_\tau)$. The latter mode is however very important, being the main source of atmospheric tau neutrinos.

Table 3: The most important (semi)leptonic decays of charmed hadrons.

Particle	Decay mode	Branching ratio
$D^{\pm}$	$e^{\pm} + \nu_e(\overline{\nu}_e)$ + hadrons	$(17.2 \pm 1.9)\%$
	$\mu^{\pm} + \nu_\mu(\overline{\nu}_\mu)$ + hadrons	–
D^0	$e^+ + \nu_e$ + hadrons	$(6.75 \pm 0.29)\%$
	$\mu^+ + \nu_\mu$ + hadrons	$(6.6 \pm 0.8)\%$
$D_s^{\pm}$	$e^{\pm} + \nu_e(\overline{\nu}_e)$ + hadrons	$\left(8^{+6}_{-5}\right)\%$
	$\mu^{\pm} + \nu_\mu(\overline{\nu}_\mu)$ + hadrons	–
	$\tau^{\pm} + \nu_\tau(\overline{\nu}_\tau)$ + hadrons	–
	$\tau^{\pm} + \nu_\tau(\overline{\nu}_\tau)$	$(7 \pm 4)\%$
Λ_c^+	$e^+ + \nu_e$ + hadrons	$(4.5 \pm 1.7)\%$
	$\mu^+ + \nu_\mu$ + hadrons	–

Calculations of the prompt neutrino flux (and even the energy ranges in which the prompt muon and electron neutrinos dominate) are very model-dependent [50, 51, 52, 53, 54]. As yet, this flux cannot be unambiguously predicted for lack of a generally accepted model for charm production at high energies.

The salient and almost model-independent features of the prompt neutrino flux are that it is practically isotropic within a wide energy range (namely, at 1 TeV $\lesssim E_\nu \lesssim 3 \times 10^3$ TeV, the maximal anisotropy is about 3–4%) and the neutrino to antineutrino ratios and the flavor ratio are close to 1. These features provide a way to discriminate the prompt neutrino contribution through the analysis of the angular distribution[6] and the relationship between "muonless" and "muonfull" neutrino events in a neutrino telescope.

[5] For muon neutrinos, at $E_\nu > 10 - 100$ TeV for vertical and at $E_\nu > 100 - 1000$ TeV for horizontal neutrino flux; for electron neutrinos at energies which are an order of magnitude less (see below).

[6] Moreover, the anisotropy of the flux of prompt muons (that is the muons originated from charm decay) for the same energy range is also very small ($\lesssim 20\%$). This fact can be a help in deciding the problem.

Table 4: Differential energy spectra of prompt $\nu_\mu + \overline{\nu}_\mu$ (in units $\mathrm{cm^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}}$) at super-high energies for vertical and horizontal directions as calculated in ref. [51] with two models of charm production.

Model:	RQPM		QGSM	
E_ν (TeV)	$\vartheta = 0°$	$\vartheta = 90°$	$\vartheta = 0°$	$\vartheta = 90°$
10^3	5.07×10^{-21}	5.24×10^{-21}	1.15×10^{-21}	1.18×10^{-21}
3×10^3	2.04×10^{-22}	2.25×10^{-22}	4.53×10^{-23}	4.88×10^{-23}
10^4	4.85×10^{-24}	6.43×10^{-24}	1.08×10^{-24}	1.35×10^{-24}
3×10^4	1.22×10^{-25}	2.27×10^{-25}	2.86×10^{-26}	4.67×10^{-26}
10^5	1.58×10^{-27}	4.94×10^{-27}	4.01×10^{-28}	1.03×10^{-27}
3×10^5	2.40×10^{-29}	1.18×10^{-28}	6.42×10^{-30}	2.61×10^{-29}
10^6	2.14×10^{-31}	1.48×10^{-30}	5.77×10^{-32}	3.55×10^{-31}

Within the region of isotropy, the differential energy spectrum of prompt neutrinos ($\nu_\mu + \overline{\nu}_\mu$) may be described by the following simple expression:

$$\frac{dF_\nu^{\mathrm{pr}}}{dE_\nu} = F_\nu^0 \left(\frac{E_\nu^0}{E_\nu}\right)^{\gamma_\nu+1} \left[1 + \left(\frac{E_\nu^0}{E_\nu}\right)^{\gamma_\nu}\right]^{-a_\nu},$$

with the model-dependent constant parameters F_ν^0, E_ν^0, γ_ν, and a_ν. As an illustration let us consider briefly the predictions derived from the two phenomenological nonperturbative models for charm production – recombination quark-parton model (RQPM) and quark-gluon string model (QGSM). According to the calculation of ref. [51],

$$F_\nu^0 = 4.65 \times 10^{-18}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}},$$
$$E_\nu^0 = 10^5\,\mathrm{GeV},\ \ \gamma_\nu = 1.96,\ \ a_\nu = 0.157$$

for RQPM and

$$F_\nu^0 = 1.19 \times 10^{-18}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}},$$
$$E_\nu^0 = 10^5\,\mathrm{GeV},\ \ \gamma_\nu = 2.01,\ \ a_\nu = 0.165$$

for QGSM. Using these values of parameters, it can be found that vertical intensities of the (π, K) muon neutrinos and prompt muon neutrinos become equal at $E_\nu \approx 32$ TeV (RQPM) or $E_\nu \approx 170$ TeV (QGSM). The corresponding energies for horizontal intensities are about 340 TeV (RQPM) and about 1700 TeV (QGSM).

A very low prompt neutrino flux was obtained in ref. [52] using a pQCD-based "state-of-the-art" model for charm production. However, a new calculation, performed in the context of pQCD with inclusion of next-to-leading order corrections [54], gives prompt muon and neutrino fluxes significantly larger than it follows from the result of ref. [52] and closer to the fluxes predicted from the QGSM.

At super-high energies, the prompt neutrino flux becomes anisotropic as evident from table 4. Thus at $E_\nu = 10^5$ TeV, the ratio of horizontal and vertical intensities of ν_μ's is about 3.1 and 2.6 for RQPM and QGSM, respectively. However, the absolute value of the flux is very small here so, it is hoped that uncertainties in the prediction for the AN angular distribution will not be a serious handicap to study extraterrestrial super-high-energy neutrinos with underwater neutrino telescopes.

4 The main sources of differences among the calculations

Disagreement between different low-energy calculations is the main obstacle for an unambiguous interpretation of the underground neutrino data. The major source of this disagreement is in the inclusive cross sections for the reaction $N + A \to \pi^{\pm} + X$ at low Feynman x [37]. It is unlikely to expect an essential progress here in the near future. Uncertainties in the primary spectrum and composition are less important but not negligible.

The comparison of the spectra of $\nu_\mu + \overline{\nu}_\mu$ and $\nu_e + \overline{\nu}_e$ at high energies calculated in the quoted papers with different models for primary spectrum, inclusive cross sections, etc., shows (fig. 3) that the discrepancy between the results for the muon neutrinos in the energy range 3–500 GeV is of the order of 30–35% for near vertical directions and ∼35–40% for near horizontal direction, while at higher energies (to a few TeV) it is less than 25% for all directions. In the case of electron neutrinos the discrepancy is about 45–50% (vertical) and 40–45% (horizontal) at 3–100 GeV (with the only exception: the $\nu_e + \overline{\nu}_e$ flux calculated in ref. [42] differs enormously from all other predictions at $E_\nu < 100$ GeV and it is in apparent contradiction with the recent Fréjus data [20]) and decreases to 20–30% at $E_\nu \approx 1$ TeV. This discrepancy is acceptable for some astrophysical experiments with the two present-day neutrino telescopes Baikal NT-200 and AMANDA-2 within the next few years, but is inadequate to analyze the current underground and future underwater experiments on neutrino oscillations by a change in the total neutrino flux [33, 34]. The possible changes in the angular distribution (due to oscillations) are vastly less model-dependent. The difference in the predictions for the multi-TeV energy range, which is most important for the calculation of the AN background for astrophysical experiments, is much bigger and drastically grows with energy ranging up to an order of magnitude in the region where the prompt neutrinos dominate (see refs. [51, 55] for a detailed comparison of different charm-production models).

The ratios $\nu_\mu/\overline{\nu}_\mu$ and (to a smaller degree) $\nu_e/\overline{\nu}_e$ for the (π, K) neutrinos are also model-dependent at $E_\nu = 10 - 10^3$ TeV[7]. For the most part this is due to our lack of understanding of the composition of primaries and mechanism of kaon production at high energies.

Increasing the accuracy in the calculation of the measurable characteristics of the AN flux will become increasingly urgent. Major input data required for calculating the AN flux are the spectrum and chemical composition of primary cosmic rays, total inelastic and differential inclusive cross sections of hadron-nucleus and nucleus-nucleus interactions over a wide kinematic range (primarily in the range of fragmentation of the projectile). The above-mentioned discrepancies in different calculations of the AN flux are due mainly to the incompleteness in the current knowledge of these input parameters. This holds, to a greater or lesser extent, for low, intermediate, high, and super-high energies. The main uncertainty in the AN flux calculation within the sub-TeV range is related to the absence of good data and theoretical models for K meson production in the fragmentation range, the so called, in cosmic ray physics, K/π problem. Within the range of prompt neutrino dominance, the principal problem is of course the charm production problem. An additional significant source of uncertainty has to do with the differential widths of inclusive semileptonic decays of charmed hadrons.

[7] Curiously, the situation is opposite for the sub-GeV energy range: here, a serious disagreement (up to about 20%) prevails in the $\nu_e/\overline{\nu}_e$ ratio predicted by different models (see refs. [35, 29]). This is one of the obstacles to an unambiguous interpretation of the R anomaly observed with underground detectors.

5 Atmospheric muons

The only tool for verification and normalization of the calculated AN flux is the flux of muons generated in the atmosphere in the same processes as neutrinos. At low energies, we need the muon data from altitudes 9-18 km above sea level (the range of effective generation both muons and neutrinos), considering that low-energy muons cannot reach the ground-level detectors due to energy loss. The recent balloon-born measurements for the muon spectra and charge ratio in the stratosphere using Matter Antimatter Spectrometer Systems (MASS and MASS 2) [56] give us the data for $E_\mu \lesssim 40$ GeV with very good statistical accuracy. But, as ill luck would have it, variations of the altitude during the flight of the apparatus reach its maximum just in the range of effective altitudes for μ, ν production, resulting in essential systematic uncertainty. Preliminary results from Isotope Matter Antimatter eXperiment (IMAX) [57] and High Energy Antimatter Telescope (HEAT) [58] for the μ^- flux and muon charge ratio (as a function of atmospheric depth) are in qualitative agreement with the MASS data in the sub-GeV energy range. It is believed that further experiments with MASS, IMAX, and HEAT will provide to make the precise normalization of the low-energy AN spectra at different geomagnetic cutoffs[8].

Large body of direct spectrometer measurements of the sea-level muon spectrum at high energies are in rather poor agreement to one another, even though each of the experiments by itself has typically quite good statistical accuracy. The most important direct and indirect data on the vertical sea-level momentum spectrum are shown in figs. 5 (for the differential spectrum) and 6 (for the integral spectrum) together with theoretical predictions from refs. [45, 55]. For a detailed discussion of these data, we refer the reader to ref. [55]. The main conclusion is however quite evident: the current sea-level data are not very useful for a precise normalization of the AN flux. It is hoped that the situation will be improved considerably in the near future, after realization of the proposal [59] to use L3 detector at LEP (CERN) for a very accurate measurement of the cosmic-ray muon spectrum between 20 GeV and 20 TeV.

At the same time, a quite representative array of data on the cosmic-ray muon depth-intensity relation (DIR) in rock (underground experiments ERPM, KGF, Baksan, Fréjus, NUSEX, SOUDAN 2, MACRO, LVD) and, to a lesser extent, in water (in particular, from Baikal, DUMAND and NESTOR prototypes) has been accumulated to date. The data available by the beginning of 1998 are compiled in ref. [55]. As an example, fig. 7 represents the most recent and responsible result from LVD experiment (Gran Sasso Laboratory) [60], together with the theoretical expectation.

As is shown in ref. [55], the currently available data of underground measurements are more self-consistent, relative to the ground-level data, at least for the depths above 5–6 km of water equivalent (corresponding roughly to 3–4 TeV of muon and neutrino energy at sea level) and hence they may be of utility to check the nuclear-cascade models at energies of primaries up to about 50–100 TeV. However, the data at depths more than 6 km of w.e. are much less reliable. Preliminary analysis suggests that, through a correlation between the measured and calculated muon intensity *vs* depth, one can attain an accuracy in the AN flux normalization better than 10%, at energies below a few TeV. Not so dusty…

The main contribution to the total systematic error in the underground measurements of the muon intensity comes from uncertainty and variability in the rock density and chemical composition above the detector.

[8] The muon momentum intervals are 0.33–40 GeV/c (μ^-) and 0.33–1.5 GeV/c (μ^+) in MASS/MASS 2 experiments, 0.42–0.47 GeV/c in IMAX, and 0.3–0.9 GeV/c in HEAT. The geomagnetic cutoffs are approximately $0.65, 4.5, 0.1$, and 4 GV/c for the MASS, MASS 2, IMAX, and HEAT, respectively.

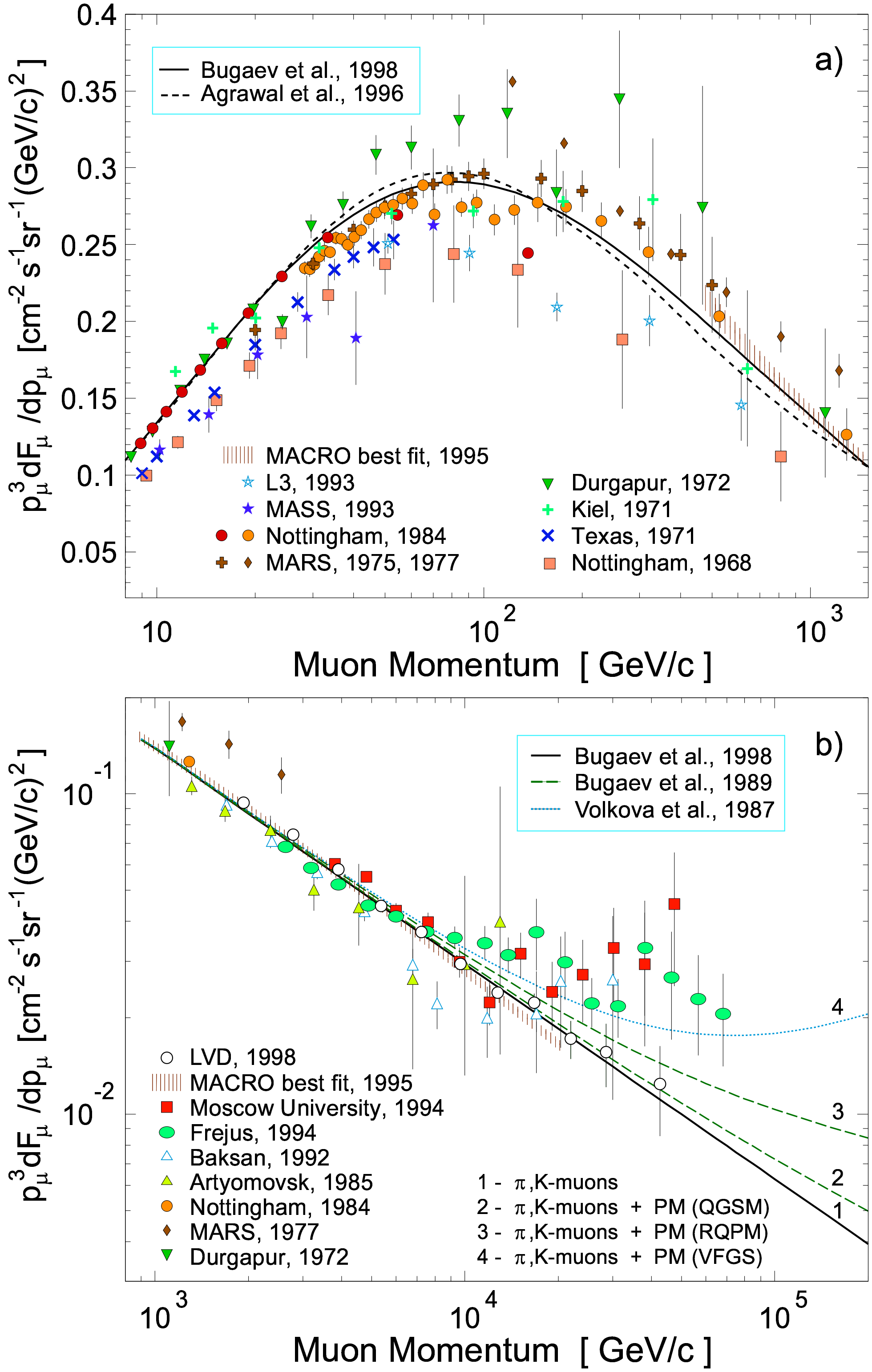

Figure 5: Vertical differential momentum spectrum of muons at sea level. The curves represent the results of refs. [45] and [55] for the conventional (π, K) differential muon spectrum and for the π, K muon spectrum plus the prompt muon contribution calculated according to QGSM, RQPM [51], and model from ref. [61] (VFGS). For the sources of the data points and further details, see ref. [55].

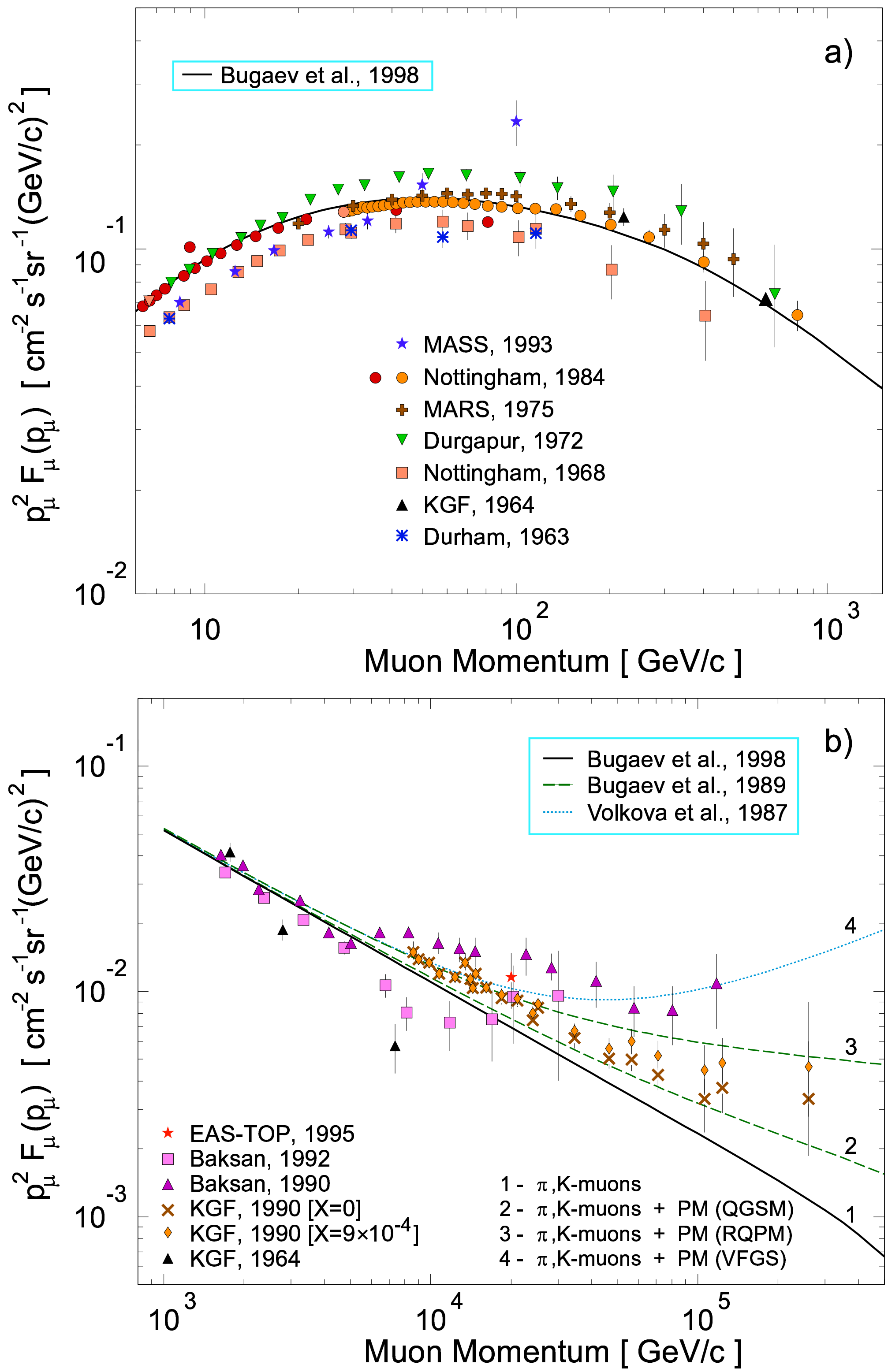

Figure 6: Vertical integral momentum spectrum of muons at sea level. The notation is the same as in fig. 5.

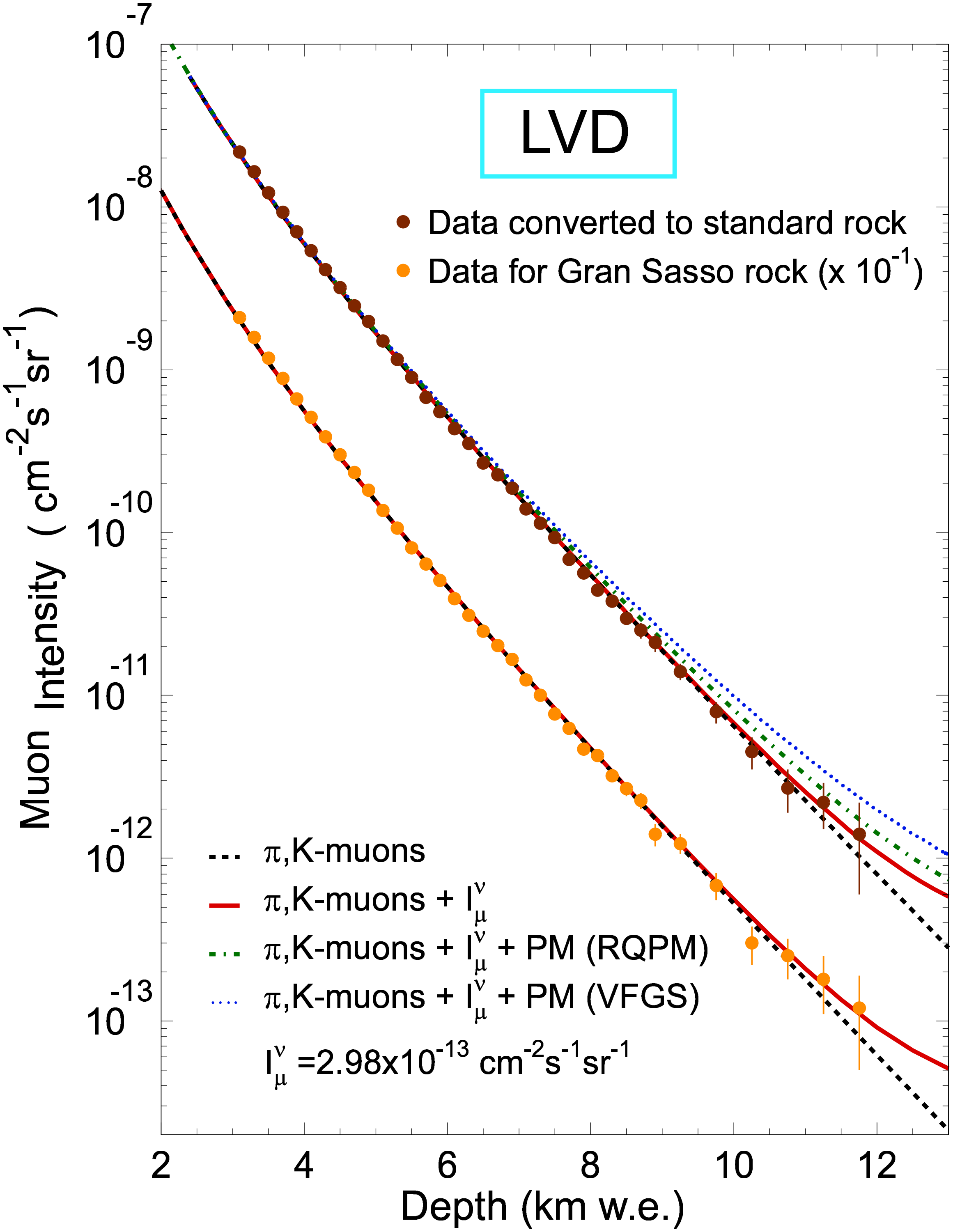

Figure 7: Muon intensity *vs* standard rock and Gran Sasso rock thickness [60]. The dashed curves represent the conventional muon "depth – intensity" relation (DIR), the solid curves represent the same plus the neutrino-induced muon background (I_μ^ν). The other curves are for the muon DIR with the prompt muon contributions calculated with two models for charm production (see caption to fig. 5) plus I_μ^ν. [Figure is taken from ref. [55].]

These problems can be overcome by subsequent measurements under water, where the matter overburden is well known and highly homogeneous. However, there is another problem common to underground and underwater muon measurements. It is the problem of muon bundles. Misidentification of a muon bundle and a single muon is an apparent source of systematics. On the other hand, the current theoretical predictions for the fraction of muon bundles *vs* depths are very model-dependent. It is beyond the scope of this paper to review this many-sided problem. Here one can do no more than call attention to its importance for the solution to the AN problem. In order to improve our knowledge at higher energies it is necessary to carry out measurements of the intensity of single muons and muon groups and also, if at all possible, of the muon energy spectra deep under water using slant directions. Needless to say these measurements are of direct interest to traditional cosmic-ray physics, and not only to neutrino astrophysics.

* * *

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