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Motivations for the Soft Wall holographic approach to strong interactions

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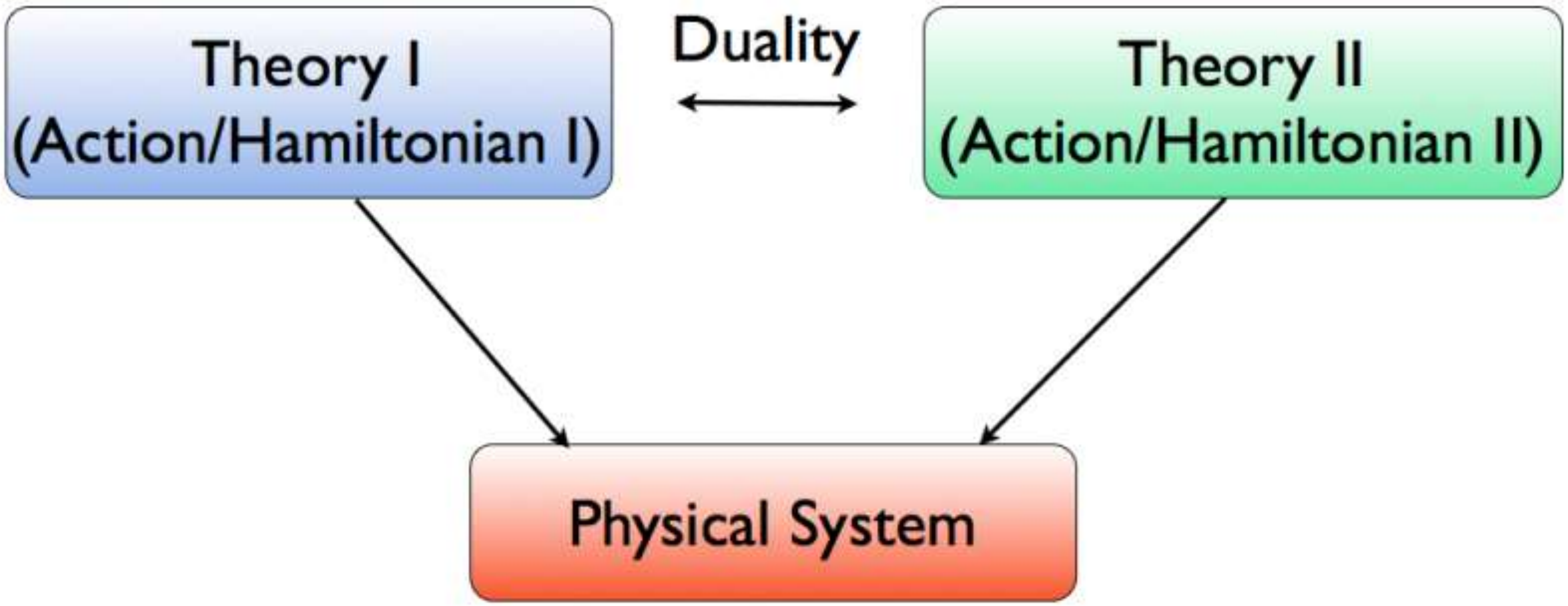
Challenge: Strong coupling problem

(QCD, Condensed matter, …)

Possible solution: Dual theory



The most interesting for applications:

<u>The strong-weak duality</u>

Examples of strong-weak duality (= S-duality)

(1). In (1+1)-dimensional space-time: The Sine-Gordon model and Thirring model

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha}{\beta^2}\left(\cos(\beta\phi) - 1\right)$$

$$\mathcal{L}_T = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_f\bar{\psi}\psi - \frac{\kappa}{2}\left(\bar{\psi}\gamma^\mu\psi\right)^2$$

are the same at quantum level if

$$\frac{4\pi}{\beta^2} = 1 + \frac{\kappa}{\pi}$$

(2). In (3+1)-dimensional space-time: Seiberg duality

It is a nonabelian extension of Montonen-Olive electric-magnetic duality

$$E_i \to B_i \qquad B_i \to -E_i$$

$$e \to e_m \Longrightarrow$$ Electrically charged fermion → Magnetic monopole

Charges are quantized: $ee_m \sim 1 \Longrightarrow e \sim \frac{1}{e_m}$

Non-perturbative strong interactions shape our world

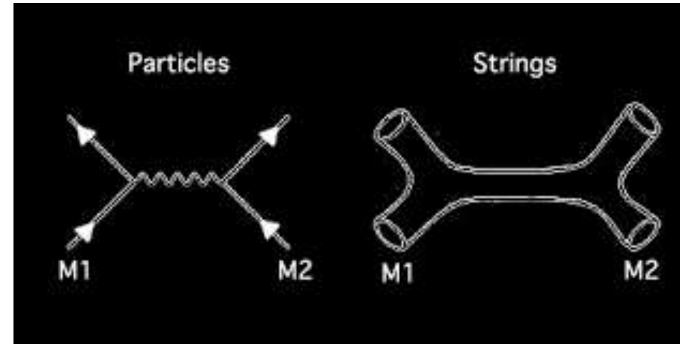
Analytical description is still not available

Analytical models?

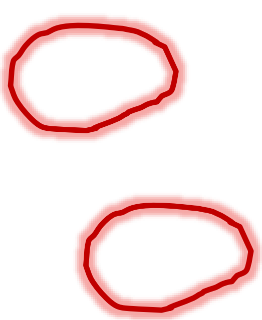
Holographic approach to QCD
(= AdS/QCD approach)

The approach is motivated by the
<u>AdS/CFT correspondence</u>
in string theory

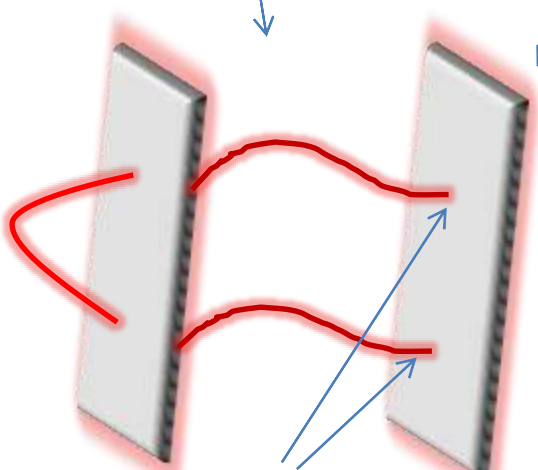
Various dualities play a decisive role in string theory



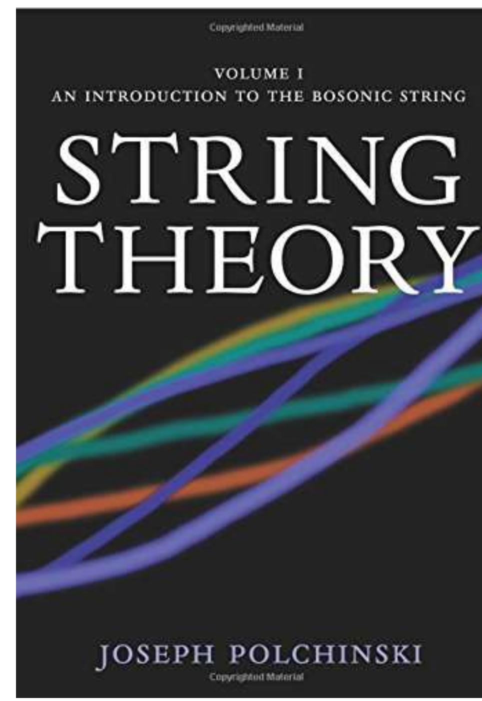
Strings: Closed and Open



Brane



Describe a gauge theory!



AdS/CFT correspondence (= gauge/gravity duality = holographic duality)

is a conjectured equivalence between a quantum gravity (in terms of string theory or M-theory) compactified on anti-de Sitter space (**AdS**) and a Conformal Field Theory (**CFT**) on AdS boundary

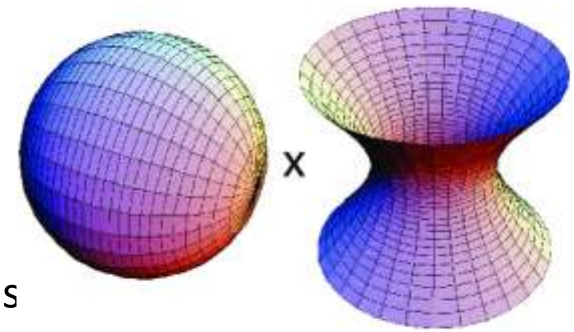
The term "Holography": Realization of the t'Hooft holographic principle (1993)

The most promoted example
(Maldacena, 1997 - *the most cited work in theoretical physics*!):

Type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}_5$ in the low-energy (i.e. supergravity) approximation	$\Leftrightarrow$	$\mathcal{N}=4$ SYM theory with *SU(N)* gauge group on AdS_5 boundary (= 4D Minkowski) in the limit $g^2 N \gg 1$

S^5: $X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2 = R^2$

AdS_5: $X_1^2 + X_2^2 - X_3^2 - X_4^2 - X_5^2 - X_6^2 = R^2$

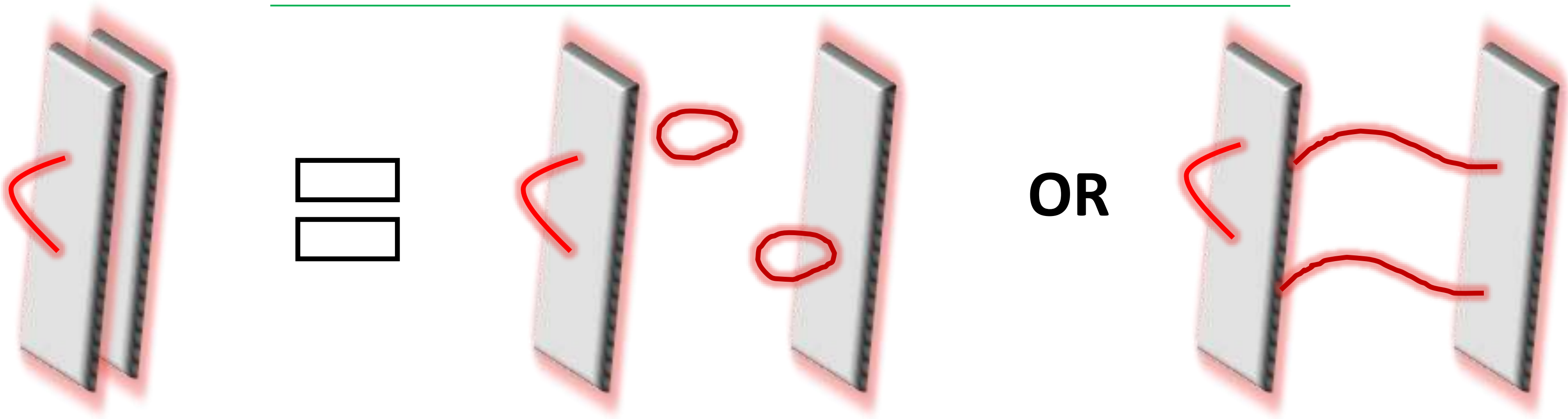


Essential ingredient: a one-to-one mapping of the global symmetries

Isometries of S^5 $\Leftrightarrow$ *SO(6) = SU(4) R*-symmetry of $\mathcal{N}=4$ Super Yang-Mills (SYM) theory

Isometries of AdS_5 $\Leftrightarrow$ Conformal group *SO(2,4)* in 4D space

THE CONCEPT

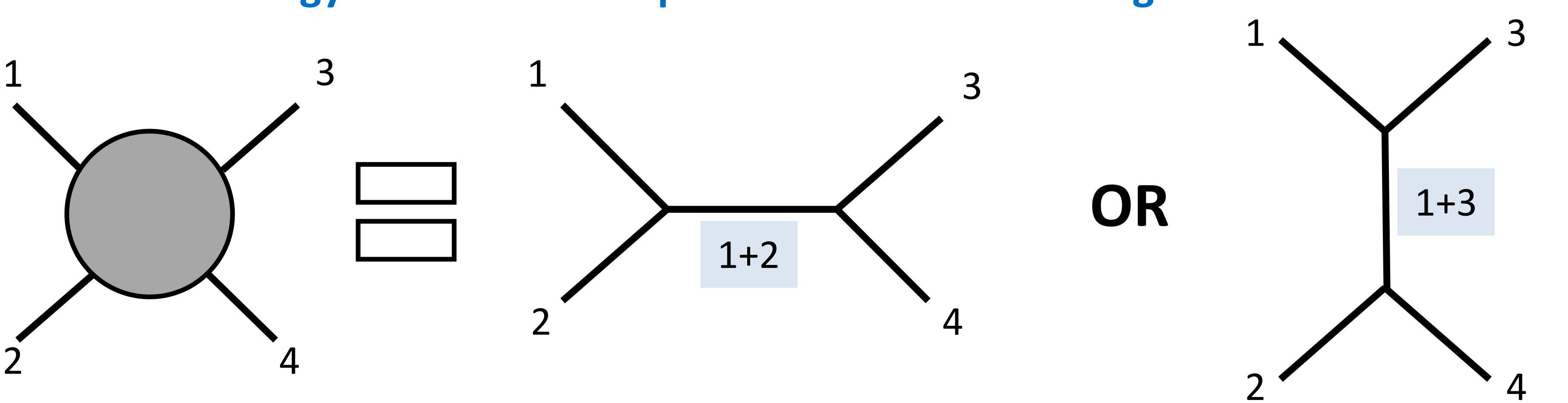


D-brane interaction (at low energies)

via closed strings (= gravitation)

via open strings (= gauge theory)

A remote analogy with dual description of hadron scattering via resonance exchange



hadron interaction (at low energies)

annihilation (= direct) channel

exchange (= cross) channel

CFT does not have ***In*** and ***Out*** asymptotic states. Observables are correlation functions of various operators. How to calculate them from the dual theory?

S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from noncritical string theory," *Phys. Lett.* **B428** (1998) 105, hep-th/9802109.

E. Witten, "Anti-de Sitter space and holography," *Adv. Theor. Math. Phys.* **2** (1998) 253, hep-th/9802150.

General idea:

$$Z_{\mathcal{N}=4} = Z_{\text{AdS}_5 \times S^5}$$

the partition function of the $\mathcal{N}=4$ SYM **=** the partition function of string theory on $\text{AdS}_5 \times S^5$

Take the supergravity (SUGRA) limit in the right side. The key proposal:

$$Z_{\mathcal{O}}[\bar{\phi}_\Delta] = \int \mathcal{D}[\text{SYM Fields}]\, e^{-S_{SYM} + \int d^4\vec{x}\, \mathcal{O}_\Delta(\vec{x})\bar{\phi}_\Delta(\vec{x})} = e^{-S_{SUGRA}[\phi[\bar{\phi}]]}$$

$S_{SUGRA}[\phi[\bar{\phi}]]$ is the supergravity action evaluated at the classical solution $\phi[\bar{\phi}]$ which has boundary value $\bar{\phi}$

Correlation functions:

$$\langle \mathcal{O}(\vec{x}_1)...\mathcal{O}(\vec{x}_n)\rangle = \frac{\delta^n}{\delta\bar{\phi}(\vec{x}_1)...\delta\bar{\phi}(\vec{x}_n)} Z_{\mathcal{O}}[\bar{\phi}]|_{\bar{\phi}=0}$$

In summary: The essence of the holographic method

$$Z_{\rm YM}[J] \equiv e^{-W_{\rm YM}[J]} = \int \mathcal{D}\phi\, e^{-S_{\rm YM} - \int d^4x\, J\mathcal{O}}$$

$$\boxed{W_{\rm YM}[J] = S_{\rm grav}[\Phi_0]\big|_{\Phi_0 = J}}$$

$$\Phi_0 \equiv \Phi_{\partial \rm AdS}$$

The output of the holographic models: **Correlation functions**

Poles of the 2-point correlator → mass spectrum

Residues of the 2-point correlator → decay constants

Residues of the 3-point correlator → transition amplitudes

Alternative way for extracting the mass spectrum is to find normalizable modes of e.o.m.

AdS/CFT dictionary

boundary: field theory	bulk: gravity
energy momentum tensor T^{ab}	metric field g_{ab}
global internal symmetry current J^a	Maxwell field A_a
order parameter/scalar operator $\mathcal{O}_{\mathrm{b}}$	scalar field ϕ
fermionic operator $\mathcal{O}_{\mathrm{f}}$	Dirac field ψ
spin/charge of the operator	spin/charge of the field
conformal dimension of the operator	mass of the field
source of the operator	boundary value of the field (leading part)
VEV of the operator	boundary value of radial momentum of the field (subleading part)
(Global aspects)	
global spacetime symmetry	local isometry
temperature	Hawking temperature
chemical potential/charge density	boundary values of the gauge potential
phase transition	Instability of black holes

Thus we have an impressive conjecture:

$$\left\{\mathcal{N}=4\ SU(N_c)\ \text{SYM theory}\right\} = \left\{\text{IIB string theory in } AdS_5 \times S_5\right\}$$

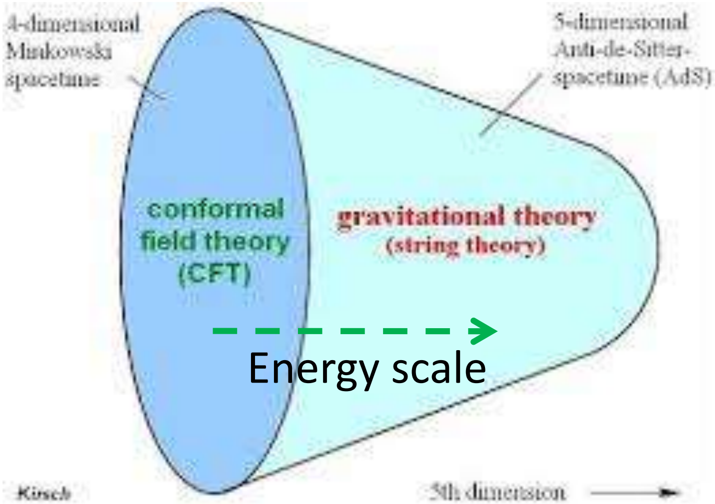
Or generally:

Strong coupling problems in non-abelian QFT	⟺	Classical Problem in a curved higher-dimensional space
g^2, N_c $\lambda \equiv g^2 N$ T'Hooft coupling		$g_s, \alpha' = l_s^2$ String coupling and string tension

Source for major inspiration!

(a great number of related models in the last 20 years)

But still to be proven…



AdS/QCD approach

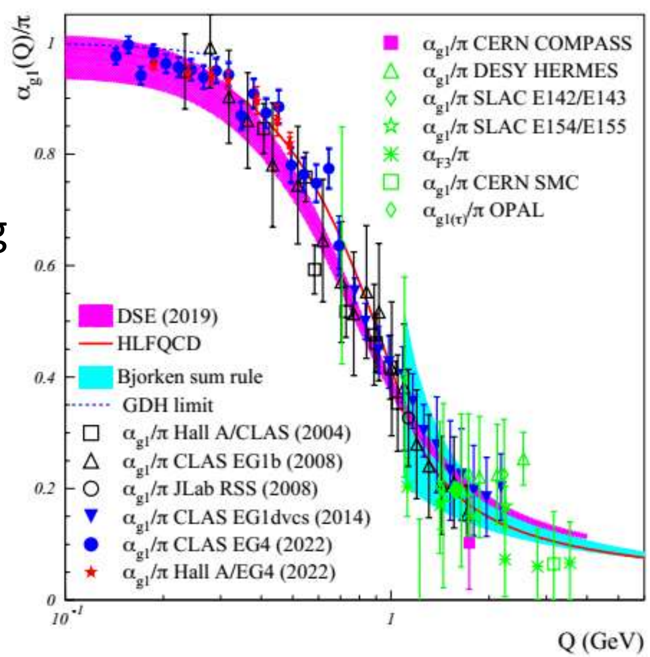
A program for implementation of holographic duality for QCD following some recipies from the AdS/CFT correspondence

String theory → **Top-down** → **QCD**

QCD → **Bottom-up** → **String theory**

Will be discussed (→ Bottom-up)

One of motivations: "freezing" of strong coupling at very low momentum



Phenomenological bottom-up AdS/QCD models

Typical ansatz:

$$S = \int d^4x dz \sqrt{g}\, F(z) \mathcal{L} \qquad F(0) = 1$$

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

- operators $\mathcal{O}(x)$ in $4D$ theory $\Leftrightarrow$ fields $\Phi(x, z)$ in $5D$ dual theory
- canonical dimension Δ of the p-form operator $\mathcal{O}(x)$ $\Leftrightarrow$ $5D$ mass of $\Phi(x, z)$: $m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$,

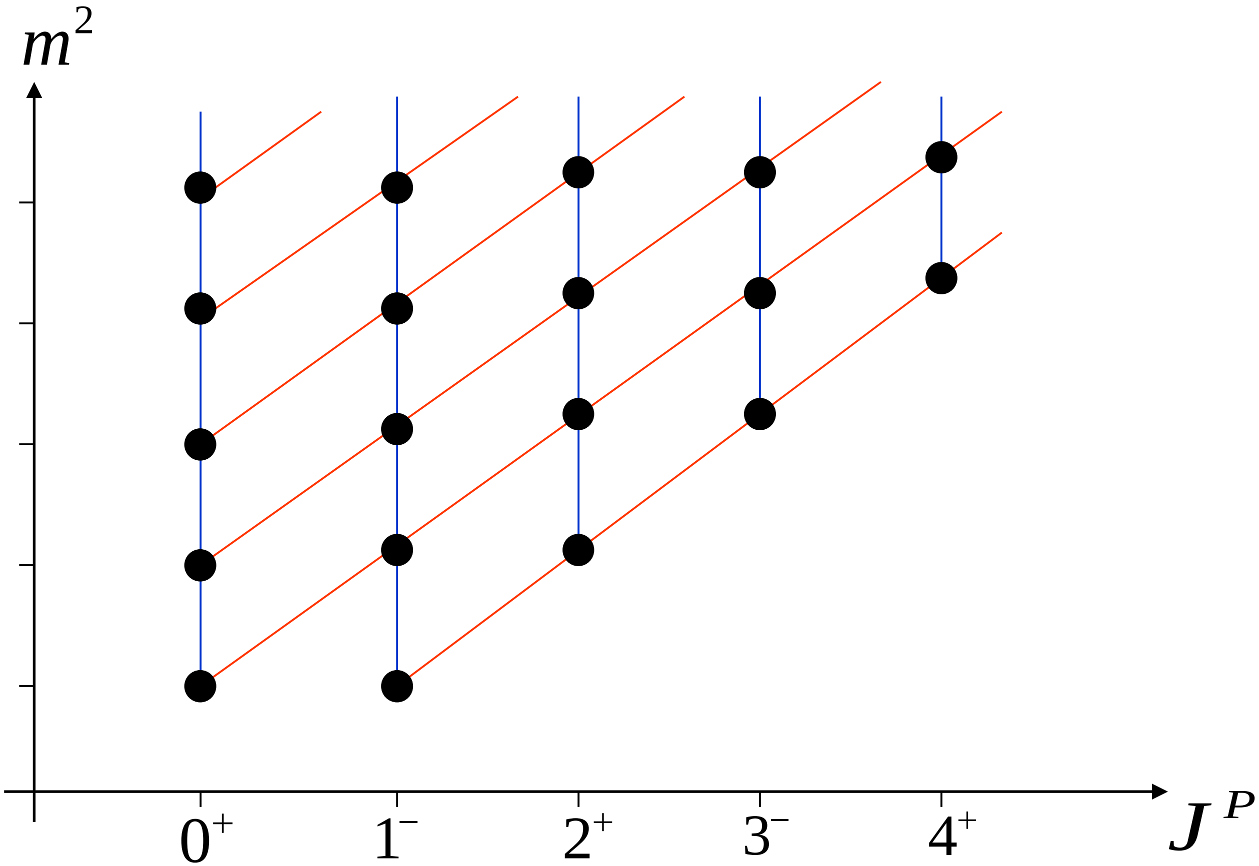
  Example 1: vector mesons $\Leftrightarrow$ $\bar{q}\gamma^\mu t^a q$ with $p = 1$ and $\Delta = 3$ $\Leftrightarrow$ $A_\mu^a(x, z)$ with $m_5^2 R^2 = 0$

  Example 2: 0^{++} glueballs $\Leftrightarrow$ $G_{\mu\nu}G^{\mu\nu}$ with $p = 0$ and $\Delta = 4$ $\Leftrightarrow$ $\varphi(x, z)$ with $m_5^2 R^2 = 0$
- the desired matter content defines $\mathcal{L}_{matter}$ in $5D$

→ radial Regge trajectories $\Leftrightarrow$ finding normalizable solutions of $5D$ equations of motion with $q^2 = M^2(n)$, $n = 0, 1, 2, \ldots$, that match certain boundary conditions,

$$\Phi(x, z) = \sum_{n=0}^{\infty} \phi_n(z)\phi^{(n)}(x)$$

Regge and radial Regge linear trajectories



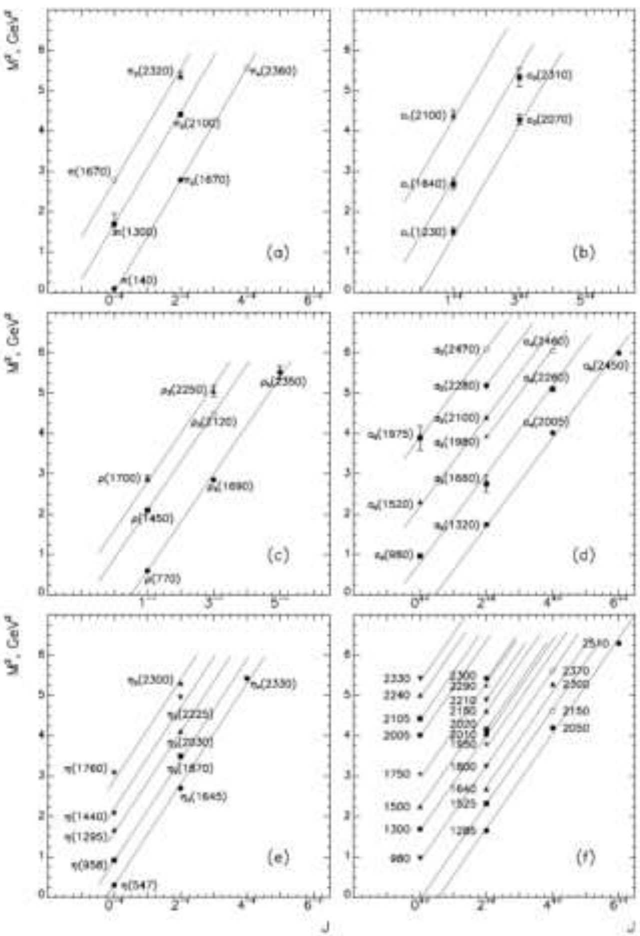
$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

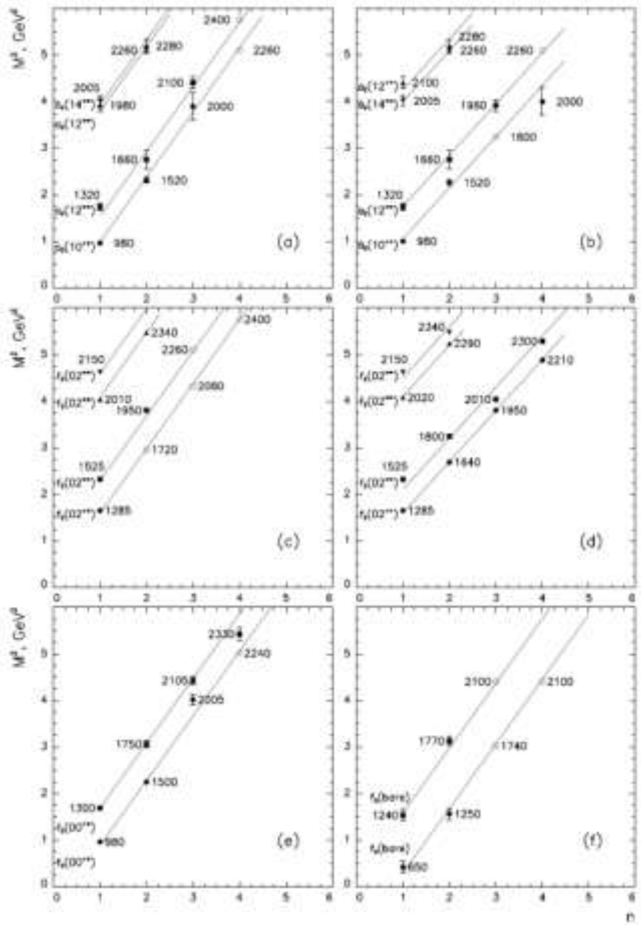
$$m^2(n) = \mu_0^2 + \alpha n$$ – Radial Regge trajectories

Linear Regge and radial trajectories: Experiment

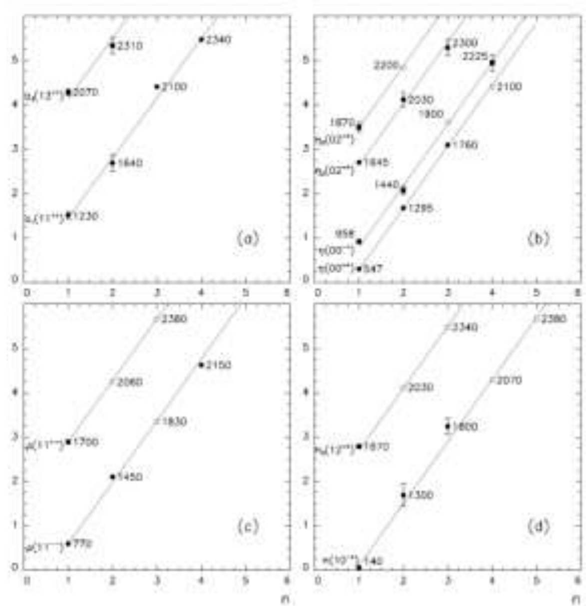
Rich source of spectral data on the light mesons – proton-antiproton annihilation

(A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, PRD (2000); D.V. Bugg, Phys. Rept. (2004))



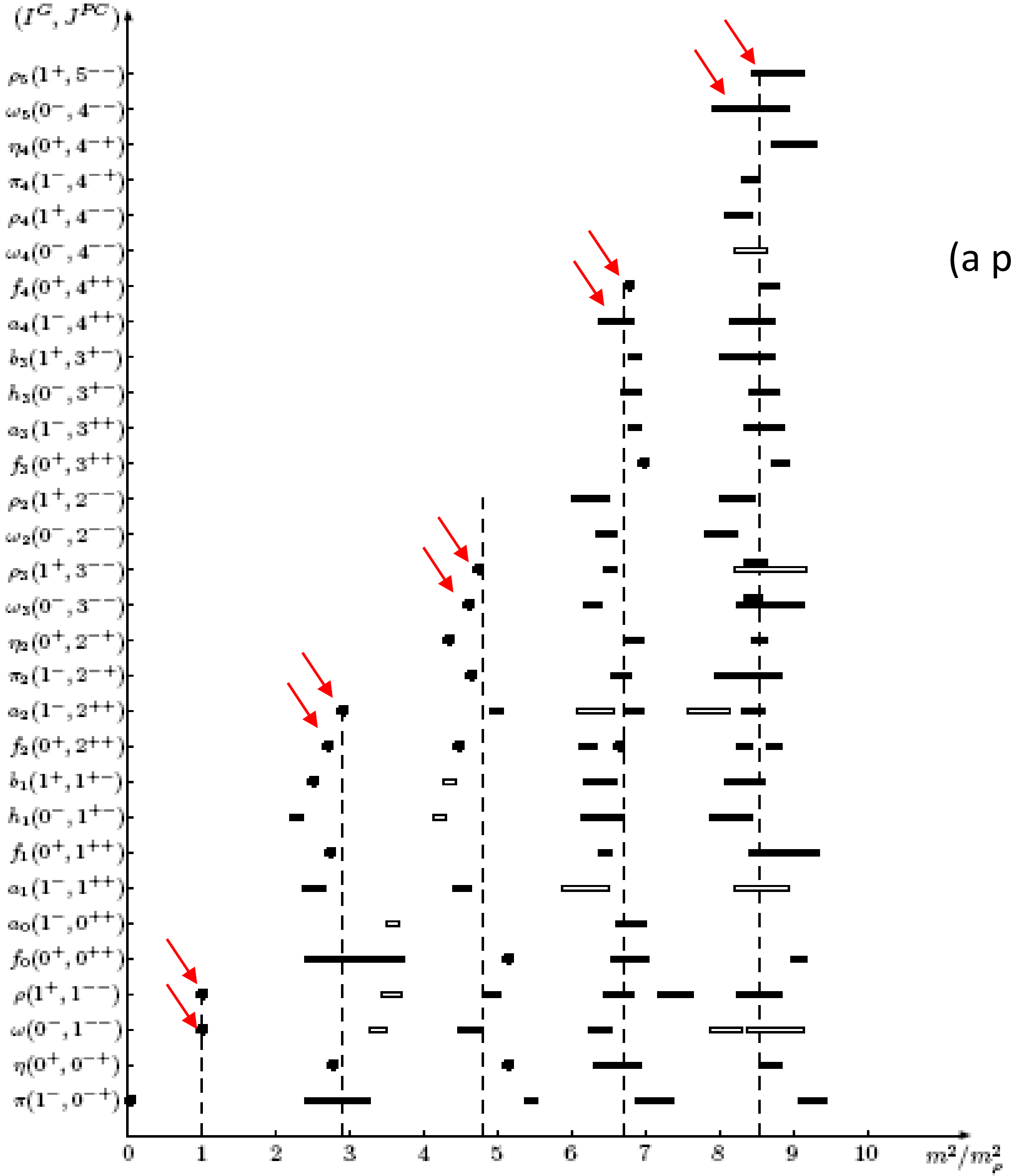


Examples



Experimental spectrum of light non-strange mesons

(a plot from S.S. Afonin, Eur. Phys. J. A 29 (2006) 327)



In average (in GeV²)

$$\bar{M}^2 \approx 1.1(N + 0.6),\ N = 0, 1, \dots$$

The major feature:
Spin-parity clustering

The spectrum of light nonstrange mesons in units of M_ρ^2. Experimental errors are indicated. Circles stay when errors are negligible. The dashed lines mark the mean mass squared in each cluster of states and the open strips and circles denote the one-star states. The arrows indicate the *J > 0* mesons which have no chiral partners (the hypothetical chiral singlets).

More exactly, ***<u>N= L + n</u>***

$$\bar{M}^2(L,n) \approx 1.1(L+n+0.6)$$

$\Longrightarrow$ The law $M^2(L,n) \sim L+n$ works!

Like in nonrelativistic hydrogen atom:

$$E(L,n) \sim \frac{1}{N^2}, \qquad N = L+n+1$$ - principal quantum number

The symmetry of the spectrum is larger than O(3), it is O(4)
(V.A. Fock, Z. Phys. 98 (1935) 145)

$\Longrightarrow$ Existence of parity (chiral) singlets
follows from the nonrelativistic definition of parity, $P = (-1)^{L+1}$

Mesons on leading Regge trajectories have *n=0*, hence, they are parity singlets

For instance: ρ-meson, (L,n)=(0,0), a_1-meson, (L,n)=(1,0), is partner for ρ´, (0,1)

<u>Potential models cannot explain the existence of “principal” quantum number!</u>

Realization of linear Regge trajectories in the bottom-up holographic approach to QCD?

Soft-wall holographic model

A. Karch, E. Katz, D. T. Son, M. A. Stephanov, PRD 74, 015005 (2006)

Example: The simplest Soft-wall model for vector mesons

$$S = -\frac{c^2}{4}\int d^4x\, dz \sqrt{g}\, e^{-az^2} F_{MN}F^{MN}$$

"Dilaton" background

$$F_{MN} = \partial_M V_N - \partial_N V_M,\; M = 0,1,2,3,4$$

Plane wave ansatz: $V_\mu(x,z) = \varepsilon_\mu e^{ipx} v(z)$ $\quad p^2 = m^2$ $\quad$ Axial gauge $V_z = 0$

After the change of variables $v_n = \sqrt{z} e^{az^2/2} \psi_n$ the e.o.m. is reduced to:

$$-\psi_n'' + U(z)\psi_n = m_n^2 \psi_n \qquad U = a^2 z^2 + \frac{3}{4z^2}$$

One has the radial Schroedinger equation for the harmonic oscillator with orbital momentum ***L=1***

$$-\psi'' + \left[z^2 + \frac{L^2 - 1/4}{z^2}\right]\psi = E\psi \qquad E = |a|m$$

The spectrum: $m_n^2 = 4|a|(n+1)$ $\qquad n = 0, 1, 2, \dots$

Spectrum from calculation of 2-point correlator

4D Fourier transform → $V^\mu(q,z)$; source → $V_0^\mu(q)$

$$V^\mu(q,z) = v(q,z)V_0^\mu(q) \qquad v(q,\epsilon) = 1$$

E.O.M.:

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v\right) + \frac{e^{-az^2}}{z}q^2 v = 0$$

Action on the solution

$$I = \int d^4x V_0^\mu V_{0\mu} \frac{e^{-az^2}}{z} v\partial_z v\Big|_{z=\epsilon}^{z=\infty}$$

$$\int d^4x e^{iqx}\langle J_\mu(x)J_\nu(0)\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(-q^2)$$

$$\Pi_V(-q^2) = c^2 \frac{\partial_z v}{q^2 z}\Big|_{z=\epsilon}$$

$$v(q,z) = \Gamma\left(1 - \frac{q^2}{4|a|}\right) e^{(a-|a|)z^2/2} U\left(\frac{-q^2}{4|a|}, 0; |a|z^2\right)$$

$$\Pi_V(-q^2) = c^2\left[\frac{a-|a|}{q^2} - \frac{1}{2}\psi\left(1-\frac{q^2}{4|a|}\right)\right] + \text{const}$$

$$\boxed{\Pi_V(-q^2) = c^2\left[\frac{a-|a|}{q^2} + \sum_{n=0}^{\infty}\frac{2|a|}{4|a|(n+1)-q^2}\right] + \text{const}}$$

$$\Pi_V(Q^2)_{Q^2\to\infty} = \frac{c^2}{2}\left[\log\left(\frac{4|a|}{Q^2}\right) - \frac{2a}{Q^2} + \frac{4a^2}{3Q^4} + \mathcal{O}\left(\frac{a^4}{Q^8}\right)\right] \qquad Q^2 = -q^2$$

$$\Pi_V(Q^2)_{\text{OPE}} = \frac{N_c}{24\pi^2}\log\left(\frac{\mu^2}{Q^2}\right) + \frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4} + \xi\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{\mu^8}{Q^8}\right)$$

$$\Rightarrow \qquad c^2 = \frac{N_c}{12\pi^2}$$

Important to make contact with QCD!

(that is why we prefer exactly linear Regge spectrum!)

Thus, the vector spectrum is $m_n^2 = 4|a|(n+1) \qquad n = 0, 1, 2, \ldots$

Generalization to the arbitrary intercept: $m_n^2 = 4|a|(n+1+\underline{b})$

$$e^{-az^2} \rightarrow \Gamma(1+b)U^2(b,0;az^2)e^{-az^2}$$

Tricomi function *(Afonin, PLB (2013))*

Extension to massless higher-spin fields

$$I = (-1)^J \frac{1}{2} \int d^4x\, dz \sqrt{g}\, e^{-az^2} \left(\nabla_N \Phi_J \nabla^N \Phi^J - m_J^2 \Phi_J \Phi^J\right)$$

$$\Phi_J \doteq \Phi_{M_1 M_2 \ldots M_J},\ M_i = 0, 1, 2, 3, 4, \qquad \partial^\mu \Phi_{\mu\ldots} = 0 \qquad \Phi_{z\ldots} = 0$$

Spectrum: $m_{n,J}^2 = 4a(n+J)$ - the spectrum of Veneziano dual amplitude (a rough model for experimental clustering in light meson resonances)

Alternative forms of the Soft-wall holographic model (without background)

1. Modified AdS metric (O. Andreev, PRD (2006))

$$g_{MN} = \frac{e^{-az^2}}{z^2}\eta_{MN}$$

(for vector SW model)

2. ***z***-dependent 5D mass

$$m_5^2(z) = A + Bz^2 + Cz^4$$

which appears after the field redefinition (S. Afonin, IJMPA (2011))

$$V_M \to e^{az^2/2}V_M$$

(it removes the “dilaton” background)

General ansatz for such holographic models

(S. A., T. Solomko, EPJC (2022) [2106.01846])

The starting point is a quadratic in fields holographic 5D action in which the Poincare invariance along the holographic coordinate ***z*** is violated in the most general way <u>compatible</u> with the linear Regge behavior of the discrete spectrum in 4 dimensions

$$S = \frac{1}{2}\int d^5x\sqrt{g}e^{cz^2}g^{M_1N_1}\dots g^{M_JN_J}\left[g^{MN}\partial_M\Phi_{M_1\dots M_J}\partial_N\Phi_{N_1\dots N_J} - \left(m_5^2 + a_1z^2 + a_2z^4\right)\Phi_{M_1\dots M_J}\Phi_{N_1\dots N_J} + bg^{zz}\Phi_{M_1\dots M_J}z\partial_z\Phi_{N_1\dots N_J}\right]$$

Projection to 4D particle states in holographic QCD: $\Phi_{z\dots} = 0$

One can further develop a general theory for the closed-form solvable holographic models possessing the Regge spectrum, demonstrate how different soft-wall like holographic models existing in the literature plus some new ones emerge from the given general setup, and study various interrelations between the emerging models.

Numerous phenomenological applications can be developed.

Possible extensions

- Various modifications of metrics and of dilaton background
- Inclusion of additional fields (to describe the chiral symmetry breaking, …)
- Inclusion of additional vertices (Chern-Simon, …)
- Account for backreaction of metrics (dynamical AdS/QCD models)
- …

Some applications

- Meson, baryon and glueball spectra
- Low-energy strong interactions (chiral dynamics)
- Hadronic formfactors
- Thermodynamic effects (QCD phase diagram)
- Description of quark-gluon plasma
- Condensed matter (high temperature superconductivity *etc.*)
- …

Deep relations with other approaches

- Light-front QCD
- QCD sum rules in the large-N_c limit
- Chiral perturbation theory supplemented by infinite number of vector mesons
- Renormalization group methods

Summary

Although the string theory is not predictive, it motivated construction of numerous phenomenological models for non-perturbative strong interactions which unexpectedly have predictability comparable with old traditional approaches

Thank you for your attention!