#### High Energy Electron Scattering from Nuclei



#### Output in the second second

- eN-scattering, kinematics, structure functions..
- A(e,e')X, PWIA, Spectral Function, 2NC..
- y-scalning, momentum distribution
- <u>FSI</u>
- Semi Inclusive quasi-elastic A(e,e'p)X
- DIS A(e,e'X) (A-1). Hadronization
- Summary

#### Mesons & baryons; static properties



### Mesons & baryons; QCD

- Field theory for strong interaction:
  - quarks interact by gluon exchange
  - quarks carry a 'colour' charge (R,B,G)
  - exchange bosons (gluons) carry colour



q/ q  $\sqrt{\alpha_s}$ q

$$\begin{aligned} \textbf{The QCD Lagrangian} \\ \textbf{L}_{QCD} &= i \sum_{q} \overline{\psi}_{q}^{j} \gamma^{\mu} (\textbf{D}_{\mu})_{jk} \psi_{q}^{k} - \sum_{q} m_{q} \overline{\psi}_{q}^{j} \psi_{q}^{k} - \frac{1}{4} G_{\mu\nu}^{a} G_{a}^{\mu\nu} \\ (j,k = 1,2,3 \text{ refer to colour; } q = u,d,s \text{ refers to flavour; } a = 1,...,8 \text{ to gluon fields} \end{aligned}$$

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$$\begin{aligned} \textbf{U}_{(j,k = 1,2,3 \text{ refer to colour; } q = u,d,s \text{ refers to flavour; } a = 1,...,8 \text{ to gluon self-energy term interaction} \end{aligned}$$

$$\begin{aligned} \textbf{U}_{(j,k = 1,2,3,a,b} \textbf{U}_{(j,k = 1,2,3,a,b)} \textbf{U}_{(j,k = 1,2,3,a,b)} \textbf{U}_{(j,k = 1,3,a,b)} \textbf{U}$$

### Mesons & baryons; QCD

- Field theory for strong interaction:
  - quarks interact by gluon exchange
  - quarks carry a 'colour' charge (R,B,G)
  - exchange bosons (gluons) carry
     colour ⇒ self-interactions (cf. QED!)
- Hadrons are colour neutral:
  - RR, BB, GG or RGB
  - leads to **Confinement**:

|q>, |qq> or |qqq̄> forbidden
Effective strength ~ gluons exch.
i) low Q<sup>2</sup>: more g's: large eff. coupling
ii) high Q<sup>2</sup>: few g's: small eff. coupling
(α<sub>s</sub>~0, asymp. freedom, pQCD)





# How to probe the quarks?



Energy  $\rightarrow$  Matter E=mc<sup>2</sup>

Bang two particles together and observe the types of particles that fly out (and their directions). In this way we can deduce the existence of new types of particles, investigate the properties of the known particles, and study the fundamental forces.

### How to probe the quarks? (Con't)

**Probes** – particles with well established structure and well known interaction with quarks – e.w. quark-lepton interaction



<u>Most</u> experiments operate at the energy frontier



Highest energy *e-p* collider: HERA at DESY in Hamburg: ~ 300 GeV

$$d_{probed} \propto \hat{\lambda} = \frac{\hbar}{p} \approx 10^{-18} \text{ m}$$

# Electron Nucleon Scattering

• kinematics:



$$L_{\mu\nu}$$
: lepton tensor  
 $W_{\mu\nu}$ : hadron tensor

Four-momentum transfer:  $q^{2} = (E - E')^{2} - (\vec{k} - \vec{k'}) \cdot (\vec{k} - \vec{k'}) =$   $= m_{a}^{2} + m_{a'}^{2} - 2(EE' - |\vec{k}| |\vec{k'}| \cos \theta) =$ 

$$\approx -4EE'\sin^2\frac{\theta}{2} \equiv -Q^2$$

• Mott Cross Section (*hc*=1):

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2 \frac{\theta}{2} \cdot \frac{E'}{E}$$

$$= \frac{4\alpha^2 E^{\prime 2}}{16E^2 E^{\prime 2} \sin^4 \frac{\theta}{2}} \cos^2 \frac{\theta}{2} \cdot \frac{1}{1 + \frac{E}{M}(1 - \cos\theta)}$$

$$= \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \cdot \frac{1}{1 + \frac{E}{M} (2\sin^2 \frac{\theta}{2})}$$

Electron scattering of a spinless point particle

# Cross Section for N(e,e')X in OPEA $d\sigma_{\text{lab}} = \frac{\delta^4 (p + Q - p_f)}{2\sqrt{\lambda(\mathbf{k}, \mathbf{p})}} \overline{\sum_{if}} |M_{fi}|^2 \left[\frac{d^3 k'}{(2\pi)^3}\right] d\tau_f$

#### **Current-Current Interaction**

$$M_{fi} = \frac{4\pi\alpha}{Q^2} \langle k'\lambda' | j_{\mu} | k\lambda \rangle \langle p | J^{\mu} | p_f \rangle$$

$$\overline{\sum_{if}} |M_{fi}|^2 \propto \underbrace{\sum_{if}} \langle k'\lambda' | j_{\mu} | k\lambda \rangle^* \langle k'\lambda' | j_{\nu} | k\lambda \rangle \frac{\alpha^2}{Q^4} \underbrace{\sum_{if}} \langle p | J^{\mu} | p_f \rangle^* \langle p | J^{\nu} | p_f \rangle$$

$$Leptonic tensor \eta_{\mu\nu}$$

$$Tr \left[ k\gamma_{\mu} k' \gamma_{\nu} \right]$$

$$\propto g_{\mu\nu} W_1(p, Q^2) + p_{\mu} p_{\nu} W_2(p, Q^2)$$



• Cross section:

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{Mott} \left( W_2(\nu) + 2W_1(\nu) \tan^2(\theta/2) \right)$$

- with
  - Mott cross section  $\sigma_{\rm Mott}$ : scattering off point charge
  - Structure functions  $W_1$ ,  $W_2$ with dimension [GeV]<sup>-1</sup>
  - Key issue: *if quark is not a fermion we will find W*<sub>1</sub>=0

#### Neutron Structure; electron scattering from nuclei



Inclusive A(e, e')XSemi-Inclusive A(e, e'p)X

<u>Semi-Exclusive</u> A(e, e'A - 1)X (detection of spectators)





•"Intermediate" distances (r ~r<sub>N</sub>) - NN-correlations?





### **Consider Unpolarized Case**

Lorentz Vectors/Scalars

6 indep. scalars:

$$p_i^2, p^2, Q^2, Q^{\bullet}p_i, Q^{\bullet}p_i, p^{\bullet}p_i$$
  
 $= m^2$ 

### Nuclear Response Tensor

$$W^{\mu\nu} = X_1 g_{\mu\nu} + X_2 p_i^{\mu} p_i^{\nu} + X_3 p^{\mu} p^{\nu} + X_4 p^{\mu} p_i^{\nu} + X_5 p_i^{\mu} p^{\nu}$$
$$+ X_6 q^{\mu} p^{\nu} + X_7 p^{\mu} q^{\nu} + X_8 q^{\mu} q^{\nu} + X_9 q^{\mu} p_i^{\nu} + X_{10} p_i^{\mu} q^{\nu}$$
$$+ (PV \text{ terms like } \epsilon_{\mu\nu\rho\sigma} q_\rho p_{\sigma})$$

Gauge invariants connervation with the fiding  $\propto j_{\mu}j_{\nu}$  )

$$q^{\mu}\eta_{\mu\nu} = q^{\nu}\eta_{\mu\nu} = 0 \quad \longrightarrow \quad X_{6..10} = 0$$

 $\eta_{\mu\nu} = \eta_{\nu\mu} \longrightarrow W_{\mu\nu} = W_{\nu\mu} \longrightarrow X_4 = X_5$ ; PV=0

4 independent responses

 $\mathbf{R}_{\mathrm{L}}, \mathbf{R}_{\mathrm{T}}, \mathbf{R}_{\mathrm{LT}}, \mathbf{R}_{\mathrm{TT}}$ 

### Putting all together ...

$$\left(\frac{\mathrm{d}^{6}\sigma}{\mathrm{d}\Omega_{\mathrm{p}}\mathrm{d}p\,\mathrm{d}v}\right)_{LAB} = \frac{pE_{p}}{\left(2\pi\right)^{3}}\sigma_{\mathrm{M}}\left[\rho_{L}R_{L}^{A} + \rho_{T}R_{T}^{A} + \rho_{LT}R_{LT}^{A}\cos\varphi_{\mathrm{x}} + \rho_{TT}R_{TT}^{A}\cos2\varphi_{\mathrm{x}}\right]$$

× 0

with 
$$\sigma_{\rm M} = \frac{\alpha^2 \cos^2 \theta/2}{4e^2 \sin^4 \theta/2}$$
  $\rho_L = \left(\frac{Q^2}{q^2}\right)^2$   $\rho_T = \frac{Q^2}{2q^2} + \tan^2 \theta/2$   
 $\rho_{TT} = \frac{Q^2}{2q^2}$   $\rho_{LT} = \frac{Q^2}{q^2} \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2}$ 

#### How to calculate the response functions?

#### **PWIA**

•The nuclear (A) current operator is the sum of one—body nucleon current operators, i.e the sum of currents for Dirac particles treated within an effective quantum field theory

$$\hat{J}^{A}_{\mu}(Q^{2}) = \sum_{N=1}^{A} \hat{J}^{N}_{\mu}(Q^{2})$$

•The final hadronic state asymptotically consists of two non interacting systems

$$|\alpha_{N}p_{1};\alpha_{A-1}P_{A-1}E_{A-1}^{f}\rangle = \hat{A}\left\{|\alpha_{N}p_{1}\rangle|\alpha_{A-1}P_{A-1}E_{A-1}^{f}\rangle\right\}$$

•The incoherent contributions leading to the emission of nucleon N, due to the interaction of  $\gamma$  with A-1, are disregarded (well justified at high Q<sup>2</sup>).





**The Spectral Function**  

$$S(\vec{k}_{1}, \mathbf{E}_{m}) = \sum_{f} \left| \left\langle \left( A - 1 \right)_{f} \left| a(\vec{k}_{1}) \right| A \right\rangle \right|^{2} \delta(\mathbf{E}_{m} - \sqrt{P_{A-1}^{2}} - M_{N} + M_{A})$$
where  $\vec{k}_{1} = -\vec{p}_{m}$  = initial momentum
$$E_{m} = E_{\min} + E_{A-1}^{*} = \nu - T_{k_{1}} - T_{A-1} = \text{missing (removal) energy}$$

### Note: S is not an observable!

$$n(\vec{k}_1) = \int_{E_{\min}}^{\infty} dE_m S(\vec{k}_1, E_m)$$
 - nuclear momentum distribution



#### The simplest nucleus: the deuteron



#### **BS** formalism

$$\begin{split} \Psi_{\mathcal{M}_{d}}^{S^{++}}(p_{1}',p_{2}') &= \mathcal{N}(\hat{k}_{1}+m)\frac{1+\gamma_{0}}{2}\hat{\xi}_{\mathcal{M}_{d}}(\hat{k}_{2}-m)\boldsymbol{\phi}_{\mathbf{S}}(p_{0},|\mathbf{p}|), \\ \Psi_{\mathcal{M}_{d}}^{D^{++}}(p_{1}',p_{2}') &= -\frac{\mathcal{N}}{\sqrt{2}}(\hat{k}_{1}+m)\frac{1+\gamma_{0}}{2}\Big(\hat{\xi}_{\mathcal{M}_{d}}+\frac{3}{2|\mathbf{p}|^{2}}(\hat{k}_{1}-\hat{k}_{2})(p\xi_{M})\Big) \\ &\times (\hat{k}_{2}-m)\boldsymbol{\phi}_{D}(p_{0},|\mathbf{p}|), \end{split}$$

$$n_D(p) \approx |\varphi_{\rm S}(\overline{p}_0, \mathbf{p})|^2 + |\varphi_{\rm D}(\overline{p}_0, \mathbf{p})|^2, \quad \overline{p}_0 = M_D / 2 - E_p$$



•Relativistic corrections are negligibly small

•Further calculations – nonrelativistic Schroedinger approach





**2NN-correlation condition** 

$$E_m \approx E_{rel} = \frac{\left(\vec{k}_2 - \vec{k}_3\right)^2}{4m} = \frac{\vec{k}_1^2}{4m}$$





 $E_{m}$  [MeV]

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<sup>3</sup>*He* Spectral Function AV18





C. Ciofi degli Atti, E. Pace and G. Salmè, Phys. Lett. 141B, 14 (1984).



Similar shapes for few-body nuclei and nuclear matter at high k (= $p_m$ ).

C. Ciofi degli Atti, E. Pace and G. Salmè, Phys. Lett. **141B**, 14 (1984).

### Some applications ....

Inclusive D(e,e')X process; y-scaling

$$\frac{d^{3}\sigma}{d\omega d\Omega_{e}} \cong \sigma_{eN} \frac{m}{|\vec{q}|} \int_{|p_{\min}|}^{\infty} p dp \ \boldsymbol{n}_{D}(p)$$

$$F(y) \equiv \frac{\frac{d^{3}\sigma}{d\omega d\Omega_{e}}}{\sigma_{eN} \frac{m}{|\vec{q}|}} = \int_{|y|}^{\infty} p dp \ \mathbf{n}_{D}(p)$$

$$F'(y)/p = n_D(p)$$



#### C. Ciofi degli Atti, L.P.K. D. Treleani Phys. Rev. C63 044601 (2001)



**PWIA** 

$$F'(y)/p = n_D(p)$$



#### **Quasielastic Electron Scattering**



R.R. Whitney et al., Phys. Rev. C 9, 2230 (1974).



## FSI

$$\begin{aligned} \mathcal{T}_{3}^{FSI}(\underline{\mathbf{p}}_{m}, E_{m}) &= \frac{1}{2J_{A}+1} \sum_{f} \left| \sum_{n=0}^{A-1} \mathcal{T}_{A}^{(n)}(\mathcal{M}_{f}, s_{1}) \right|^{2} \delta\left( E_{m} - (E_{A-1}^{f} + E_{min}) \right) \\ \mathcal{T}_{3}^{(1)} &= \int d\tau_{23} \underbrace{\frac{G_{He \to 1(23)}(k_{1}, k_{2}, k_{3}, s_{1}, s_{2}, s_{3})}{(k_{1}^{2} - M_{N}^{2})} \underbrace{\frac{f_{NN}(p_{1} - k_{1}')}{k_{1}^{2} - M_{N}^{2}}}_{(s_{2}, s_{3}|\Psi_{He}^{M_{3}}(\mathbf{k}_{1}, \mathbf{k}_{2}\mathbf{k}_{3}))} \underbrace{\frac{G_{(23) \to f}(k_{2}', k_{3}, s_{2}, s_{3})}{(k_{2}^{2} - M_{N}^{2})}}_{(s_{2}, s_{3}|\Psi_{He}^{M_{3}}(\mathbf{k}_{1}, \mathbf{k}_{2}\mathbf{k}_{3}))} \\ \mathcal{T}_{3}^{(1)} \approx \int \frac{d^{3}\kappa}{(2\pi)^{3}} \Psi_{(23)}^{f}(\mathbf{k}_{3}, \mathbf{k}_{2}'; S_{23}) \underbrace{\frac{f_{NN}(\kappa_{\perp})/4M_{N}|\mathbf{p}_{1}|}{(\kappa_{z} + \Delta_{z} + i\epsilon)}} \langle s_{1}|\Psi_{He}^{M_{3}}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3})\rangle \\ \Delta_{z} = \frac{E_{\mathbf{k}_{1}+\mathbf{q}} + E_{\mathbf{p}_{1}}}{2|\mathbf{p}_{1}|} (E_{m} - E_{\mathbf{k}_{1}+\mathbf{q}}) \\ \mathcal{T}_{3}^{(1)} = \int \frac{d^{3}\kappa}{(2\pi)^{3}} \Psi_{(23)}^{f}(\mathbf{k}_{3}, \mathbf{k}_{2}'; S_{23}) \underbrace{\frac{f_{NN}(\kappa_{\perp})/4M_{N}|\mathbf{p}_{1}|}{(\kappa_{z} + \Delta_{z} + i\epsilon)}} \langle s_{1}|\Psi_{He}^{M_{3}}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3})\rangle \\ \Delta_{z} = \frac{E_{\mathbf{k}_{1}+\mathbf{q}} + E_{\mathbf{p}_{1}}}{2|\mathbf{p}_{1}|} (E_{m} - E_{\mathbf{k}_{1}+\mathbf{q}}) \\ \langle s_{1}|\Psi_{He}^{M_{3}}(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3})\rangle \\ = \int \frac{d^{3}\kappa}{(2\pi)^{3}} \Psi_{1}^{f}(\mathbf{k}_{2}, \mathbf{k}_{2}, \mathbf{k}_{2}) \cdot \frac{d^{3}\kappa}{(\kappa_{z} + \Delta_{z} + i\epsilon)}} \\ \Delta_{z} = \frac{d^{3}\kappa}{(2\pi)^{3}} \left( \frac{d^{3}\kappa}{(2\pi)^{3}} + \frac{d^{3}\kappa}{(2\pi)^{3}} +$$

$$S_A^{FSI}(\underline{\mathbf{p}}_m, E_m) = \sum_f \int \frac{d^3 \underline{\mathbf{t}}}{(2\pi)^3} \left| \int \mathrm{e}^{i\rho \underline{\mathbf{p}}_m} \chi_{\frac{1}{2}s_1}^{\dagger} \Psi_{np}^{\underline{\mathbf{t}}\dagger}(\underline{\mathbf{r}}) \mathcal{S}_{\Delta}^{FSI}(\rho, \mathbf{r}) \Psi_{He}^{\mathcal{M}_3}(\underline{\mathbf{r}}\rho) d\rho d\mathbf{r} \right|^2 \delta \left( E - \frac{\underline{\mathbf{t}}^2}{M_N} - E_3 \right)$$

$$\mathcal{S}_{(1)}^{FSI}(\boldsymbol{\rho}, \mathbf{r}) = 1 - \sum_{i=2}^{3} \theta(z_i - z_1) \mathrm{e}^{i\Delta_z(z_i - z_1)} \Gamma(\underline{\mathrm{b}}_1 - \underline{\mathrm{b}}_i)$$

 $E_3)$ 



C. Ciofi degli Atti, L.P.K.

#### Hadronization in Deep Inelastic N(e,e')X processes



#### Nuclear size as "time filter" for hadronization









#### **Time evolution**

After  $\Delta t \approx k \approx 1 fm$  the string breaks into a qq-pair (meson) and another, less strengthened and (approx.) twice shorter string.

$$\gamma^* + N \rightarrow str \rightarrow N + str \rightarrow N + M + str \rightarrow N + 2M + str \rightarrow \dots$$
  
The break time  $t = \Delta t, 3 \Delta t, 7 \Delta t \dots$   $t = \Delta t \sum_{j=0}^{n} 2^{j} = \Delta t (2^{n+1} - 1)$ 

n+1 = the total number of produced mesond =  $n_M(t) = \frac{\ln(1 + t / \Delta t)}{\ln 2}$ 

#### The efctive nucleon-debris cross section

$$\sigma_{eff} = \sigma_{tot}^{NN} + \sigma_{tot}^{MN} n_{M}(t)$$





C.Ciofi degli Atti, L.P.K. B.Z. Kopeliovich Eur. Phys. J. A17 (2003) 133

#### Semi Exclusive DIS BONUS: D(e,e'p)X



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# **SUMMARY**

#### >PWIA

Inclusive y-scaling: direct experimental investigation of nuclear momentum distributions

>*NN* short range correlations via investigation of nuclear spectral function

➤reaction mechanism and the neutron structure

#### FSI in quasi-elastic A(e,e'p) reactions

Feynman diagramatic approach and Generalized Glauber approximation

Semi-Exclusive DIS A(e,e'X)(A-1)

- Neutron Structure Function, binding effects
- ➢ Mechanism of Hadronization

≻Color transparency etc.....

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