Twistor-String and Twistor-Beams in the Kerr-Schild Geometry.

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Black-Holes and Spinning Particles.

As it was first noticed by ‘t Hooft [G. ‘t Hooft, Nucl Phys. B 335, 138 (1990)], the recent ideas and methods in the black hole physics are based on complex analyticity and conformal field theory, which unifies the black hole physics with (super)string theory and physics of elementary particles.

Kerr-Newman solution:
as a Rotating Black-Hole and
as a Kerr Spinning Particle: Carter (1968), ($g = 2$ as that of the Dirac electron), Israel (1970), AB (1974), Lopez (1984) ...

About Forty Years of the Kerr-Schild Geometry and Twistors. They were created in the same time in the same place! (R.P. Kerr, Private talk.)
R. Penrose, J. Math. Phys.(1967).
G.C. Debney, R.P. Kerr, A.Schild, J. Math. Phys.(1969).

The Kerr Theorem.
– Twistor-String – Real and Complex Twistor-Stringy Structures of the Kerr-Schild Geometry.

– Twistor-beams. – New results on Black-Holes (AB, First Award of GRF 2009, arXive: 0903.3162).

Kerr-Schild form of the rotating black hole solutions:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \qquad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}. \qquad (1)$$

Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC).

$$k_\mu(x) = P^{-1}(du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv), \qquad (2)$$

where $Y(x) = e^{i\phi} \tan\frac{\theta}{2}$,

$$2^{\frac{1}{2}}\zeta = x + iy,\ 2^{\frac{1}{2}}\bar{\zeta} = x - iy,\ 2^{\frac{1}{2}}u = z - t,\ 2^{\frac{1}{2}}v = z + t$$

are the null Cartesian coordinates.
Congruence PNC is twosheeted and controlled by

the Kerr Theorem:

The geodesic and shear-free null congruences (GSF PNC, type D) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0\ , \qquad (3)$$

where F is an arbitrary analytic function of the *projective twistor coordinates*

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}. \qquad (4)$$

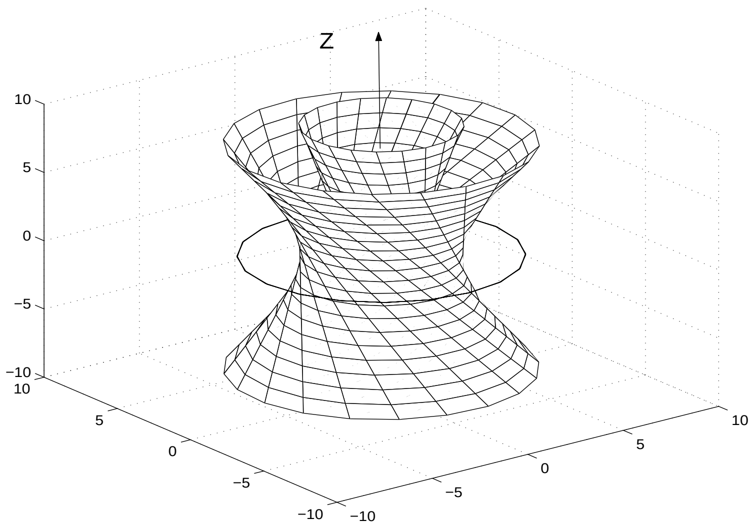

Figure 1: The Kerr singular ring and the Kerr congruence.

The Kerr singular ring $r = \cos\theta = 0$ is a branch line of space on two sheets: "negative (–)" and "positive (+)" where the fields change their directions. In particular,

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \quad g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}. \tag{5}$$

Twosheetedness! Mystery of the Kerr source!
Doubts since 1964 up to now.

a) Stringy source: E.Newman 1964, A.B. 1974, W.Israel 1975, A.B. 1975, ...

b) Rotating superconducting disk. W.Israel (1969), Hamity, I.Tiomno (1973), C.A. L'opez (1983)9; A.B. (1989,2000-2004) (Supercondacting bag $U(1)\times U(1)$ model), ... The Kerr ring as a "mirror gate" to "Alice world".
Oblate spheroidal coordinates cover spacetime twice:

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi} \sin\theta \\ z &= r\cos\theta, \qquad t = \rho + r \end{aligned} \tag{6}$$

disk $r = 0$ separates the 'out'-sheet $r > o$, from 'in'-sheet $r < o$.

New Look: Holographic BH interpretation. A.B.(2009) based on the ideas C.R. Stephens, G. t' Hooft and B.F. Whiting (1994), 't Hooft (2000), Bousso (2002).

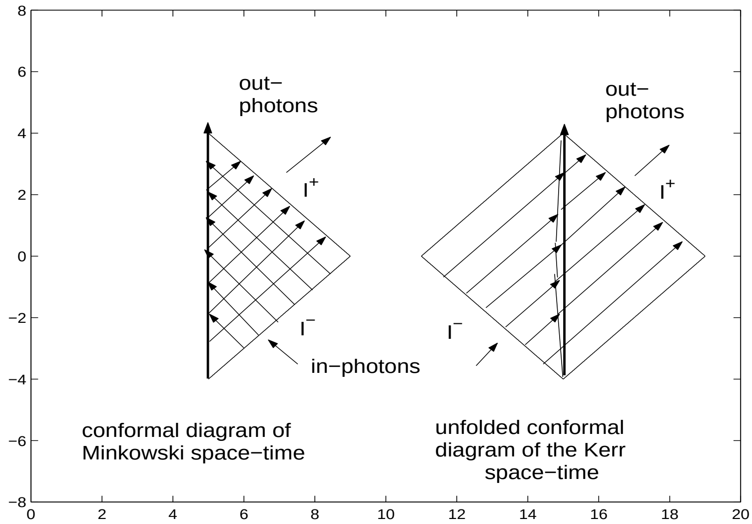

Figure 2: Penrose conformal diagrams. The unfolded Kerr-Schild spacetime corresponds to the holographic structure of a quantum black-hole spacetime.

Twosheetedness of the Kerr-Schild geometry corresponds to holographic black-hole Kerr-Schild spacetime. For stationary black hole solutions, the both: in- and out- sheets can be used as 'physical sheets'(MTW). Presence of the electromagnetic field determines out-sheet as 'physical one (out-going electromagnetic solutions far from black hole .) **Alignment** of the electromagnetic field to PNC,

$A_\mu k^\mu = 0$!

PNC is in-going at $r < 0$, passing through the Kerr ring to 'positive' sheet, $r > 0$, and turns into out-going.

Desirable structure of a quantum black hole space-time (Stephens, t' Hooft and Whiting (1994)): the in- and out-sheets separated by a (holographically dual) 2+1 boundary: $(Y,t) \in S^2 \times R$. Out-sheet is 'physical one.

For $m > 0$ the horizon exists only for $r > 0$ - out-sheet which is identified as a 'physical sheet' of the black hole. *Kerr congruence performs holographic projection of 3+1 dim bulk to 2+1 dim boundary.* "Geodesic and shear-free" (GSF) congruences:

$GSF conditions \quad \Leftrightarrow \quad Y_{,2} = Y_{,4} = 0$ provide pull-back of the conformal-analytic structure from boundary onto bulk and conformal-analytic structure of the solutions.

Complex Shift. Appel solution (1887!) and Complex Structure of the Kerr geometry.

A point-like charge e, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, -ia)$, gives a real potential

$$\phi_a = Re\ e/\tilde{r}, \tag{7}$$

where $\tilde{r}$ turns out to be a complex radial coordinate $\tilde{r} = r + i\xi$, and we obtain

$\tilde{r}^2 = r^2 - \xi^2 + 2ir\xi = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$, which leads to two equations

$$r\xi = az, \quad r^2 - \xi^2 = x^2 + y^2 + z^2 - a^2, \tag{8}$$

corresponding to the Kerr oblate spheroidal coordinates r and θ.

Starting from the usual system of angular coordinates, we would like to retain after complex shift the relation $z = r\cos\theta$, and obtain $\xi = a\cos\theta$,

$$\tilde{r} = r + ia\cos\theta, \tag{9}$$

and the equation $(r^2 + a^2)\sin^2\theta = x^2 + y^2$, which may be split into two complex conjugate equations. This splitting

$$(x \pm iy) = (r \pm ia)e^{\pm i\phi}\sin\theta \tag{10}$$

yields the Kerr-Schild Cartesian coordinate system.

Complex Light Cone. Twistors as Null Planes.

Complex Kerr-Schild null tetrad e^a, $(e^a)^2 = 0$: real directions

$$e^3 = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv$$

(PNC) and

$$e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv, \quad e^4 = dv + he^3.$$

The complex light cone with the vertex at some complex point $x_0^\mu \in CM^4$: $(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$, can be split into two families of null planes: "left" planes

$$x_L = x_0(\tau) + \alpha e^1 + \beta e^3 \tag{11}$$

spanned by null vectors e^1 and e^3, and"right"planes

$$x_R = x_0(\tau) + \alpha e^2 + \beta e^3, \tag{12}$$

spanned by null vectors e^2 and e^3.

The Kerr congruence $\mathcal{K}$ arises as a real slice of the family of the "left" null planes ($Y = const.$) of the complex light cones whose vertices lie at a complex source $x_0(\tau)$.

Complex Source of the Kerr geometry and Retarded Time

The Appel complex source $(x_o, y_o, z_o) \rightarrow (0, 0, -ia)$ can be considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ in CM^4 and parametrized by a complex time τ. There appears a complex retarded-time construction (Newman, Lind.) In the complex case there are two different ways for obtaining retarded time. For a given real point $x \in M^4$ one considers the past light cone

$\swarrow^{x}\searrow$

to obtain the root of its intersection with a given complex world-line $x_0(\tau)$. It is known that the light cone splits into

the left:

$\swarrow^{x}$

and right: ${}^{x}\searrow$ complex null planes.[1]

Correspondingly, there are two roots:

${}_{x_0}\swarrow^{x}$ and ${}^{x}\searrow_{x_0}$,

and two different (in general case) retarded times

${}_{\tau_0}\swarrow^{t}$ and ${}^{t}\searrow_{\tau_0}$

for the same complex world-line: the 'Left' and 'Right' projections.

[1] In the Kerr null tetrad e^a, they are spanned, correspondingly, on the null forms $e^3 \wedge e^1$ and $e^3 \wedge e^2$.

The real Kerr-Schild geometry appears as a real slice of this complex structure.

This construction may be super-generalized by 'super-complex translation', leading to super Kerr-Newman solution to broken N=2 supergravity (AB, Clas.Q. Grav.,2000).

Along with the considered complex world-line, there is a complex conjugate world-line, and we shall mark them the 'Left' and 'Right' ones, $X_L(\tau_L)$ and $X_R(\tau_R)$.

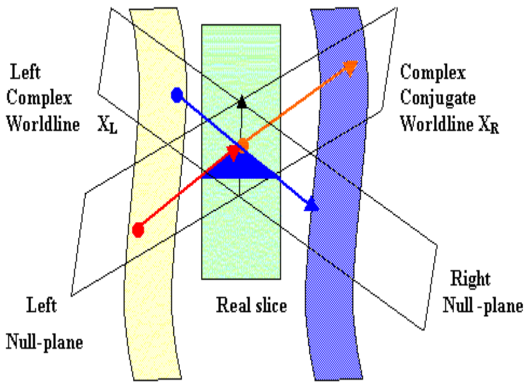

Figure 3: Complex light cone at a real point x and two null directions $e^{3(\pm)} = e^3(Y_L^{\pm}(x))$. The adjoined to congruence Left and Right complex null planes intersect the Left and Right complex world lines at four points: X_L^{adv}, X_L^{ret} and X_R^{adv}, X_R^{ret} which are related by crossing symmetry.

Complex world-line forms really a world-sheet of an Euclidean string $X_L(\tau_L) \equiv X_L^{\mu}(t_L + i\sigma_L)$. It is open string with the ends at $\sigma = \pm a$. Left and Right structures form an Orientifold ($\Omega = Antip.map + CC + Revers\ of\ time$).

Antipodal map: $Y \to -1/\bar{Y}$.

Twistor-Beams. The exact time-dependent KS solutions.

Debney, Kerr and Schild (1969). The black-hole at rest: $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, $P = 2^{-1/2}(1 + Y\bar{Y})$.
Tetrad components of electromagnetic field $\mathcal{F}_{ab} = e^\mu_a e^\nu_b \mathcal{F}_{\mu\nu}$,

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}\,, \tag{13}$$

here $Z = -P/(r + ia\cos\theta)$ is a complex expansion of the congruence. Stationarity $\Rightarrow \gamma = 0$.

Kerr-Newman solution is exclusive:

$\psi(Y) = const.$

In general case $\psi(Y)$ is an arbitrary holomorphic function of $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$, which is a projective coordinate on celestial sphere S^2,

$$A = \psi(Y)/P^2, \tag{14}$$

and there is *infinite set of the exact solutions*, in which $\psi(Y)$ is singular at the set of points $\{Y_i\}$, $\psi(Y) = \sum_i \frac{q_i}{Y(x)-Y_i}$, corresponding to angular directions $\phi_i,\ \theta_i$.

Twistor-beams. Poles at Y_i produce semi-infinite singular *lightlike beams*, supported by twistor rays of the Kerr congruence. The twistor-beams have very strong backreaction to metric $g^{\mu\nu} = \eta^{\mu\nu} - 2Hk^\mu k^\nu$, where

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2\cos^2\theta}. \tag{15}$$

How act such beams on the BH horizon?

Black holes with holes in the horizon, A.B., E.Elizalde, S.R.Hildebrandt and G.Magli, Phys. Rev. **D74** (2006) 021502(R)

Singular beams lead to formation of the holes in the black hole horizon, which opens up the interior of the "black hole" to external space.

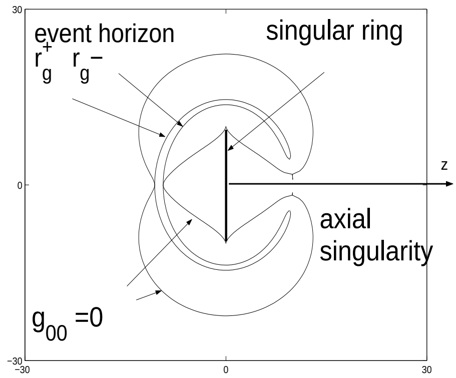

Figure 4: Near extremal black hole with a hole in the horizon. The event horizon is a closed surface surrounded by surface $g_{00} = 0$.

Twistor-beams are exact stationary and time-dependent Kerr-Schild solutions (of type D) which show that 'elementary' electromagnetic excitations have generally *singular beams* supported by twistor null lines. Interaction of a black-hole with external, *even very weak,* electromagnetic field resulted in appearance of the beams, which have very strong back reaction to metric and horizon and form a fine-grained structure of the horizon pierced by fluctuating microholes. A.B., E. Elizalde, S.R. Hildebrandt and G. Magli, Phys.Lett. **B 671** 486 (2009), arXiv:0705.3551[hep-th]; A.B., arXiv:gr-qc/0612186.

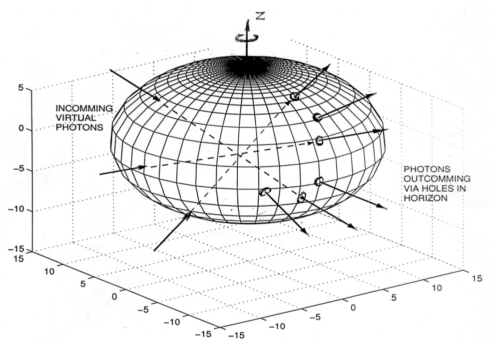

Figure 5: Excitations of a black hole by weak electromagnetic field yields twistor-beams creating a horizon covered by fluctuating micro-holes.

Time-dependent solutions of DKS equations for electromagnetic excitations, $\gamma \neq 0$, A.B. (2004-2008)

a) Exact solutions for electromagnetic field on the Kerr-Schild background, (2004),

b) Asymptotically exact wave solutions, consistent with Kerr-Schild gravity in the low frequency limit, (2006-2008)

c) Self-regularized solutions, consistent with gravity for averaged stress-energy tensor, (A.B. 2009)

Electromagnetic field is determined by functions A and γ,

$$A,_2 - 2Z^{-1}\bar{Z}Y,_3 A = 0, \quad A,_4 = 0, \tag{16}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma,_2 - Z^{-1}Y,_3 \gamma = 0, \tag{17}$$

and

Gravitational sector: has two equations for function M, which take into account the action of electromagnetic field

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = A\bar{\gamma}\bar{Z}, \tag{18}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}. \tag{19}$$

where $cD = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2$.

Similar to the exact stationary solutions, typical time-dependent (type D) solutions contain outgoing singular beam pulses which have very strong back reaction to metric and perforate horizon.

Eqs. of the electromagnetic sector were solved (2004).

GSF condition $Y,_2 = Y,_4 = 0, \Rightarrow k^\mu \partial_\mu Y = 0$.

Stationary Kerr-Schild solutions

$A = \psi/P^2$, where $\psi,_2 = \psi,_4 = 0 \Rightarrow \psi(Y) \Rightarrow$ alignment condition $k^\mu \partial_\mu \psi = 0$.

Time-dependent solutions need a *complex retarded time parameter* τ, obeying $\tau,_2 = \tau,_4 = 0$, and $\psi = \psi(Y, \tau)$.

There appears a dependence between $\dot{A}$ and γ

$(\partial_3 - Z^{-1} Y,_3 \partial_1 - \bar{Z}^{-1} \bar{Y},_3 \partial_2) A + \bar{Z}^{-1} \gamma,_2 - Z^{-1} Y,_3 \gamma = 0$.

Integration yields

$$\gamma = \frac{2^{1/2} \dot{\psi}}{P^2 Y} + \phi_0(Y, \tau)/P, \qquad (20)$$

which shows that **nonstationarity,** $\dot{\psi} = \sum_i \dot{c}_i(\tau)/(Y - Y_i) \neq 0$, **creates generally the poles** in $\gamma \sim \sum_i q_i/(Y - Y_i)$, leading to twistor-beams in directions $Y_i = e^{i\phi} \tan \frac{\theta}{2}$.

Self-regularization.

Structure of KS solutions inspire the regularization which acts immediately on the function γ. Free function $\phi_0(Y,\tau)$ of the homogenous solution may be tuned, to cancel the poles of function $\dot{\psi} = \sum_i \dot{c}_i(\tau)/(Y - Y_i)$.

i-th term

$$\gamma_{i(reg)} = \frac{2^{1/2}\dot{c}_i(\tau)}{Y(Y - Y_i)P^2} + \phi_i^{(tun)}(Y,\tau)/P. \tag{21}$$

Condition to compensate i-th pole is

$$\gamma_{i(reg)}(Y,\tau)|_{\bar{Y}=\bar{Y}_i} = 0. \tag{22}$$

We obtain

$$\phi_i^{(tun)}(Y,\tau) = -\frac{2^{1/2}\dot{c}_i(\tau)}{Y(Y - Y_i)P_i}, \tag{23}$$

where

$$P_i = P(Y,\bar{Y}_i) = 2^{-1/2}(1 + Y\bar{Y}_i) \tag{24}$$

is analytic in Y, which provides analyticity of $\phi_i^{tun}(Y,\tau)$.

As a result we obtain

$$\gamma_{i(reg)} = \frac{\dot{c}_i(\tau)(\bar{Y}_i - \bar{Y})}{P^2 P_i(Y - Y_i)}. \tag{25}$$

First gravitational DKS equation gives

$$m = m_0(Y) + \sum_{i,k} \frac{c_i \dot{\bar{c}}_k (Y_k - Y)}{(Y - Y_i)} \int_{\bar{Y}_k} \frac{d\bar{Y}}{P\bar{P}_k(\bar{Y} - \bar{Y}_k)}. \tag{26}$$

Using the Cauchy integral formula, we obtain

$$m = m_0(Y) + 2\pi i \sum_i \frac{(Y_k - Y)}{(Y - Y_i)} \sum_k \frac{c_i \dot{\bar{c}}_k}{|P_k|^2}. \tag{27}$$

Functions c_i and $\bar{c}_k$ for different beams are not correlated,

$< c_i \dot{\bar{c}}_k >= 0$. Time averaging retains only the terms with $i = k$,

$$< m >_t = m_0 - 2\pi i \sum_k \frac{c_k \dot{\bar{c}}_k}{|P_k|^2}. \tag{28}$$

Representing $c_i(\tau) = q_i(\tau)e^{-i\omega_i \tau}$ via amplitudes $q_i(\tau)$ and carrier frequencies ω_i of the beams. The mass term retains the low-frequency fluctuations and angular non-homogeneity caused by amplitudes and casual angular distribution of the beams,

$$< m >_t = m_0 + 2\pi \sum_k \omega_k \sum_k < \frac{q_k \bar{q}_k}{|P_{kk}|^2} > . \qquad (29)$$

Second gravitational DKS equations is definition of the loss of mass in radiation,

$$\dot{m} = -\frac{1}{2} P^2 \sum_{i,k} \gamma_{i(reg)} \bar{\gamma}_{k(reg)} = -\frac{1}{2} \sum_{i,k} \frac{\dot{c}_i \dot{\bar{c}}_k}{P^2 P_i \bar{P}_k} \qquad (30)$$

Time averaging removes the terms with $i \neq k$ and yields

$$< \dot{m} >_t = -\frac{1}{2} \sum_k \frac{\dot{c}_k \dot{\bar{c}}_k}{P^2 |P_k|^2}. \qquad (31)$$

In terms of the amplitudes of beams we obtain

$$< \dot{m} >_t = -\frac{1}{2} \sum_k \omega_k^2 < \frac{\dot{q}_k \bar{q}_k}{|P_{kk}|^4} >, \qquad (32)$$

which shows contribution of a single beam pulse to the total loss of mass.

The obtained solutions are consistent with the Einstein-Maxwell system of equations for the time-averaged stress-energy tensor.

Obtained results.

1) Exact time-dependent solutions for Maxwell eqs. on the Kerr-Schild background $\Rightarrow$ singular twistor-beam pulses.

2) Exact back reaction of the beams to metric and horizon $\Rightarrow$ fluctuating metric and horizon perforated by twistor-beam pulses.

3) Exact time-dependent solutions for Maxwell eqs. on the Kerr-Schild background leading to *regular*, but fluctuating radiation $\Rightarrow$ *regular* $< T^{\mu\nu} >$, but metric and horizon are covered by fluctuating twistor-beams!

4) Consistency with the corrected Einstein equations

$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} =< T^{\mu\nu} >$.

We arrive at a semiclassical (pre-quantum) geometry of fluctuating twistor-beams which takes an intermediate position between the Quantum and the usual Classical gravity.

THE END.
THANK YOU FOR YOUR ATTENTION!

Singular pp-wave solutions (A.Peres)

Self-consistent solution of the Einstein-Maxwell equations: singular plane-fronted waves (pp-waves). Kerr-Schild form with a constant vector $k_\mu = \sqrt{2} du = dz - dt$

$g_{\mu\nu} = \eta_{\mu\nu} + 2h k_\mu k_\nu$.

Function h determines the Ricci tensor

$$R^{\mu\nu} = -k^\mu k^\nu \Box h, \tag{33}$$

where $\Box$ is a flat D'Alembertian

$$\Box = 2\partial_\zeta \partial_{\bar\zeta} + 2\partial_u \partial_v \ . \tag{34}$$

The Maxwell equations take the form $\Box \mathcal{A} = J = 0$, and can easily be integrated leading to the solutions

$$\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar\zeta)] f^+(u) du, \tag{35}$$

$$\tag{36}$$

where $\Phi^\pm$ are arbitrary analytic functions, and function f^+ describes retarded waves.

The poles in $\Phi^+(\zeta)$ and $\Phi^-(\bar\zeta)$ lead to the appearance of singular lightlike beams (pp-waves) which propagate along the z^+ semi-axis.

pp-waves have very important quantum properties, being exact solutions in string theory with vanishing all quantum corrections (G.T. Horowitz, A.R. Steif, PRL **64** (1990) 260; A.A. Coley, PRL **89** (2002) 281601.)

Quadratic generating function F(Y) and interpretation of parameters. A.B. and G. Magli, Phys.Rev.**D 61**044017 (2000).

The considered in DKS function F is quadratic in Y,

$$F \equiv a_0 + a_1 Y + a_2 Y^2 + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2, \quad (37)$$

where the coefficients c and p are real constants and $a_0, a_1, a_2, q, \bar{q},$ are complex constants. The Killing vector of the solution is determined as

$$\hat{K} = c\partial_u + \bar{q}\partial_\zeta + q\partial_{\bar{\zeta}} - p\partial_v. \quad (38)$$

Writing the function F in the form

$$F = AY^2 + BY + C, \quad (39)$$

one can find two solutions of the equation $F = 0$ for the function $Y(x)$

$$Y_{1,2} = (-B \pm \Delta)/2A, \quad (40)$$

where $\Delta = (B^2 - 4AC)^{1/2}$.

We have also

$$\tilde{r} = -\partial F/\partial Y = -2AY - B, \quad (41)$$

and consequently

$$\tilde{r} = PZ^{-1} = \mp\Delta. \quad (42)$$

These two roots reflect the known twofoldedness of the Kerr geometry. They correspond to two different directions of congruence on positive and negative sheets of the Kerr space-time. In the stationary case

$$P = pY\bar{Y} + \bar{q}\bar{Y} + qY + c \ . \tag{43}$$

Link to the complex world line of the source. The stationary and boosted Kerr geometries are described by a straight complex world line with a real 3-velocity $\vec{v}$ in CM^4:

$$x_0^\mu(\tau) = x_0^\mu(0) + \xi^\mu \tau; \qquad \xi^\mu = (1, \vec{v}) \ . \tag{44}$$

The gauge of the complex parameter τ is chosen in such a way that $Re\ \tau$ corresponds to the real time t.

$\hat{K}$ is a Killing vector of the solution

$$\hat{K} = \partial_\tau x_0^\mu(\tau)\partial_\mu = \xi^\mu \partial_\mu \ . \tag{45}$$

$$P = \hat{K}\rho = \partial_\tau x_0^\mu(\tau) e_\mu^3 \ , \tag{46}$$

where

$$\rho = \lambda_2 + \bar{Y}\lambda_1 = x^\mu e_\mu^3 . \tag{47}$$

It allows one to set the relation between the parameters $p, c, q, \bar{q}$, and ξ^μ, showing that these parameters are connected with the boost of the source.

The complex initial position of the complex world line $x_0^\mu(0)$ in Eq. (44) gives six parameters for the solution, which are connected to the coefficients $a_0,\ a_1\ a_2$. It can be decomposed as $\vec{x}_0(0) = \vec{c} + i\vec{d}$, where $\vec{c}$ and $\vec{d}$ are real 3-vectors with respect to the space O(3)-rotation. The real part $\vec{c}$ defines the initial position of the source, and the imaginary part $\vec{d}$ defines the value and direction of the angular momentum (or the size and orientation of a singular ring).

It can be easily shown that in the rest frame, when $\vec{v} = 0, \quad \vec{d} = \vec{d}_0$, the singular ring lies in the orthogonal to $\vec{d}$ plane and has a radius $a = |\vec{d}_0|$. The corresponding angular momentum is $\vec{J} = m\vec{d}_0$.

Smooth and regular Kerr sources.

A.B., E. Elizalde, S.Hildebrandt and G. Magli, PRD (2002)

The Gürses and Gürsey ansatz $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$, where $h = f(r)/(r^2 + a^2 \cos^2\theta)$.

Regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = mr - e^2/2$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) a narrow intermediate region providing a smooth interpolation between i) and ii).

Non-rotating case: by $n = 4$ and $\alpha = 8\pi\Lambda/6$,

interior is a space-time of constant curvature $R = -24\alpha$.

Energy density of source $\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2$,

tangential and radial pressures $p_{rad} = -\rho, \quad p_{tan} = \rho - \frac{1}{8\pi} f''/\Sigma$, where $\Sigma = r^2$.

There is a de Sitter interior for $\alpha > 0$, and anti de Sitter interior for $\alpha < 0$. Interior is flat if $\alpha = 0$.

The resulting sources may be considered as the bags filled by a special matter with positive ($\alpha > 0$) or negative ($\alpha < 0$) energy density. The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from ‘true’ to ‘false’ vacuum. Assuming that transition region iii) is very thin, one can consider the following useful graphical representation.

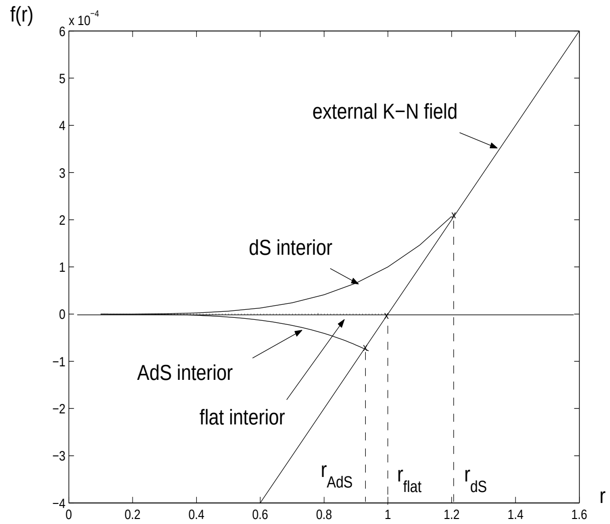

Figure 6: Regularization of the Kerr spinning particle by matching the external field with dS, flat or AdS interior.

The point of phase transition r_0 is determined by the equation $f_{int}(r_0) = f_{KN}(r_0)$ which yields

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \tag{48}$$

The first term on the right side is electromagnetic mass of a charged sphere with radius r_0, $M_{em}(r_0) = \frac{e^2}{2r_0}$, while the second term is the mass of this sphere filled by a material with a homogenous density ρ, $M_m = \frac{4}{3}\pi r_0^3 \rho$. Thus, the point of intersection r_0 acquires a deep physical meaning, providing the energy balance by the mass formation.

Transfer to rotating case. One has to set $\Sigma = r^2 + a^2 \cos^2 \theta$, and consider r and θ as the oblate spheroidal coordinates.

The Kerr source represents a disk with the boundary $r = r_0$ which rotates rigidly. In the corotating with disk coordinate system, the matter of the disk looks homogenous distributed, however, because of the relativistic effects the energy-momentum tensor increases strongly near the boundary of the disk.

In the limit of a very thin disk a stringy singularity develops on the border of disk. This case corresponds to the Israel-Hamity source 1970-1976.

The Kerr-Newman spinning particle with $J = \frac{1}{2}\hbar$, acquires the form of a relativistically rotating disk which has the form of a highly oblate ellipsoid with the thickness $r_0 \sim r_e$ and the Compton radius $a = \frac{1}{2}\hbar/m$. Interior of the disk represents a "false" vacuum having superconducting properties which are modelled by the Higgs field.

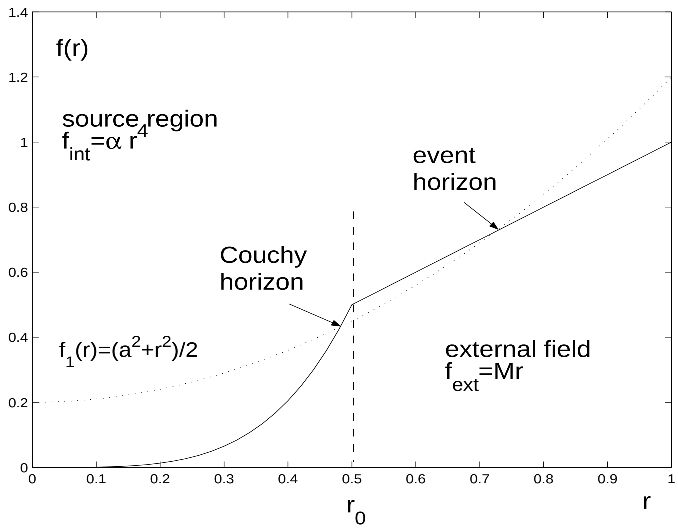

Figure 7: Matching the (rotating) internal "de Sitter" source with the external Kerr-Schild field. The dotted line $f_1(r) = (r^2 + a^2)/2$ determines graphically the position of horizons as the roots of the equation $f(r) = f_1(r)$.

Properties of the disklike Kerr source

- the disk is oblate and rigidly rotating,
- the rotation is relativistic, so the board of the disk is moving with the speed which is close to the speed of light,
- the stress-energy tensor of the matter of the disk has an exotic form resembling a special condensed vacuum state (de Sitter, flat or anti de Sitter vacua).
- electromagnetic properties of the matter of the disk are close to superconductor,

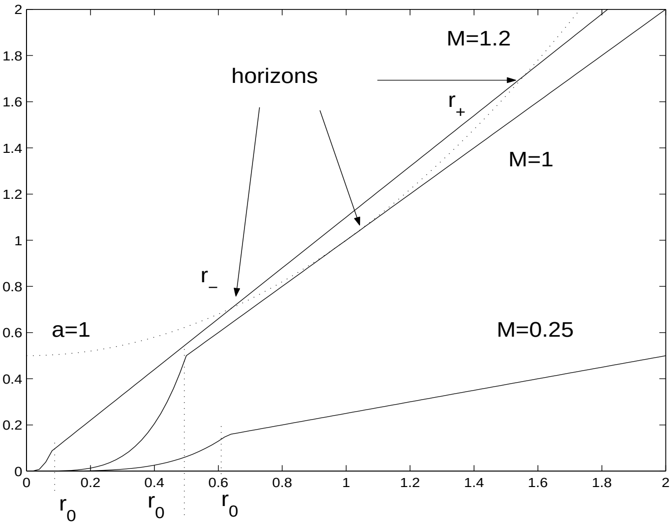

Figure 8: The sources with different masses M and matter densities ρ. Sources form the rotating disks with radius $\sim a$ and thickness $\sim r_0$ which depends on the matter density $r_0 = (\frac{3M}{4\pi\rho})^{1/3}$. The formation of the black hole horizons is shown for $a^2 < M^2$.

- the charge, strong magnetic and gravitational fields are concentrated on the stringy board of the disk, and are partially compensated from the oppositely charged part of the disk surface. It yields a very specific form of the electromagnetic field (see fig.4, and fig.5).

- finally, the main property of Kerr-Schild source - the relation $J = Ma$ between the angular momentum J, mass M, and the radius of the Kerr ring a.

Complex Kerr source, complex shift. Appel 1887!

A point-like charge e, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, ia)$, gives the real Appel potential

$$\phi_a = Re\ e/\tilde{r}, \tag{49}$$

where $\tilde{r} = r + ia \cos\theta$ is the Kerr complex radial coordinate and r and θ are the oblate spheroidal coordinates. In the Cartesian coordinates x, y, z, t

$$\tilde{r} = [(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]^{1/2} = [x^2+y^2+(z-ia)^2]^{1/2}. \tag{50}$$

Singularity of the Appel potential ϕ_a corresponds to $r = \cos\theta = 0$, and therefore, the singular ring $z = 0, \quad x^2 + y^2 = a^2$ is a branch line of space-time for two sheets, just similar to the Kerr singular ring.

Appel potential describes *exactly* the em field of the Kerr-Newman solution on the auxiliary M^4.

Complex world line and complex Kerr string.

If the Appel source is shifted to a complex point of space $(x_o, y_o, z_o) \to (0, 0, ia)$, it can be considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ in CM^4 and parametrized by a complex time τ. The complex source of the Kerr-Newman solution has just the same origin and can be described by means of a complex retarded-time construction for the Kerr geometry.

The objects described by the complex world-lines occupy an intermediate position between particle and string. Like a string they form two-dimensional surfaces or world-sheets in space-time. In many respects this source is similar to the "mysterious" $N = 2$ complex string of superstring theory.

The Kerr congruence may be understood as a track of the null planes of the family of complex light cones emanating from the points of the complex world line $x_0^\mu(\tau)$ in the retarded-time construction.

Complex retarded-time parameter.

Parameter τ may be defined for each point x of the Kerr space-time and plays the role of a complex retarded-time parameter. Its value for a given point x may be defined by L-projection, using the solution $Y(x)$ and forming the twistor parameters $\lambda_1, \quad \lambda_2$ which fix a left null plane. The points x^μ and x_0^μ are connected by the left null plane spanned by the null vectors e^1 and e^3.

The point of intersection of this plane with the complex world-line $x_0(\tau)$ gives the value of the "left" retarded time τ_L, which is in fact a complex scalar function on the (complex) space-time $\tau_L(x)$.

By using the null plane equation, one can get a retarded-advanced time equation

$$\tau = t \mp \tilde{r} + \vec{v}\vec{R}. \tag{51}$$

For the stationary Kerr solution $\tilde{r} = r + ia\cos\theta$, and one can see that the second root $Y_2(x)$ corresponds to a transfer to the negative sheet of the metric: $r \to -r; \quad \vec{R} \to -\vec{R}$, with a simultaneous complex conjugation $ia \to -ia$.

The analytical twistorial structure of the Kerr spinning particle leads to the appearance of an extra axial stringy system. As a result, the Kerr spinning particle acquires a simple stringy skeleton which is formed by a topological coupling of the Kerr circular string and the axial stringy system. The projective spinor coordinate Y is a projection of sphere on complex plane. It is singular at $\theta = \pi$, and such a singularity will be present in any holomorphic function $\psi(Y)$. Therefore, *all the aligned e.m. solutions turn out to be singular at some angular direction* θ. The simplest modes

$$\psi_n = qY^n \exp i\omega_n\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau) \quad (52)$$

can be numbered by index $n = \pm 1, \pm 2, ...$

The leading wave terms are

$$\mathcal{F}|_{wave} = f_R \, d\zeta \wedge du + f_L \, d\bar{\zeta} \wedge dv, \quad (53)$$

where

$$f_R = (AZ)_{,1}\,; \qquad f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1} \quad (54)$$

are the factors describing the "left" and "right" waves propagating along the z^- and z^+ semi-axis correspondingly.

The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values $\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia.$

The leading wave for $n = 1$,
$\mathcal{F}^-_1 = \frac{4qe^{i2\phi+i\omega_1\tau_-}}{\rho^2}\, d\bar{\zeta} \wedge dv$,
propagates to $z = -\infty$ along the z^- semi-axis.
The leading wave for $n = -1$,
$\mathcal{F}^+_{-1} = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du$,
is singular at z^+ semi-axis and propagates to $z = +\infty$.

The $n = \pm 1$ partial solutions represent asymptotically the singular plane-fronted e.m. waves propagating without damping.

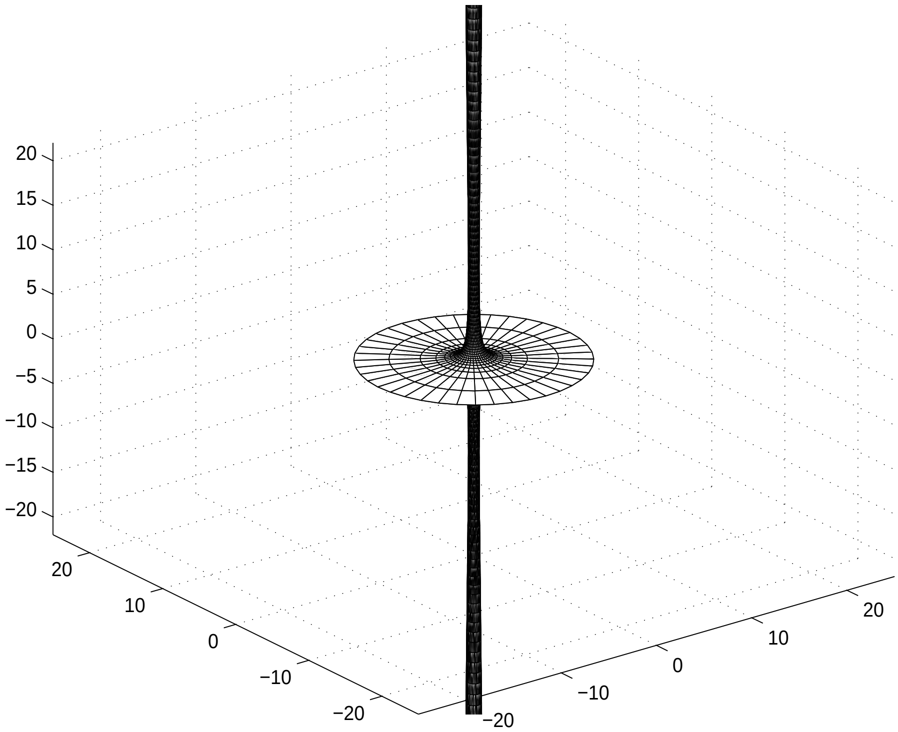

Figure 9: The Kerr disk-like source and two axial semi-infinite beams.

Wonderful Consequences of the Kerr Theorem

Kerr's multi-particle solution is obtained on the base of the Kerr theorem. Choosing generating function of the Kerr theorem F as a product of partial functions F_i for spinning particles i=1,...k, we obtain a multi-sheeted, multi-twistorial space-time over M^4 possessing unusual properties. Twistorial structures of the i-th and j-th particles do not feel each other, forming a type of its internal space. Gravitation and electromagnetic interaction of the particles occurs via a singular twistor line which is common for twistorial structures of interacting particles. The obtained multi-particle Kerr-Newman solution turns out to be 'dressed' by singular twistor lines linked to surrounding particles. We conjecture that this structure of space-time has the relation to a stringy structure of vacuum and opens a geometrical way to quantum gravity.

The Kerr theorem generating function $F(T^a)$ has to be at most quadratic in Y to provide singular lines to be confined in a restricted region, which corresponds to the Kerr PNC up to the Lorentz boosts, orientations of angular momenta and the shifts of origin.

Another form for this function is $F = (\lambda_1 - \lambda_1^0)\check{K}\lambda_2 - (\lambda_2 - \lambda_2^0)\check{K}\lambda_1$ which is related to the Newman-initiated complex representation of the Kerr geometry. In this case, function $F(Y)$ can be expressed via the set of parameters q which determine the motion and orientation of the Kerr spinning particle and takes the form

$$F(Y|q) = A(x|q)Y^2 + B(x|q)Y + C(x|q). \qquad (55)$$

This equations can be resolved explicitly, leading to two roots $Y = Y^{\pm}(x|q)$ which correspond to two sheets of the Kerr space-time. The root $Y^+(x)$ determines via (**??**) the out-going congruence on the (+)-sheet, while the root $Y^-(x)$ gives the in-going congruence on the (−)-sheet. Therefore, function F may be represented in the form $F(Y|q) = A(x|q)(Y - Y^+)(Y - Y^-)$, which allows one to obtain all the required functions of the Kerr solution in explicit form. The detailed form of $Y^{\pm}(x|q)$ is not important for our treatment.

Multi-twistorial space-time. Selecting an isolated i-th particle with parameters q_i, one can obtain the roots $Y_i^{\pm}(x)$ of the equation $F_i(Y|q_i) = 0$ and express F_i in the form

$$F_i(Y) = A_i(x)(Y - Y_i^{+})(Y - Y_i^{-}). \tag{56}$$

Then, substituting the $(+)$ or $(-)$ roots $Y_i^{\pm}(x)$ in the relation (**??**), one determines congruence $k_\mu^{(i)}(x)$ and consequently, the Kerr-Schild ansatz (**??**) for metric

$$g_{\mu\nu}^{(i)} = \eta_{\mu\nu} + 2h^{(i)} k_\mu^{(i)} k_\nu^{(i)}, \tag{57}$$

and finally, the function $h^{(i)}(x)$ may be expressed in terms of $\tilde{r}_i = -d_Y F_i$, (**??**), as follows

$$h^{(i)} = \frac{m}{2}\left(\frac{1}{\tilde{r}_i} + \frac{1}{\tilde{r}_i^*}\right) + \frac{e^2}{2|\tilde{r}_i|^2}. \tag{58}$$

Electromagnetic field is given by the vector potential

$$A_\mu^{(i)} = \Re e(e/\tilde{r}_i) k_\mu^{(i)}. \tag{59}$$

What happens if we have a system of k particles? One can form the function F as a product of the known blocks $F_i(Y)$,

$$F(Y) \equiv \prod_{i=1}^{k} F_i(Y). \tag{60}$$

The solution of the equation $F = 0$ acquires $2k$ roots $Y_i^{\pm}$, and the twistorial space turns out to be multi-sheeted.

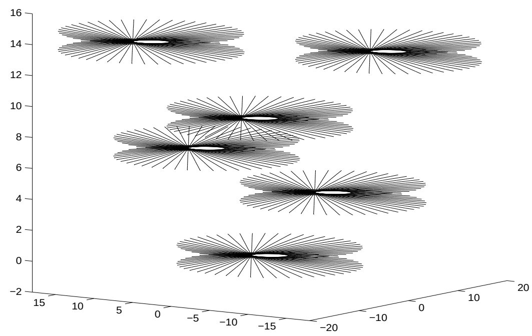

Figure 10: Multi-sheeted twistor space over the auxiliary Minkowski space-time of the multi-particle Kerr-Schild solution. Each particle has twofold structure.

The twistorial structure on the i-th $(+)$ or $(-)$ sheet is determined by the equation $F_i = 0$ and does not depend on the other functions $F_j, \quad j \neq i$. Therefore, the particle i does not feel the twistorial structures of other particles. Similar, the condition for singular lines $F = 0, \ d_Y F = 0$ acquires the form

$$\prod_{l=1}^{k} F_l = 0, \qquad \sum_{i=1}^{k} \prod_{l \neq i}^{k} F_l d_Y F_i = 0 \tag{61}$$

and splits into k independent relations

$$F_i = 0, \quad \prod_{l \neq i}^{k} F_l d_Y F_i = 0. \tag{62}$$

One sees, that i-th particle does not feel also singular lines of other particles. The space-time splits on the independent twistorial sheets, and therefore, the twistorial structure related to the i-th particle plays the role of its "internal space".

It looks wonderful. However, it is a direct generalization of the well known twofoldedness of the Kerr space-time which remains one of the mysteries of the Kerr solution for the very long time.

Multi-particle Kerr-Schild solution. Using the Kerr-Schild formalism with the considered above generating functions $\prod_{i=1}^k F_i(Y) = 0$, one can obtain the exact asymptotically flat multi-particle solutions of the Einstein-Maxwell field equations. Since congruences are independent on the different sheets, the congruence on the i-th sheet retains to be geodesic and shear-free, and one can use the standard Kerr-Schild algorithm of the paper [**?**]. One could expect that result for the i-th sheet will be in this case the same as the known solution for isolated particle. Unexpectedly, there appears a new feature having a very important consequence.

Formally, we have only to replace F_i by $F = \prod_{i=1}^k F_i(Y) = \mu_i F_i(Y)$, where $\mu_i = \prod_{j\neq i}^k F_j(Y)$ is a normalizing factor which takes into account the external particles. However, in accordance with (**??**) this factor μ_i will appear also in the function $\tilde{r} = -d_Y F = -\mu_i d_Y F_i$, and in the function $P = \mu_i P_i$.

So, we obtain the different result

$$h_i = \frac{m_i(Y)}{2\mu_i^3}(\frac{1}{\tilde{r}_i} + \frac{1}{\tilde{r}_i^*}) + \frac{(e/\mu_i)^2}{2|\tilde{r}_i|^2}, \tag{63}$$

$$A_\mu^{(i)} = \Re e \frac{e}{\mu_i \tilde{r}_i} k_\mu^{(i)} \tag{64}$$

which looks like a renormalization of the mass m and charge e.[2]

This fact turns out to be still more intriguing if we note that μ_i is not constant, but a function of Y_i. We can specify its form by using the known structure of blocks F_i

$$\mu_i(Y_i) = \prod_{j\neq i} A_j(x)(Y_i - Y_j^+)(Y_i - Y_j^-). \qquad (65)$$

The roots Y_i and $Y_j^{\pm}$ may coincide for some values of Y_i, which selects a common twistor for the sheets i and j. Assuming that we are on the i-th (+)-sheet, where congruence is out-going, this twistor line will also belong to the in-going ($-$)-sheet of the particle j . The metric and electromagnetic field will be singular along this twistor line, because of the pole $\mu_i \sim A(x)(Y_i^+ - Y_j^-)$. Therefore, interaction occurs along a light-like Schild string which is common for twistorial structures of both particles. The field structure of this string is similar to the well known structure of pp-wave solutions.

These equations give the *exact multi-particle solution of the Einstein-Maxwell field equations.* It follows from the fact that the equations were fully integrated out in [**?**] and expressed via functions P and Z before (without) concretization of the form of congruence, under the only

[2]Function $m_i(Y)$ is free and satisfies the condition $(m_i)_{,\bar{Y}} = 0$. It and has to be chosen in the form $m_i(Y) = m_0\mu_i^3$ to provide reality of metric.

condition that it is geodesic and shear free. In the same time the Kerr theorem determines the functions P and Z via generating function F, eq.(**??**), and the condition of reality for metric may be provided by a special choice of the free function $m(Y)$.

The obtained multi-particle solutions show us that, in addition to the usual Kerr-Newman solution for an isolated spinning particle, there is a series of the exact 'dressed' Kerr-Newman solutions which take into account surrounding particles and differ by the appearance of singular twistor strings connecting the selected particle to external particles. This is a new gravitational phenomena which points out on a probable stringy (twistorial) texture of vacuum and may open a geometrical way to quantum gravity.

The number of surrounding particles and number of blocks in the generating function F may be assumed countable. In this case the multi-sheeted twistorial space-time will possess the properties of the multi-particle Fock space.

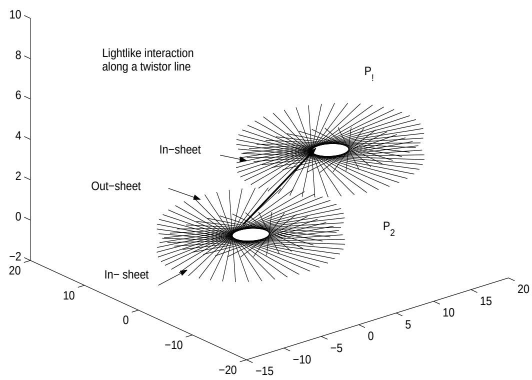

Figure 11: Schematic representation of the lightlike interaction via a common twistor line connecting out-sheet of one particle to in-sheet of another.