

Kerr-Schild geometry and twistorial structure of spinning particles.

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ZHETP 1974, Phys.Let B 1989, Phys.Rev.D
1995,1998,2002,2003...

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Spinning particles and twistors are traditional subjects of string theory and SQS-conference.

Traditional treatment - pointlike (or stringlike) objects in (super)space-time.

Kerr-Schild spinning particle is EXTENDED (soliton-like) space-time object.

Kerr-Newman solution has gyromagnetic ratio, $g = 2$, as that of the Dirac electron (Carter 1969).

Frame (skeleton) of the Kerr spinning particle is twistorial structure of the rotating black hole solution.

This twistorial frame determines structure of surrounding (super)fields: spinor, electromagnetic, gravitational, (axion and dilaton).

The Dirac theory of electron neglects gravity.
Is it small? NO!

Kerr geometry shows that gravity acts actively
in the Compton region!

Spin of electron $S = 1/2$ is very high with respect to mass $m \approx 10^{-22}$ (in the units $c = \hbar = G = 1$).

Gravitational field $\Rightarrow$ the **Kerr, or Kerr-Newman soln**, $S = ma$, where a is radius of the Kerr singular ring of the **Compton size**,

$$a = S/m = \hbar/2m \sim 10^{22} = 10^{-11} \text{ cm.}$$

Closed string of the Compton size. The field around the Kerr string is similar to the field around a heterotic string (A.B. PRD52,1995, PRD57,1998).

This string polarizes strongly electromagnetic field in the Compton region of electron.

Structure of the Kerr-Schild geometry.

The Kerr-Newman metric may be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} - 2Hk_{\mu}k_{\nu}, \quad (1)$$

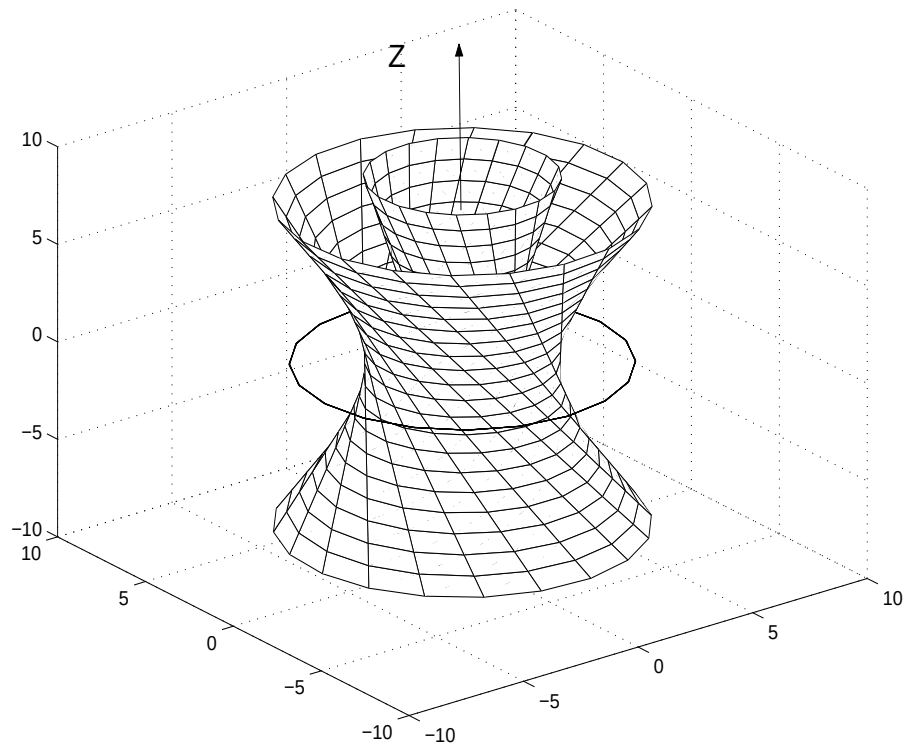
where $\eta_{\mu\nu}$ is auxiliary Minkowski metric and

$$H = \frac{mr - q^2/2}{r^2 + a^2 \cos^2 \theta},$$

So, metric is Minkowskian almost everywhere, for exclusion of a small neighborhood of the Kerr string.

Congruence k_{μ} is a twisting family of null rays, forming a vortex which is described by **the Kerr theorem** in twistor terms. It plays very important role, since the field k_{μ} **polarizes space-time** and determines not only the form of Kerr-Newman metric, but also the Kerr-Newman electromagnetic vector potential $A_{\mu} \sim k_{\mu}$, and the flow of radiation $T^{\mu\nu} \sim \Phi(x)k^{\mu}k^{\nu}$.

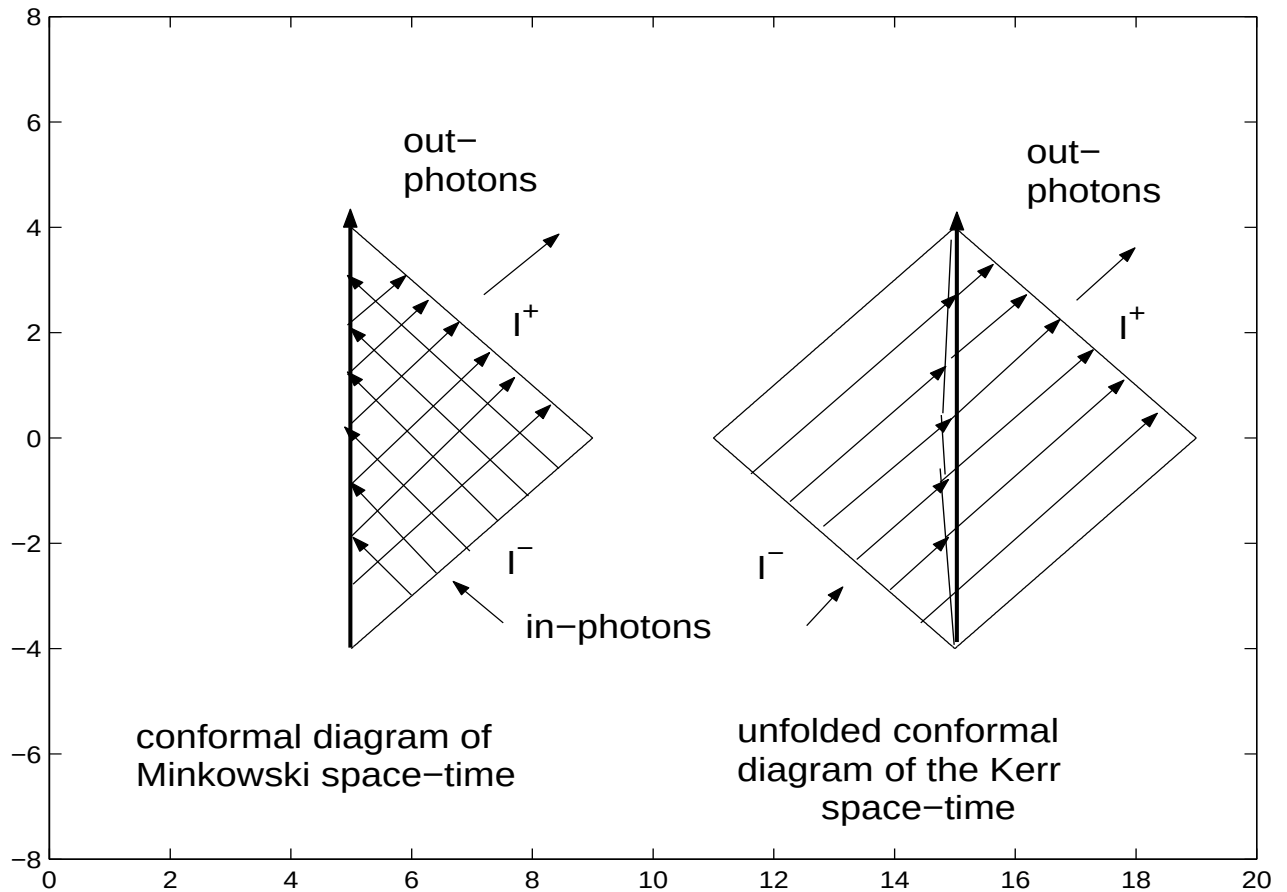
The Kerr congruence is vortex of null lines (twistors)



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The Kerr singular ring and the Kerr congruence.

The Kerr singular ring is a branch line of space on two sheets: "negative" and "positive" where the fields change their signs and directions. Congruence covers the space-time twice.



Twosheetedness of the Kerr space-time. The 'in' and 'out' electromagnetic fields are positioned on different sheets, they are **aligned to Kerr congruence** and don't interact with each other.

The Kerr string has very strong dragging effect leading to a very specific polarization of the electromagnetic field which has to be **aligned with the Kerr principal null congruence**,

$$k^\mu F_{\mu\nu} = 0! \quad (2)$$

As a result, the electromagnetic field of the corresponding Kerr-Newman solution $F_{\mu\nu}$ is singular in the Compton region !

The Question: “Why Quantum Theory does not feel such drastic changes in the structure of space time and fields on the Compton distances?”

Problem of the Kerr source. W.Israel (1970), A.B.(1974,1988,2000), M. Gürses and F.Gürsey (1975) V. Hamity (1976), A. Krasinski (1978), C.A. López(1984), ...

Alternatives:

a/ Alice string + twosheetedness,

b/ disklike source.

Compromise: **rotating disk of superconducting material.**

The Kerr ring represents a “mirror gates” in the “Alice mirror world” where the mass and charge have another signs and fields have different directions.

Similar twovaluedness appears in the models of the cosmic “Alice” strings which are connected with superconducting properties of the source. The “negative” sheet looks as a mirror image of the “positive” one.

The Kerr spinning particle:

Parameters of electron:

$$a \sim 10^{22}, \quad m \sim 10^{-22}, \quad ma = 1/2, \\ e^2 \approx 137^{-1}, \quad g = 2 \quad (G = c = \hbar = 1).$$

$a \gg m \Rightarrow$ no horizons.

Source - relativistically rotating disk:

Compton radius $a = \frac{\hbar}{2m}$ 'and

'Classical' thickness $r_e = \frac{e^2}{2m}$. Very thin $r_e/ae^2 \approx 137^{-1}$.

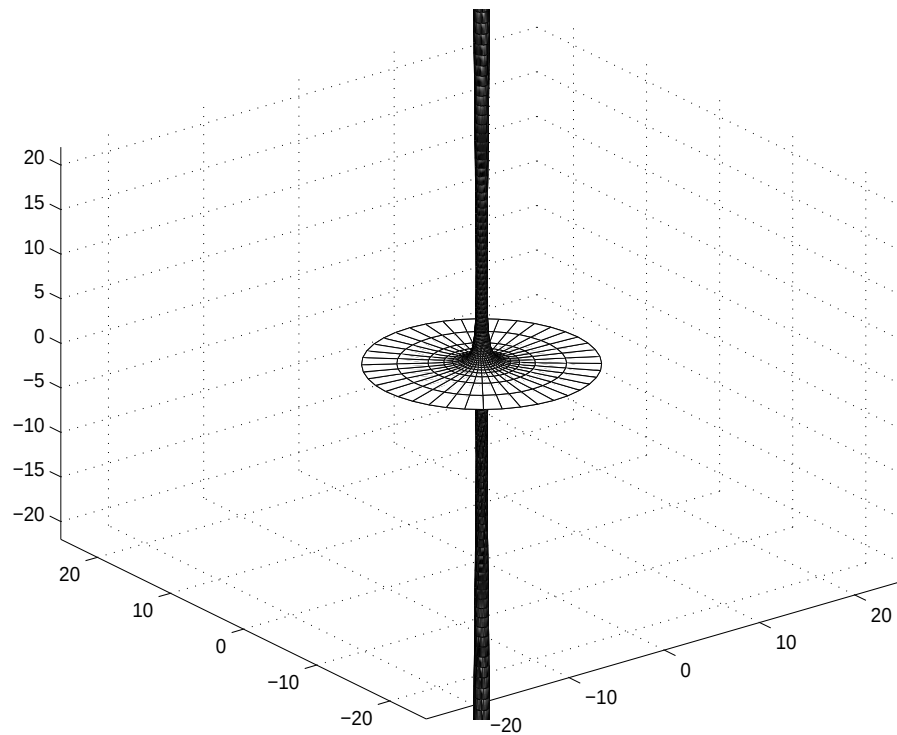
Wave excitations of the Kerr string $\Rightarrow$ the wave function.

The aligned wave excitations generate **light-like beams**. The simplest modes of excitations

$$\psi_n = qY^n \exp i\omega_n\tau \equiv q\left(\tan \frac{\theta}{2}\right)^n \exp i(n\phi + \omega_n\tau) \quad (3)$$

$n = \pm 1, \pm 2, \dots$ lead to extra 'axial' stringy system.

For $n = \pm 1$, the leading wave terms describe the “left” and “right” singular e.m. waves propagating along the z^- and z^+ semi-axis correspondingly. Asymptotically, they are pp-waves propagating without damping - exact solution of the Einstein-Maxwell field equations (A.Peres).



Disk-like source has classical thickness $r_{cl} = e^2/2m$ and Compton radius $\hbar/2m$. Two axial beams are modulated by de Broglie wave.

Complex Kerr source

Complex shift $(x, y, z) \rightarrow (x, y, z + ia)$ of the Coulomb solution $\phi = q/r$, (Appel, 1887 !)

$$\phi(x, y, z) = \Re e \ q/\tilde{r} \quad (4)$$

$\tilde{r} = \sqrt{x^2 + y^2 + (z - ia)^2}$ - complex radial coordinate.

It is exactly the Kerr-Newman source - *complex world-line* ${}^\mu(\tau)$.

$$\tilde{r} = r - ia \cos \theta, \quad (5)$$

r and θ are the oblate spheroidal coordinates.

The Kerr-Schild coordinates θ, ϕ, ρ fixe null rays in M^4 (twistors) which is parametrized by coordinate r . The complex retarded time is determined in analogy with the real one, but is based on the complex null cones.

Complex world-line is a complex half-string.

The left complex world line $X_L(\tau_L)$ is parametrized by complex parameter $\tau_L = \rho_L + i\sigma_L$ - world-sheet of a *complex string* which is extended in the complex time direction σ .

A fixed value of σ_L corresponds to the fixed value of $\cos \theta_L$, and selects a set of null ray of the Kerr congruence (twistors).

Since $\sigma \in [-a, a]$, this complex string is open with the ends at $\sigma = \pm ia$.

Two boundary points of the complex world line have the real tracks: two twistors –

axial semi-infinite half-strings: left and right.

Orientifold.

The left half-string $X_L(\tau_L)$ and right one $X_R(\tau_R)$ are joined by orientifold, forming one closed but folded string.

The interval $[-a, a]$ is covered by parameter σ twice: from left to right for left half-string, and with opposite orientation for the right half-string.

Generating function of the Kerr theorem $F(Y)$
(DKS 1969)

$$F \equiv \phi(Y) + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2 \quad (6)$$

where $\phi = a_0 + a_1Y + a_2Y^2$.

a_0, a_1, a_2 define orientation of angular momentum, while $p, q, \bar{q}, c$ determine Killing vector and function $P = pY\bar{Y} + qY + \bar{q}\bar{Y} + c$.

For the source at rest $p = c = 2^{-1/2}$, $q = \bar{q} = 0$ and $P = 2^{-1/2}(1 + Y\bar{Y})$.

The light-like solution with the boost oriented in z-direction $c = 1$, $p = q = \bar{q} = 0$, $P = 1$,

$$F = -iaY + (\zeta - Yv) . \quad (7)$$

The complex radial distance

$$\tilde{r} = -\partial_Y F = v + ia. \quad (8)$$

Solution of the equation $F = 0$ yields

$$Y = \zeta/(ia + v) , \quad (9)$$

and function $Y(x)$ determines the principal null congruence e^3 .

$$Z = \theta + i\omega = -\tilde{r}^{-1} = (v - ia)/[v^2 + a^2] . \quad (10)$$

Expansion θ is maximal at $v^2 = a^2$ and tends to zero near the front plane $v = 0$, where the twist ω is maximal. In the vicinity of the axis z and far from the front plane, congruence tends to simple form $e^3 = du$ corresponding to pp-wave solutions.

For spinning solutions, ($a \neq 0$) the front surface $v = 0$ is smooth.

As a result, metric is determined by KS ansatz $g^{\mu\nu} = \eta^{\mu\nu} + h k^\mu k^\nu$ with function h given by

$$h = [mv - A(Y)\bar{A}(\bar{Y})]/(v^2 + a^2). \quad (11)$$

Electromagnetic field is given by

$$\mathcal{F}_{12} = A/[v^2 + a^2]; \quad \mathcal{F}_{31} = -A'_Y(Y)/[v^2 + a^2]. \quad (12)$$

In the case $A = \text{const.}$ this solution is *the spinning light-like limit of the Kerr-Newman solution*. The solutions with $A = Y^n$, $n = -1, -2, \dots$ are singular by $Y \rightarrow 0$. Similar to

‘gyrons’, they represent the spinning light-like beams with singular strings positioned along the z -axis. Near this string, $Y \rightarrow 0$, the Kerr congruence takes the constant direction $e^3 = du$, and solution tends to the Peres pp-wave solutions.

The degree of function F changes by jump in the ultrarelativistic limit. Position of the “negative” sheet of the Kerr solution corresponds to the negative values of the radial distance $Re \tilde{r} = 2^{-1/2}(z - t)$.

The “positive” and “negative” Kerr’s sheets are placed on the different half-spaces divided by the front-surface $z = t$. The “negative” sheet becomes the sheet of advanced fields placing before the front of solution.

To keep during the limit the constant relativistic value of spin J , one has to set a constant

value for parameter $a = a_0 \sqrt{1 - (v/c)^2}$. Therefore, a_0 has to tend to ∞ by $v \rightarrow c$.

In the light-like limit the Kerr singular ring turns out to be shifted to infinity.

The parameters m and a in the light-like solutions correspond to their relativistic values, i.e. $m = E$ and $a = J/m$.

It yields $J = ma \Rightarrow E = J/a$, and setting $J = \hbar$, one obtains $E = \hbar/a$, therefore

a corresponds to de Broglie wavelength of the photon, i.e.

$$J = ma = \hbar \Rightarrow E = \hbar\nu.$$

This relativistic parameter a determines complex shift of radial distance $\tilde{r} = v + ia$ and controls twist of the null congruence.

Conclusions

The Kerr geometry may be hidden beyond the Quantum Theory of spinning particles!

Spinning particles have extended twistorial structure of the Kerr-Schild geometry.

The Kerr closed string and twistorial polarization of the Kerr-Schild space-time determines Quantum Processes in Compton region.

The Dirac equation may be considered as a master equation controlling the motion of the extended twistorial structure of electron, hep-th/0507109.

The Kerr-Schild metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he_{\mu}^3e_{\nu}^3, \quad \sqrt{-g} = 1. \quad (13)$$

$\eta_{\mu\nu}$ - auxiliary Minkowski space-time.

Principal null direction

$$e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv \quad (14)$$

is expressed via complex function $Y(x)$ in the null Cartesian coordinates $\sqrt{2}\zeta = x+iy$, $\sqrt{2}\bar{\zeta} = x - iy$, $\sqrt{2}u = z - t$, $\sqrt{2}v = z + t$.

Null tetrad $e^a, a = 1, 2, 3, 4$. Real directions $e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv$ and $e^4 = dv + he^3$, and two complex conjugate directions

$$e^1 = d\zeta - Ydv, \quad e^2 = d\bar{\zeta} - \bar{Y}dv.$$

The Kerr theorem.

The geodesic and shearfree (GSF) null congruences $e^3(Y)$ satisfy the conditions

$$Y_{,2} = Y_{,4} = 0.$$

GSF congruences are determined by $Y(x)$ which is a solution of the equation

$$F(Y, l_1, l_2) = 0 , \quad (15)$$

where F is an arbitrary analytic function of the projective twistor coordinates

$$Y, \quad l_1 = \zeta - Yv, \quad l_2 = u + Y\bar{\zeta} . \quad (16)$$

Integration of the Einstein-Maxwell field equations for the geodesic and shear-free congruences was fulfilled in DKS, leading to the following form of the function

$$h = \frac{1}{2}M(Z + \bar{Z}) - \frac{1}{2}A\bar{A}Z\bar{Z} , \quad (17)$$

of the Kerr-Schild ansatz:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he_{\mu}^3e_{\nu}^3. \quad (18)$$

Necessary functions $Z = P/\tilde{r}$, $Y(x)$ and parameters are determined by the generating function F .

$$PZ^{-1} = \tilde{r} = - \quad dF/dY \quad (19)$$

is complex radial distance, factor P is connected with Killing vector or the boost of the source.

There was obtained a system of differential equations for functions A , and M .

Electromagnetic sector:

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3} A = 0, \quad A_{,4} = 0, \quad (20)$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0, \quad \gamma_{,4} = 0, \quad (21)$$

where $\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2$.

The strength tensor of self-dual electromagnetic field is given by the tetrad components

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1} . \quad (22)$$

Gravitational sector:

Real function M .

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3} M = A\bar{\gamma}\bar{Z}, \quad (23)$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad M_{,4} = 0. \quad (24)$$

*For any holomorphic $F(Y) \Rightarrow$ GSF congruence
 $\Rightarrow$ algebraically special solution.*

Final integration of eqs. I and II was given only for $\gamma = 0$ and quadratic in Y function $F(Y)$, (Debney, Kerr and Schild 1969) $\Rightarrow$ a broad class of exact solutions containing **Kerr-Newman solution** as a very important particular case:

Metric $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_{\mu}k_{\nu}$, where $h = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}$.

Electromagnetic field $A_{\mu} = \frac{er}{r^2 + a^2 \cos^2 \theta} k_{\mu}$.

It is remarkable simple form. All the complications are included in the form of the the **Kerr congruence** $k_{\mu}(x) = e_{\mu}^3(Y)\sqrt{2}/(1 + Y\bar{Y})$ which is determined by function $Y(x)$ (solution of the eq. $F(Y, x) = 0$).

Singularity is by $r + ia \cos \theta \equiv \partial_Y F = 0 \Rightarrow r = \cos \theta = 0$.

The simplest class of the exact stationary Kerr-Schild solutions.

Electromagnetic field is **aligned with the Kerr congruence**:

$$F^{\mu\nu}k_{\mu} = 0. \quad (25)$$

Kerr -Schild metric $g^{\mu\nu} = \eta^{\mu\nu} - 2hk^{\mu}k^{\nu}$, where

$$h = m(Z + \bar{Z})/(2P^3) - A\bar{A}Z\bar{Z}/2. \quad (26)$$

Stationary case, $P = 2^{-1/2}(1 + Y\bar{Y})$ and A has the general form

$$A = \psi(Y)/P^2, \quad (27)$$

and ψ is an arbitrary holomorphic function of Y .

Kerr-Newman solution is very particular case:
 $\psi(Y) = e = \text{const.}$

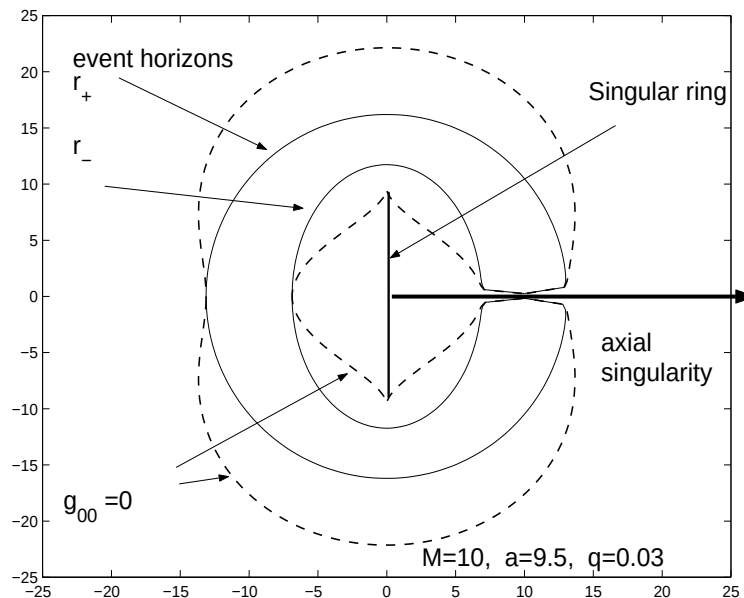
In general case function $Y(x) = e^{i\phi} \tan \frac{\theta}{2} \in CP^1$ is coordinate on projective sphere, and there is an infinite set of the *exact solutions*, in which function $\psi(Y)$ is holomorphic on the punctured sphere at the set of points $\{Y_i, i = 1, 2, \dots\}$, $\psi(Y) = \sum_i \frac{q_i}{Y(x) - Y_i}$.

In these solutions $\psi(Y)$ is singular at a set of angular directions ϕ_i, θ_i , and there appear semi-infinite lightlike beams, (singular pp-strings) along some of the null rays of the Kerr congruence. **How act such beams on the BH horizon?**

Singular beams lead to formation of the holes in the black hole horizon, which opens up the interior of the “black hole” to external space.

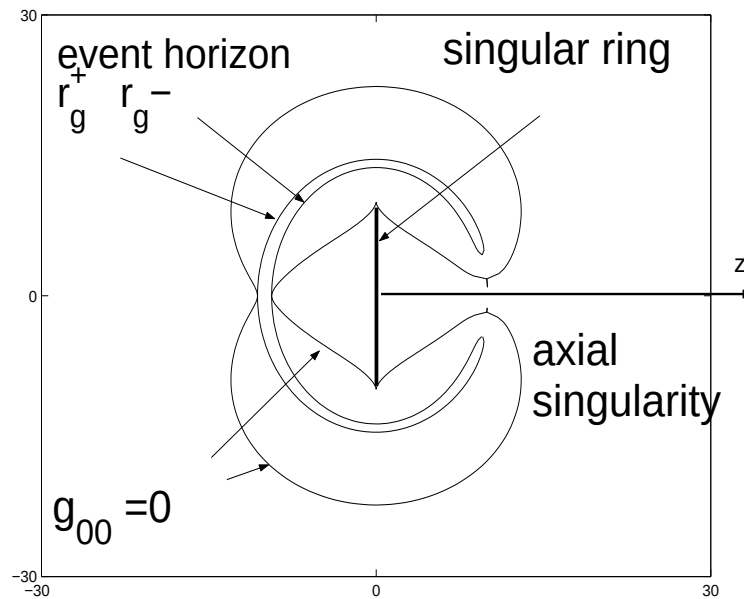
Black holes with holes in the horizon

A.B., E.Elizalde, S.R.Hildebrandt and G.Magli,
Phys. Rev. **D74** (2006) 021502(R)



Singular beam forms a small hole in the horizon.

The boundaries of **ergosphere** (punctured) are determined by $g_{00} = 0$. The **event horizons**, r_+ and r_- , are null surfaces obeying $(\partial_r S)^2 \{r^2 + a^2 + (q/\tan \frac{\theta}{2})^2 - 2Mr\} - (\partial_\theta S)^2 = 0$. They are joined by a tunnel, by forming a simply connected surface.



Near extremal black hole with a hole in the horizon, caused by a lightlike singular beam. The r^+ and r^- surfaces are joined, by forming a simply connected surface.

Wave and nonstationary electromagnetic excitations.

Exact *self-consistent* solutions with $\gamma \neq 0$ are absent, however, EM eqs. may be solved. (Eq.I) $A = \psi/P^2$, where $\psi_{,2} = \psi_{,4} = 0$, but now $\psi = \psi(Y, \tau)$, where τ is complex retarded time obeying $\tau_{,2} = \tau_{,4} = 0$.

(Eq.II) $DA + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0$,
where $D = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2$.

Integration yields

$$\gamma = \frac{2^{1/2}\dot{\psi}}{P^2Y} + \phi(Y, \tau)/P, \quad (28)$$

By any nonstationarity, $\dot{\psi} \neq 0$, there appears a pole in ψ or γ , and there appears inevitably a singular beam.

It was shown that for the slowly varying EM field $\gamma \rightarrow 0$, and the approximate KS solutions containing singular beams tend to the corresponding exact stationary Kerr-Schild solutions.

The real structure of the Kerr-Newman solution.

The Kerr-Newman metric may be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} - 2Hk_{\mu}k_{\nu}, \quad (29)$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski metric and

$$H = \frac{mr - q^2/2}{r^2 + a^2 \cos^2 \theta},$$

one sees that metric is Minkowskian almost everywhere, for exclusion of a small neighborhood of the Kerr string. However, this string has very strong dragging effect which polarizes space-time leading to a very specific polarization of the electromagnetic fields. As a result, the electromagnetic field of the corresponding Kerr-Newman solution $F_{\mu\nu}$, which cannot be considered as a weak one for parameters of charged particles.

It is aligned with the Kerr principal null congruence $k_\mu(x)$, $k^\mu F_{\mu\nu} = 0$ (!) The explicit form of the field k_μ is determined by the Kerr theorem.

It is a twisting family of null rays, fig.1, forming a vortex which is described by the Kerr theorem in twistor terms. *

PNC plays very important role, since the field k_μ determines not only form of Kerr-Newman metric, but also the Kerr-Newman electromagnetic vector potential $A_\mu = \frac{qr}{r^2 + a^2 \cos^2 \theta}$ and the flow of radiation (in the radiative rotating solutions).

There appears the Question: "Why Quantum Theory does not feel such drastic changes in

*Complicate form of the field $k_\mu(x)$ determines the complicate form of the Kerr metric, contrary to the extremely simple Kerr-Schild form of metric.

the structure of space time on the Compton distances?"

How can such drastic changes in the structure of space-time and electromagnetic field be experimentally unobservable and theoretically ignorable in QED?

The negative sheet of Kerr geometry may be truncated along the disk $r = 0$. In this case, inserting the truncated space-time into the Einstein-Maxwell equation, one obtains on the ‘right’ side of the equations the source with a disk-like support. This source has a specific matter with superconducting properties $[0, 0]$. The ‘negative’ sheet of space appears now as a mirror image of the positive one, so the Kerr singular ring is an ‘Alice’ string related to the mirror world. Such a modification changes interpretation, but does not simplify problem, since it means that Quantum Theory does not feel this ‘giant’ mirror of the Compton size, while the virtual charges have to be very sensitive to it.[†]

The assumption, that QED has to be corrected taking into account the peculiarities of

[†]Note, that this disk is relativistically rotating and has a thickness of the order of classical size of electron, $r_e = e^2/2m, [0, 0]$.

the space-time caused by the Kerr geometry, may not be considered as reasonable because of the extraordinary exactness of the QED.

There is apparently the unique way to resolve this contradiction: to conjecture that the Kerr geometry is hidden beyond the Quantum Theory, i.e. is already taken into account and play there essential role.

From this point of view there is no need to quantize gravity, since the Kerr geometry may be the source of some quantum properties, i.e. may be primary with respect to the Quantum Theory.

Microgeon with spin.

Let us consider the Wheeler's model of **Mass Without Mass – 'Geon'**. The photons are moving along the ring-like orbits, being bound

by the own gravitational field. Such field configuration may generate the particle-like object having the mass and angular momentum. Could such construction be realized with an unique photon? In general, of course - not, because of the weakness of gravitational field. However, similar idea on 'mass without mass' is realized in the theory of massless relativistic strings and may be realized due to the stringy properties of the Kerr solution with $a \gg m$. In the Kerr geometry, one can excite the Kerr circular string by an electromagnetic field propagating along this singular string as along of a waveguide. Electromagnetic excitations of the Kerr source with $a \gg m$ has the stringy structure, and leads to a contribution to the mass and spin. In particular, the model of microgeon with spin turns out to be self-consistent [0, 0, 0].

Analysis of the exact *aligned* electromagnetic excitations on the Kerr background shows an

unexpected peculiarity $[0, 0]$: the inevitable appearance of two axial singular semi-infinite half-strings of opposite chiralities. There appears the following stringy skeleton of a spinning particle, fig. 2.

[ht]

Skeleton of the Kerr Spinning Particle.

The spin of this microgeon may be interpreted as excitation of the Kerr string by a photon moving along a circular orbit, which is reminiscent of the electron self-energy diagram in QED.

In the Kerr's gravity, the virtual photon line of this diagram does not leave the Compton region of the particle due to the Kerr stringy

waveguide. As it was shown in [0], the axial half-strings are the null-strings (the Schild, or the pp-wave strings) and may be described by the bilinear spinor combinations formed from the solutions of the Dirac equation.[‡]

Moreover, there is a wonderful fact, that the basic quantum relation $E = \hbar\omega$ is already contained in the basic relation of the Kerr geometry $J = ma$, (). Indeed, setting $J = \frac{\hbar}{2}$, one writes () as $a = \frac{\hbar}{2m}$.

So far, we considered the constant $\hbar$ not as a quantum constant, but as an experimentally constant characterizing the spin of electron. Let us consider now the classical fields propagating along the Kerr ring with speed of the light and with the winding number of phase

[‡]The axial and circular singularities form a specific multsheeted topology of space-time, which admits the spinor two-valuedness.

$n = 1/2$. The corresponding length of wave will be $\lambda = 2\pi a/n = 2\pi\hbar/m$ and the corresponding frequency $\omega = 2\pi c/\lambda = cm/\hbar$. It yields

$$\frac{E}{c} \equiv mc = \hbar\omega. \quad (30)$$

Up to now, we have not used the quantum operators at all. We have used only the topological properties providing the two-valued representations by rotations, or the classical quantization of phase (winding number). As a result, we have obtained the quantum relation (28) from the classical Kerr relation $J = ma$:

$$J = ma \quad \Rightarrow \quad E = \hbar\omega. \quad (31)$$

It suggests that *'Kerr's geometry' may cause the origin of Quantum properties,*

Dirac equation and the complex Kerr geometry.

Dirac equation in the Weyl basis splits:

$$\sigma_{\alpha\dot{\alpha}}^{\mu}(i\partial_{\mu}+eA_{\mu})\chi^{\dot{\alpha}}=m\phi_{\alpha}, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}(i\partial_{\mu}+eA_{\mu})\phi_{\alpha}=m\chi^{\dot{\alpha}}$$

and the Dirac spinors form a null tetrad. The null vectors $k_L^{\mu} = \bar{\chi}\sigma^{\mu}\chi$ $k_R^{\mu} = \bar{\phi}\bar{\sigma}^{\mu}\phi$, characterize polarization of a free electron in the state with a definite projection of angular momentum on axis z , [0]. Two others null vectors $m^{\mu} = \phi\sigma^{\mu}\chi$, and $\bar{m}^{\mu} = (\phi\sigma^{\mu}\chi)^+$ are controlled by the phase of wave function and set a synchronization of the null tetrad in the surrounding space-time, playing the role of an order parameter which carries the de Broglie wave.

It is well known [?] that the Kerr-Newman solution has the same gyromagnetic ratio ($g = 2$), as that of the Dirac electron. There appears a natural question: is it an incident or there is a deep relationship between the Dirac equation and the Kerr-Newman geometry? This problem is related to the problem of description of

electron in coordinate representation, and to the problem of localized states in the Dirac theory [0, ?].

Problem of the coordinate description

It is known that operator of coordinate $\hat{x} = \nabla_{\vec{p}}$ is not Hermitean in any relativistic theory, $(\Psi, \hat{x}\Phi) \neq (\hat{x}\Psi, \Phi)$.

It suggests that coordinate of electron may be complex. In the terms of the null vectors $k_L = (1, \vec{k}_L) = (1, 0, 0, 1)$ and $k_R = (1, \vec{k}_R) = (1, 0, 0, -1)$, it takes the form

$$(\bar{\Psi} \hat{X} \Psi) = x + ia(k_L + k_R), \quad (32)$$

where x is a center of mass and $a = \frac{\hbar c}{2m}$ is the Compton length. In the Weil representation, the vectors k_L and k_R transform independently by Lorentz transformations and transfer to each other by the space reflection (inversion) $P = \eta_P \gamma_4$, $|\eta_P| = 1$. It gives a hint

that the Dirac particle may be formed by two complex point-like particles $X = \frac{1}{2}(X_L + X_R)$ propagating along the complex world-lines

$$X_L^\mu(t) = x^\mu(t) + ia(1, 0, 0, 1) \quad X_R^\mu(t) = x^\mu(t) + ia(1, 0, 0, -1) \quad (33)$$

Such a representation turns out to be close related to the complex representation of the Kerr geometry $[0, 0, 0, 0, 0]$.

Complex representation of the Kerr geometry

In 1887 (!) Appel [0] consider a simple complex transformation of the Coulomb solution $\phi = q/r$, a complex shift $(x, y, z) \rightarrow (x, y, z + ia)$ of the origin $(x_0, y_0, z_0) = (0, 0, 0)$ to the point $(0, 0, ia)$. On the real section $(\text{real}(x, y, z))$, the resulting solution

$$\phi(x, y, z) = \Re q/\tilde{r} \quad (34)$$

acquires a complex radial coordinate $\tilde{r} = \sqrt{x^2 + y^2 + z^2}$.
Representing $\tilde{r}$ in the form

$$\tilde{r} = r - ia \cos \theta \quad (35)$$

one obtains for $\tilde{r}^2$

$$r^2 - a^2 \cos^2 \theta - 2iar \cos \theta = x^2 + y^2 + z^2 - a^2 - 2iaz. \quad (36)$$

Imaginary part of this equation gives $z = r \cos \theta$, which may be substituted back in the real part of (34). It leads to the equation $x^2 + y^2 = (r^2 + a^2) \sin^2 \theta$, which may be split into two conjugate equations $x \pm iy = (r \pm ia)e^{\pm i\phi} \sin \theta$. Therefore, we obtain the transfer from the complex coordinate $\tilde{r}$ to the Kerr-Schild coordinate system

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi} \sin \theta, \\ z &= r \cos \theta, \\ t &= r + \rho. \end{aligned} \quad (37)$$

Here r and θ are the oblate spheroidal coordinates, and the last relation is a definition of the real retarded-time coordinate ρ . The Kerr-Schild coordinates θ, ϕ, ρ fixe a null ray in M^4 (twistor) which is parametrized by coordinate r .

One sees, that after complex shift, the singular point-like source of the Coulomb solution turns into a singular ring corresponding to $\tilde{r} = 0$, or $r = \cos \theta = 0$. This ring has radius a and lies in the plane $z = 0$. The space-time is foliated on the null congruence of twistor lines, shown on fig. 1. It is twofolded having the ring-like singularity as the branch line. Therefore, for the each real point $(t, x, y, z) \in \mathbb{M}^4$ we have two points, one of them is lying on the positive sheet of space, corresponding to $r > 0$, and another one lies on the negative sheet, where $r < 0$.

It was obtained that the Appel potential corresponds exactly to electromagnetic field of the Kerr-Newman solution written on the auxiliary Minkowski space of the Kerr-Schild metric (27), [0]. The vector of complex shift $\vec{a} = (a_x, a_y, a_z)$ corresponds to direction of the angular momentum J of the Kerr solution, and $|a| = J/m$.

Newman and Lind [0] suggested a description of the Kerr-Newman geometry in the form of a retarded-time construction, in which it is generated by a complex source, propagating along a *complex world line* ${}^\mu(\tau)$ in a complexified Minkowski space-time $\mathbf{CM}^4$. The rigorous description of this representation was given in the Kerr-Schild approach [0] based on the Kerr theorem and the Kerr-Schild form of metric (27)[§] The complex retarded time is determined

[§]It is related to the existence of auxiliary Minkowski metric $\eta^{\mu\nu}$, [0, 0].

in analogy with the real one, but is to be based on the complex null cones $[0, 0, 0]$.

Let's consider the complex radial distance from a real point x to a complex point X_L of the 'left' complex world-line

$$\tilde{r}_L = \sqrt{(\vec{x} - \vec{X}_L)^2} = r_L - ia \cos \theta_L. \quad (38)$$

To determine a retarded-time parameter τ_L one has to write down the light-cone equation $ds^2 = 0$, or

$$\tilde{r}_L^2 - (t - \tau_L)^2 = 0 \quad (39)$$

It may be split into two retarded-advanced-time equations $t - \tau_L = \pm \tilde{r}_L$. The retarded-time equation corresponds to the sign $+$ and, due to (33), leads to relation

$$\tau_L = t - r_L + ia \cos \theta_L. \quad (40)$$

One sees that τ_L turns out to be complex

$$\tau_L = \rho_L + i\sigma_L, \quad \sigma_L = a \cos \theta_L. \quad (41)$$

The complex worldline as a string

In the complex retarded-time construction, the left complex world line $X_L(\tau_L)$ has to be parametrized by complex parameter $\tau_L = \rho_L + i\sigma_L$. It has a few important consequences.

i/ Being parametrized by two parameters ρ and σ , the complex world-line is really a world-sheet and corresponds to a *complex string*. This string is very specific, since it is extended in the complex time direction σ .

ii/ A fixed value of σ_L corresponds to the fixed value of $\cos \theta_L$, and, in accordance with (36),

together with the fixed parameter ϕ , it selects a null ray of the Kerr congruence (twistor).

iii/ Since $|\cos \theta| \leq 1$, parameter σ is restricted by interval $\sigma \in [-a, a]$, i.e. complex string is open and the points $\rho \pm ia$ are positioned at its ends. The world-sheet represents an infinite strip: $(t, \sigma) : -\infty < t < \infty, \sigma \in [-a, a]$.

iv/ From (31) and (38) one sees that the left complex point of the Dirac x-coordinate $X_L = ia(1, 0, 0, 1)$ has $\Im m \tau_L = ia \cos \theta_L$, which yields $\cos \theta_L = 1$.

Therefore, this is the boundary point of the complex world line and coordinate relations (36) show that the family of complex light cones positioned at this boundary have the real tracks along the axial null line $z = r, \quad x = y = 0$.

Similar treatment for the right complex point of the Dirac x-coordinate $X_R = ia(1, 0, 0, -1)$

show that it is also placed on the same boundary of the stringy strip (the same timelike component ia), however, $\Im m \tau_R = -ia \cos \theta$, which yields $\cos \theta_R = -1$ and corresponds to the axial null line propagating in opposite direction $z = -r, \quad x = y = 0$.

Therefore, two complex sources of the Dirac operator of coordinate have the real image in the real space-time in the form of the considered above two axial semi-infinite half-strings: left and right.[¶]

Note, that there is an asymmetry in the complex left and right coordinates $X_L = ia(1, 0, 0, 1)$ and $X_R = ia(1, 0, 0, -1)$. The time-like components of the both sources are adjoined to the same right end of the complex string interval $[-ia, ia]$. This asymmetry is removed by

[¶]For more details see [0, 0, 0].

a remarkable stringy construction - orientifold $[0, 0, 0]$.

Orientifold.

The models of relativistic strings contains usually two stringy modes: left and right. So, the modes $X_L(\tau_L)$ and $X_R(\tau_R)$ represent only the half-strings on the interval $\sigma \in [-a, a]$. Orientifold is formed from two open half-strings which are joined forming one closed string, and this closed string is to be folded one. The interval $[-a, a]$ is covered by parameter σ twice: the first time from left to right, and, say the left half-string has the usual parametrization. While the interval $[-a, a]$ is reversed and covers the original one in opposite orientation for the right half-string. Therefore, the parameter σ covers interval twice and string turns out to be closed, but folded. The right and left

string modes are flipping between the the initiate and the reversed intervals. One sees that for the complex interval the revers is equivalent to complex conjugation of the parameter τ . So, one has to put $\tau_R = \bar{\tau}_L$.^{||} After orientifolding, the complex timelike coordinates of the points X_L and $\bar{X}_R$ turns out to be sitting on the opposite ends of the interval $[-a, a]$, while their imaginary space-like coordinates will be coinciding, which corresponds to one of the necessary orientifold condition $X_L(\tau_L) = \bar{X}_R(\bar{\tau}_R)$.

Conclusion

The above treatment shows that the electron may possess the nontrivial real and complex structures which are related to the real and complex structures of the Kerr geometry. The Dirac equation works apparently in the complex Minkowski space-time, and the space-time

^{||}Details of this construction may be found in [?, 0, 0, 0].

source of the naked electron is not elementary, but represents a specific complex string with two quark-like sources sitting on the ends of this string. While, after orientifolding this string, the space coordinates of these sources are merging, turning into a complex point shifted in the imaginary direction on the Compton distance a . This complex position of the source is, apparently, the origin of the problems with localized states and with the operator of coordinate in the Dirac theory.

The obtained recently multiparticle Kerr-Schild solutions [0] shed some light on the multiparticle structure of the dressed electron considered in QED. This treatment is based on the remarkable properties of the Kerr theorem. There is also remarkable renormalization of the Kerr singularity by gravitational field [0]. However, these questions go out of the frame of this paper.

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