Quantum gravity model with fundamental spinor fields

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Outline

Introduction

- Outstanding problem of modern physics: *status of quantum gravity*
- Quantized Einstein's theory is non-renormalizable
- Maybe metric (tetrad, connection) is not fundamental?
- Long history of attempts to construct *gravity models* from more fundamental constituents: to mention but a few
- De Broglie-Vigier "fusion" theory: particles of higher spins arise as union (fusion) of spin $1/2$ particles [L. De Broglie, *Introduction to the Vigier theory of elementary particles* (Elsevier, 1963)]
- Nonlinear spinor theory of Heisenberg [W. Heisenberg, *Einführung in die einheitliche Feldtheorie der Elementarteilchen* (Hirzel, 1967)]

- *Pregeometry* models
  Terazawa (1980): metric from scalar fields

$$g_{ij} = \partial_i \Phi \, \partial_j \Phi$$

  Akama (1978): tetrad from fermion fields

$$e_i^\alpha = \frac{i}{2} \left(\overline{\Psi} \gamma^\alpha \partial_i \Psi - \partial_i \overline{\Psi} \gamma^\alpha \Psi \right)$$

- Most recent: Diakonov model [D. Diakonov, *Towards lattice-regularized quantum gravity*, arXiv: 1109.0091; Talk at Köln Colloquium (9 December 2011)]
- Advantages of Diakonov gravity model:
- Gauge-theoretic (a la Yang-Mills) approach to gravity
- Explicit Lorentz invariance (improving Akama)
- Our main aim: understanding Diakonov's model using tools of modern differential geometry and gauge theory; extending model also to connection

Gauge approach to gravity

- [F.W. Hehl, et al, Metric-affine gauge theory of gravity: Field equations, Noether identities, world spinors, and breaking of dilation invariance, Phys. Repts. 258 (1995) 1; Yu. N. Obukhov, Poincaré gauge gravity: selected topics, Int. J. Geom. Meth. Mod. Phys. 3 (2006) 95; V. N. Ponomarev, A.O. Barvinsky, Yu. N. Obukhov, Geometodynamical methods and gauge approach in the theory of gravitational interactions (Moscow, 1985)]
- Gravity described by orthonormal *coframe* (tetrad) and Lorentz connection

$$\vartheta^\alpha = e_i{}^\alpha dx^i, \qquad \Gamma^{\alpha\beta} = \Gamma_i{}^{\alpha\beta} dx^i = -\Gamma^{\beta\alpha}$$

  (*translational* potential and *Lorentz* potential, Kibble 1961)
- Field strengths: torsion and curvature

$$T^\alpha = D\vartheta^\alpha, \qquad R^{\alpha\beta} = -R^{\beta\alpha} = \text{``}D\Gamma^{\alpha\beta}\text{''}$$

Diakonov's model of gravity

- Original field variables of Diakonov's theory are

$$(\vartheta^\alpha, \Gamma^{\alpha\beta}, \psi, \overline{\psi})$$

- Fermion field ψ is treated as *fundamental*
- Coframe arises as $\vartheta^\alpha = \vartheta^\alpha(\psi, \overline{\psi})$:

$$\ell^{-4}\vartheta^\alpha = \frac{i}{2}\left(\overline{\psi}\gamma^\alpha D\psi - D\overline{\psi}\gamma^\alpha\psi\right)$$

- In Poincaré gauge gravity framework, *Diakonov's Lagrangian* reads (with $\eta_{\alpha\beta} = {}^\star(\vartheta_{\alpha\beta})$, $\eta = {}^\star 1$)

$$L = V + L_{\rm mat} = -\frac{1}{2\kappa}\left(a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\Lambda\,\eta\right) + L_{\rm mat}$$

- "Matter" Lagrangian is a constraint

$$L_{\rm mat} := \varphi^\alpha \wedge \lambda_\alpha, \qquad \varphi^\alpha = \frac{i}{2}\left(\overline{\psi}\gamma^\alpha D\psi - D\overline{\psi}\gamma^\alpha\psi\right) - \ell^{-4}\vartheta^\alpha$$

- Diakonov is left with variables $\Gamma^{\alpha\beta}, \psi, \overline{\psi}$ and λ_α
- Field equations (derived for the first time)

$$\begin{aligned}
\ell^{-4}\lambda_\alpha &= -\frac{a_0}{2\kappa}\, R^{\rho\sigma}\wedge\eta_{\alpha\rho\sigma} + \frac{\Lambda}{\kappa}\,\eta_\alpha,\\
\frac{a_0}{2\kappa}\, T^\rho\wedge\eta_{\alpha\beta\rho} &= -\frac{1}{8}\,\overline{\psi}(\gamma^\rho\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\gamma^\rho)\psi\,\lambda_\rho,\\
i\gamma^\alpha\lambda_\alpha\wedge D\psi &- \frac{i}{2}\gamma^\alpha(D\lambda_\alpha)\,\psi = 0
\end{aligned}$$

- Lagrange multiplier found in terms of Einstein's 3-form,

$$D\lambda_\alpha = -\frac{\ell^4}{\kappa}\left(\frac{a_0}{2}\,\eta_{\alpha\beta\rho\sigma}R^{\rho\sigma} - \Lambda\eta_{\alpha\beta}\right)\wedge T^\beta$$

- Meaning of constraint$\varphi^\alpha = 0$ that eliminates tetrad:
- It's Hooke's type *constitutive law* in Cosserat elasticity

$$\Sigma^{\rm D}_\alpha = \ell^{-4} g_{\alpha\beta}\,\widehat{{}^\star\vartheta^\beta}\,.$$

(force stress $\Sigma^{\rm D}_\alpha$)$\propto$(distortion ϑ) with (el.moduli $\ell^{-4}g_{\alpha\beta}$)

Extension of Diakonov's model

- Although using gauge approach, Diakonov went half-way in recasting gravitational variables in terms of fermions
- Coframe ϑ^α is translational potential; Σ_α its current
- Analogously: connection $\Gamma^{\alpha\beta}$ is rotational (Lorentz) potential; spin moment (torque) $\tau_{\alpha\beta}$ its current
- Natural extension: add to translational constitutive law $\varphi^\alpha = 0$ its rotational counterpart

$$\tau^{\rm D}_{\alpha\beta} = {\rm L}^{-2} {\rm g}_{\alpha\gamma} {\rm g}_{\beta\delta} \, {}^\star{\rm K}^{\gamma\delta}$$

- Here $K^{\alpha\beta} = \widetilde{\Gamma}^{\alpha\beta} - \Gamma^{\alpha\beta}$ is contortion
- We thus have additional constraint

$$\varphi^{\alpha\beta} := \frac{1}{4} {}^\star\!\left(\vartheta^{\alpha\beta} \wedge \overline{\psi}\gamma\gamma_5\psi\right) - {\rm L}^{-2}\left(\widetilde{\Gamma}^{\alpha\beta}(\vartheta, {\rm d}\vartheta) - \Gamma^{\alpha\beta}\right)$$

to be added with Lagrange multiplier, $+\varphi^{\alpha\beta} \wedge \lambda_{\alpha\beta}$

Field equations of extended model

- We derive complete set of equations

$$\frac{a_0}{2\kappa} R^{\rho\sigma} \wedge \eta_{\alpha\rho\sigma} - \frac{\Lambda}{\kappa}\eta_\alpha = -\ell^{-4}\lambda_\alpha + \mathrm{L}^{-2}\mathrm{D}\xi_\alpha - \mathrm{L}^{-2}(\mathrm{e}_\alpha\rfloor \mathrm{T}^\beta)\wedge\xi_\beta,$$
$$\frac{a_0}{2\kappa} T^\rho \wedge \eta_{\alpha\beta\rho} = -\frac{1}{8}\overline{\psi}(\gamma^\rho\sigma_{\alpha\beta} + \sigma_{\alpha\beta}\gamma^\rho)\psi\,\lambda_\rho + \mathrm{L}^{-2}\lambda_{\alpha\beta},$$
$$i\gamma^\alpha\lambda_\alpha \wedge D\psi - \frac{i}{2}\gamma^\alpha(D\lambda_\alpha)\,\psi + \frac{1}{4}\vartheta^{\alpha\beta}\wedge {}^\star\lambda_{\alpha\beta}\wedge\gamma\gamma_5\psi = 0$$

- Here $\xi_\alpha = 2e^\rho\rfloor\lambda_{\rho\alpha} + \frac{1}{2}\vartheta_\alpha\wedge(e^\rho\rfloor e^\sigma\rfloor\lambda_{\rho\sigma})$
- Dynamics is contained in *non-linear spinor equation*
- Following Akama and Diakonov, we choose $a_0 = 0$; that reduces gravitational Lagrangian to volume of spacetime

$$L = \frac{\Lambda}{\kappa}\eta + \varphi^\alpha\wedge\lambda_\alpha + \varphi^{\alpha\beta}\wedge\lambda_{\alpha\beta}.$$

- Resulting theory is highly nonlinear, also the system of equations for Lagrange multipliers
- Solving these iteratively, in first approximation we find

$$\lambda_\alpha = \frac{\Lambda\ell^4}{\kappa}\,\eta_\alpha, \qquad {}^\star\lambda_{\alpha\beta} = -\frac{\Lambda\ell^4 \mathrm{L}^2}{4\kappa}\,\eta_{\alpha\beta\rho\sigma}\vartheta^\rho\,\overline{\psi}\gamma^\sigma\gamma_5\psi$$

- As a consequence, spinor equation simplifies to

$$i\gamma^\alpha D_\alpha\psi - \frac{3}{8}\,\mathrm{L}^2\,(\overline{\psi}\gamma^\alpha\gamma_5\psi)\,\gamma_\alpha\gamma_5\psi = 0$$

- Thus, in lowest approximation, extended Diakonov model reduces to *Heisenberg's nonlinear spinor theory*, one of most advanced models attempted to describe all physical interactions in terms of fundamental fermions
- Now: with gravity included!

Conclusions and Outlook

- Diakonov's grand challenge: quantizing this model by evaluating partition function (path integral) using methods of lattice field theory
- Such a quantum model is well defined, well-behaved in ultraviolet, explicitly Lorentz and diffeomorphism invariant in continuum limit
- Our results: Extended model includes *all* gravitational potentials (coframe and connection) as constructs of fundamental fermions
- Physical basis: complete set of constitutive laws for elastic Cosserat continuum (model of spacetime)
- Heisenberg's nonlinear spinor theory found in lowest order
- Work was done together with Friedrich Hehl (Cologne): Phys. Lett. **B713** (2012) 321.

Thanks !