Exploring the nucleon: a fundamental quest

Know what we are made of!

Understand the strong force: “QCD”

Use protons as tool for discovery (e.g. LHC)
Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

Changing $x$ we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

See talk by Peter Kroll

These distributions are also referred to as 3D (three-dimensional) distributions
Virtual photon serves as a microscopic probe of the nucleon:

Larger $Q^2$ probe smaller distances – DGLAP evolution

Plot from EIC whitepaper
Virtual photon serves as a microscopic probe of the nucleon:

Fixing $Q^2$ and changing the energy we probe BFKL evolution.
Virtual photon serves as a microscopic probe of the nucleon:

Recombination of gluons leads to non linear effects – BK/JIMWLK evolution and phenomenon of saturation. Dilute vs dense regime of QCD.
“Experiments with spin have killed more theories than any other single physical parameter”

Elliot Leader, Spin in Particle Physics, Cambridge U. Press (2001)

“Polarisation data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.”

Consider $A_N$ in hadron hadron collision:

$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$
QCD had a very simple prediction:

Helicity flip is proportional to the small mass of the quark, thus

\[ A_N \simeq 0.001 \]

Kane, Pumplin and Repko (1978)
Experiment proved this prediction wrong

$A_N \approx 40\%$

$E704 \ (1991)$

Alexei Prokudin
Spin and QCD

Asymmetry survives with energy

$$\sqrt{s} = 62 \text{ GeV}$$

**PHENIX Preliminary**
20% polarization uncertainty on $A_N$ scale
10% energy calibration uncertainty on $x_F$ scale

$p^+ + p \rightarrow \pi^o + X$ at $\sqrt{s} = 200 \text{ GeV}$

**STAR PRELIMINARY**

2.5 < $\eta$ < 4

**RHIC: STAR, BRAHMS and PHENIX**

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Asymmetry survives with energy

\[
\sin (\phi_h - \phi_S) = A_{UL} \frac{1}{z}
\]

HERMES and COMPASS

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Failure of QCD?
Not at all: better understanding of QCD
Better understanding of QCD

\[ \sigma(Q, \tilde{s}) \propto \left( p, \tilde{s} \right) + k \leftarrow t \sim 1/Q + \ldots + \frac{Q_s}{Q} \]

Multy parton correlations contribute to the cross section.

These correlations are called **Efremov-Teryaev-Qiu-Sterman** matrix elements, They appear at twist-3 level in cross section.

\[ \sigma = \sigma^{LT} + \frac{Q_s}{Q} \sigma^{HT} + \ldots \]

\[ = H^{LT} \otimes f_2 \otimes f_2 + \frac{Q_s}{Q} H^{HT} \otimes f_3 \otimes f_2 + \ldots \]

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Better understanding of QCD

If only one large scale is present in the process, then

$$A_N \propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp)$$
$$\propto T^{(3)}(x, x, S_\perp) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x, S_\perp) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, \bar{z}) + ...$$

Leading power cancels

Twist-3 parton correlation functions

$$T^{(3)}(x, x, S_\perp) \propto$$

Twist-3 parton fragmentation functions

$$D^{(3)}(z, \bar{z}) \propto$$

No probability interpretation!

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Evolution of twist-3 matrix elements

One starts from factorization

\[ \Delta \sigma = \frac{1}{Q} H \left( \frac{Q}{\mu_F}, \alpha_s \right) f_2(\mu_F) f_3(\mu_F) \]

- Calculate directly
  - Kang, Qiu 2009
  - Yuan, Zhou 2009

- Calculate from scale dependence of hard part
  - Yuan, Vogelsang 2009

- Renormalization of twist-3 operators
  - Braun et al 2009

Good agreement of results!

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Collinear vs TMD factorization

We can consider two different kinematical regions

\[ Q_1, Q_2, \ldots \gg \Lambda_{QCD} \quad \text{Collinear} \]
\[ Q_1 \gg Q_2 > \Lambda_{QCD} \quad \text{TMD} \]

- Twist-3 – integration over parton momenta
- TMD – direct information on partonic transverse motion

\[ A_N(Q^2, p_T) \]

Ji, Qiu, Yuan, Vogelsang, Koike

Consistent in the overlap region!

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**Transverse Momentum Dependent distributions**

**SIDIS**

If produced hadron has low transverse momentum \( P_{hT} \sim \Lambda_{QCD} \ll Q \)

it will be sensitive to quark transverse momentum \( k_{\perp} \)

TMD factorization proven in QCD

Ji, Ma, Yuan (2002)

Collins (2011)

For processes with two hadrons:

SIDIS, DY, e+e-

\[
\Phi_{ij}(x, k_{\perp}) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_{\perp}}{(2\pi)^2} e^{ixP^+\xi^- - ik_{\perp}\xi_{\perp}} \langle P, S_P | \bar{\psi}_j(0) U(0, \xi) \psi_i(\xi) | P, S_P \rangle
\]

Gauge Invariant

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Transverse Momentum Dependent distributions

\[ \Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - ik_\perp\xi_\perp} \langle P, S_P | \bar{\psi}_j(0) U(0, \xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+ = 0} \]

SIDIS in IMF:

Gauge link

\[ U(a, b; n) = e^{-ig \int_a^b d\lambda n \cdot A_\alpha(\lambda n) t_\alpha} \]

Ensures gauge invariance of the distribution, cannot be canceled by gauge choice

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8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied


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Correlation of transverse quark motion and the nucleon spin – Sivers function

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}(x, k_T^2) \frac{k_x}{M} \]

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

No polarisation:

Polarisation:

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al., 2011;
EIC Whitepaper

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Tomographic scan of the nucleon

$\mathbf{x f_1(x, S_T, k_T)}$

$u$ quark

$d$ quark

$k_x$ (GeV)

$k_y$

Anselmino et al 2009

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Transverse Momentum Dependent distributions

\[ W(x, b_{\perp}, k_{\perp}) \]

\[ \int d^2 b_{\perp} \]

\[ \int d^2 k_{\perp} \]

\[ f(x, b_{\perp}, k_{\perp}) \]

Generalized Parton Distributions

\[ H(x, \xi, t) \]

Form Factors

\[ \int d^2 k_{\perp} \rightarrow \int dx \]

\[ f(x) \rightarrow F(Q^2) \]

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Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance. Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)

\[ f_{1T}^{\perp \text{SIDIS}} = -f_{1T}^{\perp \text{DY}} \]

One of the main goals is to verify this relation. It goes beyond “just” check of TMD factorization. Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou

See talk by Michela Chiosso
Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.

Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)

\[ f_{1T}^{\perp \text{SIDIS}} \]

Drell-Yan is at much different resolution scale $Q$. EIC will operate at higher $Q$. What do we know about evolution of TMDs?

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AnDY, COMPASS, JPARC, PAX etc
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SIDIS and parton model

“QCD improved” parton model:

Radiation of gluons create transverse momenta

Terms like this appear

\[(\alpha_s)^n \left( \ln \frac{Q^2}{P_T^2} \right)^m\]

Result needs to be resummed for small \( P_T \)

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985
Koike, Nagashima, Vogelsang
Kang, Xiao, Yuan

Implementation of resummation in QCD
Resummation

Resummation (CSS) is in configuration space. Fourier transform is needed for observables.

For Drell-Yan

\[
\frac{d\sigma}{dq_T} \sim \int d^2b_T e^{iq_T \cdot b_T} \hat{W}(x_1, x_2, b_T)e^{-S(b_T, Q)} + Y(q_T, Q)
\]

A lot of phenomenology done. Energies from 20 GeV to 2 TeV.

Brock, Landry, Nadolsky, Yuan 2003
Qiu, Zhang 2001

Drawbacks:
• Process dependent fits
• No direct connection to TMDs
• Designed for large energies
Evolution of TMDs

One needs a unique definition of TMDs

\[ W^{\mu\nu} = \sum_f |H_f(Q^2, \mu)|^{\mu\nu} \]
\[ \times \int d^2k_{1T} d^2k_{2T} F_{f/P_1}(x_1, k_{1T}; \mu, \zeta_F) F_{\bar{f}/P_1}(x_2, k_{2T}; \mu, \zeta_F) \]
\[ \times \delta^{(2)}(k_{1T} + k_{2T} - q_T) + Y(q_T, Q) \]

\[ F_{f/P_1}(x_1, k_{1T}; \mu, \zeta_F) \]

TMD distribution of partons in a hadron

Renorm group (RG) renormalization

Rapidity divergence regulator

Foundations of perturbative QCD
Collins 2011
Evolution of TMDs

Evolution of TMDs is done in coordinate space $b_T$

$$F_{f/P}(x, k_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2b_T e^{ik_T \cdot b_T} \tilde{F}_{f/P}(x, b_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Why coordinate space?

Convolutions become simple products:

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu)|^{\mu\nu} \times \int d^2b_T e^{ib_T q_T} \tilde{F}_{f/P_1}(x_1, b_T; \mu, \zeta_F) \tilde{F}_{f/P_1}(x_2, b_T; \mu, \zeta_F)$$

Collins, Soper 1982
Collins, Soper, Sterman 1985
Idilbi, Ji, Ma, Yuan 2004
Boer, Gamberg, Musch, AP 2011

In principle experimental study of functions in coordinate space is possible

Boer, Gamberg, Musch, AP 2011
TMD evolution

Energy evolution

\[ \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu) \]

Collins-Soper kernel in coordinate space

Renormalization group equations

\[ \frac{d \tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \]

\[ \frac{d \ln F(x, b_T, \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu), \zeta) \]

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011
TMD evolution

Energy evolution

\[ \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu) \]

Collins-Soper kernel in coordinate space

At small \( b_T \) perturbative treatment is possible

\[ \tilde{K}(b_T, \mu) = -\frac{\alpha_s C_F}{\pi} \left( \ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_E \right) + \mathcal{O}(\alpha_s^2) \]

Large \( b_T \) nonperturbative – matching via \( b_* \)

\[ b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2 / b_{max}^2}} \]

Brock, Landry, Nadolsky, Yuan 2003

Other methods of matching are available i.e. Vogelsang et al
TMD evolution

Solution Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

\[ \tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = F_{f/P}(x; Q_0) \exp \left( - \left[ \frac{\langle k_T^2 \rangle}{4} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right] b_T^2 \right) \]

Non perturbative

Gaussian behaviour is appropriate only in a limited range

TMDs change with energy and resolution scale

JLab Energies

Tevatron Energies

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Can we see signs of evolution in the experimental data?

Aybat, AP, Rogers 2011

COMPASS data is at
\[ \langle Q^2 \rangle \simeq 3.6 \text{ (GeV}^2) \]

HERMES data is at
\[ \langle Q^2 \rangle \simeq 2.4 \text{ (GeV}^2) \]
Can we explain the experimental data?

Full TMD evolution is needed!

COMPASS dashed line

\[ \langle Q^2 \rangle \simeq 3.6 \ (GeV^2) \]

HERMES solid line

\[ \langle Q^2 \rangle \simeq 2.4 \ (GeV^2) \]
TMD evolution

This is the first implementation of TMD evolution for observable

Functions change with energy

Phenomenological analysis with evolution is now possible

Aybat, AP, Rogers 2011

Asymmetry changes with $Q^2$

Anselmino, Boglione, Melis 2012

Solid line – TMD evolution fit
Dashed line – DGLAP fit

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Helicity structure

\[ \Delta q(x) = \left| \begin{array}{c} P, + \overrightarrow{\times} \\overrightarrow{\times} \{ xP \} X \end{array} \right|^2 - \left| \begin{array}{c} P, + \overrightarrow{\times} \{ xP \} X \end{array} \right|^2 \]

\[ \Delta g(x) = \left| \begin{array}{c} P, + \overrightarrow{\times} \overrightarrow{\times} \{ xP \} X \end{array} \right|^2 - \left| \begin{array}{c} P, + \overrightarrow{\times} \{ xP \} X \end{array} \right|^2 \]

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g \]

\[ \Delta \Sigma(Q^2) = \int_0^1 dx \left[ \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} \right] (x, Q^2) \]

\[ \Delta G(Q^2) = \int_0^1 dx \Delta g(x, Q^2) \]

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We still have a lot to learn about strangeness:

\[ x(s + \bar{s}) \]

**HERMES**

**ATLAS**

\[ r_s = 0.5 \frac{s + \bar{s}}{d} \]

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New developments on $\Delta g$: de Florian, Sassot, Stratmann, Vogelsang

$$A_{LL} = \frac{\sigma_{\leftrightarrow} - \sigma_{\rightarrow\rightarrow}}{\sigma_{\leftrightarrow} + \sigma_{\rightarrow\rightarrow}}$$

$\sqrt{s}=200$ GeV $\hat{p}+\hat{p} \rightarrow$ jet+$X$ $|\eta|<1$

$\pm 8.8\%$ scale uncertainty from polarization not shown

Gives gluon with $\int_{0.05}^{0.2} dx \Delta g \approx 0.1$

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TMDs and twist-3 are related

At operator level:

$$T_F(x, x) = - \int d^2 \vec{k}_\perp \frac{k_\perp^2}{M} (f_{1T}(x, k_\perp))_{SIDIS}$$

Boer, Mulders, Pijlman, Ji, Qiu, Vogelsang, Yuan, Koike, Vogelsang, Yuan, Zhou, Yuan, Liang, Bacchetta, Boer, Diehl, Mulders

We can compare results from SIDIS and PP:


Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)

→ a sign puzzle

Kang, Qiu, Vogelsang, Yuan (2011)
Conclusion?

Inconsistency in QCD formalism for single-spin?

Collins-type effect dominant in \( pp \rightarrow \pi X \) ?

Can one describes it with nodes in \( x \) or \( kt \)?
Joint fit to SIDIS and pp data:

Kang, AP (2011)

SIDIS  pp (STAR)

Conclusion?
Works (reasonably) well for SIDIS and STAR, but fails for BRAHMS!
Has ramifications for DY spin asymmetry: Kang, AP (2011)

Strengthens case for study of DY “sign change”!

AnDY, COMPASS, E906, W bosons at RHIC

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Direct (prompt) photon $A_N$

Kang, Gamberg (2012)
Kanazawa, Koike (2012)

$T_F(x, x)$ from pp

Sensitivity to Sivers type of contribution only.

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Collins contribution can be substantial

Anselmnino, Boglione, D'Alesio, Leader (5), Melis, Murgia, AP (2012)

Collins from SIDIS

Collins contribution can be substantial

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TMDs for LHC

Boer, den Dunnen, Pisano, Schlegel, Vogelsang (2012)

Parity of the Higgs!

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CONCLUSIONS

A lot of new results will be presented this week!

Enjoy SPIN 2012!
Spares to follow
New developments on $\Delta g$ : de Florian, Sassot, Stratmann, Vogelsang

$$A_{LL} = \frac{\sigma\rightarrow - \sigma\rightarrow\rightarrow}{\sigma\rightarrow\leftrightarrow + \sigma\rightarrow\rightarrow}$$

$\sqrt{s}=200$ GeV $\vec{p}+\vec{p} \rightarrow$ jet+$X \ |\eta|<1$

$\pm 8.8\%$ scale uncertainty from polarization not shown

gives gluon with $\int_{0.05}^{0.2} dx \Delta g \approx 0.1$

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Drell Yan

Analysis in hadronic cm frame

\[ A_N = \frac{\sum_q f_{1T}^q(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T) \sigma_{q\bar{q}}}{\sum_q f_1^q(x_2, p_T) \otimes f_1^{\bar{q}}(x_1, p_T) \sigma_{q\bar{q}}} \]

With TMD evolution

No TMD evolution

\[ \sin(\phi_{1r} - \phi_s) \]

\[ A_{UT} \]

Asymmetry is supposed with respect to LO analysis

Result with TMD Evolution depends on the choice of \( g_K(b_T) \)

Unpolarised x-sections should be reanalyzed

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Fragmentation FF: BELLE (M. Leitgab at DIS 2012)

- marks new era of precision fragmentation functions.
  Evolution $\Rightarrow D^h_g$ $e^+e^-$ vs. RHIC(pp)

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TMD evolution

Relation to collinear treatment:

\[
\tilde{F}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f} \left( \frac{x}{\hat{x}}, b_T, \mu, \zeta \right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD} b_T)
\]

Valid at small \( b_T \), lowest order:

\[
\tilde{C}_{j/f} \left( \frac{x}{\hat{x}}, b_T, \mu, \zeta \right) = \delta_{jf} \delta \left( \frac{x}{\hat{x}} - 1 \right) + \mathcal{O}(\alpha_s)
\]

Higher order for TMD PDFs

Aybat Rogers 2011

Higher order for Sivers function

Kang, Xiao, Yuan 2011

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\[ \tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) \]
\[ \times \exp \left[ -g_K(b_T) \ln \frac{Q}{Q_0} \right] \]
\[ \times \exp \left[ \ln \frac{Q}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu')); 1 \right) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \]
\[ + \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{Q}{Q_0} \gamma_K(g(\mu')) \]

Typically for TMDs:
\[ \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) = F_{f/P}(x; Q_0) \exp \left( - \frac{\langle k_T^2 \rangle}{4} b_T^2 \right) \]