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DSPIN-07

Elastic proton-proton and proton-antiproton scattering: analysis of complete set of helicity amplitudes

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Outline

- ***Physics motivation***
  - **Direct reconstruction of the scattering matrix elements.**
  - **Investigation of spin-effects in the extraction of forward scattering amplitude.**

- ***Preliminary results***
  - **Phenomenological parameterizations of the amplitudes for elastic proton-proton collisions.**
  - **Full set of helicity amplitudes for proton-antiproton collisions.**
  - **Definition of free parameters for proton-proton amplitudes:**
    - **Slope parameters**
    - **Global scattering characteristics**
  - **Dependences on t-invariant for elastic scattering for:**
    - **Proton-proton differential cross-section**
    - **Proton-antiproton differential cross-section**

- ***Summary and plans***

Physics motivation

- **Definition of the scattering matrix is the main goal of the complete experiment with proton-proton and/or proton-antiproton scattering**
  - The method of the direct reconstruction is valid at any initial energies

- **Scattering amplitudes**
  - Additional new experiments are necessary in particular with proton-antiproton beams in order to
    - **obtain important information about helicity amplitudes;**
    - **cancel some uncertainties in the phenomenological parameterizations of amplitudes;**
    - **improve the precision of the experimental data**
  - Many analytic parameterizations are very close for high energies, it turns that they may be differ markedly at low and medium energy. Therefore new experiments are necessary at wide energy region, in particular at low and medium energies.

- **Elastic scattering is (in some sense) one of the most fundamental type of reaction (but it's also the most difficult to understand theoretically).**

Proton-proton scattering: amplitude parameterization

- We suggest the following ***analytical*** phenomenological parameterization for proton-proton spin non-flip amplitude

$$\left[\Phi_1(S,t)\right]_j = A_1 \exp\left(\frac{B_1(S,t)}{2}t\right) + A_2 \exp\left[\left(\frac{B_2(S,t)}{2} - i\beta\frac{\pi}{2}\right)t\right] + \xi_j, \quad \xi_j = A_3 \begin{cases} \exp\left(\frac{B_3(S,t)}{2}t\right), & j=1 \\ t^{-\delta}, & j=2. \end{cases}$$

low t region: $10^{-2} \le -t \le 0.12$ GeV^2

intermediate t region: $0.10 \le -t \le 0.85$ GeV^2 and interference dip range

large t region: $-t \ge 1.5$ GeV^2

- We have to use
  - derivative relations $\Phi_5(S,t) = C_1 \frac{\partial \Phi_1(S,t)}{\partial\left(\sqrt{t}\right)}; \Phi_4(S,t) = C_2 \frac{\partial^2 \Phi_1(S,t)}{\partial\left(\sqrt{t}\right)^2}.$
    - Let's note: A_k, k=1-3; C_l, l=1,2 – free complex (in general) parameters above
  - some additional suggestions

1-st version	2-d version
$\Phi_1 = \Phi_3$	
$\Phi_2 = 0$	$\Phi_2 = -\Phi_4$

in order to calculate (spin-dependent) observables for proton-proton as well as for proton-antiproton elastic scattering

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Proton-antiproton scattering: amplitude parameterization

I-st method for amplitude parameterization in proton-antiproton collisions:

We suppose that the proton-antiproton spin non-flip amplitudes is described by same functional dependences as well as for proton-proton scattering but with different parameter values.

S.B.Nurushev, V.A.Okorokov et al. Proceedings of the SPIN-2001 Workshop, E1,2-2002-103, JINR, Dubna, 2002, pp.302-304.

V.A.Okorokov S.B.Nurushev, et al. Proceedings of the SPIN-2005 Workshop, JINR, Dubna, 2006, pp.401-406.

II-d method for amplitude parameterization in proton-antiproton collisions:

cross-symmetry relations are used for definition proton-antiproton helicity amplitudes via known helicity amplitudes of the proton-proton scattering .

Preliminary results

1-st version

$$\left.\begin{aligned}
\bar\Phi_1 &= 1/2\left[2\Phi_1 \sin^2\psi + \left(1+\cos^2\psi\right)\Phi_4\right];\\
\bar\Phi_2 &= 1/2\sin^2\psi\left[2\Phi_1 - \Phi_4\right];\\
\bar\Phi_3 &= 1/2\left[-\Phi_4 \sin^2\psi - 2\Phi_1\cos^2\psi\right];\\
\bar\Phi_4 &= 1/2\left[\Phi_4 \sin^2\psi + 2\Phi_1\cos^2\psi\right];\\
\bar\Phi_5 &= 1/2\left[\sin\psi\cos\psi\left(2\Phi_1 - \Phi_4\right) + 2\Phi_5\right].
\end{aligned}\right\}(II.1)$$

2-d version

$$\left.\begin{aligned}
\bar\Phi_1 &= \Phi_1 \sin^2\psi + \Phi_4\cos^2\psi;\\
\bar\Phi_2 &= \Phi_1 \sin^2\psi + \Phi_4\cos^2\psi;\\
\bar\Phi_3 &= -\left[\Phi_4 \sin^2\psi + \Phi_1\cos^2\psi\right];\\
\bar\Phi_4 &= \Phi_4 \sin^2\psi + \Phi_1\cos^2\psi;\\
\bar\Phi_5 &= \cos\psi\sin\psi\left(\Phi_1 - \Phi_4\right) + \Phi_5.
\end{aligned}\right\}(II.2)$$

$$\cos\psi = \sqrt{\frac{St}{\left(S-4m_p^2\right)\left(t-4m_p^2\right)}};\ \sin\psi = 2m_p\sqrt{\frac{4m_p^2 - S - t}{\left(S-4m_p^2\right)\left(t-4m_p^2\right)}}.$$

we obtain analytical expressions for all amplitudes for proton-antiproton collisions via known amplitudes of the proton-proton scattering and can study different spin-dependent observables.

Spin-dependent observables

The differential cross-section, polarization, elements of the spin tensors and Wolfenstein parameters are under study for elastic proton-antiproton scattering at low and medium energies.

1-st version

$$
\left.\begin{array}{l}
d\sigma/dt \equiv \sigma_0 = |F_0|^2; \\
P\sigma_0 = \mathrm{Im}\left[(2F_0 - F_2)^* F_1\right]; \\
(1 - D_{nn})\sigma_0 = 1/2\,|F_0|^2; \\
(1 - C_{nn})\sigma_0 = 1/2\left[|F_0|^2 + |F_0 + F_2|^2\right]; \\
D_t = C_{nn}; \\
C_{kp}\sigma_0 = 1/4\left[-|F_0|^2 + |F_0 - F_2|^2\right]\sin\theta - \mathrm{Re}\left(F_1^* F_2\right)\cos\theta; \\
R\sigma_0 = |F_0|^2 \cos(\theta/2) - \mathrm{Re}\left[F_1^*(2F_0 + F_2)\right]\sin(\theta/2); \\
A\sigma_0 = 1/2\left[2|F_0|^2 - |F_2|^2\right]\sin(\theta/2) + \mathrm{Re}\left[F_1^*(2F_0 + F_2)\right]\cos(\theta/2); \\
R'\sigma_0 = -|F_0|^2 \sin(\theta/2) - \mathrm{Re}\left[F_1^*(2F_0 + F_2)\right]\cos(\theta/2); \\
A'\sigma_0 = 1/2\left[2|F_0|^2 - |F_2|^2\right]\cos(\theta/2) - \mathrm{Re}\left[F_1^*(2F_0 + F_2)\right]\sin(\theta/2).
\end{array}\right\} (1)
$$

2-d version

$$
\left.\begin{array}{l}
d\sigma/dt \equiv \sigma_0 = |F_0|^2; \\
P\sigma_0 = 2\,\mathrm{Im}\left[(F_0 - F_2)^* F_1\right]; \\
D_{nn} \equiv 1; \\
(1 - C_{nn})\sigma_0 = |F_0 + F_2|^2; \\
D_t = C_{nn}; \\
C_{kp} \equiv 0; \\
R\sigma_0 = \left(|F_0|^2 - |F_2|^2\right)\cos(\theta/2) - 2\,\mathrm{Re}\left[F_1 (F_0 + F_2)^*\right]\sin(\theta/2); \\
A\sigma_0 = \left[|F_0|^2 - |F_2|^2\right]\sin(\theta/2) + \mathrm{Re}\left[F_1 (F_0 + F_2)^*\right]\cos(\theta/2); \\
R'\sigma_0 = -\left(|F_0|^2 - |F_2|^2\right)\sin(\theta/2) - \mathrm{Re}\left[F_1^* (F_0 + F_2)\right]\cos(\theta/2) = -A; \\
A'\sigma_0 = \left[|F_0|^2 - |F_2|^2\right]\cos(\theta/2) - 2\,\mathrm{Re}\left[F_1^* (F_0 + F_2)\right]\sin(\theta/2) = R.
\end{array}\right\} (2)
$$

θ **is the scattering angle in center-of-mass system of proton-proton (antiproton) collisions**

Slope parameter $B_1(S,t)$: low t region

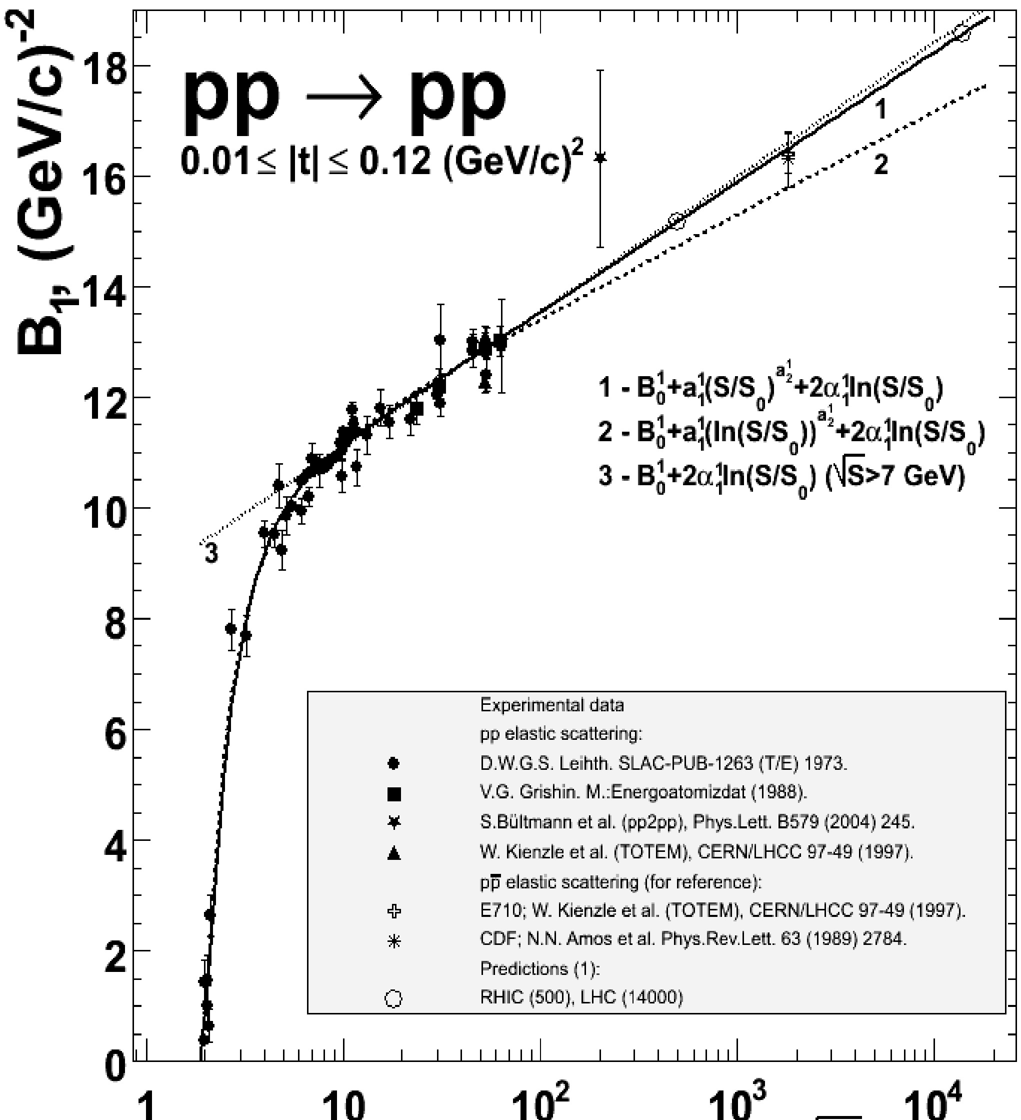

Fig.1. Energy dependence of the slope parameter at low |t| for elastic pp scattering.

We have considered the standard linear fit as well as two new functions for approximation the slope parameter energy dependence for low t range

(fit parameter values – see Table I).

The parameterization (2) underestimates the value of slope parameter at high energies

Table I. Fit parameters for various approximation functions (1) – (3)

Parameter	Approximation function		
	(1)	(2)	(3) ($\sqrt{S}$ > 7 GeV)
B_0^1	8.83 ± 0.04	9.8 ± 0.2	8.68 ± 0.09
a_1^1	-80.4 ± 0.4	-23 ± 2	---
a_2^1	-1.567 ± 0.004	-2.50 ± 0.15	---
α_1^1	0.255 ± 0.004	0.199 ± 0.015	0.264 ± 0.008
χ^2/ndf	2.54	2.50	2.14

$\alpha_1^{(1)} \approx \alpha_P$ as expected

Parameterization (1) describes well the slope parameter at low energies and predicts the reasonable values of the slope parameter in high energy domain.

Therefore we will use the parameterization (1) in our further analysis.

Slope parameter $B_2(S,t)$: intermediate t region

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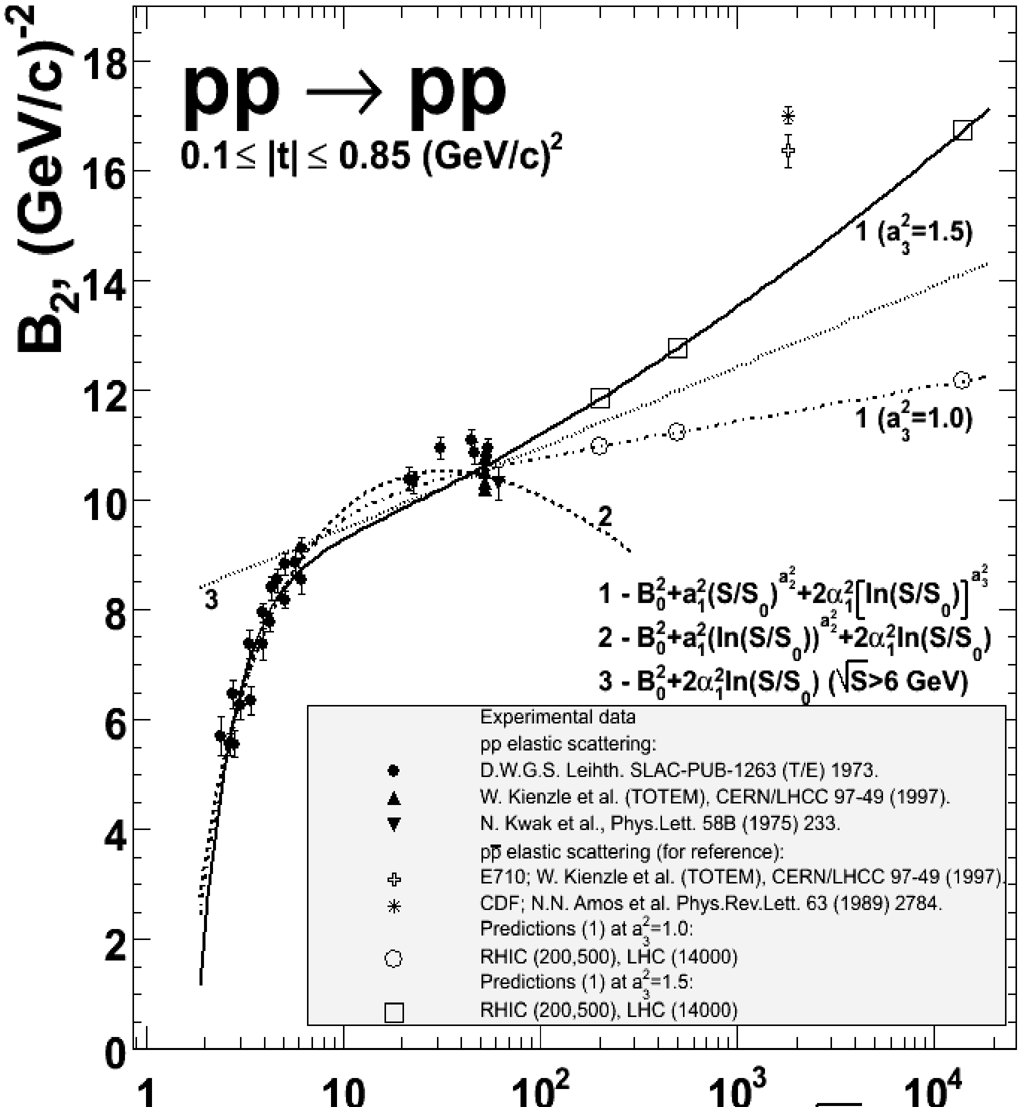

Fig.2. Energy dependence of the slope parameter at intermediate |t| for elastic pp scattering.

- We have considered the standard linear fit as well as two new functions for approximation the slope parameter energy dependence for low t range
- We have fitted only proton-proton data

  (fit parameter values – see Table II).
- The best fit quality is for parameterization (2) but (2) shows decreasing of slope parameter at high energy.

Table II. Fit parameters for various approximation functions (1) – (3)

Parameter	Approximation function			
	(1)		(2)	(3) ($\sqrt{S}$ > 6 GeV)
B_0^2	9.49 ± 0.05	8.39 ± 0.04	-32.7 ± 0.3	8.0 ± 0.3
a_1^2	-20 ± 2	-38 ± 7	-36.0 ± 0.3	---
a_2^2	-0.80 ± 0.05	-1.29 ± 0.09	0.228 ± 0.004	---
α_1^2	0.07 ± 0.01	0.050 ± 0.001	-0.92 ± 0.03	0.16 ± 0.02
a_3^2	1 (fixed)	1.500 ± 0.005	---	---
χ^2/ndf	3.44	4.66	3.14	5.56

- Parameterization (1.2) = (1) @ a_3^2=1.5 shows some poor fit quality. But
- Parameterization (1.2) describes well the slope parameter at low energies and predicts the reasonable values of the slope parameter in high energy domain.
- Therefore we will use the parameterization (1.2) in our further analysis.

Global scattering parameter (1): total cross-section

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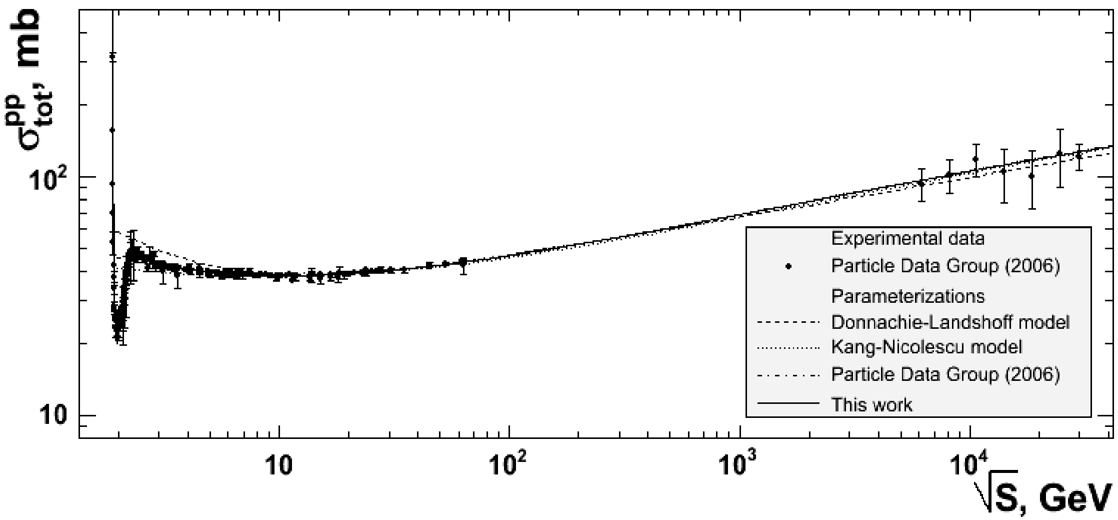

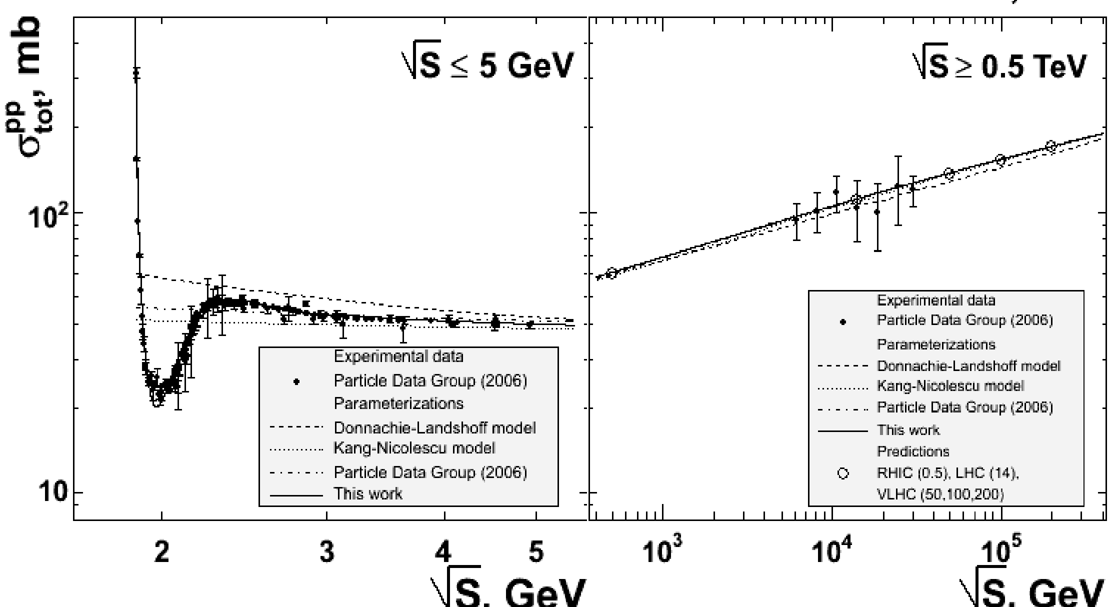

Fig.3. Energy dependence of the proton-proton total cross-section.
Bottom line: total cross-section at low and (ultra) high energies.

- **We have considered the most successful ($\sqrt{S}$>5 GeV) parameterizations for description of energy total cross-section dependence:**
  - **DL (Pomeron-Reggeon); KN (maximal Odderon); PDG (Pomeron-Reggeon with triple pole).**
- **We suppose the new parameterization for low energy data approximation (down to low energy limit)**

$$\sigma^{pp}_{tot}(S) = \sum_{i=1}^{3} \sigma^{i}_{tot},$$

$$\sigma^{1}_{tot} = a_1 \left(\frac{S_1}{S - 4m_p^2} \right)^{a_2}, \quad \sigma^{2}_{tot} = a_3 \frac{J_1\left(a_4\left(S/S_1 - a_5\right)\right)}{\left(a_4\left(S/S_1 - a_5\right)\right)^{a_6}},$$

$$\sigma^{3}_{tot} = Z^{pp} + B\ln^2\left(S/S_0\right) + Y_1^{pp}\left(S_1/S\right)^{\eta_1} - Y_2^{pp}\left(S_1/S\right)^{\eta_2},$$

- **first term describes very sharp behavior at energies close to low limit ($\sqrt{S} \approx 2m_p$), second term – low and intermediate energy domain, last term – standard PDG parameterization.**
- **Parameterization describes well the energy dependence of total cross-section at all available energies at qualitative level but χ^2/ndf=6.95 seems large so far** **(more detail discussion – see below slide # 11; fit parameter values – add slides).**
- **We will use this parameterization in our further analysis.**

Global scattering parameter (2): ρ=[Re Φ(S, t=0)]/[Im Φ(S, t=0)]

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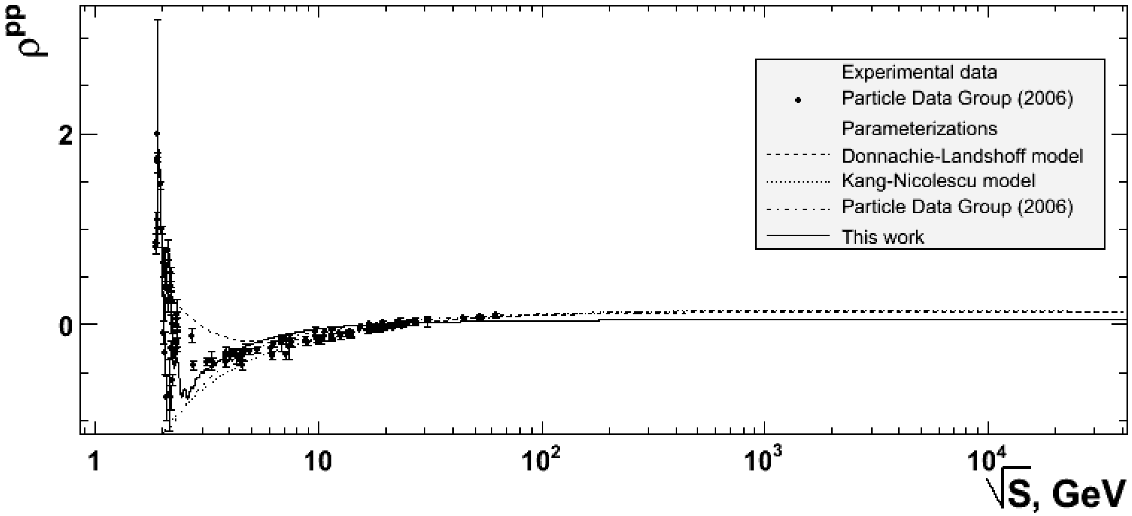

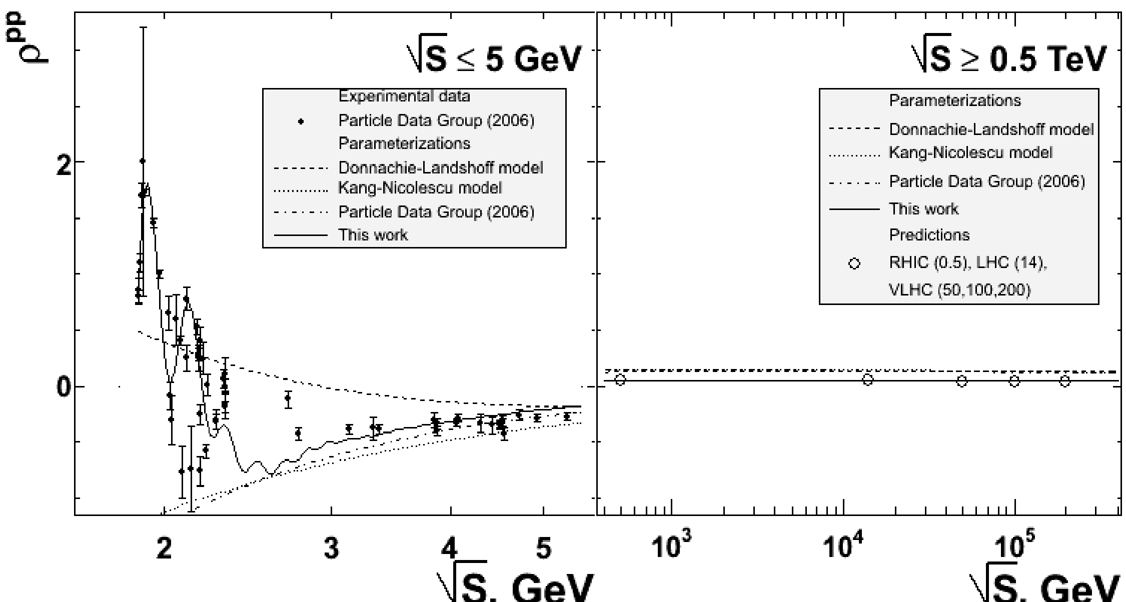

Fig.4. Energy dependence of the ρ parameter for proton-proton scattering. Bottom line: ρ parameter at low and (ultra) high energies.

- **The most analytic approach for ρ based on analyticity relations. Corresponding parameterization for DL and KN models are shown on the left pictures.**
- **We use the simplified analyticity relations in order to derive the ρ(S) for PDG parameterization of total cross-section.**
- **We suppose to use the modified analyticity relations in order to approximate all available data for ρ based on the new parameterization for σ_{tot}**

$$\rho_{pp} \simeq \frac{1}{2\sigma_{tot}^{pp}}\left[2\sigma_{tot}^{pp}\Lambda + \sum_{i=1}^{3}\left(\frac{K_i}{S} + \pi\delta_i \frac{d\left(\sigma_{tot}^{pp}\right)_i}{d\ln\left(S/S_1\right)}\right)\right],$$

$$\Lambda = \lambda_1 \frac{J_1\left(\lambda_2\left(S/S_1-\lambda_3\right)\right)}{\left(\lambda_2\left(S/S_1-\lambda_3\right)\right)^{\lambda_4}},$$

- **the first term describes the high irregular behavior of ρ at low energies**
- **This parameterization describes the energy dependence of ρ parameter at all available energies at qualitative level but χ^2/ndf=7.8 seems large so far**
  (more detail discussion – see below slide # 11; fit parameter values – add slides).
- **We will use this parameterization in our analysis.**

Discussion

1) Slope parameters:

- We have fitted the $B_1(S,t)$ slope parameter well at quantitative level but the new experimental data are essential at higher energies in order to obtain approximation $B_2(S,t)$ more unambiguously.

2) Total cross-section and ρ-parameter:

- The present different models predict quite similar results for total cross-section and ρ at high energies, but they valid only above 10 GeV or so. These models differ at low energies $\sqrt{S} < 5$ GeV dramatically.

-We approximated the global scattering parameters at qualitative level for all available energy domain. One needs to stress that our approach results are very close to the results of other models at (ultra) high energies for total cross-section but data of our approach some smaller than that for other models for ρ-parameters:

$\rho \approx 0.04$ for our analysis

$\rho \approx 0.1$ in other models

at (ultra) high energies $\sqrt{S} > 0.5$ TeV.

3) Based on the optical theorem and on these results we can decrease the number of free parameters for parameterization of spin non-flip amplitude:

$$\operatorname{Im} A_1 = \frac{\sigma_{tot}^{pp}}{4\sqrt{\pi}} - \operatorname{Im} A_2 - \operatorname{Im} A_3; \quad \operatorname{Re} A_1 = \frac{\sigma_{tot}^{pp}\rho^{pp}}{4\sqrt{\pi}} - \operatorname{Re} A_2 - \operatorname{Re} A_3.$$

4) The rest parameters are defined by fit of experimental proton-proton data for differential cross-section $d\sigma/dt$, in particular.

Proton-proton scattering: differential cross-section

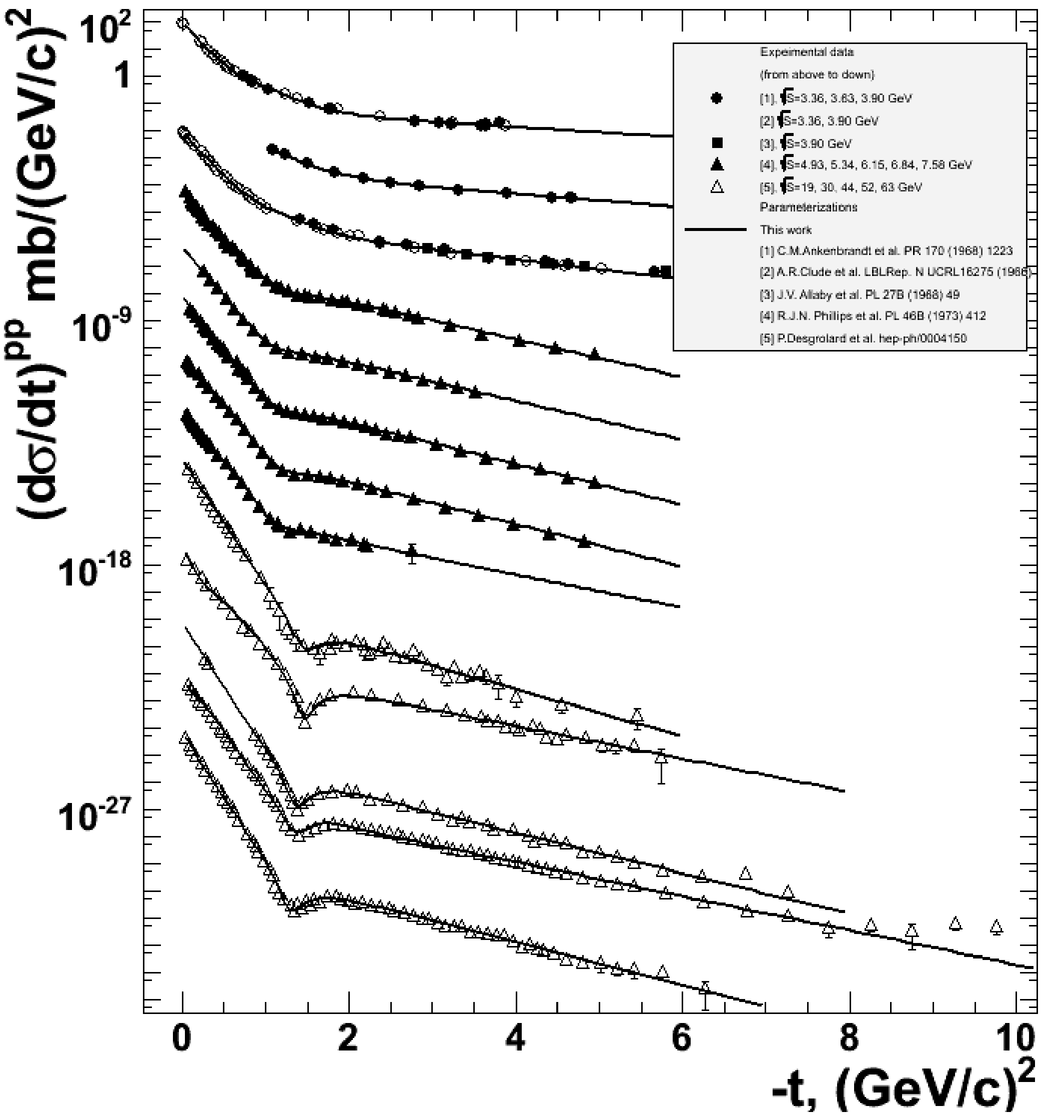

Fig.5. Differential cross section for proton-proton interactions. A factor 0.01 between each successive energy is omitted.

- We have considered the data for pp differential cross-section in wide energy domain ($\sqrt{S} \approx 2 - 62$ GeV) and for t range $t \approx 10^{-2}$ - 10 GeV.
- Experimental data were fitted based on our analytic parameterization of full set of helicity amplitudes for proton-proton collisions.
- Experimental dependences and corresponding fits are shown on the Fig.5 for some initial energies.
- Our parameterization describes experimental points well at any energies understudy and up to $t \sim 9$ GeV at quantitative level.
- Disagreement the experimental data and approximation curves at high t is expected: the high t domain is described by power dependence inspired pQCD.
- We plan to study the parameterization for spin non-flip helicity amplitude with power term for high t domain in future.

Proton-antiproton scattering: differential cross-section

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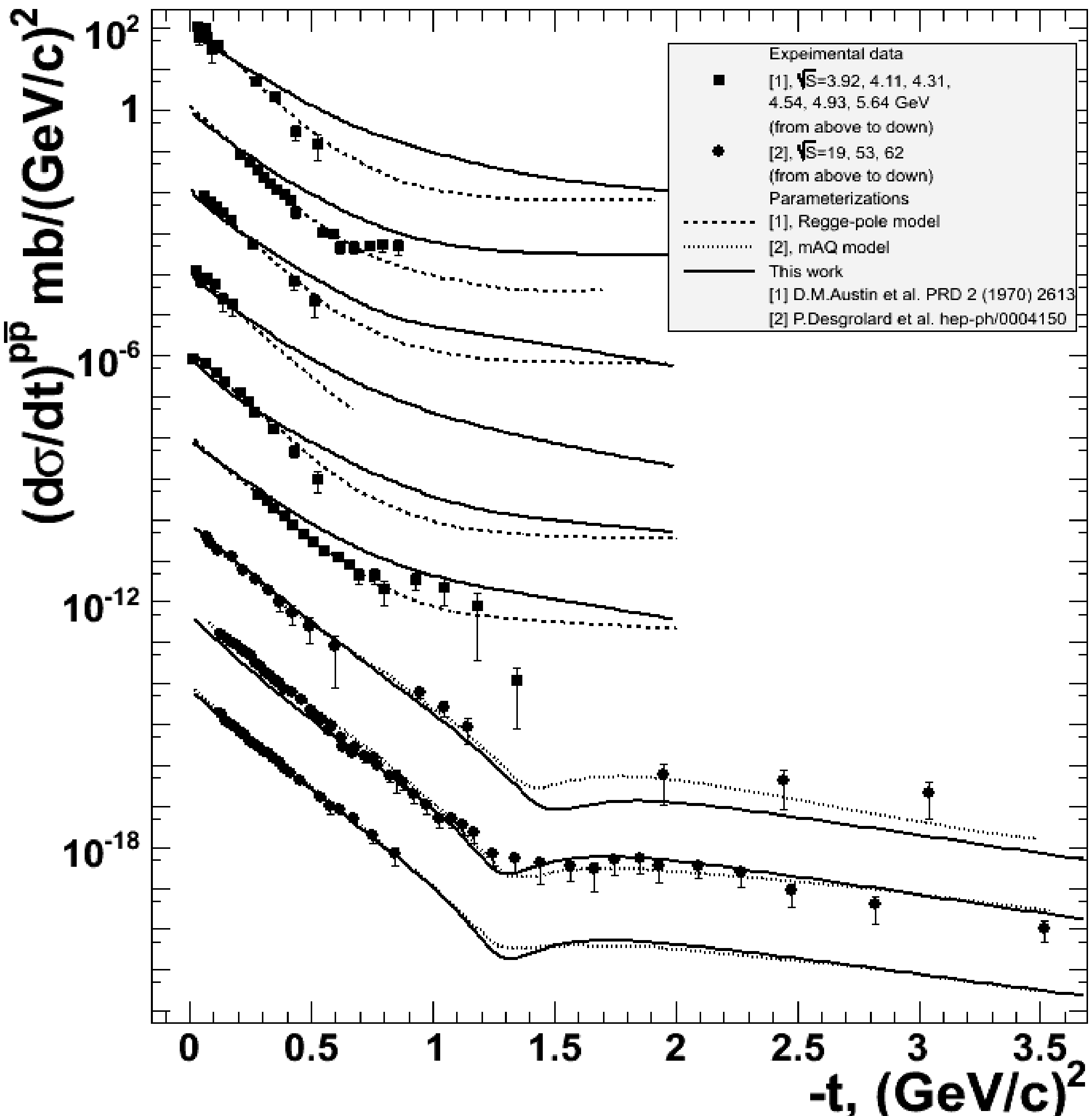

Fig.6. Differential cross section for proton-antiproton interactions. A factor 0.01 between each successive energy is omitted.

- **We have considered the available experimental data for ppbar differential cross-section at low and intermediate energies t ≈ 10^{-2} - 3 GeV.**
- **Analytic curves contradict with experimental data and some other models.**
- **Our approach describes experimental data well at energies $\sqrt{S} \geq 19$ GeV at all t values and good corresponds the mAQ model. The agreement of our approach and both experimental data and mAQ model improves with energy increasing.**
- **But our approach contraries to experimental data and Regge model predictions at low energies**

Summary & plans

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- **New analytic parameterization is suggested for full set of helicity amplitudes for elastic proton-proton collisions.**
  - **Energy dependences of global scattering characteristics (σ_{tot} & ρ) are approximated at qualitative level for proton-proton collisions.**
  - **The new parameterization of full set of helicity amplitudes allows to describe well proton-proton experimental differential cross section at $\sqrt{S} \approx 2 - 62$ GeV and in wide range of square of momentum transfer (up to 9 GeV) .**

- **Full set of helicity amplitudes for proton-antiproton elastic scattering is derived based on the known helicity amplitude parameterization and crossing-symmetry.**
  - **The t-dependences are obtained for differential proton-antiproton differential cross-sections for wide energy domain and t-ranges.**
  - **Analytic parameterization describes experimental data well at $\sqrt{S} \geq 19$ GeV and for low and intermediate t value ($t < 1.5$ GeV). At intermediate energies analytic parameterization some under predicts $d\sigma/dt$ at high t values.**
  - **At low energies analytic parameterization contradicts experimental data for elastic proton-antiproton scattering.**

- **We plan to continue study both variants of the analytical parameterization of amplitudes and spin-dependent parameters for both proton-proton and proton-antiproton scattering in wide energy domain.**

Backup slides

Fit parameters for new approximations of σ_{tot} & ρ

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- We have fitted the energy dependence of total cross-section by new parameterization at all available energies (didn't fixed parameters in the third term, i.e. didn't use the PDG values). Values of fit parameters are shown in the Table III.
- We fitted the ρ energy dependence for all available data. Values of approximation parameters are shown in the Table IV.

Table III. Fit parameters for pp total cross-section approximation

Parameter	Approximation function σ^i_{tot}		
	σ^1_{tot}	σ^2_{tot}	σ^3_{tot}
a_1, mb	6.818 ± 0.225	---	---
a_2	0.918 ± 0.009	---	---
a_3, mb	---	$(1000.0 \pm 0.9) \cdot 10^3$	---
a_4	---	1.489 ± 0.011	---
a_5	---	0.646 ± 0.028	---
a_6	---	5.641 ± 0.009	---
Z^{pp}, mb	---	---	36.95 ± 0.08
B, mb	---	---	0.3127 ± 0.0062
S_0, GeV^2	---	---	41.980 ± 0.005
Y_1^{pp}, mb	---	---	42.68 ± 0.03
η_1	---	---	0.5500 ± 0.0001
Y_2^{pp}, mb	---	---	28.59 ± 0.24
η_2	---	---	0.5324 ± 0.0046
χ^2/ndf	$1730/249\ (6.95)$		

Table IV. Fit parameter values for proton-proton ρ

Parameters	Approximation function	
	Λ	$\sum_{i=1}^{3}\left(\frac{K_i}{S} + \pi\delta_i \frac{d\left(\sigma_{tot}^{pp}\right)_i}{d\ln\left(S/S_1\right)}\right)$
λ_1	-200 ± 34	---
λ_2	7.34 ± 0.07	---
λ_3	3.000 ± 0.009	---
λ_4	1.93 ± 0.11	
K_1	---	29 ± 4
δ_1	---	$(-2.0 \pm 0.6) \cdot 10^{-3}$
K_2	---	-81 ± 4
δ_2	---	0.239 ± 0.005
K_3	---	-321 ± 4
δ_3	---	0.32 ± 0.05
χ^2/ndf	$828/106\ (7.8)$	