TENSOR MAGNETIC POLARIZABILITY OF THE DEUTERON IN STORAGE-RING EXPERIMENTS

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1 INTRODUCTION

Magnetic polarizability is an important property of deuteron and other nuclei. Tensor magnetic polarizability is defined by spin interactions of nucleons. Measurement of the tensor magnetic polarizability of the deuteron gives an important information about an interaction between spins of nucleons and provides a good possibility to examine the theory of spin-dependent nuclear forces.

For polarized deuteron beams in storage rings, the main effects caused by the tensor magnetic polarizability have been investigated by V. Baryshevsky and co-workers [1, 2]. The tensor magnetic polarizability, β_T, conditions the spin rotation with two frequencies instead of one and therefore occasions beating with the frequency proportional to β_T [1, 2]. This effect makes it possible to measure the tensor magnetic polarizability of the deuteron in storage ring experiments.

We propose to use the tensor-polarized beam for measuring the tensor magnetic polarizability of the deuteron. If the initial vector polarization of such a beam in zero, the interaction of the magnetic moment of the deuteron with external fields cannot lead to the appearance of any vector polarization. However, the tensor interactions cause nonzero final vector polarization of the beam. According to estimates, the final vector polarization can be of order of 1%. Such a polarization can be measured. In the present work, we derive general formulae describing the effects caused by the tensor magnetic polarizability of deuteron in storage rings. To check previously obtained results and develop a more general theory, we follow the theory of spin amplitudes (see Refs. [3, 4]) which is partially changed. We use the matrix Hamiltonian for determining an evolution of spin wave function.

2 HAMILTONIAN APPROACH IN THE METHOD OF SPIN AMPLITUDES

The method of spin amplitudes uses quantum mechanics formalism to more easily describe spin dynamics (see Refs. [3, 4]).

The spin rotation can be exhaustively described with the polarization vector $\boldsymbol{P}$ which is defined by

$$P_i = \frac{< S_i >}{S}, \quad i, j = x, y, z, \tag{1}$$

where S_i are corresponding spin matrices and S is the spin quantum number.

Particles with spin $S \geq 1$ also possess a tensor polarization. Main characteristics of such a polarization are specified by the polarization tensor P_{ij} which is given by [5]

$$P_{ij} = \frac{3 < S_i S_j + S_j S_i > -2S(S+1)\delta_{ij}}{2S(2S-1)}, \quad i,j = x,y,z. \tag{2}$$

The polarization tensor satisfies the conditions $P_{ij} = P_{ji}$ and $P_{xx} + P_{yy} + P_{zz} = 1$ and therefore has five independent components. Additional tensors composed of products of three or more spin matrices are needed only for the exhaustive description of polarization of particles/nuclei with spin $S \geq 3/2$.

The spin matrices for spin-1 particles have the form

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3}$$

The nontrivial spin dynamics predicted in Refs. [1, 2] and conditioned by the tensor magnetic polarizability of deuteron is the example of importance of spin tensor interactions in the physics of polarized beams. Tensor interactions of deuteron can also be described with the method of spin amplitudes. In this case, three-component spinors and 3×3 matrices should be used. The method of spin amplitudes is mathematically advantageous because transporting the three-component spinor is much simpler than transporting the three-dimensional polarization vector $\boldsymbol{P}$ and five independent components of the polarization tensor P_{ij} together.

When the deuteron's spin projection onto the direction defined by the spherical angles θ, ψ is equal to unit $(\lambda = 1)$, the components of the polarization vector and the polarization tensor are given by

$$\boldsymbol{P} = \begin{pmatrix} \sin\theta\cos\psi \\ \sin\theta\sin\psi \\ \cos\theta \end{pmatrix}, \tag{4}$$

$$P_{ij} = \frac{3}{2} \begin{pmatrix} \sin^2\theta\cos^2\psi - \frac{1}{3} & \sin^2\theta\sin\psi\cos\psi & \sin\theta\cos\theta\cos\psi \\ \sin^2\theta\sin\psi\cos\psi & \sin^2\theta\sin^2\psi - \frac{1}{3} & \sin\theta\cos\theta\sin\psi \\ \sin\theta\cos\theta\cos\psi & \sin\theta\cos\theta\sin\psi & \cos^2\theta - \frac{1}{3} \end{pmatrix}. \quad (5)$$

When $\lambda = 0$,

$$\boldsymbol{P} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (6)$$

$$P_{ij} = -3 \begin{pmatrix} \sin^2\theta\cos^2\psi - \frac{1}{3} & \sin^2\theta\sin\psi\cos\psi & \sin\theta\cos\theta\cos\psi \\ \sin^2\theta\sin\psi\cos\psi & \sin^2\theta\sin^2\psi - \frac{1}{3} & \sin\theta\cos\theta\sin\psi \\ \sin\theta\cos\theta\cos\psi & \sin\theta\cos\theta\sin\psi & \cos^2\theta - \frac{1}{3} \end{pmatrix}. \quad (7)$$

We follow the traditional quantum mechanical approach perfectly expounded by R. Feynman [6] and use the matrix Hamilton equation and the matrix Hamiltonian H for determining an evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \quad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}, \tag{8}$$

where H is 3×3 matrix, Ψ is the three-component spin wave function (spinor), $H_{ij} = H^*_{ji}$ and $i, j = 1, 0, -1$.

A determination of spin dynamics can be divided into several stages, namely

i) a solution of Hamilton equation (8) and a determination of eigenvalues and eigenvectors of the Hamilton matrix H;

ii) a derivation of spin wave function consisting in a solution of a set of three linear algebraic equations;

iii) a calculation of time evolution of polarization vector and polarization tensor.

3 HAMILTON OPERATOR IN A CYLINDRICAL COORDINATE SYSTEM

The spin dynamics can be analytically calculated when a storage ring is either circular or divided into circular sectors by empty spaces. In this case, the use of cylindrical coordinates can be very successful. Equation of spin motion in storage rings in a cylindrical coordinate system has the form [7]

$$\begin{gathered}\frac{d\boldsymbol{S}}{dt}=\boldsymbol{\omega}_a\times\boldsymbol{S},\quad \boldsymbol{\omega}_a=-\frac{e}{m}\Biggl\{a\boldsymbol{B}-\frac{a\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B})\\ +\left(\frac{1}{\gamma^2-1}-a\right)(\boldsymbol{\beta}\times\boldsymbol{E})+\frac{1}{\gamma}\left[\boldsymbol{B}_{\|}-\frac{1}{\beta^2}(\boldsymbol{\beta}\times\boldsymbol{E})_{\|}\right]\\ +\frac{\eta}{2}\left(\boldsymbol{E}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})+\boldsymbol{\beta}\times\boldsymbol{B}\right)\Biggr\},\end{gathered}\qquad (9)$$

where $a=(g-2)/2,\ g=2\mu m/(eS)$, $\eta=2dm/(eS)$, and d is the EDM. The sign $\|$ means a horizontal projection for any vector.

$$\boldsymbol{\omega}_a=\boldsymbol{\Omega}-\dot{\phi}\boldsymbol{e}_z, \tag{10}$$

where $\boldsymbol{\Omega}$ is the Thomas-Bargmann-Michel-Telegdi (T-BMT) frequency [8] corrected for the EDM [7, 9, 10, 11] and $\dot{\phi}\boldsymbol{e}_z$ is the instantaneous angular frequency of orbital revolution.

The Hamiltonian in the rotating frame has the form

$$\mathcal{H}=\mathcal{H}_0+\boldsymbol{S}\cdot\boldsymbol{\omega}_a, \tag{11}$$

where $\boldsymbol{\omega}_a$ is defined by Eq. (9).

The particle in the rotating frame is localized and ideally is in rest. Therefore, we can direct the x- and y-axes in this frame along the radial and longitudinal axes, respectively. This procedure is commonly used (see Refs. [3, 4, 5]) and results in the direct substitution of spin matrices (3) for S_ρ and S_ϕ:

$$S_\rho = S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_\phi = S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}. \qquad (12)$$

The matrix S_z remains unchanged.

4 CORRECTIONS TO THE HAMILTON OPERATOR FOR TENSOR POLARIZABILITIES OF DEUTERON

Correction to the Hamilton operator for the magnetic polarizability of the deuteron contains scalar and tensor parts. The scalar part is spin-independent and can be disregarded.

The interaction Hamiltonian depending on the magnetic polarizability is given by

$$V = -\frac{1}{2}\beta_{ik} B'_i B'_k, \qquad (13)$$

where β_{ik} is the tensor of magnetic polarizability, $\boldsymbol{B'}$ is the effective field acting on a particle (fields in the particle's rest frame, i.e., in the rotating frame). The spin-dependent part of the Hamiltonian defined by the tensor magnetic polarizability is equal to [1, 2]

$$V = -\beta_T(\boldsymbol{S}\cdot\boldsymbol{B'})^2, \tag{14}$$

where β_T is the tensor magnetic polarizability.

The correction to the Hamilton operator in the rotating frame is equal to

$$V = -\frac{1}{2\gamma}\boldsymbol{m'}\cdot\boldsymbol{B'} = -\frac{\beta_T}{\gamma}(\boldsymbol{S}\cdot\boldsymbol{B'})^2. \tag{15}$$

Eq. (15) can be transformed to the form

$$V = -\beta_T\gamma B_z^2 S_z^2. \tag{16}$$

5 MEASUREMENT OF TENSOR MAGNETIC POLARIZABILITY OF THE DEUTERON IN STORAGE RINGS

Baryshevsky et al. [1, 2] have shown the tensor magnetic polarizability causes the spin rotation with two frequencies, ω_1 and ω_2, instead of ω_0 and therefore experiences beating with the frequency $\omega_1 - \omega_2 \approx \beta_T B^2$. This effect can be discovered in storage ring experiments. In this section, we derive general formulae describing spin dynamics and consider the possibility to measure the tensor magnetic polarizability of the deuteron.

Let us consider spin dynamics of deuteron beam in a uniform magnetic field. In this case, the matrix Hamiltonian has the form

$$H = \begin{pmatrix} E_0 + \omega_0 + \mathcal{B} & 0 & 0 \\ 0 & E_0 & 0 \\ 0 & 0 & E_0 - \omega_0 + \mathcal{B} \end{pmatrix}, \qquad (17)$$

where

$$\mathcal{B} = -\beta_T \gamma_0 B_z^2, \tag{18}$$

ω_0 is the angular frequency of spin rotation (g−2 frequency), and E_0 is the zero energy level.

If the deuteron beam is vector-polarized and the direction of its polarization is defined by the spherical angles θ and ψ, the evolution of three components of polarization vector is given by

$$\begin{gathered} P_\rho(t) = \sin\theta\cos(\omega_0 t + \psi)\cos(b_0 t) - \sin\theta\cos\theta\sin(\omega_0 t + \psi)\sin(b_0 t), \\ P_\phi(t) = \sin\theta\sin(\omega_0 t + \psi)\cos(b_0 t) + \sin\theta\cos\theta\cos(\omega_0 t + \psi)\sin(b_0 t), \\ P_z(t) = P_z(0), \end{gathered} \tag{19}$$

where $\Delta\omega = \omega_0 - \omega$ and the initial vertical polarization is defined by

$$P_z(0) = \cos\theta, \quad P_{zz}(0) = \frac{1}{2}(3\cos^2\theta - 1). \tag{20}$$

These equations confirm the conclusion given by Baryshevsky *at* *al.* [1, 2] that the tensor magnetic polarizability of the deuteron causes the spin rotation with two frequencies. This effect is rather small but not negligible.

When $|\Delta\omega|t \ll 1, \ |b_0|t \ll 1$, Eq. (19) takes the form

$$
\begin{aligned}
P_\rho(t) &= \sin\theta\cos(\omega_0 t + \psi) - b_0 t \sin\theta\cos\theta\sin(\omega_0 t + \psi), \\
P_\phi(t) &= \sin\theta\sin(\omega_0 t + \psi) + b_0 t \sin\theta\cos\theta\cos(\omega_0 t + \psi), \\
P_z(t) &= \cos\theta.
\end{aligned} \tag{21}
$$

Since $b_0 \sim 10^{-5}\ \text{s}^{-1}$ and the expected duration of measurement $t \sim 10^3$ s, $b_0 t \sim 10^{-2}$. Therefore, the effect of the tensor magnetic polarizability on the spin rotation in the horizontal plane can be observed.

We propose the significant improvement of precision of a possible experiment. Conditions of observation of effect conditioned by the tensor magnetic polarizability can be much better with the use of a tensor-polarized deuteron beam. If the initial vector polarization of such a beam in zero, the interaction of the magnetic moment of the deuteron cannot lead to the appearance of any vector polarization. Therefore, nonzero vector polarization of the beam can be conditioned by nothing but the tensor interactions. When the projection of the deuteron spin onto the direction defined by the spherical angles θ and ψ is fixed and is equal to zero, the time dependence of the polarization vector is given by

$$
\begin{gathered}
P_\rho(t) = 2\sin\theta\cos\theta\sin(\omega_0 t + \psi)\sin(b_0 t), \\
P_\phi(t) = -2\sin\theta\cos\theta\cos(\omega_0 t + \psi)\sin(b_0 t), \\
P_z(t) = 0.
\end{gathered}
\tag{22}
$$

In the considered case

$$
P_\rho(0) = P_\phi(0) = P_z(0) = 0, \quad P_{zz}(0) = -3\cos^2\theta + 1. \tag{23}
$$

When $|\Delta\omega|t \ll 1,\ |b_0|t \ll 1$, Eq. (22) takes the form

$$\begin{aligned} P_\rho(t) &= 2b_0 t \sin\theta\cos\theta\sin\left(\omega_0 t + \psi\right), \\ P_\phi(t) &= -2b_0 t \sin\theta\cos\theta\cos\left(\omega_0 t + \psi\right), \\ P_z(t) &= 0. \end{aligned} \tag{24}$$

Eq. (24) shows the possibility of measurement of the tensor magnetic polarizability of the deuteron in storage ring experiments.

6 TENSOR MAGNETIC POLARIZABILITY OF THE DEUTERON IN THE EDM EXPERIMENT

Tensor magnetic polarizability of the deuteron also affects the spin dynamics in the deuteron EDM experiment in storage rings [1, 2]. In this section, an influence of the tensor magnetic polarizability of the deuteron on the spin motion in the EDM experiment is calculated in detail. We take into consideration the EDM and the tensor magnetic polarizability of the deuteron.

In this case, the matrix Hamiltonian is given by

$$H = \begin{pmatrix} E_0 + \omega_0 + \mathcal{B} & \mathcal{E} & 0 \\ \mathcal{E}^* & E_0 & \mathcal{E} \\ 0 & \mathcal{E}^* & E_0 - \omega_0 + \mathcal{B} \end{pmatrix}, \tag{25}$$

where

$$\begin{gathered} \mathcal{B} = b_0 + b_1 \cos{(\omega t + \varphi)} + b_2 \cos{[2(\omega t + \varphi)]}, \\ b_0 = -\beta_T B_z^2 \gamma_0 \left[1 + \frac{1}{4}(1 + 3\beta_0^2\gamma_0^2)\gamma_0^2(\Delta\beta_0)^2\right], \\ b_1 = -\beta_T B_z^2 \beta_0 \gamma_0^3 \cdot \Delta\beta_0, \\ b_2 = -\frac{1}{4}\beta_T B_z^2 (1 + 3\beta_0^2\gamma_0^2)\gamma_0^3(\Delta\beta_0)^2, \\ \mathcal{E} = \mathcal{E}_0 \exp{[-i(\omega t + \varphi)]}, \quad \mathcal{E}_0 = \frac{e\eta\omega_c}{8\sqrt{2}\pi p_0} E_0 l \left(\frac{\omega_0}{a\gamma_0^2\omega} \pm 1\right). \end{gathered} \tag{26}$$

If the deuteron beam is vector-polarized and the direction of its polarization is defined by the spherical angles θ and ψ, the evolution of three components of polarization vector is given by

$$
\begin{aligned}
P_\rho(t) &= \sin\theta\cos(\omega_0 t+\psi)\cos(b_0 t) - \sin\theta\cos\theta\sin(\omega_0 t+\psi)\sin(b_0 t) \\
&+\sqrt{2}[P_{zz}(0)+P_z(0)]\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\
&-\sqrt{2}[P_{zz}(0)-P_z(0)]\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t \\
&+\frac{1}{\sqrt{2}}\sin^2\theta[\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t \\
&-\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t],
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
P_\phi(t) &= \sin\theta \sin(\omega_0 t + \psi)\cos(b_0 t) + \sin\theta\cos\theta\cos(\omega_0 t + \psi)\sin(b_0 t) \\
&-\sqrt{2}[P_{zz}(0) + P_z(0)]\frac{\mathcal{E}_0}{\Delta\omega + b_0}\cos\left(\frac{\omega_0 + \omega + b_0}{2}t + \varphi\right)\sin\frac{\Delta\omega + b_0}{2}t \\
&+\sqrt{2}[P_{zz}(0) - P_z(0)]\frac{\mathcal{E}_0}{\Delta\omega - b_0}\cos\left(\frac{\omega_0 + \omega - b_0}{2}t + \varphi\right)\sin\frac{\Delta\omega - b_0}{2}t \\
&+\frac{1}{\sqrt{2}}\sin^2\theta\Big[\frac{\mathcal{E}_0}{\Delta\omega + b_0}\sin\left(\frac{3\omega_0 - \omega - b_0}{2}t + 2\psi - \varphi\right)\sin\frac{\Delta\omega + b_0}{2}t \\
&-\frac{\mathcal{E}_0}{\Delta\omega - b_0}\cos\left(\frac{3\omega_0 - \omega + b_0}{2}t + 2\psi - \varphi\right)\sin\frac{\Delta\omega - b_0}{2}t\Big], \\
P_z(t) &= P_z(0) \\
&+\sqrt{2}\sin\theta(1 + \cos\theta)\frac{\mathcal{E}_0}{\Delta\omega + b_0}\sin\left(\frac{\Delta\omega + b_0}{2}t + \psi - \varphi\right)\sin\frac{\Delta\omega + b_0}{2}t \\
&+\sqrt{2}\sin\theta(1 - \cos\theta)\frac{\mathcal{E}_0}{\Delta\omega - b_0}\sin\left(\frac{\Delta\omega - b_0}{2}t + \psi - \varphi\right)\sin\frac{\Delta\omega - b_0}{2}t,
\end{aligned}
\tag{28}
$$

where $\Delta\omega = \omega_0 - \omega$ and the initial vertical polarization is defined by

$$
\begin{aligned}
P_z(0) = \cos\theta, \quad P_{zz}(0) &= \frac{1}{2}(3\cos^2\theta - 1), \\
P_{zz}(0) + P_z(0) &= \frac{1}{2}(1+\cos\theta)(3\cos\theta - 1), \\
P_{zz}(0) - P_z(0) &= -\frac{1}{2}(1-\cos\theta)(1+3\cos\theta).
\end{aligned} \tag{29}
$$

These equations confirm the conclusion given by Baryshevsky *at al.* [1, 2]. The oscillation with two frequencies takes place not only for the horizontal components of the polarization vector, but also for the vertical component [1, 2].

When $|\Delta\omega|t \ll 1,\ |b_0|t \ll 1$, Eqs. (27),(28) take the form

$$\begin{aligned} P_\rho(t) = {} & \sin\theta\cos\left(\omega_0 t+\psi\right) - b_0 t\sin\theta\cos\theta\sin\left(\omega_0 t+\psi\right) \\ & +\sqrt{2}\mathcal{E}_0 t\cos\theta\sin\left(\frac{\omega_0+\omega}{2}t+\varphi\right) \\ & +\frac{1}{\sqrt{2}}P_{zz}(0)\mathcal{E}_0 b_0 t^2\cos\left(\omega_0 t+\varphi\right) \\ & +\frac{1}{2\sqrt{2}}\mathcal{E}_0 b_0 t^2\sin^2\theta\cos\left(\omega_0 t+2\psi-\varphi\right), \end{aligned} \tag{30}$$

$$\begin{aligned} P_\phi(t) = {} & \sin\theta\sin\left(\omega_0 t+\psi\right) + b_0 t\sin\theta\cos\theta\cos\left(\omega_0 t+\psi\right) \\ & -\sqrt{2}\mathcal{E}_0 t\cos\theta\cos\left(\frac{\omega_0+\omega}{2}t+\varphi\right) \\ & +\frac{1}{\sqrt{2}}P_{zz}(0)\mathcal{E}_0 b_0 t^2\sin\left(\omega_0 t+\varphi\right) \\ & +\frac{1}{2\sqrt{2}}\mathcal{E}_0 b_0 t^2\sin^2\theta\sin\left(\omega_0 t+2\psi-\varphi\right), \end{aligned} \tag{31}$$

$$P_z(t) = \cos\theta + \sqrt{2}\mathcal{E}_0 t\sin\theta\sin\left(\psi-\varphi\right).$$

Since $b_0 \sim 10^{-5}\ \mathrm{s}^{-1}$ and the expected duration of measurement

$t \sim 10^3$ s, $b_0 t \sim 10^{-2}$.

The effect of the tensor magnetic polarizability on the spin rotation in the horizontal plane can be observed. However, Eqs. (28),(31) show the effect of the tensor magnetic polarizability on the buildup of the vertical polarization in the EDM experiment is negligible. Maximum corrections to the main result caused by this effect are of order of $b_0 t$. Therefore, the tensor magnetic polarizability of the deuteron need not be taken into account in the EDM experiment.

If the initial vector polarization of the deuteron beam in zero, the nonzero vector polarization can be conditioned by nothing but the tensor interactions. When the projection of the deuteron spin onto the direction defined by the spherical angles θ and ψ is fixed and is equal to zero, the time dependence of the polarization vector is given by

$$
\begin{aligned}
P_\rho(t) &= 2\sin\theta\cos\theta\sin\left(\omega_0 t+\psi\right)\sin\left(b_0 t\right) \\
&+\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\
&-\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t \\
&+\sqrt{2}\sin^2\theta[\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\
&-\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t],
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
P_\phi(t) &= -2\sin\theta\cos\theta\cos\left(\omega_0 t+\psi\right)\sin\left(b_0 t\right)\\
&-\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega+b_0}\cos\left(\frac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t\\
&+\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\frac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t\\
&+\sqrt{2}\sin^2\theta[\frac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\frac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t\\
&-\frac{\mathcal{E}_0}{\Delta\omega+b_0}\cos\left(\frac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t],\\
P_z(t) &= -2\sqrt{2}\sin\theta\cos\theta\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{\Delta\omega+b_0}{2}t+\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t\\
&+2\sqrt{2}\sin\theta\cos\theta\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{\Delta\omega-b_0}{2}t+\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t.
\end{aligned}
\tag{33}
$$

In the considered case

$$
P_\rho(0)=P_\phi(0)=P_z(0)=0,\qquad P_{zz}(0)=-3\cos^2\theta+1. \tag{34}
$$

When $|\Delta\omega| t \ll 1,\ |b_0| t \ll 1$, Eqs. (32),(33) take the form

$$
\begin{aligned}
P_\rho(t) &= 2b_0 t \sin\theta\cos\theta\sin\left(\omega_0 t + \psi\right) \\
&+ \frac{1}{\sqrt{2}} P_{zz}(0)\mathcal{E}_0 b_0 t^2 \cos\left(\omega_0 t + \varphi\right) \\
&- \frac{1}{\sqrt{2}}\mathcal{E}_0 b_0 t^2 \sin^2\theta\cos\left(\omega_0 t + 2\psi - \varphi\right),
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
P_\phi(t) &= -2b_0 t \sin\theta\cos\theta\cos\left(\omega_0 t + \psi\right) \\
&+ \frac{1}{\sqrt{2}} P_{zz}(0)\mathcal{E}_0 b_0 t^2 \sin\left(\omega_0 t + \varphi\right) \\
&- \frac{1}{\sqrt{2}}\mathcal{E}_0 b_0 t^2 \sin^2\theta\sin\left(\omega_0 t + 2\psi - \varphi\right),
\end{aligned} \tag{36}
$$

$$
P_z(t) = 0.
$$

7 CONCLUSIONS

- **The previous results by Baryshevsky *at al.* [1, 2] have been confirmed. The tensor magnetic polarizability of the deuteron causes the spin rotation with two frequencies and experiences beating that frequency is proportional to the tensor magnetic polarizability.**

- **The tensor magnetic polarizability of the deuteron can be measured in storage ring experiments. For this purpose, we propose to use the tensor polarized beam. The final vector polarization can be of order of 1%.**

- **The effect of the tensor magnetic polarizability on the buildup of the vertical polarization in the planned deuteron EDM experiment is negligible.**

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