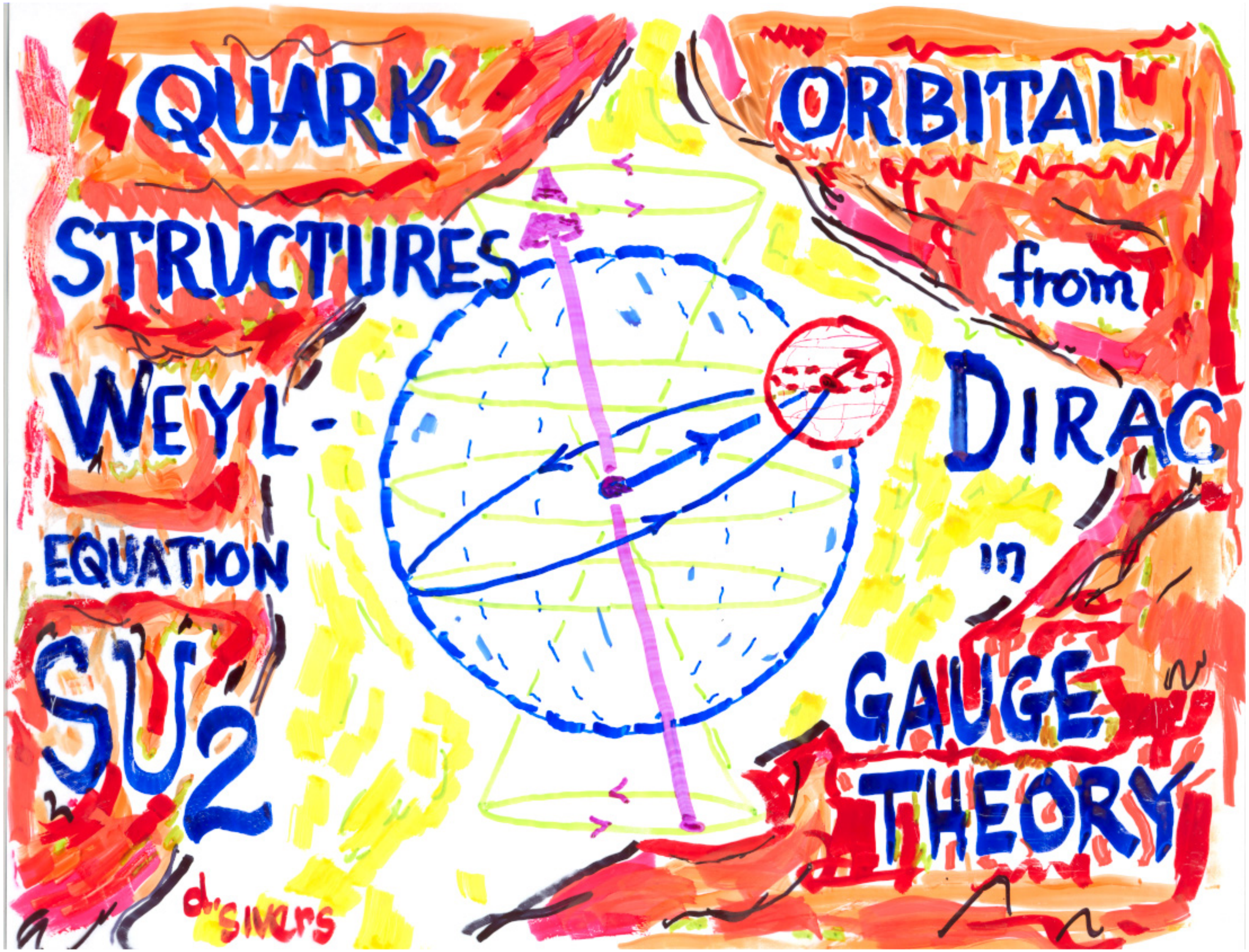

QUARK
ORBITAL
STRUCTURES
from
WEYL-
DIRAC
EQUATION
in
SU2
GAUGE
THEORY
d. sivers

OUTLINE

I. A_τ-odd Quantum Structures

a.) Mulders, Tangerman Classification

2 Dstn. fcns   2 fragmentation fcns.

Spin/Orbit Dynamics

b.) Origin of A_τ-odd dynamics

Confinement ⇄ Chiral Dynamics

II. The Collins Functions

Spin/orbit Dynamics in Fragmentation & Fracture functions

II. The Chiral Quark Model

Spin Orbit Dynamics & Normalization of Orbital Distributions & Boer-Mulders functions for the nucleon

III. SU_2 Gauge Thy.

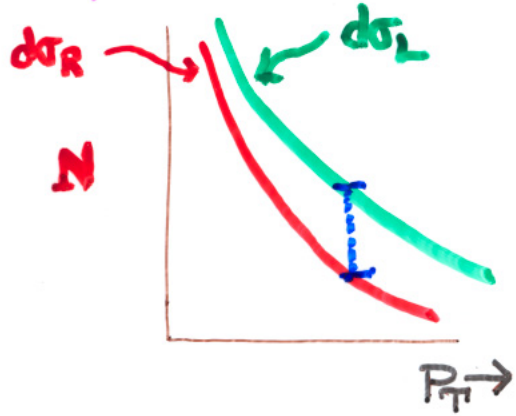
a). Spherical ansatz → 2-dim Abelian Higgs model

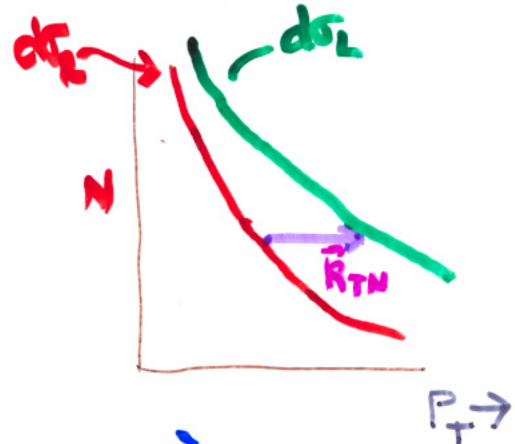
b). Weyl-Dirac eqn. & A_T-odd solutions

Process Dependence & Wilson Operators

SPIN-ORIENTED MOMENTUM

Single-Spin Asymmetries always involve a spin-oriented momentum



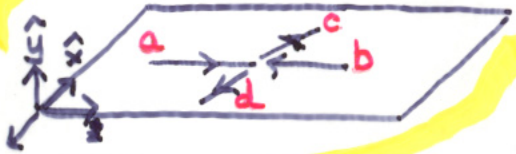


$$A_N \, d\sigma = d\sigma_L - d\sigma_R$$

2 equivalent descriptions

$\vec{k}_{TN}$ = momentum transferred by spin orientation in scattering

The form is highly constrained by finite symmetries and rotational invariance



	Σ_x	Σ_y	Σ_z
C	+	+	−
P	−	+	−
T	−	+	+
(CPT)	+	+	+
O	−	−	−
A_τ	+	−	+

Parity - Hodge*

$$P(A,V) \to (A,-V)$$
$$*(A,V) \to (V,A)$$
$$O(A,V) \to (-A,V)$$

$$A_\tau = PO$$
$$PA_\tau = O$$
$$A_\tau O = P$$

The Operators $\mathbb{1}, \mathbb{P}, \Theta, A_\tau$ form Group

$\mathbb{P}^2 = \Theta^2 = A_\tau^2 = 1 \Rightarrow$ all single spin obs.

fall into one of 2 categories:

1. $\mathbb{P}$-odd, A_τ-even
2. $\mathbb{P}$-even, A_τ-odd

QCD pert. theory $\mathbb{P}$-conserved, A_τ only broken by mass effects.

The Idempotent Operator $\Pi_A^- = \left(\frac{1 - A_\tau}{2}\right)$ projects spin/orbit dynamics & hence spin/oriented momentum

$\mathcal{T}$ vs. A_τ

$\mathcal{T}$ antiunitary $\quad \mathcal{T}|a\rangle_{in} = |a_t\rangle_{out}$

A_τ unitary $\quad A_\tau|a\rangle_{in} = |a_{A_\tau}\rangle_{in}$

$\mathcal{T}: (\vec{p}_i, \vec{\sigma}_i) \to (-p_i, -\sigma_i)$ $\quad A_\tau: (+\vec{p}_i, +\vec{\sigma}_i) \to (-\vec{p}_i, -\vec{\sigma}_i)$

only for single-particle
non-interacting states
can they be confused

Time reflection reverses the order of operators!!

$$\mathcal{T}(\vec{\sigma}\cdot(\vec{k}_1 \times \vec{k}_2)) = \vec{\sigma}\cdot(\vec{k}_1 \times \vec{k}_2)$$

$$A_\tau(\vec{\sigma}\cdot(\vec{k}_1 \times \vec{k}_2)) = -\vec{\sigma}\cdot(\vec{k}_1 \times \vec{k}_2)$$

Cross products that are connected to spin/orbit dynamics have been called $\mathcal{T}$-odd

4 "Leading-Twist" Quantum Structures

MULDERS – TANGERMAN (A_γ'odd)

$\Delta^N D_{\pi/q\uparrow}(z, k_{TN}; \mu^2)$

Collins Fcns.

$\Delta^N D_{\Lambda\uparrow q}(z, k_{TN}; \mu^2)$

Polarizing Fragmentation Fcns.

$\Delta^N G^{front}_{q\uparrow/p}(x, k_{TN}(x); \mu^2)$

Boer-Mulders Fcns.

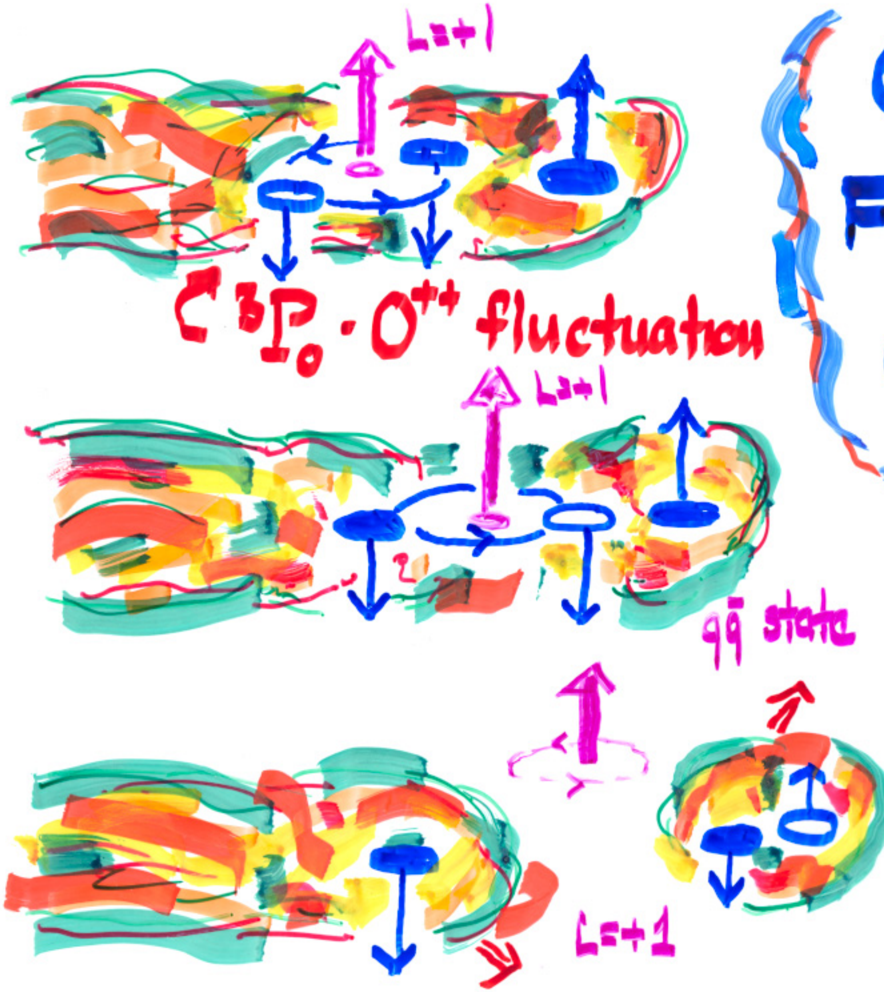
$\Delta^N G^{front}_{q/p\uparrow}(x, k_{TN}(x); \mu^2)$

Orbital Distribution Fcns

Quark Transverse Spin
"Chiral" Observables

Hadronic Transverse Spin

These functions have a common Dynamical origin



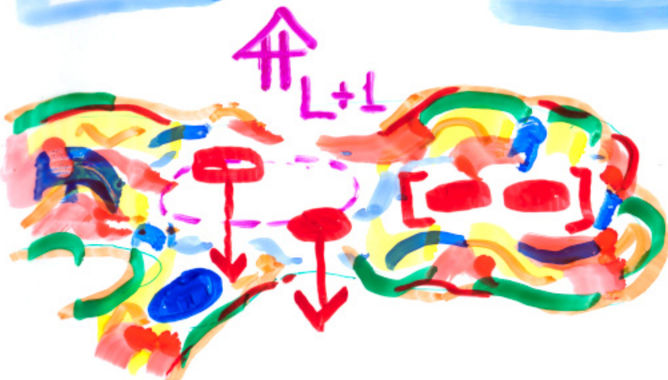
COLLINS FUNCTION

$q_i\uparrow \to q\downarrow_j \pi_{i\bar{j}}$

final state directly reflects orbital angular momentum

$\langle \vec{L}\cdot\hat{\sigma}_q \rangle$

SOME ASSEMBLY REQUIRED



[● ●] favored $I^S = 0^+$ diquark

Interplay of confinement & chiral mechanisms on display in polarizing Fracture & Fragmentation Fcn's for Baryons

Validates DeGrand Miettinen phenomenology for hyperon Polarization Data

Virtual

$\vec{L} \cdot \sigma_p$ Components Provide Spin/Oriented Momentum

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For Distribution Functions

FACTORIZATION but not UNIVERSALITY

Orbital Distributions & Boer-Mulders Distributions Measure an Intrinsic Property of the Nucleon $\langle \vec{L} \cdot \hat{\sigma}_p \rangle$ and $\langle \vec{L}_q \cdot \hat{\sigma}_q \rangle$

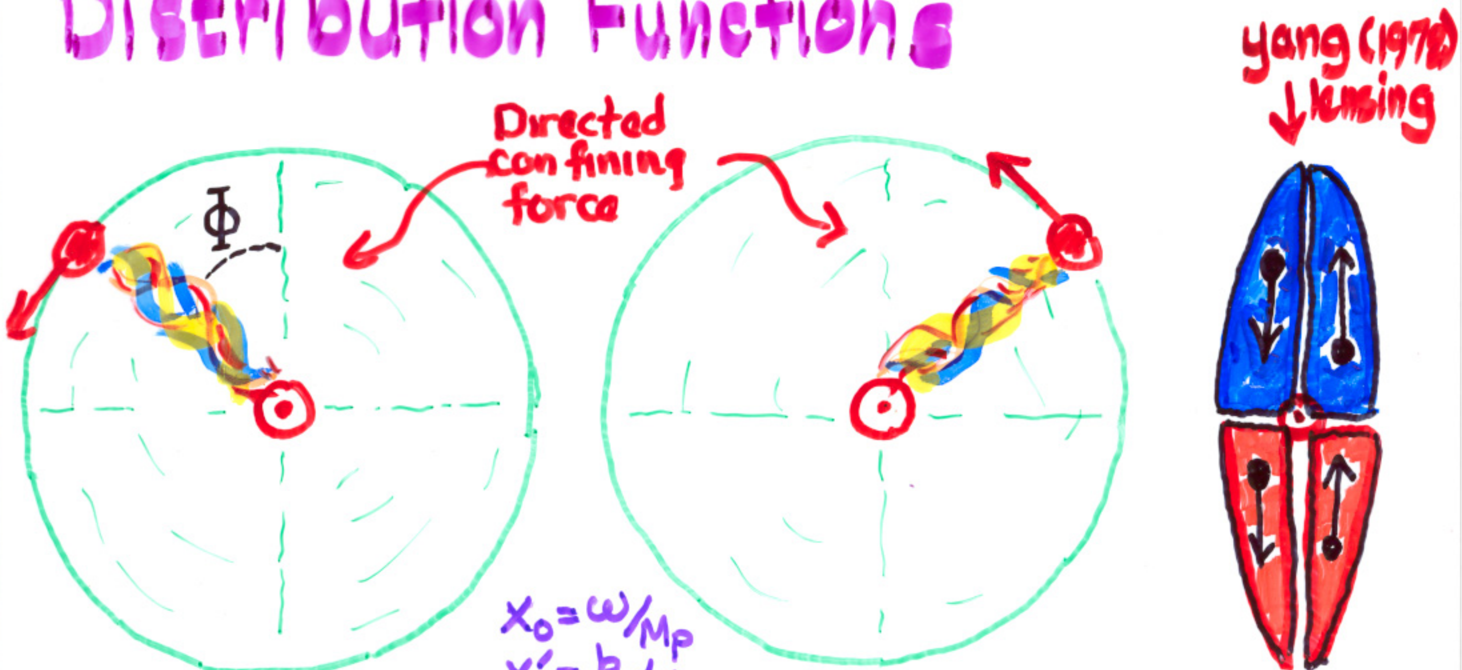
All A_τ-odd dynamics can be factorized into $\Delta^N G^{front}_{q/p\uparrow}(x, k_{TN}(x); \mu^2)$; $\Delta^N G^{front}_{q\uparrow/p}(x, k_{TN}(x); \mu^2)$

but significant Process-Dependence must occur!

Rotating Constituents:

Distribution Functions

$dN = \langle \vec{L}\cdot\hat{\sigma}\rangle \, d\phi$



$x_0 = \omega / M_p$

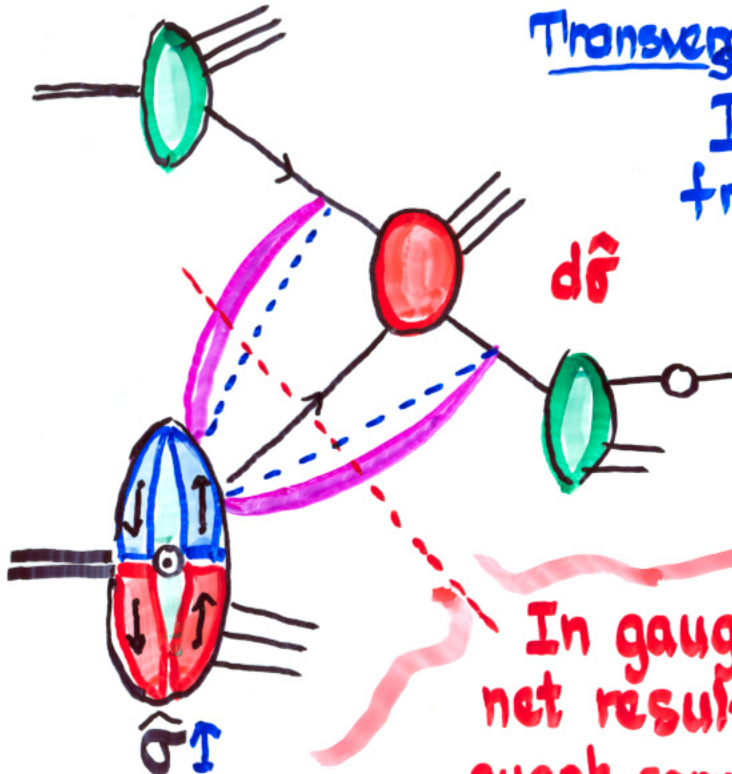
$x' = k_0 / M_p$

$$k_M = (\omega, -k_0\sin\phi, 0, -k_0\cos\phi)$$

$$\omega = (m_q^2 + \langle k_y^2\rangle + k_0^2)^{1/2}$$

$$x = \frac{k_-}{P_-} = x_0 + x'\cos\phi \qquad k_{TN} = -M_p x'\sin\phi = -M_p\left[x'^2 - (x-x_0)^2\right]^{1/2}$$

Hard-Scattering Factorization



Transverse Projection

Preferential Scattering from Segment of orbit influenced by ISI & FSI --- (local)

In gauge-link formulation net result given by nonlocal quark correlator $\propto k_{TN}$

$$e^{ip\xi}\langle p,S|\bar{\psi}(0)\exp\{i\int A_a^{\mu}T^a d\lambda_{\mu}\}\,\Gamma\,\exp\{-i\int A_a^{\mu}T^a d\lambda_{\mu}\}\,\psi(\xi)|pS\rangle$$

[Deconstructing ISI & FSI in SINGLE SPIN OBSERVABLES gives information on The Intrinsic]

Information ---

correlation of orbital angular momentum and proton spin for each **quark flavor**

$\langle \vec{L}_i \cdot \hat{\sigma}_p \rangle$

antiquark flavor

gluons

and self correlation $\langle \vec{L}_i \cdot \hat{\sigma}_{q_i} \rangle$ for quarks

Orbital Dist'ns

$$\Delta^N G^{front}_{q/p\uparrow} = \frac{\langle \vec{L}_q \cdot \hat{S}_p \rangle}{2\pi} \frac{M_p}{k_{TN}(x)}$$

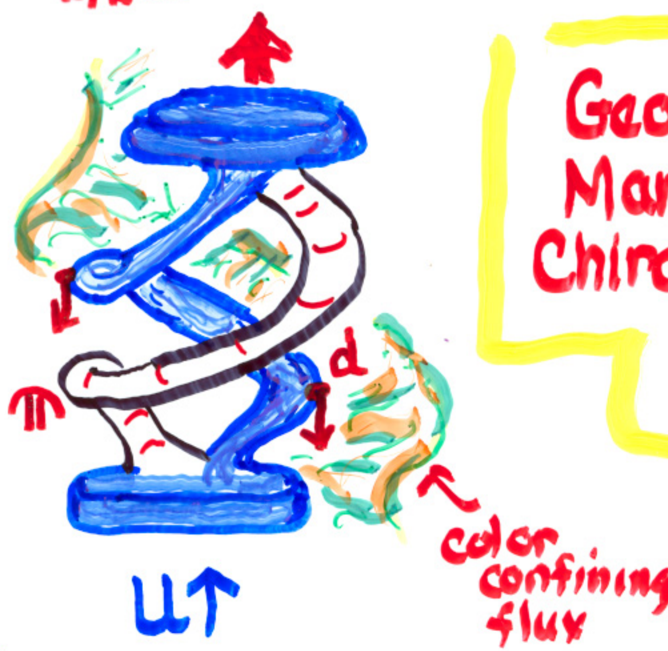
$$\Delta^N G^{front}_{g/p\uparrow} = \frac{\langle \vec{J}_g \cdot \hat{S} \rangle}{2\pi} \frac{M_p}{k_{TN}(x)}$$

Boer-Mulders Dist'ns

$$\Delta^N G^{front}_{q\uparrow/p} = \frac{\langle \vec{L}_q \cdot \vec{S}_q \rangle}{2\pi} \frac{M_p}{k_{TN}(x)}$$

$u\uparrow \to d\downarrow \pi^+(u\bar{d})$

$L = +1$



Georgi Manohar Chiral Quark Model

Existing Model Normalizes all A_T-odd Distributions (Same as Collins fcns. $q\uparrow \to q'\downarrow(\bar{q}q)$)

GEORGI - MANOHAR CHIRAL QUARK MODEL

an effective field theory that gives resolution structures for <u>Constituent</u> <u>Quarks</u>

$$U\uparrow \rightarrow [1-\eta_B-\alpha_c(1+\varepsilon_s+\varepsilon_0)]\,u\uparrow \quad (L=0, A_\gamma=+)$$

$$+[\eta_B u\downarrow + \alpha_c[d\downarrow(\bar{d}u)_{\pi^+} + \varepsilon_s s\downarrow(\bar{s}u)_{K^+} + \varepsilon_0 u\downarrow(\bar{u}u)_{\pi^0+\eta+\eta'}]] \left(\begin{matrix} L=1 \\ A_\gamma=- \end{matrix}\right)$$

$$D\downarrow \rightarrow [1-\eta_B-\alpha_c(1+\varepsilon_s+\varepsilon_0)]\,d\downarrow \quad (L=0, A_\gamma=+)$$

$$+[\eta_B d\uparrow + \alpha_c[u\uparrow(\bar{u}d)_{\pi^-} + \varepsilon_s s\uparrow(\bar{s}d)_{K^0} + \varepsilon_0 d\uparrow(\bar{d}d)_{\pi^0}]] \quad (L=1, A_\gamma=-)$$

$$+U\downarrow(L=0,-1) \quad + D\uparrow(L=+1,0)$$

couplings fixed by low-energy properties

After Some Algebra

$\langle \vec{J}_g \cdot \hat{\sigma}_p(\mu^2) \rangle = 0.10 \pm 0.02$

$\langle \vec{L}_q \cdot \hat{\sigma}_p(\mu^2) \rangle = \begin{cases} u: 0.197 \pm 0.02 \\ d: -0.012 \pm 0.01 \\ s: 0.15 \pm 0.01 \end{cases}$

Gluon & Quark Orbital Dstn's

$\langle \vec{L}_{\bar{q}} \cdot \hat{\sigma}_p(\mu^2) \rangle = \begin{cases} \bar{u}: 0.015 \pm 0.005 \\ \bar{d}: 0.053 \pm 0.006 \\ \bar{s}: 0.023 \pm 0.003 \end{cases}$

Antiquark Orbital Distributions

$\langle \vec{L}_q \cdot \hat{\sigma}_q(\mu^2) \rangle = \begin{cases} u: -0.160 \pm 0.02 \\ d: -0.125 \pm 0.02 \\ s: -0.045 \pm 0.01 \end{cases}$

Boer-Mulders functions normalized

Large and negative

all at $\mu^2 \simeq 1-2\ GeV^2$

$\sum_{i=q,\bar{q}} \langle \vec{L}_i \cdot \hat{\sigma}_p(\mu^2) \rangle = 0.292 \pm 0.03$

$\Sigma_b = 0.215 \pm 0.02$

$\Sigma_T = 0.215 \pm 0.02$

$\langle J_q \cdot \hat{\sigma}_p(\mu^2) \rangle = 0.40 \pm 0.02$

$g_A/g_V = 1.25 \pm 0.05$

Combined normalization of all orbital distributions & Boer-Mulders fcns from a model not designed to describe Spin asymmetries

SU_2 Gauge Thy. in Spherical Coordinates

Jacob, Wudka
Ralston, Sinez

$$gA_0^a(r,t) = A_0(r,t)\,\hat{r}_a$$

$$gA_i^a(r,t) = A_1(r,t)\,\rho_{ia} + \frac{a(r,t)\sin\beta(r,t)}{r}\,\delta_{ia}^T + \frac{a(r,t)\cos\beta(r,t) - 1}{r}\,\varepsilon_{ia}^T$$

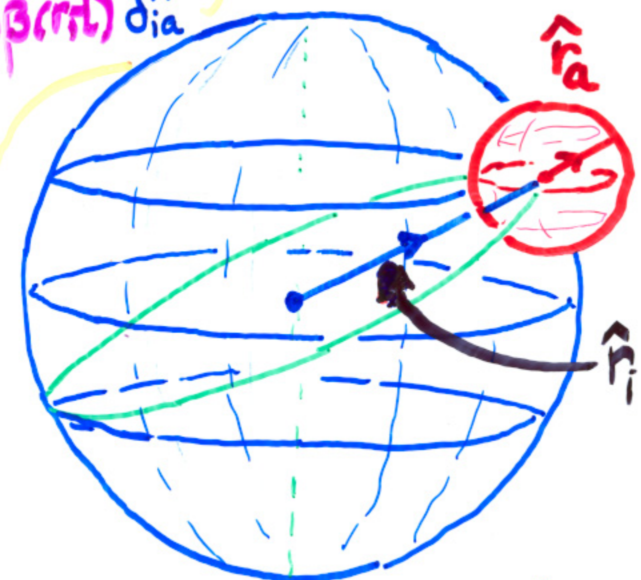
$$\rho_{ia} = \hat{r}_i\,\hat{r}_a$$

$$\delta_{ia}^T = \delta_{ia} - \hat{r}_i\hat{r}_a = \hat{\theta}_i\hat{\theta}_a + \hat{\phi}_i\hat{\phi}_a$$

$$\varepsilon_{ia}^T = \varepsilon_{ial}\,\hat{r}_l = \hat{\phi}_i\hat{\theta}_a - \hat{\theta}_i\,\phi_a$$

$$\frac{1}{r}\delta_{ia}^T = \partial_i\hat{r}_a$$

$$\frac{1}{r}\varepsilon_{ia}^T = -i[\hat{r}, \partial_i\hat{r}]_a$$



use gauge freedom to orient

$(\hat{r}_i, \hat{\theta}_i, \hat{\phi}_i) - (\hat{r}_a, \hat{\theta}_a, \hat{\phi}_a)$

3 space   group Space

$$\varepsilon^S_{ia}(\beta) = \delta^T_{ia}\cos\beta - \epsilon^T_{ia}\sin\beta$$

$$\varepsilon^A_{ia}(\beta) = \delta^T_{ia}\sin\beta - \varepsilon^T_{ia}\cos\beta$$

Rotations · Gauge Trs.
F preserved

$$R_{ij}(\alpha) = \delta_{ij} + \varepsilon^S_{ij}(-\alpha)$$

$$\Omega_{ab}(\psi) = \delta_{ab} + \varepsilon^S_{ab}(-\psi)$$

FIELD STRENGTHS

$$E_L = \frac{\partial A_0}{\partial r} - \frac{\partial A_1}{\partial t} \qquad B_L = \frac{a^2-1}{r^2}$$

$$E_A = -\frac{1}{r}\partial a/\partial t \qquad B_A = -\frac{a}{r}\left(\frac{\partial\beta}{\partial r} - A_1\right)$$

$$E_S = -\frac{a}{r}(\partial\beta/\partial t - A_0) \qquad B_S = -\frac{1}{r}\partial a/\partial r$$

2 Dim Abelian Higgs

$$F_{\ell m} = \partial_\ell A_m - \partial_m A_\ell$$

$$\Phi = a e^{i\beta}$$

$$D_\ell = \partial_\ell - i A_\ell$$

$$r^2 \mathcal{L}_g = r^2(F_{\ell m}F^{\ell m}) + 2D^\ell\phi\, D_\ell\phi^* + \frac{1}{r^2}(|\phi|^2 - 1)$$

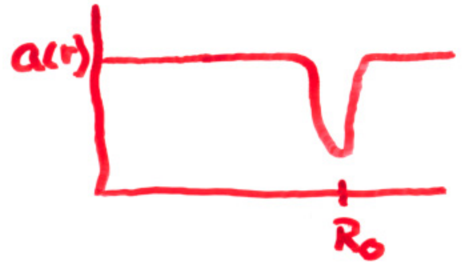
non-trivial topological current

$$K_0 = (a^2-1)A_1 - a^2\partial\beta/\partial r \qquad \partial^\ell K_\ell = g^2 r^2 E^a_i B^a_i$$

$$K_1 = -(a^2-1)A_0 + a^2\partial\beta/\partial t$$

ORBITAL SOLUTIONS

Coordinate gauge: $A_1 \to 0$



$E_L = \sigma \to A_0 = \sigma r$

$B_L = \frac{a^2 - 1}{r^2}$

$E_S = -\frac{a}{r}\,\partial\beta/\partial t - \sigma r$

$B_S = -\frac{1}{r}\,\partial a/\partial r$

orbital current at $r = R_0$

SU_2 color currents in "corridor around $r = R_0$

Electric Confining force

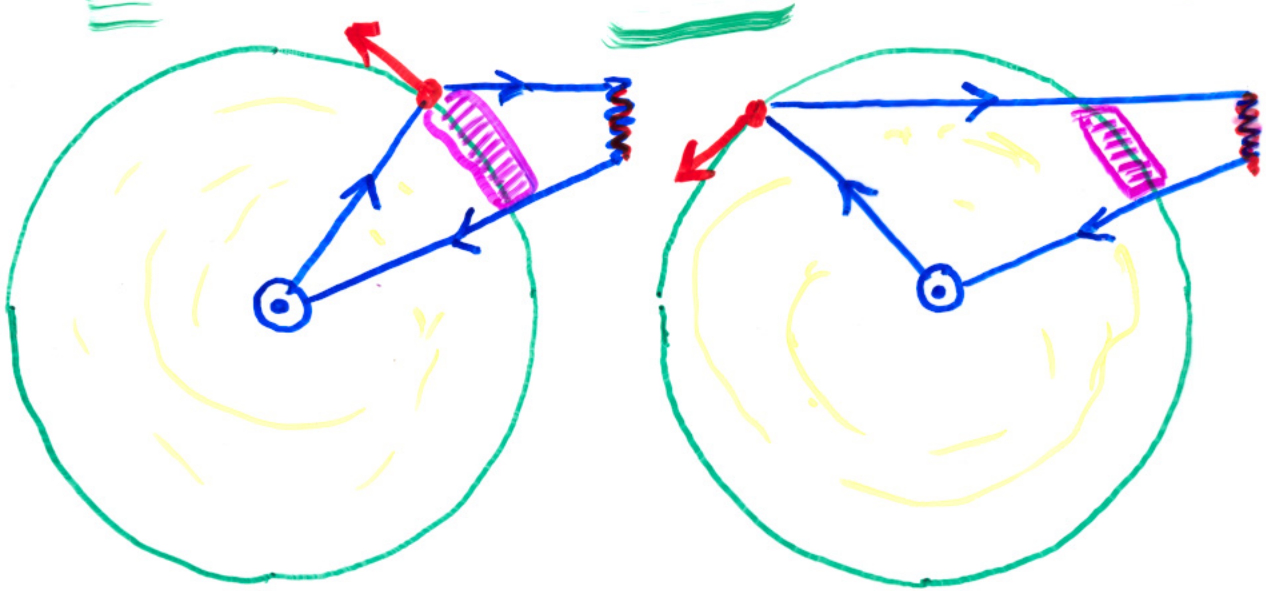
The process-depedence uncovered by Initial-State and Final-State interactions is key to understanding spin-orbit dynamics

Unlocking the connections to Generalized Parton Distributions through

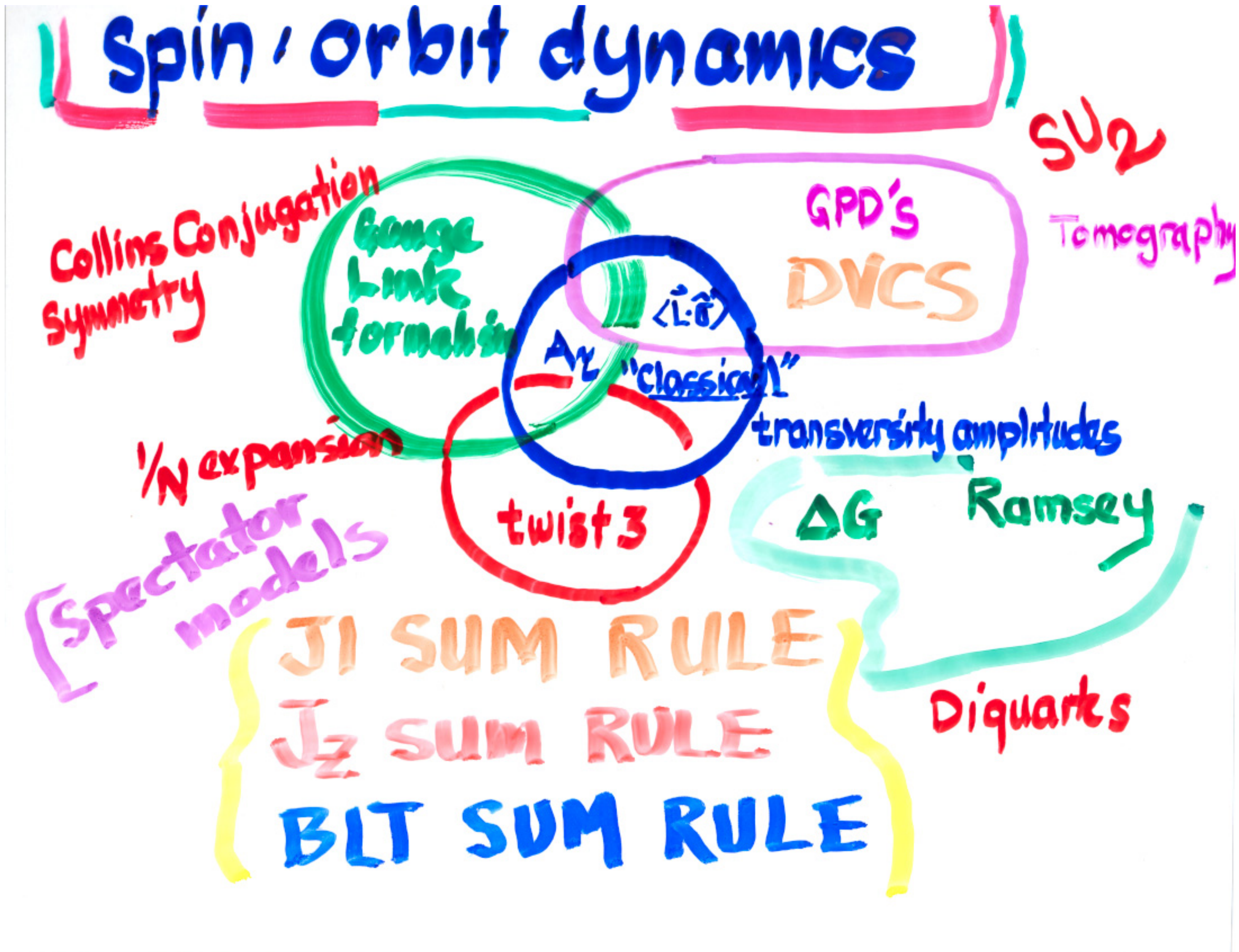
$\langle \vec{L}_q \cdot \hat{\sigma}_p \rangle$ $\langle \vec{L}_g \cdot \hat{\sigma}_p \rangle$ orbital dstn's

$\langle \vec{L}_q \cdot \hat{\sigma}_{q\uparrow} \rangle$ Boer-Mulders dstn's

Final State Interactions Beyond the One-Gluon Level



$\exp\{ig\int A^{\mu}_{a}T_{a}\cdot dl_{\mu}\}$ explicitly dominated by "confinement effects" in coordinate gauge in SU_2


Spin / orbit dynamics
SU2
GPD's
DVCS
Tomography
Collins Conjugation Symmetry
Gauge Link formalism
⟨L·S⟩
"classical"
twist 3
transversity amplitudes
1/N expansion
ΔG
Ramsey
Spectator models
JI SUM RULE
Jz SUM RULE
BLT SUM RULE
Diquarks