Colour Modification of Factorisation in Single-Spin Asymmetries

Philip G. Ratcliffe[1,2] & Oleg Teryaev[3]

[1] Dip.to di Fisica e Matematica, Università dell'Insubria, Como

[2] Istituto Nazionale di Fisica Nucleare, Sezione di Milano–Bicocca

[3] Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna

Outline

Acknowledgments

First of all, many thanks to the organisers for inviting me:

Acknowledgments

First of all, many thanks to the organisers for inviting me:
this is my first time at a DSPIN workshop and also in Dubna.

Acknowledgments

First of all, many thanks to the organisers for inviting me:
this is my first time at a DSPIN workshop and also in Dubna.

The work presented here has been performed in collaboration with Oleg Teryaev (PGR and Teryaev, 2007 and work in progress).

Acknowledgments

First of all, many thanks to the organisers for inviting me:
this is my first time at a DSPIN workshop and also in Dubna.

The work presented here has been performed in collaboration with Oleg Teryaev (PGR and Teryaev, 2007 and work in progress).

The support for the visits of Teryaev to Como was provided in part by the Landau Network (Como) and also by the recently completed (and hopefully to be renewed) Italian ministry-funded PRIN2006 on Transversity.

Acknowledgments

First of all, many thanks to the organisers for inviting me:
this is my first time at a DSPIN workshop and also in Dubna.

The work presented here has been performed in collaboration with Oleg Teryaev (PGR and Teryaev, 2007 and work in progress).

The support for the visits of Teryaev to Como was provided in part by the Landau Network (Como) and also by the recently completed (and hopefully to be renewed) Italian ministry-funded PRIN2006 on Transversity.

Some of the ideas have already been presented at other workshops (Teryaev and PGR, 2008a,b; PGR and Teryaev, 2009b).

Motivation & Interest

The concept of a single-spin asymmetry (SSA) in high-energy hadronic physics has long been an enigma.

Motivation & Interest

The concept of a single-spin asymmetry (SSA) in high-energy hadronic physics has long been an enigma.

Before being first observed experimentally, hadronic SSA's were predicted to be very small—for a variety of reasons.

Motivation & Interest

The concept of a single-spin asymmetry (SSA) in high-energy hadronic physics has long been an enigma.

Before being first observed experimentally, hadronic SSA's were predicted to be very small—for a variety of reasons.

Experimentally, however, SSA's turn out to be very large indeed in many hadronic processes.

Motivation & Interest

The concept of a single-spin asymmetry (SSA) in high-energy hadronic physics has long been an enigma.

Before being first observed experimentally, hadronic SSA's were predicted to be very small—for a variety of reasons.

Experimentally, however, SSA's turn out to be very large indeed in many hadronic processes.

It was also long held that SSA's should eventually vanish with growing CoM energy and/or p_T.

Motivation & Interest

The concept of a single-spin asymmetry (SSA) in high-energy hadronic physics has long been an enigma.

Before being first observed experimentally, hadronic SSA's were predicted to be very small—for a variety of reasons.

Experimentally, however, SSA's turn out to be very large indeed in many hadronic processes.

It was also long held that SSA's should eventually vanish with growing CoM energy and/or p_T.

Again, however, many large SSA's observed so far show no signs of any particular high-energy or p_T suppression.

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_\wedge \vec{k}\right)$$

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_{\wedge} \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector,

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p} _\wedge \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector, while
$\vec{p}$ and $\vec{k}$ are initial/final particle/jet momenta.

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_{\wedge} \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector, while

$\vec{p}$ and $\vec{k}$ are initial/final particle/jet momenta.

(Such a combination is dictated by parity conservation.)

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_\wedge \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector, while
$\vec{p}$ and $\vec{k}$ are initial/final particle/jet momenta.

(Such a combination is dictated by parity conservation.)

A typical example might be

$\vec{p}$ = beam direction

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_{\wedge} \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector, while
$\vec{p}$ and $\vec{k}$ are initial/final particle/jet momenta.

(Such a combination is dictated by parity conservation.)

A typical example might be

$\vec{p}$ = beam direction
$\vec{s}$ = target polarisation (transverse w.r.t. $\vec{p}$)

Single-Spin Asymmetries

Generically, SSA's reflect correlations of the form

$$\vec{s} \cdot \left(\vec{p}_\wedge \vec{k} \right)$$

where

$\vec{s}$ is some particle polarisation vector, while
$\vec{p}$ and $\vec{k}$ are initial/final particle/jet momenta.

(Such a combination is dictated by parity conservation.)

A typical example might be

$\vec{p}$ = beam direction
$\vec{s}$ = target polarisation (transverse w.r.t. $\vec{p}$)
$\vec{k}$ = final-state particle direction

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse . . .

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse
(there are, however, exceptions).

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse
(there are, however, exceptions).

Transforming the spin basis from transversity

$$|\uparrow/\downarrow\rangle$$

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse (there are, however, exceptions).

Transforming the spin basis from transversity to helicity

$$|\uparrow/\downarrow\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \pm \mathrm{i}|-\rangle \right]$$

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse (there are, however, exceptions).

Transforming the spin basis from transversity to helicity

$$|\uparrow/\downarrow\rangle = \frac{1}{\sqrt{2}} \left[\, |+\rangle \pm \mathrm{i} \, |-\rangle \right]$$

any such asymmetry (schematically)

$$\mathcal{A}_N \sim \frac{\langle\uparrow|\uparrow\rangle - \langle\downarrow|\downarrow\rangle}{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle}$$

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse (there are, however, exceptions).

Transforming the spin basis from transversity to helicity

$$|\uparrow/\downarrow\rangle = \tfrac{1}{\sqrt{2}} \left[\, |+\rangle \pm \mathrm{i} |-\rangle \right]$$

any such asymmetry (schematically) takes on the form

$$\mathcal{A}_N \sim \frac{\langle\uparrow|\uparrow\rangle - \langle\downarrow|\downarrow\rangle}{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle} \sim \frac{2\,\mathrm{Im}\,\langle +|-\rangle}{\langle +|+\rangle + \langle -|-\rangle}$$

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse (there are, however, exceptions).

Transforming the spin basis from transversity to helicity

$$|\uparrow/\downarrow\rangle = \frac{1}{\sqrt{2}} \left[|+\rangle \pm \mathrm{i}|-\rangle \right]$$

any such asymmetry (schematically) takes on the form

$$\mathcal{A}_N \sim \frac{\langle\uparrow|\uparrow\rangle - \langle\downarrow|\downarrow\rangle}{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle} \sim \frac{2\,\mathrm{Im}\,\langle+|-\rangle}{\langle+|+\rangle + \langle-|-\rangle}$$

The appearance of both $|+\rangle$ and $|-\rangle$ in the numerator indicates the presence of a helicity-flip amplitude.

Single-Spin Asymmetries

Usually then, polarisations in SSA's must be transverse (there are, however, exceptions).

Transforming the spin basis from transversity to helicity

$$|\uparrow/\downarrow\rangle = \tfrac{1}{\sqrt{2}}\left[\,|+\rangle \pm \mathrm{i}\,|-\rangle\right]$$

any such asymmetry (schematically) takes on the form

$$\mathcal{A}_N \sim \frac{\langle\uparrow|\uparrow\rangle - \langle\downarrow|\downarrow\rangle}{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle} \sim \frac{2\,\mathrm{Im}\,\langle+|-\rangle}{\langle+|+\rangle + \langle-|-\rangle}$$

The appearance of both $|+\rangle$ and $|-\rangle$ in the numerator indicates the presence of a helicity-flip amplitude.

The imaginary phase implies *naïvely* T-odd processes.

Single-Spin Asymmetries

Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:

Single-Spin Asymmetries

Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:

- massless fermion helicity is conserved
- tree diagram amplitudes are always real

Single-Spin Asymmetries

* Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:
  - massless fermion helicity is conserved
  - tree diagram amplitudes are always real

* "*. . . observation of significant polarizations in the above reactions would contradict either QCD or its applicability.*"

Single-Spin Asymmetries

* Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:

  - massless fermion helicity is conserved
  - tree diagram amplitudes are always real

* "... *observation of significant polarizations in the above reactions would contradict either QCD or its applicability.*"

  The existence of transverse polarisation itself does not depend on particle masses

Single-Spin Asymmetries

* Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:

    - massless fermion helicity is conserved
    - tree diagram amplitudes are always real

* "... *observation of significant polarizations in the above reactions would contradict either QCD or its applicability.*"

  The existence of transverse polarisation itself does not depend on particle masses—*cf.*, the natural ($\sim 9\,\%$) LEP beam polarisation.

Single-Spin Asymmetries

* Kane, Pumplin and Repko (1978) realised that in the high-energy or massless limit and the Born approximation a gauge theory (such as QCD) cannot furnish either requirement:

  - massless fermion helicity is conserved
  - tree diagram amplitudes are always real

* "*. . . observation of significant polarizations in the above reactions would contradict either QCD or its applicability.*"

  The existence of transverse polarisation itself does not depend on particle masses—*cf.*, the natural ($\sim 9\,\%$) LEP beam polarisation.

  The problem of the (small) quark masses does arise when we seek measurable transverse-spin effects, which usually require spin flip.

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

In 1985 Efremov and Teryaev discovered one way out:

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

In 1985 Efremov and Teryaev discovered one way out:

Consideration of the three-parton correlators involved in, *e.g.*, g_2, leads to the following observations:

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

In 1985 Efremov and Teryaev discovered one way out:

Consideration of the three-parton correlators involved in, *e.g.*, g_2, leads to the following observations:

- the relevant mass scale here is not that of the current quark, but of the hadron;

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

In 1985 Efremov and Teryaev discovered one way out:

Consideration of the three-parton correlators involved in, *e.g.*, g_2, leads to the following observations:

- the relevant mass scale here is not that of the current quark, but of the hadron;
- the pseudo-two-loop nature of the diagrams leads to an imaginary part in certain regions of partonic phase space.

Single-Spin Asymmetries

As we know, large asymmetries were found . . . but QCD survived!

In 1985 Efremov and Teryaev discovered one way out:

Consideration of the three-parton correlators involved in, *e.g.*, g_2, leads to the following observations:

- the relevant mass scale here is not that of the current quark, but of the hadron;
- the pseudo-two-loop nature of the diagrams leads to an imaginary part in certain regions of partonic phase space.

However, it took some years before progress was made and the richness of the available structure was fully exploited—see Qiu and Sterman (1991; 1992).

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

- Twist-3, three-parton correlators: transverse-spin dependent (*cf.*, g_2) – the pseudo two-loop nature provides effective spin flip and imaginary part via pole terms.

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

- Twist-3, three-parton correlators: transverse-spin dependent (*cf.*, g_2) – the pseudo two-loop nature provides effective spin flip and imaginary part via pole terms.
- Internal quark motion: correlation between the transverse polarisation of a quark and its own transverse momentum. Requires orbital angular momentum and soft-gluon exchange.

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

- Twist-3, three-parton correlators: transverse-spin dependent (*cf.*, g_2) – the pseudo two-loop nature provides effective spin flip and imaginary part via pole terms.

- Internal quark motion: correlation between the transverse polarisation of a quark and its own transverse momentum. Requires orbital angular momentum and soft-gluon exchange. The Sivers function.

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

- Twist-3, three-parton correlators: transverse-spin dependent (*cf.*, g_2) – the pseudo two-loop nature provides effective spin flip and imaginary part via pole terms.
- Internal quark motion: correlation between the transverse polarisation of a quark and its own transverse momentum. Requires orbital angular momentum and soft-gluon exchange. The Sivers function.
- Transversity: hadron helicity flip – correlated with quark flip. Chirality conservation requires it be combined with another T-odd (distribution or fragmentation) function.

Single-Spin Asymmetries

It turns out that there are many ways to produce SSA's.

- Twist-3, three-parton correlators: transverse-spin dependent (*cf.*, g_2) – the pseudo two-loop nature provides effective spin flip and imaginary part via pole terms.

- Internal quark motion: correlation between the transverse polarisation of a quark and its own transverse momentum. Requires orbital angular momentum and soft-gluon exchange. The Sivers function.

- Transversity: hadron helicity flip – correlated with quark flip. Chirality conservation requires it be combined with another T-odd (distribution or fragmentation) function.

The first and second mechanisms turn out to be related . . .

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,
- k_T-dependent distribution and fragmentation functions,

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,
- k_T-dependent distribution and fragmentation functions,
- interference fragmentation functions,

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,
- k_T-dependent distribution and fragmentation functions,
- interference fragmentation functions,
- higher-spin functions, *e.g.*, vector-meson fragmentation functions.

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,
- k_T-dependent distribution and fragmentation functions,
- interference fragmentation functions,
- higher-spin functions, *e.g.*, vector-meson fragmentation functions.
- ...

Single-Spin Asymmetries

There are various types of distribution and fragmentation functions that can be active in producing SSA's:

- higher-twist distribution and fragmentation functions,
- k_T-dependent distribution and fragmentation functions,
- interference fragmentation functions,
- higher-spin functions, *e.g.*, vector-meson fragmentation functions.
- ...

We shall examine the first two here and only for distribution functions.

Single-Hadron Production

Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \;\to\; h(P_h) + X$$

Single-Hadron Production

Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \rightarrow h(P_h) + X$$

A is transversely polarised. B is not polarised.

Single-Hadron Production

Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \rightarrow h(P_h) + X$$

A is transversely polarised. B is not polarised.
The unpolarised (or even spinless) hadron h is produced with large transverse momentum $\boldsymbol{P}_{hT}$.

Single-Hadron Production

Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \to h(P_h) + X$$

A is transversely polarised. B is not polarised.
The unpolarised (or even spinless) hadron h is produced with large transverse momentum $\boldsymbol{P}_{hT}$.
PQCD is therefore applicable.

Single-Hadron Production

Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \rightarrow h(P_h) + X$$

A is transversely polarised. B is not polarised.
The unpolarised (or even spinless) hadron h is produced with large transverse momentum $\boldsymbol{P}_{hT}$.
PQCD is therefore applicable.
Typically, A and B are protons while h may be a pion or kaon *etc*.

Single-Hadron Production

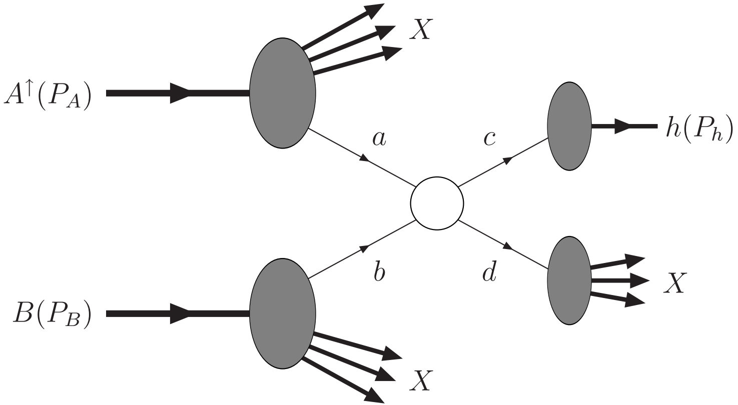
Single-hadron production (in hadron–hadron scattering) with a single transversely polarised hadron:

$$A^{\uparrow}(P_A) + B(P_B) \;\to\; h(P_h) + X$$

A is transversely polarised. B is not polarised.
The unpolarised (or even spinless) hadron h is produced with large transverse momentum $\boldsymbol{P}_{hT}$.
PQCD is therefore applicable.
Typically, A and B are protons while h may be a pion or kaon *etc.*
One measures the following SSA:

$$A_T^h = \frac{\mathrm{d}\sigma(\boldsymbol{S}_T) - \mathrm{d}\sigma(-\boldsymbol{S}_T)}{\mathrm{d}\sigma(\boldsymbol{S}_T) + \mathrm{d}\sigma(-\boldsymbol{S}_T)}$$

Single-Hadron Production



Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$d\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha} f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$d\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha} \, f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

where f_a (f_b) is the density of parton type a (b) in hadron A (B),

Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$d\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^a_{\alpha'\alpha} \, f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

where f_a (f_b) is the density of parton type a (b) in hadron A (B), $\rho^a_{\alpha\alpha'}$ is the spin density matrix for parton a,

Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$\mathrm{d}\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha}\, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

where f_a (f_b) is the density of parton type a (b) in hadron A (B),
$\rho^{a}_{\alpha\alpha'}$ is the spin density matrix for parton a,
$\mathcal{D}^{\gamma\gamma'}_{h/c}$ is the fragmentation matrix of parton c into final hadron h

Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$\mathrm{d}\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha} \, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

where f_a (f_b) is the density of parton type a (b) in hadron A (B),
$\rho^{a}_{\alpha\alpha'}$ is the spin density matrix for parton a,
$\mathcal{D}^{\gamma\gamma'}_{h/c}$ is the fragmentation matrix of parton c into final hadron h
and $\mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'}$ is the elementary cross-section:

Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

$$\mathrm{d}\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^{a}_{\alpha'\alpha}\, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z),$$

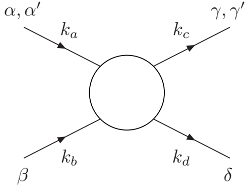
where f_a (f_b) is the density of parton type a (b) in hadron A (B),
$\rho^{a}_{\alpha\alpha'}$ is the spin density matrix for parton a,
$\mathcal{D}^{\gamma\gamma'}_{h/c}$ is the fragmentation matrix of parton c into final hadron h
and $\mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'}$ is the elementary cross-section:

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\gamma\gamma'} = \frac{1}{16\pi\hat{s}^2}\,\frac{1}{2}\sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta}\,\mathcal{M}^{*}_{\alpha'\beta\gamma'\delta}$$

where $\mathcal{M}_{\alpha\beta\gamma\delta}$ is the amplitude for the hard partonic process.

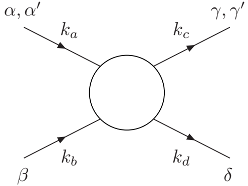
Single-Hadron Production

$$\mathcal{M}_{\alpha\beta\gamma\delta},\ \mathcal{M}_{\alpha'\beta\gamma'\delta} \quad =$$



Single-Hadron Production

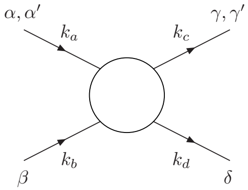
$\mathcal{M}_{\alpha\beta\gamma\delta}, \ \mathcal{M}_{\alpha'\beta\gamma'\delta} \quad =$



For an unpolarised (or spinless) collinearly produced hadron, the off-diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ vanish; *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} \propto \delta_{\gamma\gamma'}$.

Single-Hadron Production

$\mathcal{M}_{\alpha\beta\gamma\delta},\ \mathcal{M}_{\alpha'\beta\gamma'\delta} \quad =$

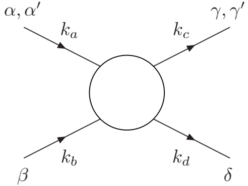


For an unpolarised (or spinless) collinearly produced hadron, the off-diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ vanish; *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} \propto \delta_{\gamma\gamma'}$.

Then (schematically) helicity conservation implies $\alpha = \alpha'$ and there is no dependence on the spin of hadron *A*, so all SSA's are zero.

Single-Hadron Production

$$\mathcal{M}_{\alpha\beta\gamma\delta},\ \mathcal{M}_{\alpha'\beta\gamma'\delta} \quad =$$



For an unpolarised (or spinless) collinearly produced hadron, the off-diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ vanish; *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} \propto \delta_{\gamma\gamma'}$.

Then (schematically) helicity conservation implies $\alpha = \alpha'$ and there is no dependence on the spin of hadron A, so all SSA's are zero.

To avoid this conclusion, either intrinsic quark transverse motion or higher-twist effects must be considered . . .

Transverse Motion and SSA

Intrinsic quark transverse motion can generate SSA's in three essentially different ways (a necessarily T-odd effect):

Transverse Motion and SSA

Intrinsic quark transverse motion can generate SSA's in three essentially different ways (a necessarily T-odd effect):

1. $\boldsymbol{k}_T$ in hadron A requires $f_a(x_a)$ to be replaced by $\mathcal{P}_a(x_a, \boldsymbol{k}_T)$, which may depend on the spin of A
(distribution level).

Transverse Motion and SSA

Intrinsic quark transverse motion can generate SSA's in three essentially different ways (a necessarily T-odd effect):

1. $\boldsymbol{k}_T$ in hadron A requires $f_a(x_a)$ to be replaced by $\mathcal{P}_a(x_a, \boldsymbol{k}_T)$, which may depend on the spin of A (distribution level).
2. $\boldsymbol{\kappa}_T$ in hadron h allows $\mathcal{D}_{h/c}^{\gamma\gamma'}$ to be non-diagonal (fragmentation level).

Transverse Motion and SSA

Intrinsic quark transverse motion can generate SSA's in three essentially different ways (a necessarily T-odd effect):

1. $\boldsymbol{k}_T$ in hadron A requires $f_a(x_a)$ to be replaced by $\mathcal{P}_a(x_a, \boldsymbol{k}_T)$, which may depend on the spin of A (distribution level).
2. $\boldsymbol{\kappa}_T$ in hadron h allows $\mathcal{D}_{h/c}^{\gamma\gamma'}$ to be non-diagonal (fragmentation level).
3. $\boldsymbol{k}'_T$ in hadron B requires $f_b(x_b)$ to be replaced by $\mathcal{P}_b(x_b, \boldsymbol{k}'_T)$. The transverse spin of parton b in the unpolarised B may then couple both to the transverse spin of a and $\boldsymbol{k}'_T$) (distribution level).

Transverse Motion and SSA

The three corresponding mechanisms are:

1. the Sivers effect (1990)

Transverse Motion and SSA

The three corresponding mechanisms are:

1. the Sivers effect (1990)
2. the Collins effect (1993)

Transverse Motion and SSA

The three corresponding mechanisms are:

1. the Sivers effect (1990)
2. the Collins effect (1993)
3. an effect studied by Boer (1999) in Drell–Yan

Transverse Motion and SSA

The three corresponding mechanisms are:

1. the Sivers effect (1990)
2. the Collins effect (1993)
3. an effect studied by Boer (1999) in Drell–Yan

Note:

All such intrinsic-$\boldsymbol{k}_T$, -$\boldsymbol{\kappa}_T$ or -$\boldsymbol{k}'_T$ effects are T-odd; they require initial- or final-state interactions.

Transverse Motion and SSA

The three corresponding mechanisms are:

1. the Sivers effect (1990)
2. the Collins effect (1993)
3. an effect studied by Boer (1999) in Drell–Yan

Note:

All such intrinsic-$\boldsymbol{k}_T$, -$\boldsymbol{\kappa}_T$ or -$\boldsymbol{k}'_T$ effects are T-odd; they require initial- or final-state interactions.

When quark transverse motion is included, the QCD factorisation theorem is not completely proven (but see Ji, Ma and Yuan, 2005).

Transverse Motion and SSA

The Sivers effect relies on T-odd k_T-dependent distribution functions and predicts SSA's of the form

$$E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$

$$= |\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \mathrm{d}x_b \int \mathrm{d}^2\boldsymbol{k}_T \, \frac{1}{\pi z} \times \Delta_0^T f_a(x_a, \boldsymbol{k}_T) \, f_b(x_b) \, \frac{\mathrm{d}\hat{\sigma}(x_a, x_b, \boldsymbol{k}_T)}{\mathrm{d}\hat{t}} \, D_{h/c}(z)$$

where $\Delta_0^T f$ (related to $f_{1T}^{\perp}$) is a T-odd distribution.

Higher-Twist and SSA

Efremov and Teryaev (1985) first pointed out that non-vanishing SSA's can also be obtained in PQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators.

Higher-Twist and SSA

Efremov and Teryaev (1985) first pointed out that non-vanishing SSA's can also be obtained in PQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators.

Such asymmetries were later evaluated in the context of QCD factorisation by Qiu and Sterman, who studied direct photon production (1991; 1992) and later hadron production (1998).

Higher-Twist and SSA

Efremov and Teryaev (1985) first pointed out that non-vanishing SSA's can also be obtained in PQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators.

Such asymmetries were later evaluated in the context of QCD factorisation by Qiu and Sterman, who studied direct photon production (1991; 1992) and later hadron production (1998).

This program has also been extended to cover the chirally-odd contributions by Kanazawa and Koike (2000a; 2000b).

Higher-Twist and SSA

Efremov and Teryaev (1985) first pointed out that non-vanishing SSA's can also be obtained in PQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators.

Such asymmetries were later evaluated in the context of QCD factorisation by Qiu and Sterman, who studied direct photon production (1991; 1992) and later hadron production (1998).

This program has also been extended to cover the chirally-odd contributions by Kanazawa and Koike (2000a; 2000b).

And more has been done by others since.

Higher-Twist and SSA

There are three different possible higher-twist contributions:

$$\begin{aligned} d\sigma = \sum_{abc} \Big\{ & G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes d\hat{\sigma} \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes d\hat{\sigma}' \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\} \end{aligned}$$

Higher-Twist and SSA

There are three different possible higher-twist contributions:

$$d\sigma = \sum_{abc} \Big\{ G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes d\hat{\sigma} \otimes D_{h/c}(z) + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes d\hat{\sigma}' \otimes D_{h/c}(z) + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\}$$

The 1st. term is the chirally-even mechanism proposed by Efremov and Teryaev and developed by Qiu and Sterman.

Higher-Twist and SSA

There are three different possible higher-twist contributions:

$$
\begin{aligned}
d\sigma = \sum_{abc} \Big\{ & G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes d\hat{\sigma} \otimes D_{h/c}(z) \\
& + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes d\hat{\sigma}' \otimes D_{h/c}(z) \\
& + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\}
\end{aligned}
$$

The 1st. term is the chirally-even mechanism proposed by Efremov and Teryaev and developed by Qiu and Sterman.
The 2nd. contains transversity and is the chirally-odd contribution analysed by Kanazawa and Koike.

Higher-Twist and SSA

There are three different possible higher-twist contributions:

$$d\sigma = \sum_{abc} \Big\{ G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes d\hat{\sigma} \otimes D_{h/c}(z) + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes d\hat{\sigma}' \otimes D_{h/c}(z) + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes d\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\}$$

The 1st. term is the chirally-even mechanism proposed by Efremov and Teryaev and developed by Qiu and Sterman.
The 2nd. contains transversity and is the chirally-odd contribution analysed by Kanazawa and Koike.
The 3rd. also contains transversity but additionally requires a twist-3 fragmentation function $D_{h/c}^{(3)}$.

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

They cannot, however, differentiate between types of contributions.

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

They cannot, however, differentiate between types of contributions.

The Qiu–Sterman calculations (based on three-parton correlators) are somewhat opaque, involving many diagrams, complicated momentum flow, colour and spin structure.

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

They cannot, however, differentiate between types of contributions.

The Qiu–Sterman calculations (based on three-parton correlators) are somewhat opaque, involving many diagrams, complicated momentum flow, colour and spin structure.

They also compare well with the data.

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

They cannot, however, differentiate between types of contributions.

The Qiu–Sterman calculations (based on three-parton correlators) are somewhat opaque, involving many diagrams, complicated momentum flow, colour and spin structure.

They also compare well with the data.

However, the twist-3 correlators (as in g_2) obey constraining relations with k_T-dependent densities—so, are they equivalent?

Phenomenology

Anselmino *et al.* (2003a) have compared the data with different models inspired by the various possible k_T-dependent mechanisms described earlier and find good descriptions.

They cannot, however, differentiate between types of contributions.

The Qiu–Sterman calculations (based on three-parton correlators) are somewhat opaque, involving many diagrams, complicated momentum flow, colour and spin structure.

They also compare well with the data.

However, the twist-3 correlators (as in g_2) obey constraining relations with k_T-dependent densities—so, are they equivalent?

They also exhibit a special convenient factorisation property.

Pole Factorisation

Efremov and Teryaev noticed that twist-3 diagrams involving three-parton correlators can supply the necessary imaginary part via a pole term; spin-flip is implicit (related to the gluon).

Pole Factorisation

Efremov and Teryaev noticed that twist-3 diagrams involving three-parton correlators can supply the necessary imaginary part via a pole term; spin-flip is implicit (related to the gluon).

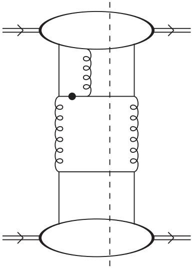
The standard $\mathrm{i}\varepsilon$ propagator prescription

$$\frac{1}{k^2 \pm \mathrm{i}\varepsilon} = \mathrm{P}\,\frac{1}{k^2} \mp \mathrm{i}\pi\delta(k^2)$$

leads to an imaginary contribution for $k^2 \to 0$.

Pole Factorisation

Efremov and Teryaev noticed that twist-3 diagrams involving three-parton correlators can supply the necessary imaginary part via a pole term; spin-flip is implicit (related to the gluon).



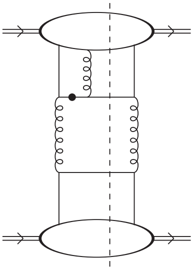
The standard $\mathrm{i}\varepsilon$ propagator prescription (in —•—),

$$\frac{1}{k^2 \pm \mathrm{i}\varepsilon} = \mathrm{P}\,\frac{1}{k^2} \mp \mathrm{i}\pi\delta(k^2)$$

leads to an imaginary contribution for $k^2 \to 0$.

Pole Factorisation

Efremov and Teryaev noticed that twist-3 diagrams involving three-parton correlators can supply the necessary imaginary part via a pole term; spin-flip is implicit (related to the gluon).



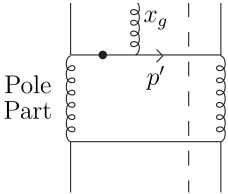
The standard $\mathrm{i}\varepsilon$ propagator prescription (in —•—),

$$\frac{1}{k^2 \pm \mathrm{i}\varepsilon} = \mathrm{P}\,\frac{1}{k^2} \mp \mathrm{i}\pi\delta(k^2)$$

leads to an imaginary contribution for $k^2 \to 0$.

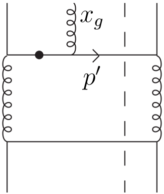
A gluon with $x_g p$ inserted into an (initial or final) external line p' sets $k = p' - x_g p$ and thus $x_g \to 0 \Leftrightarrow k^2 \to 0$.

Pole Factorisation



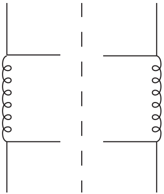
Pole Factorisation

Pole Part



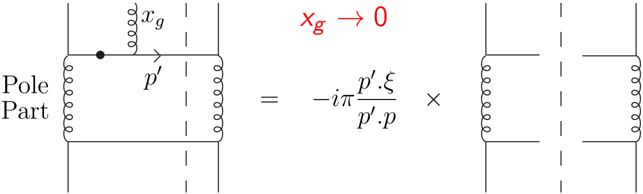
$x_g \to 0$

$$= \; -i\pi \frac{p'.\xi}{p'.p} \; \times$$



p is the incoming proton momentum, p' the outgoing hadron and ξ is the gluon polarisation vector (lying in the transverse plane).

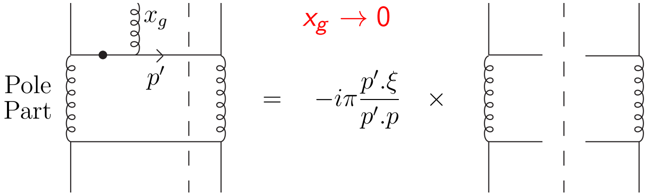
Pole Factorisation



p is the incoming proton momentum, p' the outgoing hadron and ξ is the gluon polarisation vector (lying in the transverse plane).

This can be performed systematically for all soft poles (gluon and fermion): *i.e.*, on all external legs with all insertions (PGR, 1999).

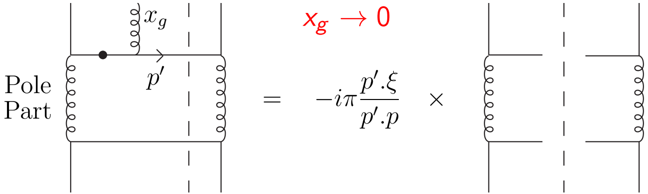
Pole Factorisation



p is the incoming proton momentum, p' the outgoing hadron and ξ is the gluon polarisation vector (lying in the transverse plane).

This can be performed systematically for all soft poles (gluon and fermion): *i.e.*, on all external legs with all insertions (PGR, 1999). The case of so-called hard poles has not yet been examined.

Pole Factorisation



p is the incoming proton momentum, p' the outgoing hadron and ξ is the gluon polarisation vector (lying in the transverse plane).

This can be performed systematically for all soft poles (gluon and fermion): *i.e.*, on all external legs with all insertions (PGR, 1999). The case of so-called hard poles has not yet been examined.

The structures are still complex: for a given correlator there are many insertions, with different signs and momentum dependence.

Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . .

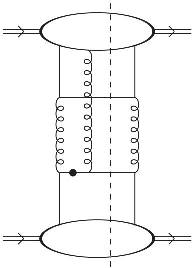
Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . . I shall return to this . . .

Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . . I shall return to this . . .

In all cases (examined) it turns out that just one diagram dominates in the large-N_c limit.

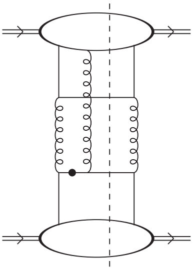


Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . . I shall return to this . . .

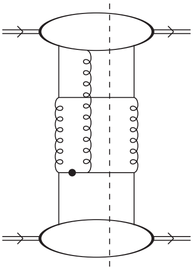
In all cases (examined) it turns out that just one diagram dominates in the large-N_c limit.

All other insertions into external (on-shell) legs are relatively suppressed by $1/N_c{}^2$.



Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . . I shall return to this . . .



In all cases (examined) it turns out that just one diagram dominates in the large-N_c limit.

All other insertions into external (on-shell) legs are relatively suppressed by $1/N_c{}^2$.

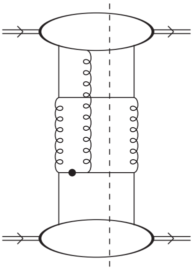
This has been examined in detail by Ramilli (Insubria U. Masters thesis, 2007): the leading diagrams provide a good approximation.

Large N_c

The colour structure of the various diagrams (with the different soft insertions) is also very different . . . I shall return to this . . .

In all cases (examined) it turns out that just one diagram dominates in the large-N_c limit.

All other insertions into external (on-shell) legs are relatively suppressed by $1/N_c{}^2$.



This has been examined in detail by Ramilli (Insubria U. Masters thesis, 2007): the leading diagrams provide a good approximation.

It still needs to be repeated for the other twist-3 contributions (*e.g.*, also in fragmentation).

Link Between k_T and $\tau = 3$

The question now arises: what is the relationship between twist-3 and k_T-dependent mechanisms?

Link Between k_T and $\tau = 3$

The question now arises: what is the relationship between twist-3 and k_T-dependent mechanisms?

It might be hoped that, via the equations of motion *etc*., by linking the (Efremov–Teryaev) higher-twist (three-parton) mechanisms to the (*e.g*., Sivers-like) k_T-dependent mechanisms, one could arrive at unique predictions for SSA's.

Link Between k_T and $\tau = 3$

The question now arises: what is the relationship between twist-3 and k_T-dependent mechanisms?

It might be hoped that, via the equations of motion *etc.*, by linking the (Efremov–Teryaev) higher-twist (three-parton) mechanisms to the (*e.g.*, Sivers-like) k_T-dependent mechanisms, one could arrive at unique predictions for SSA's.

Ma and Wang (2003) made a first attempt for DY processes, but the predictions were found not to be unique.

Link Between k_T and $\tau = 3$

The question now arises: what is the relationship between twist-3 and k_T-dependent mechanisms?

It might be hoped that, via the equations of motion *etc*., by linking the (Efremov–Teryaev) higher-twist (three-parton) mechanisms to the (*e.g*., Sivers-like) k_T-dependent mechanisms, one could arrive at unique predictions for SSA's.

Ma and Wang (2003) made a first attempt for DY processes, but the predictions were found not to be unique.

Ji *et al*. (2006a,b) have also examined the relationships between k_T-dependent and higher-twist mechanisms by matching in the common intermediate k_T region—their results are positive.

Colour Modification

PGR and Teryaev (2007) provide an *a posteriori* proof of the relation between twist three and the Sivers function.

Colour Modification

PGR and Teryaev (2007) provide an *a posteriori* proof of the relation between twist three and the Sivers function.

The starting point is the Sivers function factorised formula:

$$d\Delta\sigma \sim \int d^2k_T \, dx \, f_S(x, k_T) \; \mathrm{Tr}\left[\gamma_\rho \, H(xP, k_T)\right] \, \epsilon^{\rho s P k_T}$$

Colour Modification

PGR and Teryaev (2007) provide an *a posteriori* proof of the relation between twist three and the Sivers function.

The starting point is the Sivers function factorised formula:

$$d\Delta\sigma \sim \int d^2k_T \, dx \; f_S(x, k_T) \; \mathrm{Tr}\left[\gamma_\rho \, H(xP, k_T)\right] \epsilon^{\rho s P k_T}$$

We expand the subprocess coefficient function H in powers of k_T:

$$\sim \int d^2k_T \, dx \; f_S(x, k_T) \; \mathrm{Tr}\left[\gamma_\rho \, \frac{\partial H(xP, k_T)}{\partial k_T^\alpha}\right]_{k_T=0} k_T^\alpha \, \epsilon^{\rho s P k_T},$$

keeping the first non-vanishing term.

Colour Modification

Using various identities and the fact that there are other momenta involved, this can be rearranged into the following form:

$$d\Delta\sigma \sim M \int dx\, f_S^{(1)}(x)\ \mathrm{Tr}\left[\not{P} \, \frac{\partial H(xP, k_T)}{\partial k_T^\alpha} \right]_{k_T=0} \epsilon^{\alpha s P n}$$

Colour Modification

Using various identities and the fact that there are other momenta involved, this can be rearranged into the following form:

$$d\Delta\sigma \sim M \int dx \, f_S^{(1)}(x) \; \mathrm{Tr}\left[\not{P} \, \frac{\partial H(xP, k_T)}{\partial k_T^\alpha} \right]_{k_T=0} \epsilon^{\alpha s P n}$$

where

$$f_S^{(1)}(x) = \int d^2 k_T \, f_S(x, k_T) \, \frac{k_T^2}{2M^2}.$$

Colour Modification

Using various identities and the fact that there are other momenta involved, this can be rearranged into the following form:

$$d\Delta\sigma \sim M \int dx\, f_S^{(1)}(x)\ \mathrm{Tr}\left[\not{P} \frac{\partial H(xP, k_T)}{\partial k_T^\alpha} \right]_{k_T=0} \epsilon^{\alpha s P n}$$

where

$$f_S^{(1)}(x) = \int d^2 k_T\, f_S(x, k_T)\, \frac{k_T^2}{2M^2}.$$

This coincides with the master formula of Koike *et al.* (2007b,a) for twist-3 gluonic poles in high-p_T processes.

Colour Modification

Using various identities and the fact that there are other momenta involved, this can be rearranged into the following form:

$$d\Delta\sigma \sim M \int dx\, f_S^{(1)}(x)\ \mathrm{Tr}\left[\not{P}\, \frac{\partial H(xP, k_T)}{\partial k_T^\alpha} \right]_{k_T=0} \epsilon^{\alpha sPn}$$

where

$$f_S^{(1)}(x) = \int d^2k_T\, f_S(x, k_T)\, \frac{k_T^2}{2M^2}.$$

This coincides with the master formula of Koike *et al.* (2007b,a) for twist-3 gluonic poles in high-p_T processes.

The Sivers function can thus be identified with the gluonic-pole strength $T(x, x)$ multiplied by a process-dependent colour factor.

Colour Modification

The sign of the Sivers function is determined by whether ISI or FSI is relevant:

$$f_S^{(1)}(x) = \sum_i C_i \, \frac{1}{2M} \, T(x,x)$$

Colour Modification

The sign of the Sivers function is determined by whether ISI or FSI is relevant:

$$f_S^{(1)}(x) = \sum_i C_i \, \frac{1}{2M} \, T(x,x)$$

C_i is a relative colour factor defined w.r.t. an Abelian subprocess.

Colour Modification

The sign of the Sivers function is determined by whether ISI or FSI is relevant:

$$f_S^{(1)}(x) = \sum_i C_i \, \frac{1}{2M} \, T(x,x)$$

C_i is a relative colour factor defined w.r.t. an Abelian subprocess.

Now, to generate high p_T, the emission of an extra hard gluon is necessary.

Colour Modification

The sign of the Sivers function is determined by whether ISI or FSI is relevant:

$$f_S^{(1)}(x) = \sum_i C_i \frac{1}{2M} T(x,x)$$

C_i is a relative colour factor defined w.r.t. an Abelian subprocess.

Now, to generate high p_T, the emission of an extra hard gluon is necessary.

According to the process under consideration, the FSI may occur *before* or *after* emission of this extra gluon, again leading to different colour factors.

Colour Modification

The sign of the Sivers function is determined by whether ISI or FSI is relevant:

$$f_S^{(1)}(x) = \sum_i C_i \, \frac{1}{2M} \, T(x,x)$$

C_i is a relative colour factor defined w.r.t. an Abelian subprocess.

Now, to generate high p_T, the emission of an extra hard gluon is necessary.

According to the process under consideration, the FSI may occur *before* or *after* emission of this extra gluon, again leading to different colour factors.

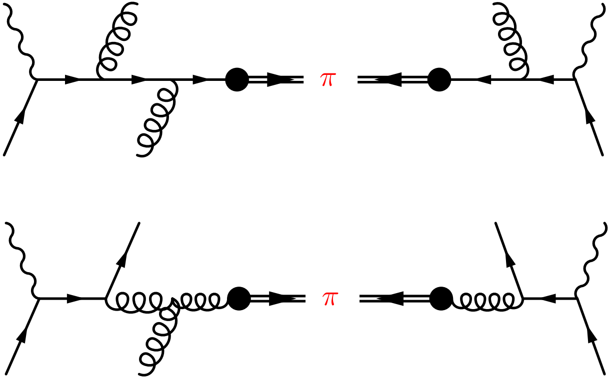
In this sense, factorisation is broken in SIDIS, although in a simple and accountable manner.

Colour Modification

Consider the particular application of this relation to high-p_T SIDIS:

Colour Modification

Consider the particular application of this relation to high-p_T SIDIS:



Twist-3 SIDIS π production via quark and gluon fragmentation.

Large x

The relation between gluonic poles (*e.g.*, the Sivers function) and T-even transverse-spin effects (*e.g.*, g_2—Shuryak *et al*. 1982; Bukhvostov *et al*. 1983; Efremov *et al*. 1984; PGR 1986; Balitsky *et al*. 1989) remains unclear.

Large x

The relation between gluonic poles (*e.g.*, the Sivers function) and T-even transverse-spin effects (*e.g.*, g_2—Shuryak *et al.* 1982; Bukhvostov *et al.* 1983; Efremov *et al.* 1984; PGR 1986; Balitsky *et al.* 1989) remains unclear.

There are model-based estimates and approximate sum rules.

Large x

The relation between gluonic poles (*e.g.*, the Sivers function) and T-even transverse-spin effects (*e.g.*, g_2—Shuryak *et al.* 1982; Bukhvostov *et al.* 1983; Efremov *et al.* 1984; PGR 1986; Balitsky *et al.* 1989) remains unclear.

There are model-based estimates and approximate sum rules.

The compatibility of twist-3 evolution with dedicated studies of gluonic-pole evolution (Kang *et al.* 2009; Zhou *et al.* 2009 and at NLO Vogelsang *et al.* 2009) is however still unproven.

Large x

The relation between gluonic poles (*e.g.*, the Sivers function) and T-even transverse-spin effects (*e.g.*, g_2—Shuryak *et al*. 1982; Bukhvostov *et al*. 1983; Efremov *et al*. 1984; PGR 1986; Balitsky *et al*. 1989) remains unclear.

There are model-based estimates and approximate sum rules.

The compatibility of twist-3 evolution with dedicated studies of gluonic-pole evolution (Kang *et al*. 2009; Zhou *et al*. 2009 and at NLO Vogelsang *et al*. 2009) is however still unproven.

For large-x the g_2 evolution equations simplify: they diagonalise in the double-moment arguments (Ali *et al*., 1991).

Large x

The relation between gluonic poles (*e.g.*, the Sivers function) and T-even transverse-spin effects (*e.g.*, g_2—Shuryak *et al*. 1982; Bukhvostov *et al*. 1983; Efremov *et al*. 1984; PGR 1986; Balitsky *et al*. 1989) remains unclear.

There are model-based estimates and approximate sum rules.

The compatibility of twist-3 evolution with dedicated studies of gluonic-pole evolution (Kang *et al*. 2009; Zhou *et al*. 2009 and at NLO Vogelsang *et al*. 2009) is however still unproven.

For large-x the g_2 evolution equations simplify: they diagonalise in the double-moment arguments (Ali *et al*., 1991).

For the Sivers function and gluonic poles, this is the important kinematical region: SSA's grow (Qiu *et al*., 1991).

Large x

The gluonic-pole strength $T(x)$, corresponds to a specific matrix element (Qiu *et al.*, 1991).

Large x

The gluonic-pole strength $T(x)$, corresponds to a specific matrix element (Qiu *et al.*, 1991).

It is also the residue of a general qqg vector correlator $b_V(x_1, x_2)$ (Teryaev, 1995; Korotkiian *et al.*, 1995):

$$b_V(x_1, x_2) = \frac{T(\frac{x_1+x_2}{2})}{x_1 - x_2} + \text{regular part}$$

Large x

The gluonic-pole strength $T(x)$, corresponds to a specific matrix element (Qiu *et al.*, 1991).

It is also the residue of a general qqg vector correlator $b_V(x_1, x_2)$ (Teryaev, 1995; Korotkiian *et al.*, 1995):

$$b_V(x_1, x_2) = \frac{T(\frac{x_1+x_2}{2})}{x_1 - x_2} + \text{regular part}$$

defined as

$$b_V(x_1, x_2) = \frac{i}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} \, e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \times \epsilon^{\mu s p_1 n} \langle p_1, s | \bar{\psi}(0) \not{n} D_\mu(\lambda_1) \psi(\lambda_2) | p_1, s \rangle$$

Large x

There also exists another correlator, projected onto an axial rather than vector Dirac matrix:

$$b_A(x_1, x_2) = \frac{1}{M} \int \frac{d\lambda_1 d\lambda_2}{\pi} \, e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \times \langle p_1, s|\bar{\psi}(0)\not{n}\gamma^5 s\cdot D(\lambda_1)\psi(\lambda_2)|p_1, s\rangle,$$

Large x

There also exists another correlator, projected onto an axial rather than vector Dirac matrix:

$$b_A(x_1, x_2) = \frac{1}{M} \int \frac{d\lambda_1 d\lambda_2}{\pi} \, e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \times \langle p_1, s | \bar{\psi}(0) \not{n} \gamma^5 s \cdot D(\lambda_1) \psi(\lambda_2) | p_1, s \rangle,$$

which is required for the complete description of transverse-spin asymmetries—both SSA's and g_2.

Large x

There also exists another correlator, projected onto an axial rather than vector Dirac matrix:

$$b_A(x_1,x_2) = \frac{1}{M}\int \frac{d\lambda_1 d\lambda_2}{\pi}\, e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \\ \times \langle p_1,s|\bar{\psi}(0)\not{n}\gamma^5 s\cdot D(\lambda_1)\psi(\lambda_2)|p_1,s\rangle,$$

which is required for the complete description of transverse-spin asymmetries—both SSA's and g_2.

The two correlators have opposite symmetry properties for $x_1 \leftrightarrow x_2$ (determined by T invariance):

$$b_A(x_1,x_2) = b_A(x_2,x_1), \quad b_V(x_1,x_2) = -b_V(x_2,x_1).$$

Large x

In DIS and SSA's only a particular combination appears (Efremov *et al.*, 1984):

$$b_{-}(x_1, x_2) = b_A(x_2, x_1) - b_V(x_1, x_2).$$

Large x

In DIS and SSA's only a particular combination appears (Efremov *et al.*, 1984):

$$b_-(x_1, x_2) = b_A(x_2, x_1) - b_V(x_1, x_2).$$

The evolution equations (Bukhvostov *et al.*, 1983; PGR, 1986; Balitsky *et al.*, 1989) are written in terms of another quantity, which is expressed as matrix elements of the gluon field strength:

$$Y(x_1, x_2) = (x_1 - x_2)\, b_-(x_1, x_2).$$

Large x

In DIS and SSA's only a particular combination appears (Efremov *et al.*, 1984):

$$b_-(x_1, x_2) = b_A(x_2, x_1) - b_V(x_1, x_2).$$

The evolution equations (Bukhvostov *et al.*, 1983; PGR, 1986; Balitsky *et al.*, 1989) are written in terms of another quantity, which is expressed as matrix elements of the gluon field strength:

$$Y(x_1, x_2) = (x_1 - x_2)\, b_-(x_1, x_2).$$

Thus, since it should be safe to assume that $b_-(x_1, x_2)$ has no double pole, we see

$$T(x) = Y(x, x).$$

Large x

The evolution is easiest to study in Mellin-moment form and for $Y(x,y)$ these become double moments:

$$Y^{mn} = \int dx\, dy\, x^m\, y^n\, Y(x,y),$$

Large x

The evolution is easiest to study in Mellin-moment form and for $Y(x,y)$ these become double moments:

$$Y^{mn} = \int dx\, dy\, x^m\, y^n\, Y(x,y),$$

where the variables are restricted to $|x|$, $|y|$ & $|x-y| < 1$.

Large x

The evolution is easiest to study in Mellin-moment form and for $Y(x, y)$ these become double moments:

$$Y^{mn} = \int dx\, dy\, x^m\, y^n\, Y(x, y),$$

where the variables are restricted to $|x|$, $|y|$ & $|x - y| < 1$.

We wish to examine the behaviour for x and y both close to unity and therefore close to each other.

Large x

The evolution is easiest to study in Mellin-moment form and for $Y(x,y)$ these become double moments:

$$Y^{mn} = \int dx\, dy\, x^m\, y^n\, Y(x,y),$$

where the variables are restricted to $|x|$, $|y|$ & $|x-y| < 1$.

We wish to examine the behaviour for x and y both close to unity and therefore close to each other.

Thus, the gluonic pole provides the dominant contribution:

$$\lim_{x,y \to 1} Y(x,y) = T\left(\tfrac{x+y}{2}\right) + O(x-y).$$

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_{\mathrm{F}} + \frac{C_{\mathrm{A}}}{2}\right) \ln n \; Y^{nn},$$

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_\mathrm{F} + \frac{C_\mathrm{A}}{2}\right)\ln n \; Y^{nn},$$

where the derivative (dot) is w.r.t. $s = \beta_0^{-1} \ln \ln Q^2$.

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_F + \frac{C_A}{2}\right) \ln n \; Y^{nn},$$

where the derivative (dot) is w.r.t. $s = \beta_0^{-1} \ln\ln Q^2$.
In terms of $T(x)$ this translates back into

$$\dot{T}(x) = 4\left(C_F + \frac{C_A}{2}\right) \int_x^1 dz \, \frac{(1-z)}{(1-x)} \frac{1}{(z-x)_+} \, T(z),$$

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_F + \frac{C_A}{2}\right) \ln n \; Y^{nn},$$

where the derivative (dot) is w.r.t. $s = \beta_0^{-1} \ln \ln Q^2$.
In terms of $T(x)$ this translates back into

$$\dot{T}(x) = 4\left(C_F + \frac{C_A}{2}\right) \int_x^1 dz \, \frac{(1-z)}{(1-x)} \frac{1}{(z-x)_+} \, T(z),$$

which is very similar to the unpolarised case,

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_\mathrm{F} + \frac{C_\mathrm{A}}{2}\right) \ln n \; Y^{nn},$$

where the derivative (dot) is w.r.t. $s = \beta_0^{-1} \ln \ln Q^2$.
In terms of $T(x)$ this translates back into

$$\dot{T}(x) = 4\left(C_\mathrm{F} + \frac{C_\mathrm{A}}{2}\right) \int_x^1 dz \, \frac{(1-z)}{(1-x)} \frac{1}{(z-x)_+} \, T(z),$$

which is very similar to the unpolarised case, but differs by

- a colour factor $(C_\mathrm{F} + C_\mathrm{A}/2)$,

Large x

In this approximation (which now becomes large $m = n$) the LO evolution equations simplify:

$$\dot{Y}^{nn} = 4\left(C_\mathrm{F} + \frac{C_\mathrm{A}}{2}\right) \ln n \; Y^{nn},$$

where the derivative (dot) is w.r.t. $s = \beta_0^{-1} \ln\ln Q^2$.
In terms of $T(x)$ this translates back into

$$\dot{T}(x) = 4\left(C_\mathrm{F} + \frac{C_\mathrm{A}}{2}\right) \int_x^1 dz \, \frac{(1-z)}{(1-x)} \frac{1}{(z-x)_+} \, T(z),$$

which is very similar to the unpolarised case, but differs by

- a colour factor $(C_\mathrm{F} + C_\mathrm{A}/2)$,
- a softening factor $(1-z)/(1-x)$.

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

The softening factor is inessential to the asymptotic solution: it just implies standard evolution for the function $(1-x)T(x)$.

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

The softening factor is inessential to the asymptotic solution: it just implies standard evolution for the function $(1-x)T(x)$.

The asymptotic solution for an initial $f(x, Q_0^2) \propto (1-x)^a$ has the same form (Gross, 1974) but modified with $a \to a(s)$:

$$a(s) = a + 4\,(C_F + C_A/2)\,s$$

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

The softening factor is inessential to the asymptotic solution: it just implies standard evolution for the function $(1-x)T(x)$.

The asymptotic solution for an initial $f(x, Q_0^2) \propto (1-x)^a$ has the same form (Gross, 1974) but modified with $a \to a(s)$:

$$a(s) = a + 4\,(C_F + C_A/2)\,s$$

and for $T(x)$, a is shifted to $a-1$; also in the evolution.

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

The softening factor is inessential to the asymptotic solution: it just implies standard evolution for the function $(1-x)T(x)$.

The asymptotic solution for an initial $f(x, Q_0^2) \propto (1-x)^a$ has the same form (Gross, 1974) but modified with $a \to a(s)$:

$$a(s) = a + 4\left(C_F + C_A/2\right)s$$

and for $T(x)$, a is shifted to $a-1$; also in the evolution.

So, the unpolarised NS asymptotic solutions are valid for $T(x)$ too.

Large x

Thus, w.r.t. unpolarised evolution, the three-parton kernel pole structure is identical, but the effective colour charge of the extra gluon reflects in an extra piece in the colour factor: $C_A/2$.

The softening factor is inessential to the asymptotic solution: it just implies standard evolution for the function $(1-x)T(x)$.

The asymptotic solution for an initial $f(x, Q_0^2) \propto (1-x)^a$ has the same form (Gross, 1974) but modified with $a \to a(s)$:

$$a(s) = a + 4\,(C_F + C_A/2)\,s$$

and for $T(x)$, a is shifted to $a-1$; also in the evolution.

So, the unpolarised NS asymptotic solutions are valid for $T(x)$ too.

This large-x limit coincides with recent studies of gluonic-pole evolution (Kang *et al.*; Zhou *et al.*; Vogelsang *et al.*, 2009).

Summary & Conclusions

Viewing the Sivers function as a twist-3 gluonic-pole contribution, we see that it is process dependent: besides a sign (ISI *vs.* FSI), there is a process-dependent colour factor.

Summary & Conclusions

Viewing the Sivers function as a twist-3 gluonic-pole contribution, we see that it is process dependent: besides a sign (ISI *vs.* FSI), there is a process-dependent colour factor.

This colour factor is defined by the colour charge of the initial and final partons. It gives the sign difference between SIDIS and DY at low p_T, but at high p_T in hadronic reactions it is more complicated.

Summary & Conclusions

Viewing the Sivers function as a twist-3 gluonic-pole contribution, we see that it is process dependent: besides a sign (ISI *vs.* FSI), there is a process-dependent colour factor.

This colour factor is defined by the colour charge of the initial and final partons. It gives the sign difference between SIDIS and DY at low p_T, but at high p_T in hadronic reactions it is more complicated.

Such a picture is complementary to the matching in the region of common validity. Such matching between various p_T regions now takes the form of a p_T-dependent colour factor.

Summary & Conclusions

Viewing the Sivers function as a twist-3 gluonic-pole contribution, we see that it is process dependent: besides a sign (ISI *vs.* FSI), there is a process-dependent colour factor.

This colour factor is defined by the colour charge of the initial and final partons. It gives the sign difference between SIDIS and DY at low p_T, but at high p_T in hadronic reactions it is more complicated.

Such a picture is complementary to the matching in the region of common validity. Such matching between various p_T regions now takes the form of a p_T-dependent colour factor.

It does, however, also lend some justification to the feasibility of global Sivers-function fits (Teryaev, 2006).

Summary & Conclusions

We have also shown that the evolution of such a Sivers function is governed by generic twist-3 evolution equations.

Summary & Conclusions

We have also shown that the evolution of such a Sivers function is governed by generic twist-3 evolution equations.

Its effective nature, allows us to relate the PQCD evolution of T-odd (Sivers function) and T-even (gluonic pole) quantities.

Summary & Conclusions

We have also shown that the evolution of such a Sivers function is governed by generic twist-3 evolution equations.

Its effective nature, allows us to relate the PQCD evolution of T-odd (Sivers function) and T-even (gluonic pole) quantities.

An important ingredient here is the large-x approximation, in which gluonic-poles dominate and the evolution simplifies.

Summary & Conclusions

We have also shown that the evolution of such a Sivers function is governed by generic twist-3 evolution equations.

Its effective nature, allows us to relate the PQCD evolution of T-odd (Sivers function) and T-even (gluonic pole) quantities.

An important ingredient here is the large-x approximation, in which gluonic-poles dominate and the evolution simplifies.

We have found that the Sivers function evolution is multiplicative and described by the usual twist-2 spin-averaged kernel, modified by a specific colour factor.

Thank you!

Ali, A., Braun, V.M. and Hiller, G. (1991), *Phys. Lett.* **B266**, 117.

Anselmino, M., D'Alesio, U. and Murgia, F. (2003a), *Phys. Rev.* **D67**, 074010.

Anselmino, M., D'Alesio, U. and Murgia, F. (2003b), in proc. of the *XV Int. Symp. on High-Energy Spin Physics—SPIN 2002* (Upton/Danvers, Sept. 2002), eds. Y.I. Makdisi, A.U. Luccio and W.W. Mackay; *AIP Conf. Proc.* **675**, 474.

Balitsky, I.I. and Braun, V.M. (1989), *Nucl. Phys.* **B311**, 541.

Boer, D. (1999), *Phys. Rev.* **D60**, 014012.

Bukhvostov, A.P., Kuraev, É.A. and Lipatov, L.N. (1983), *JETP Lett.* **37**, 482.

Collins, J.C. (1993), *Nucl. Phys.* **B396**, 161.

Efremov, A.V. and Teryaev, O.V. (1982), *Sov. J. Nucl. Phys.* **36**, 140.

Efremov, A.V. and Teryaev, O.V. (1984), *Sov. J. Nucl. Phys.* **39**, 962.

Efremov, A.V. and Teryaev, O.V. (1985), *Phys. Lett.* **B150**, 383.

Gross, D.J. (1974), *Phys. Rev. Lett.* **32**, 1071.

Ji, X., Ma, J.-P. and Yuan, F. (2005), *Phys. Rev.* **D71**, 034005.

Ji, X., Qiu, J.-W., Vogelsang, W. and Yuan, F. (2006a), *Phys. Rev. Lett.* **97**, 082002.

Ji, X., Qiu, J.-W., Vogelsang, W. and Yuan, F. (2006b), *Phys. Lett.* **B638**, 178.

Kanazawa, Y. and Koike, Y. (2000a), *Phys. Lett.* **B478**, 121.

Kanazawa, Y. and Koike, Y. (2000b), *Phys. Lett.* **B490**, 99.

Kane, G.L., Pumplin, J. and Repko, W. (1978), *Phys. Rev. Lett.* **41**, 1689.

Kang, Z.-B. and Qiu, J.-W. (2009), *Phys. Rev.* **D79**, 016003.

Koike, Y. and Tanaka, K. (2007a), *Phys. Rev.* **D76**, 011502.

Koike, Y. and Tanaka, K. (2007b), *Phys. Lett.* **B646**, 232; *erratum ibid.* **B668** (2008) 458.

Korotkiian, V.M. and Teryaev, O.V. (1995), *Phys. Rev.* **D52**, 4775.

Ma, J.-P. and Wang, Q. (2003), *Eur. Phys. J.* **C37**, 293.

Qiu, J. and Sterman, G. (1991), *Phys. Rev. Lett.* **67**, 2264.

Qiu, J. and Sterman, G. (1992), *Nucl. Phys.* **B378**, 52.

Qiu, J. and Sterman, G. (1998), *Phys. Rev.* **D59**, 014004.

Ramilli, M. (2007), M.Sc. thesis, Dept. of Phys. and Math., Insubria U. in Como.

Ratcliffe, P.G. (1986), *Nucl. Phys.* **B264**, 493.

Ratcliffe, P.G. (1999), *Eur. Phys. J.* **C8**, 403.

Ratcliffe, P.G. and Teryaev, O.V. (2007), hep-ph/0703293.

Ratcliffe, P.G. and Teryaev, O.V. (2009a), in progress.

Ratcliffe, P.G. and Teryaev, O.V., invited talk to appear in proc. of the *ECT* Workshop on Recent Advances in Perturbative QCD and Hadronic Physics* (Trento, July 2009b), eds. A. Belitsky, K. Goeke, D. Müller, A. Radyushkin and O. Teryaev.

Shuryak, E.V. and Vainshtein, A.I. (1982), *Nucl. Phys.* **B201**, 141.

Sivers, D. (1990), *Phys. Rev.* **D41**, 83.

Teryaev, O.V. (1995), in proc. of the *Workshop on the Prospects of Spin Physics at HERA* (Zeuthen, Aug. 1995), eds. J. Blümlein and W.-D. Nowak (DESY 95-200), p. 132.

Teryaev, O.V. (2006), in proc. of the *Int. Workshop on Transverse Polarisation Phenomena in Hard Processes—Transversity 2005* (Como, Sept. 2005), eds. V. Barone and P.G. Ratcliffe (World Sci.), p. 276.

Teryaev, O.V. and Ratcliffe, P.G. (2008a), invited talk in proc. of the *XII Adv. Res. Workshop on High Energy Spin Physics—DSPIN-07* (Dubna, Sept. 2007), eds. A.V. Efremov and S.V. Goloskokov (Joint Inst. for Nuclear Research), p. 182.

Teryaev, O.V. and Ratcliffe, P.G. (2008b), invited talk presented at the *Second Int. Workshop on Transverse Polarisation Phenomena in Hard Processes—Transversity 2008* (Ferrara, May 2008).

Vogelsang, W. and Yuan, F. (2009), *Phys. Rev.* **D**, to appear; arXiv:0904.0410 [hep-ph].

Zhou, J., Yuan, F. and Liang, Z.-T. (2009), *Phys. Rev.* **D79**, 114022.