Super-(dS+AdS) and double Super-Poincare

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Abstract

It is well-known that anti-de Sitter Lie algebra $\mathfrak{o}(2,3)$ has a standard $\mathbb{Z}_2$-graded superextension. Recently it was shown that de Sitter Lie algebra $\mathfrak{o}(1,4)$ admits a superextension based on the $\mathbb{Z}_2\times\mathbb{Z}_2$-grading. A main aim of this talk is to show that there exits a nontrivial superextention of de Sitter and anti-de Sitter Lie algebras simultaneously. Using the standard contraction procedure for this superextension we obtain an $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebra which contains simultaneously standard ($\mathbb{Z}_2$-graded) and alternative ($\mathbb{Z}_2\times\mathbb{Z}_2$-graded) super-Poincaré algebras with $N=2,4,\ldots$ supercharge multiplets.

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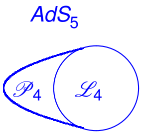
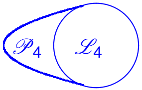


$$g_{ab} = \mathrm{diag}\,(1,-1,-1,-1,1)$$

$$[\mathscr{L}_4,\mathscr{P}_4] = \mathscr{P}_4,\;\; [\mathscr{P}_4,\mathscr{P}_4] = \mathscr{L}_4$$

AdS_5



$$g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,1)$$

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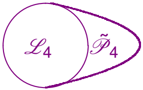
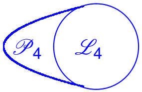
dS_5



$$g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,-1)$$

$$[\mathscr{L}_4,\tilde{\mathscr{P}}_4]=\tilde{\mathscr{P}}_4,\;\;[\tilde{\mathscr{P}}_4,\tilde{\mathscr{P}}_4]=\mathscr{L}_4$$

AdS_5



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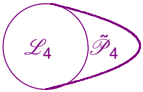
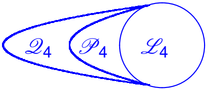
dS_5



$$g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,-1)$$

$$[\mathscr{L}_4,\tilde{\mathscr{P}}_4]=\tilde{\mathscr{P}}_4,\;\;[\tilde{\mathscr{P}}_4,\tilde{\mathscr{P}}_4]=\mathscr{L}_4$$

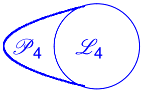
$SAdS_5$



$$g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,1)$$

$$[\mathscr{P}_4,\mathscr{Q}_4]=\mathscr{Q}_4,\;\;\{\mathscr{Q}_4,\mathscr{Q}_4\}=AdS_5$$

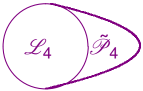
AdS_5



$g_{ab} = \mathrm{diag}\,(1,-1,-1,-1,1)$

$[\mathscr{L}_4, \mathscr{P}_4] = \mathscr{P}_4, \;\; [\mathscr{P}_4, \mathscr{P}_4] = \mathscr{L}_4$

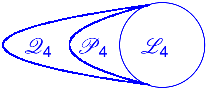
dS_5



$g_{ab} = \mathrm{diag}\,(1,-1,-1,-1,-1)$

$[\mathscr{L}_4, \tilde{\mathscr{P}}_4] = \tilde{\mathscr{P}}_4, \;\; [\tilde{\mathscr{P}}_4, \tilde{\mathscr{P}}_4] = \mathscr{L}_4$

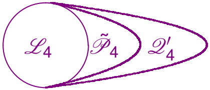
$SAdS_5$



$g_{ab} = \mathrm{diag}\,(1,-1,-1,-1,1)$

$[\mathscr{P}_4, \mathscr{Q}_4] = \mathscr{Q}_4, \;\; \{\mathscr{Q}_4, \mathscr{Q}_4\} = AdS_5$

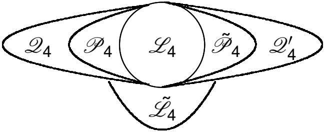
SdS_5



$g_{ab} = \mathrm{diag}\,(1,-1,-1,-1,-1)$

$\{\tilde{\mathscr{P}}_4, \mathscr{Q}'_4\} = \mathscr{Q}'_4, \;\; [[\mathscr{Q}'_4, \mathscr{Q}'_4]] = dS_5$

Super-(AdS_5+dS_5)



$$(1).\ [\mathscr{L}_4,\tilde{\mathscr{L}}_4]=\tilde{\mathscr{L}}_4,\ [\mathscr{L}_4,\mathscr{P}_4]=\mathscr{P}_4,\ [\mathscr{L}_4,\tilde{\mathscr{P}}_4]=\tilde{\mathscr{P}}_4,\ [\mathscr{L}_4,\mathscr{Q}_4]=\mathscr{Q}_4,\ [\mathscr{L}_4,\mathscr{Q}'_4]=\mathscr{Q}'_4,$$

$$(2).\ [\tilde{\mathscr{L}}_4,\tilde{\mathscr{L}}_4]=\mathscr{L}_4,\ [\tilde{\mathscr{L}}_4,\mathscr{P}_4]=\tilde{\mathscr{P}}_4,\ [\tilde{\mathscr{L}}_4,\tilde{\mathscr{P}}'_4]=\mathscr{P}_4\}\ \{\tilde{\mathscr{L}},\mathscr{Q}_4\}=\mathscr{Q}'_4,\ \{\tilde{\mathscr{L}}_4,\mathscr{Q}'_4\}=\mathscr{Q}_4,$$

$$(3).\ [\mathscr{P}_4,\mathscr{P}_4]=\mathscr{L}_4,\ [\mathscr{P}_4,\tilde{\mathscr{P}}_4]=\tilde{\mathscr{L}}_4,\ [\mathscr{P}_4,\mathscr{Q}_4]=\mathscr{Q}_4,\ [\mathscr{P}_4,\mathscr{Q}'_4]=\mathscr{Q}'_4,$$

$$(4).\ [\tilde{\mathscr{P}}_4,\tilde{\mathscr{P}}_4]=\mathscr{L}_4,\ \{\tilde{\mathscr{P}}_4,\mathscr{Q}_4\}=\mathscr{Q}'_4,\ \{\mathscr{P}_4,\mathscr{Q}'_4\}=\mathscr{Q}_4,\ \{\mathscr{P}_4,\mathscr{Q}'_4\}=\mathscr{Q}'_4,$$

$$(5).\ \{\mathscr{Q}_4,\mathscr{Q}_4\}=\mathscr{L}_4\oplus\mathscr{P},\ \{\mathscr{Q}_4,\mathscr{Q}'_4\}=\tilde{\mathscr{L}}_4\oplus\tilde{\mathscr{P}},\ \{\mathscr{Q}'_4,\mathscr{Q}'_4\}=\mathscr{L}_4\oplus\tilde{\mathscr{P}}.$$

Contruction limit : *Super-*(AdS_5+dS_5) $\Longrightarrow$ *Double super-Poincare*

(a) the same relations (1), (2), and

(b) to replace $\mathscr{L}_4$ and $\tilde{\mathscr{L}}_4$ by $\mathbf{0}$ in the left-side of the relations (3)–(5)

The $\mathbb{Z}_2$-graded superalgebra

A $\mathbb{Z}_2$-graded Lie superalgebra (LSA) $\mathfrak{g}$, as a linear space, is a direct sum of two graded components

$$\mathfrak{g} \quad = \quad \bigoplus_{a=0,1} \mathfrak{g}_a = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \tag{1}$$

with a bilinear operation (the general Lie bracket), $[\![\cdot,\cdot]\!]$, satisfying the identities:

$$\deg([\![x_a,y_b]\!]) \quad = \quad \deg(x_a)+\deg(y_b) = a+b \pmod 2, \tag{2}$$

$$[\![x_a,y_b]\!] \quad = \quad -(-1)^{ab}[\![y_b,x_a]\!], \tag{3}$$

$$[\![x_a,[\![y_b,z]\!]]\!] \quad = \quad [\![[\![x_a,y_b]\!],z]\!]+(-1)^{ab}[\![y_b,[\![x_a,z]\!]]\!], \tag{4}$$

where the elements x_a and y_b are homogeneous, $x_a\in\mathfrak{g}_a$, $y_b\in\mathfrak{g}_b$, and the element $z\in\mathfrak{g}$ is not necessarily homogeneous. The grading function $\deg(\cdot)$ is defined for homogeneous elements of the subspaces $\mathfrak{g}_0$ and $\mathfrak{g}_1$ modulo 2, $\deg(\mathfrak{g}_0)=0$, $\deg(\mathfrak{g}_1)=1$. The first identity (2) is called the grading condition, the second identity (3) is called the symmetry property and the condition (4) is the Jacobi identity. It follows from (2) that $\mathfrak{g}_0$ is a Lie subalgebra in $\mathfrak{g}$, and $\mathfrak{g}_1$ is a $\mathfrak{g}_0$-module. It follows from (2) and (3) that the general Lie bracket $[\![\cdot,\cdot]\!]$ for homogeneous elements posses two values: commutator $[\cdot,\cdot]$ and anticommutator $\{\cdot,\cdot\}$.

The $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebra

$\mathbb{Z}_2\times\mathbb{Z}_2$-graded LSA $\tilde{\mathfrak{g}}$, as a linear space, is a direct sum of four graded components

$$\tilde{\mathfrak{g}} \;=\; \bigoplus_{\mathbf{a}=(a_1,a_2)} \mathfrak{g}_{\mathbf{a}} \;=\; \mathfrak{g}_{(0,0)}\oplus\mathfrak{g}_{(1,1)}\oplus\mathfrak{g}_{(1,0)}\oplus\mathfrak{g}_{(0,1)} \tag{5}$$

with a bilinear operation $[\![\cdot,\cdot]\!]$ satisfying the identities (grading, symmetry, Jacobi):

$$\deg([\![x_{\mathbf{a}},y_{\mathbf{b}}]\!]) \;=\; \deg(x_{\mathbf{a}})+\deg(y_{\mathbf{b}}) = \mathbf{a}+\mathbf{b} = (a_1+b_1,a_2+b_2), \tag{6}$$

$$[\![x_{\mathbf{a}},y_{\mathbf{b}}]\!] \;=\; -(-1)^{\mathbf{ab}}[\![y_{\mathbf{b}},x_{\mathbf{a}}]\!], \tag{7}$$

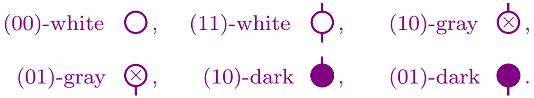
$$[\![x_{\mathbf{a}},[\![y_{\mathbf{b}},z]\!]]\!] \;=\; [\![[\![x_{\mathbf{a}},y_{\mathbf{b}}]\!],z]\!]+(-1)^{\mathbf{ab}}[\![y_{\mathbf{b}},[\![x_{\mathbf{a}},z]\!]]\!], \tag{8}$$

where the vector (a_1+b_1,a_2+b_2) is defined $\mod(2,2)$ and $\mathbf{ab}=a_1b_1+a_2b_2$. Here in (6)-(8) $x_{\mathbf{a}}\in\tilde{\mathfrak{g}}_{\mathbf{a}}$, $y_{\mathbf{b}}\in\tilde{\mathfrak{g}}_{\mathbf{b}}$, and the element $z\in\tilde{\mathfrak{g}}$ is not necessarily homogeneous. It follows from (6) that $\tilde{\mathfrak{g}}_{(0,0)}$ is a Lie subalgebra in $\tilde{\mathfrak{g}}$, and the subspaces $\tilde{\mathfrak{g}}_{(1,1)}$, $\tilde{\mathfrak{g}}_{(1,0)}$ and $\tilde{\mathfrak{g}}_{(0,1)}$ are $\tilde{\mathfrak{g}}_{(0,0)}$-modules. It should be noted that $\tilde{\mathfrak{g}}_{(0,0)}\oplus\tilde{\mathfrak{g}}_{(1,1)}$ is a Lie subalgebra in $\tilde{\mathfrak{g}}$ and the subspace $\tilde{\mathfrak{g}}_{(1,0)}\oplus\tilde{\mathfrak{g}}_{(0,1)}$ is a $\tilde{\mathfrak{g}}_{(0,0)}\oplus\tilde{\mathfrak{g}}_{(1,1)}$-module, and moreover $\{\tilde{\mathfrak{g}}_{(1,1)},\tilde{\mathfrak{g}}_{(1,0)}\}\subset\tilde{\mathfrak{g}}_{(0,1)}$ and vice versa $\{\tilde{\mathfrak{g}}_{(1,1)},\tilde{\mathfrak{g}}_{(0,1)}\}\subset\tilde{\mathfrak{g}}_{(1,0)}$. It follows from (6) and (7) that the general Lie bracket $[\![\cdot,\cdot]\!]$ for homogeneous elements posses two values: commutator $[\cdot,\cdot]$ and anticommutator $\{\cdot,\cdot\}$ as well as in the previous $\mathbb{Z}_2$-case.

Analysis of matrix realizations of the basis $\mathbb{Z}_2\times\mathbb{Z}_2$-graded Lie superalgebras shows that these superalgebras(as well as $\mathbb{Z}_2$-graded Lie superalgebras) have Cartan-Weyl and Chevalle bases, Weyl groups, Dynkin diagrams, etc. However these structures have a specific characteristics for $\mathbb{Z}_2$- and $\mathbb{Z}_2\times\mathbb{Z}_2$-graded cases. Let us to consider, for example, the Dynkin diagrams. In the case of $\mathbb{Z}_2$-graded superalgebras the nodes of the Dynkin diagram and corresponding simple roots occur only three types:

white ○, gray ⊗, dark ●.

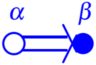
while in the case of $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebras we have six types of nodes:



Now I would like to consider in detail two basic superalgebras of rank 2: the orthosymplectic $\mathbb{Z}_2$-graded superalgebra $\mathfrak{osp}(1|4)$ and the orthosymplectic $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebra $\mathfrak{osp}(1|2,2):=\mathfrak{osp}(1,0|2,2)$. It will be shown that their real forms, which contain the Lorentz subalgebra $\mathfrak{o}(1,3)$, give us the super-anti-de Sitter (in the $\mathbb{Z}_2$-graded case) and super-de Sitter (in the $\mathbb{Z}_2\times\mathbb{Z}_2$-graded case) Lie superalgebras.

The orthosymplectic $\mathbb{Z}_2$-graded superalgebra $\mathfrak{osp}(1|4)$.

The Dynkin diagram:

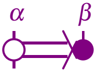


The Serre relations: $[e_{\pm\alpha},[e_{\pm\alpha},e_{\pm\beta}]]=0,\quad [\{[e_{\pm\alpha},e_{\pm\beta}],e_{\pm\beta}\},e_{\pm\beta}]=0.$

The root system Δ_+: $\underbrace{2\beta,2\alpha+2\beta,\alpha,\alpha+2\beta}_{\deg(\cdot)=0},\ \underbrace{\beta,\alpha+\beta}_{\deg(\cdot)=1}.$

The orthosymplectic $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebra $\mathfrak{osp}(1|2,2)$.

The Dynkin diagram:



The Serre relations: $\{e_{\pm\alpha},\{e_{\pm\alpha},e_{\pm\beta}\}\}=0,\quad \{[\{e_{\pm\alpha},e_{\pm\beta}\},e_{\pm\beta}],e_{\pm\beta}\}=0.$

The root system Δ_+: $\underbrace{2\beta,2\alpha+2\beta}_{\deg(\cdot)=(00)},\ \underbrace{\alpha,\alpha+2\beta}_{\deg(\cdot)=(11)},\ \underbrace{\beta}_{\deg(\cdot)=(10)},\ \underbrace{\alpha+\beta}_{\deg(\cdot)=(01)}.$

Commutation relations, which contain Cartan elements, are the same for the $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$ superalgebras and they are:

$$\begin{aligned} [\![e_{\gamma}, e_{-\gamma'}]\!] &= \delta_{\gamma,\gamma'} h_{\gamma}, \\ [h_{\gamma}, e_{\gamma'}] &= (\gamma,\gamma')e_{\gamma'} \end{aligned} \tag{1}$$

for $\gamma,\gamma' \in \{\alpha,\beta\}$. These relations together with the Serre relations for the superalgebras $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$ are called the defining relations of these superalgebras.

It is easy to check that these defining relations are invariant with respect to the non-graded Cartan involution $(^{\dagger})$ $((x^{\dagger})^{\dagger}=x$, $[\![x,y]\!]^{\dagger}=[\![y^{\dagger},x^{\dagger}]\!]$ for any homogenous elements x and y):

$$e^{\dagger}_{\pm\gamma} = e_{\mp\gamma}, \qquad h^{\dagger}_{\gamma} = h_{\gamma}. \tag{2}$$

The composite root vectors $e_{\pm\gamma}$ $(\gamma\in\Delta_{+})$ for $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$ are defined as follows

$$\begin{aligned} e_{\alpha+\beta} &:= [\![e_{\alpha}, e_{\beta}]\!], & e_{\alpha+2\beta} &:= [\![e_{\alpha+\beta}, e_{\beta}]\!], \\ e_{2\alpha+2\beta} &:= \frac{1}{\sqrt{2}}\{e_{\alpha+\beta}, e_{\alpha+\beta}\}, & e_{2\beta} &:= \frac{1}{\sqrt{2}}\{e_{\beta}, e_{\beta}\}, \\ e_{-\gamma} &:= e^{\dagger}_{\gamma}\,. \end{aligned} \tag{3}$$

These root vectors satisfy the non-vanishing relations:

$$\begin{array}{rclrcl}
[e_{\alpha},e_{\alpha+2\beta}] &=& (-1)^{\deg\alpha\cdot\deg\beta}\sqrt{2}e_{2\alpha+2\beta}, & [e_{\alpha},e_{2\beta}] &=& \sqrt{2}\,e_{\alpha+2\beta},\\
[\![e_{\alpha+\beta},e_{-\alpha}]\!] &=& -(-1)^{\deg\alpha\cdot\deg\beta}e_{\beta}, & [e_{\alpha+2\beta},e_{-\alpha}] &=& -\sqrt{2}\,e_{2\beta},\\
[e_{2\alpha+2\beta},e_{-\alpha}] &=& -(-1)^{\deg\alpha\cdot\deg\beta}\sqrt{2}e_{\alpha+2\beta}, & [e_{2\beta},e_{-\beta}] &=& -\sqrt{2}\,e_{\beta},\\
[\![e_{\alpha+2\beta},e_{-\alpha-\beta}]\!] &=& -(-1)^{\deg\alpha\cdot\deg\beta}e_{\beta}, & [\![e_{\beta},e_{-\alpha-\beta}]\!] &=& e_{-\alpha},\\
[\![e_{\beta},e_{-\alpha-2\beta}]\!] &=& -e_{-\alpha-\beta}, & [e_{2\alpha+2\beta},e_{-\alpha-\beta}] &=& -\sqrt{2}e_{\alpha+\beta},\\
[e_{\alpha+2\beta},e_{-2\alpha-2\beta}] &=& -(-1)^{\deg\alpha\cdot\deg\beta}\sqrt{2}e_{-\alpha}, & [e_{2\beta},e_{-\alpha-2\beta}] &=& -\sqrt{2}\,e_{-\alpha},\\
\{e_{\alpha+\beta},e_{-\alpha-\beta}\} &=& h_{\alpha}+h_{\beta}, & [e_{\alpha+2\beta},e_{-\alpha-2\beta}] &=& -h_{\alpha}-2h_{\beta},\\
[e_{2\beta},e_{-2\beta}] &=& -2h_{\beta}, & [e_{2\alpha+2\beta},e_{-2\alpha-2\beta}] &=& -2h_{\alpha}-2h_{\beta}.
\end{array} \quad (4)$$

The rest of non-zero relations is obtained by applying the non-graded Cartan involution (†) to these relations.

Now we find real forms of $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$, which contain the real Lorentz subalgebra $\mathfrak{so}(1,3)$.
It is not difficult to check that the antilinear mapping $(^*)$ $((x^*)^* = x$, $[\![x,y]\!]^* = [\![y^*,x^*]\!]$ for any homogenous elements x and y) given by

$$\begin{aligned}
e^*_{\pm\alpha} &= -(-1)^{\deg\alpha\cdot\deg\beta}e_{\mp\alpha}, & e^*_{\pm\beta} &= -ie_{\pm(\alpha+\beta)}, \\
e^*_{\pm2\beta} &= -e_{\pm(2\alpha+2\beta)}, & e^*_{\pm(\alpha+2\beta)} &= -e_{\pm(\alpha+2\beta)}, \qquad (5)\\
h^*_{\alpha} &= h_{\alpha}, & h^*_{\beta} &= -h_{\alpha}-h_{\beta}.
\end{aligned}$$

is an antiinvolution and the desired real form with respect to the antiinvolution is presented as follows.

The Lorentz algebra $\mathfrak{o}(1,3)$:

$$\begin{aligned}
L_{12} &= -\frac{1}{2}h_{\alpha},\\
L_{13} &= -\frac{i}{2\sqrt{2}}\Big(e_{2\beta}+e_{2\alpha+2\beta}+e_{-2\beta}+e_{-2\alpha-2\beta}\Big),\\
L_{23} &= -\frac{1}{2\sqrt{2}}\Big(e_{2\beta}-e_{2\alpha+2\beta}-e_{-2\beta}+e_{-2\alpha-2\beta}\Big),\\
L_{01} &= \frac{i}{2\sqrt{2}}\Big(e_{2\beta}+e_{2\alpha+2\beta}-e_{-2\beta}-e_{-2\alpha-2\beta}\Big),\\
L_{02} &= \frac{1}{2\sqrt{2}}\Big(e_{2\beta}-e_{2\alpha+2\beta}+e_{-2\beta}-e_{-2\alpha-2\beta}\Big),\\
L_{03} &= -\frac{i}{2}(h_{\alpha}+2h_{\beta}).
\end{aligned} \tag{6}$$

The generators $L_{\mu 4}$ (curved four-momentum):

$$\begin{aligned}
L_{04} &= -\frac{i}{2}\Big(e_{\alpha+2\beta}+(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha-2\beta}\Big),\\
L_{14} &= -\frac{i}{2}\Big(e_{\alpha}+(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha}\Big),\\
L_{24} &= \frac{1}{2}\Big(e_{\alpha}-(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha}\Big),\\
L_{34} &= -\frac{i}{2}\Big(e_{\alpha+2\beta}-(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha-2\beta}\Big).
\end{aligned} \tag{7}$$

Here are: $\deg\alpha=0, \deg\beta=1$, i.e. $(-1)^{\deg\alpha\cdot\deg\beta}=1$, for the case of the $\mathbb{Z}_2$-grading; $\deg\alpha=(1,1), \deg\beta=(1,0)$, i.e. $(-1)^{\deg\alpha\cdot\deg\beta}=-1$, for the case of the $\mathbb{Z}_2\times\mathbb{Z}_2$-grading.

The all elements L_{ab} $(a,b=0,1,2,3,4)$ satisfy the relations

$$[L_{ab},L_{cd}] \;=\; i(g_{bc}L_{ad}-g_{bd}L_{ac}+g_{ad}L_{bc}-g_{ac}L_{bd}), \qquad (8)$$

$$L_{ab} \;=\; -L_{ba}, \qquad L_{ab}^{\star}=L_{ab}, \qquad (9)$$

where the metric tensor g_{ab} is given by

$$\begin{aligned} g_{ab} &= \mathrm{diag}\,(1,-1,-1,-1,g_{44}^{(\alpha)}), \\ g_{44}^{(\alpha)} &= (-1)^{\deg\alpha\cdot\deg\beta}. \end{aligned} \qquad (10)$$

Thus we see that

(a) in the case of the $\mathbb{Z}_2$-grading, $(-1)^{\deg\alpha\cdot\deg\beta}=1$, the generators L_{ab} $(a,b=0,1,2,3,4)$ generate the anti-de-Sitter algebra $\mathfrak{o}(2,3)$, and

(b) in the case of the $\mathbb{Z}_2\times\mathbb{Z}_2$-grading, $(-1)^{\deg\alpha\cdot\deg\beta}=-1$, the generators L_{ab} $(a,b=0,1,2,3,4)$ generate the de-Sitter algebra $\mathfrak{o}(1,4)$.

Finally we introduce the "supercharges":

$$\begin{aligned} Q_1 &:= \sqrt{2}\exp\Big(-\frac{i\pi}{4}\Big)\, e_{\alpha+\beta}\ , \qquad & Q_2 &:= \sqrt{2}\exp\Big(-\frac{i\pi}{4}\Big)\, e_{-\alpha-\beta}\ , \\ \bar{Q}_{\dot 1} &:= \sqrt{2}\exp\Big(-\frac{i\pi}{4}\Big)\, e_{\beta}\ , \qquad & \bar{Q}_{\dot 2} &:= \sqrt{2}\exp\Big(-\frac{i\pi}{4}\Big)\, e_{-\beta}\ . \end{aligned} \tag{11}$$

that have the following commutation relations between themselves:

$$\begin{aligned} \{Q_1, Q_1\} &= -i2\sqrt{2}e_{2\alpha+2\beta} = 2(L_{13}-iL_{23}-L_{01}+iL_{02})\ , \\ \{Q_2, Q_2\} &= -i2\sqrt{2}e_{-2\alpha-2\beta} = 2(L_{13}+iL_{23}-L_{01}-iL_{02})\ , \\ \{Q_1, Q_2\} &= -i2(h_\alpha+h_\beta) = 2(L_{03}+iL_{12})\ , \\ \{\bar{Q}_{\dot\eta}, \bar{Q}_{\dot\zeta}\} &= \{Q_\zeta, Q_\eta\}^* \qquad (\bar{Q}_{\dot\eta} = Q^*_\eta \text{ for } \eta=1,2;\ \dot\eta=\dot 1,\dot 2), \end{aligned} \tag{12}$$

$$\begin{aligned} [\![Q_1, \bar{Q}_{\dot 1}]\!] &= -i2e_{\alpha+2\beta} = 2(L_{04}+L_{34}), \\ [\![Q_1, \bar{Q}_{\dot 2}]\!] &= -i2e_{\alpha} = 2(L_{14}-iL_{24}), \\ [\![Q_2, \bar{Q}_{\dot 1}]\!] &= -i2(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha} = 2(L_{14}+iL_{24}), \\ [\![Q_2, \bar{Q}_{\dot 2}]\!] &= -i2(-1)^{\deg\alpha\cdot\deg\beta}e_{-\alpha-2\beta} = 2(L_{04}-L_{34}). \end{aligned} \tag{13}$$

Here $[\![\cdot,\cdot]\!]\equiv\{\cdot,\cdot\}$ for the $\mathbb{Z}_2$-case and $[\![\cdot,\cdot]\!]\equiv[\cdot,\cdot]$ for the $\mathbb{Z}_2\times\mathbb{Z}_2$-case. We can also calculate commutation relations between the operators L_{ab} and the supercharges Q's and $\bar{Q}$'s .

Using the standard contraction procedure: $L_{\mu 4}=R\,P_\mu$ $(\mu=0,1,2,3)$, $Q_\alpha\to\sqrt{R}\,Q_\alpha$ and $\bar{Q}_{\dot\alpha}\to\sqrt{R}\,\bar{Q}_{\dot\alpha}$ $(\alpha=1,2;$ $\dot\alpha=\dot1,\dot2)$ for $R\to\infty$ we obtain the super-Poincaré algebras (standard $\mathbb{Z}_2$-graded and alternative $\mathbb{Z}_2\times\mathbb{Z}_2$-graded) which are generated by $L_{\mu\nu}$, P_μ, Q_α, $\bar{Q}_{\dot\alpha}$ where $\mu,\nu=0,1,2,3$; $\alpha=1,2$; $\dot\alpha=\dot1,\dot2$, with the relations (we write down only those which are distinguished in the $\mathbb{Z}_2$- and $\mathbb{Z}_2\times\mathbb{Z}_2$-cases).

(I) For the $\mathbb{Z}_2$-graded Poincaré SUSY:

$$[P_\mu,Q_\alpha] \;=\; [P_\mu,\bar{Q}_{\dot\alpha}]=0,\quad \{Q_\alpha,\bar{Q}_{\dot\beta}\} \;=\; 2\sigma^\mu_{\alpha\dot\beta}P_\mu. \qquad (14)$$

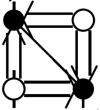
(II) For the $\mathbb{Z}_2\times\mathbb{Z}_2$-graded Poincaré SUSY:

$$\{\tilde{P}_\mu,Q'_\alpha\} \;=\; \{\tilde{P}_\mu,\bar{Q}'_{\dot\alpha}\}=0,\quad [Q'_\alpha,\bar{Q}'_{\dot\beta}] \;=\; 2\sigma^\mu_{\alpha\dot\beta}\tilde{P}_\mu, \qquad (15)$$

Now I would like to construct a $\mathbb{Z}_2\times\mathbb{Z}_2$-graded SUSY which contains as subalgebras the super-anti-de Sitter $\mathfrak{osp}(1|4)$ and the super-de Sitter $\mathfrak{osp}(1|2,2)$ in nontrivial way, that is this SUSY is not a direct sum of $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$. Let us consider : (i) two $\mathbb{Z}_2$-graded superalgebras $\mathfrak{osp}(1|4)$ with the Dynkin diagrams:

(ii) two $\mathbb{Z}_2\times\mathbb{Z}_2$-graded superalgebras $\mathfrak{osp}(1|2,2)$ with the Dynkin diagrams:

We identify the same nodes of these four Dynkin diagrams and as a consequence the resulting Dynkin diagram is given by



The roots α, $\tilde{\alpha}$ and also β', β'' differ from each other only by grading: $\deg(\alpha)=(0,0)$, $\deg(\tilde{\alpha})=(1,1)$; $\deg(\beta')=(1,0)$, $\deg(\beta'')=(0,1)$.

A $\mathbb{Z}_2\times\mathbb{Z}_2$-graded contragredient superalgebra with such Dynkin diagram does indeed exist and its the positive root system Δ_+ is given by

(a) even roots with $\deg(\cdot)=(0,0)$: $2\beta,\ 2\alpha+2\beta,\ \alpha,\ \alpha+2\beta,$

$(\tilde{a})$ even roots with $\deg(\cdot)=(1,1)$: $\widetilde{2\beta},\ \widetilde{2\alpha+2\beta},\ \tilde{\alpha},\ \widetilde{\alpha+2\beta},$

(b') odd roots with $\deg(\cdot)=(1,0)$: $\beta',\ (\alpha+\beta)',$

(b'') odd roots with $\deg(\cdot)=(0,1)$: $\beta'',\ (\alpha+\beta)''.$

We have the following structure of subalgebras:

(i) the double roots 2β, $2\alpha+2\beta$ is of the complex Lorentz algebra $\mathfrak{o}(5)\cong\mathfrak{sl}(2)\oplus\mathfrak{sl}(2)$,

(ii) the root system (a) is of $\mathfrak{o}(5)$ or $\mathfrak{sp}(4)$,

(iii) the root system (a) together (b') or (b'') is the positive root system of the orthosymplectic superalgebra $\mathfrak{osp}(1|4)$,

(iv) the root system (i) together with $\tilde{\alpha}$, $\widetilde{\alpha+2\beta}$ is the positive root system of $\mathfrak{o}(5)$ or $\mathfrak{sp}(4)$ too,

(v) the root system (iv) together with the roots β', $(\alpha+\beta)''$ or β'', $(\alpha+\beta)'$ is the positive root system of the orthosymplectic superalgebra $\mathfrak{osp}(1|2,2)$,

(vi) the roots $(\tilde{a})$ is not any positive root system of subalgebras.

Because the constructed $\mathbb{Z}_2\times\mathbb{Z}_2$-graded Lie superalgebra contains the orthosymplectic superalgebras $\mathfrak{osp}(1|4)$ and $\mathfrak{osp}(1|2,2)$ it is called Doudle Orthosymplectic Superalgebra $\mathfrak{dosp}(1|4;1|2,2)$.

Now we construct a real form real form of the doudle orthosymplectic superalgebra $\mathfrak{dosp}(1|4;1|2,2)$, which contain the real Lorentz subalgebra $\mathfrak{so}(1,3)$. This real form, *Super-*(AdS_5+dS_5), generates by the following elements:

$$L_{ab},\;\; \tilde{L}_{ab},\;\; Q^{'}_{\alpha},\;\; \bar{Q}^{'}_{\dot{\alpha}},\;\; Q^{''}_{\alpha},\;\; \bar{Q}^{''}_{\dot{\alpha}}$$

where $a,b=0,1,2,3,4;\ \alpha=1,2;\ \dot{\alpha}=\dot{1},\dot{2}$.

Algebraic structure of *Super-*(AdS_5+dS_5):

(1) $\mathscr{L}_4=\{L_{\mu,\nu}|\mu,\nu=0,1,2,3\}$, $\deg(\mathscr{L}_4)=(0,0)$, - Lorentz Lie algebra,

(2) $\tilde{\mathscr{L}}_4=\{\tilde{L}_{\mu,\nu}|\mu,\nu=0,1,2,3\}$, $\deg(\tilde{\mathscr{L}}_4)=(1,1)$, - graded Lorentz subspace,

$$[L_{\mu,\nu},L_{\mu',\nu'}]=[\tilde{L}_{\mu,\nu},\tilde{L}_{\mu',\nu'}]=i(g_{\nu,\mu'}L_{\mu,\nu'}-g_{\nu,\nu'}L_{\mu,\nu}+g_{\mu,\nu'}L_{\mu,\nu'}-g_{\mu,\mu'}L_{\nu,\nu'}),$$

$$[L_{\mu,\nu},\tilde{L}_{\mu',\nu'}]=i(g_{\nu,\mu'}\tilde{L}_{\mu,\nu'}-g_{\nu,\nu'}\tilde{L}_{\mu,\nu}+g_{\mu,\nu'}\tilde{L}_{\mu,\nu'}-g_{\mu,\mu'}\tilde{L}_{\nu,\nu'}),$$

$$g_{\mu,\nu}=\mathrm{diag}\,(1,-1,-1,-1)$$

(3) $\mathscr{P}_4=\{L_{\mu,4}|\mu=0,1,2,3\}$, $\deg(\mathscr{P}_4)=(0,0)$, - curved AdeS four-momenta,

(4) $\tilde{\mathscr{P}}_4=\{\tilde{L}_{\mu,4}|\mu=0,1,2,3\}$, $\deg(\mathscr{P}_4)=(1,1)$, - curved deS four-momenta,

(5) $AdS_5=\mathscr{L}_4\oplus\mathscr{P}_4$ - Anti-de Sitter Lie algebra,

(6) $dS_5=\mathscr{L}_4\oplus\tilde{\mathscr{P}}_4$ - de Sitter Lie algebra,

(7) $\mathscr{Q}'_4=\{Q'_\alpha,\bar{Q}'_{\dot\alpha}|\alpha=1,2;\dot\alpha=\dot1,\dot2\}$, $\deg(\mathscr{Q}'_4)=(1,0)$,

(8) $\mathscr{Q}''_4=\{Q''_\alpha,\bar{Q}''_{\dot\alpha}|\alpha=1,2;\dot\alpha=\dot1,\dot2\}$, $\deg(\mathscr{Q}''_4)=(0,1)$,

(9) $SAdS_5=AdS_5\oplus\mathscr{Q}'_4$ - Super-Anti-de Sitter,

(10) $SAdS_5=AdS_5\oplus\mathscr{Q}''_4$ - Super-Anti-de Sitter,

(11) $\tilde{\mathscr{Q}}'_4=\{Q'_\alpha,\bar{Q}''_{\dot\alpha}|\alpha=1,2;\dot\alpha=\dot1,\dot2\}$, $\deg(Q'_\alpha)=(1,0)$, $\deg(\bar{Q}''_\alpha)=(0,1)$,

(12) $\tilde{\mathscr{Q}}''_4=\{Q''_\alpha,\bar{Q}'_{\dot\alpha}|\alpha=1,2;\dot\alpha=\dot1,\dot2\}$,$\deg(Q''_\alpha)=(1,0)$, $\deg(\bar{Q}''_\alpha)=(0,1)$.

(13) $SdS_5=dS_5\oplus\tilde{\mathscr{Q}}'_4$ - Super-de Sitter,

(14) $SAdS_5=dS_5\oplus\tilde{\mathscr{Q}}''_4$ - Super-de Sitter.

Using the stand. contraction procedure: $L_{\mu 4}=R\,P_{\mu}$, $\tilde{L}_{\mu 4}=R\,\tilde{P}_{\mu}$ $(\mu=0,1,2,3)$, and $Q_{\alpha}^{'}\to\sqrt{R}\,Q_{\alpha}^{'}$, $\bar{Q}_{\dot\alpha}^{'}\to\sqrt{R}\,\bar{Q}_{\dot\alpha}^{'}$, and $Q_{\alpha}^{''}\to\sqrt{R}\,Q_{\alpha}^{''}$ and $\bar{Q}_{\dot\alpha}^{''}\to\sqrt{R}\,\bar{Q}_{\dot\alpha}^{''}$ $(\alpha=1,2;\ \dot\alpha=\dot1,\dot2)$ for $R\to\infty$ we obtain the double Poincaré and alternative Poincaré superalgebra which is generated by $L_{\mu\nu}$, P_{μ}, $\tilde{L}_{\mu\nu}$, $\tilde{P}_{\mu}$, $Q_{\alpha}^{'}$, $\bar{Q}_{\dot\alpha}^{'}$, $Q_{\alpha}^{''}$, $\bar{Q}_{\dot\alpha}^{''}$ where $\mu,\nu=0,1,2,3$; $\alpha=1,2$; $\dot\alpha=\dot1,\dot2$.

This real double $\mathbb{Z}_2\times\mathbb{Z}_2$-graded Poincaré SUSY have the structure of subalgebras:

(i) Lorentz algebra is generated by the elements $L_{\mu\nu}$ $(\mu,\nu=0,1,2,3)$,

(ii) standard Poincaré algebra is generated by Lorentz algebra and the four-momenta P_{μ}

(iii) alternative Poincaré algebra is generated by Lorentz algebra and the four-momenta $\tilde{P}_{\mu}$,

(iv) standard $N=1$ super-Poincaré algebra is generated by the standard Poincaré and supergarges $Q_{\alpha}^{'}$, $\bar{Q}_{\dot\alpha}^{'}$,

(v) standard $N=1$ super-Poincaré algebra is generated by the standard Poincaré and supergarges $Q_{\alpha}^{''}$, $\bar{Q}_{\dot\alpha}^{''}$,

(iv) alternative $N=1$ super-Poincaré algebra is generated by the altrnative Poincaré and supergarges $Q_{\alpha}^{'}$, $\bar{Q}_{\dot\alpha}^{''}$,

(v) alternative $N=1$ super-Poincaré algebra is generated by the altrnative Poincaré and supergarges $Q_{\alpha}^{''}$, $\bar{Q}_{\dot\alpha}^{'}$.

We write down commutation relations only for the symmetry algebra, i.e. for the superalgebra generated by the supergarges $Q^{'}_{\alpha}$, $\bar{Q}^{'}_{\dot{\alpha}}$, $Q^{''}_{\alpha}$, $\bar{Q}^{''}_{\dot{\alpha}}$ and the fourmomenta P_{μ}, $\tilde{P}_{\mu}$.

(I) Standard commutation relations:

$$[P_{\mu},Q^{'}_{\alpha}] \;=\; [P_{\mu},\bar{Q}^{'}_{\dot{\alpha}}]=0, \quad \{Q^{'}_{\alpha},\bar{Q}^{'}_{\dot{\beta}}\} \;=\; 2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu}, \qquad (1)$$

$$[P_{\mu},Q^{''}_{\alpha}] \;=\; [P_{\mu},\bar{Q}^{''}_{\dot{\alpha}}]=0, \quad \{Q^{''}_{\alpha},\bar{Q}^{''}_{\dot{\beta}}\} \;=\; 2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu}, \qquad (2)$$

(II) Alternative commutation relations:

$$\{\tilde{P}_{\mu},Q^{'}_{\alpha}\} \;=\; \{\tilde{P}_{\mu},\bar{Q}^{''}_{\dot{\alpha}}\}=0, \quad [Q^{'}_{\alpha},\bar{Q}^{''}_{\dot{\beta}}] \;=\; 2\sigma^{\mu}_{\alpha\dot{\beta}}\tilde{P}_{\mu}, \qquad (3)$$

$$\{\tilde{P}_{\mu},Q^{''}_{\alpha}\} \;=\; \{\tilde{P}_{\mu},\bar{Q}^{'}_{\dot{\alpha}}\}=0, \quad [Q^{''}_{\alpha},\bar{Q}^{'}_{\dot{\beta}}] \;=\; 2\sigma^{\mu}_{\alpha\dot{\beta}}\tilde{P}_{\mu}. \qquad (4)$$

Let us consider the supergroups associated the double super-Poincaré. A group element g is given by the exponential of the double super-Poincaré generators, namely

$$\begin{aligned} g(\omega^{\mu\nu},\tilde{\omega}^{\mu\nu},x^{\mu},\tilde{x}^{\mu},\theta'^{\alpha},\bar{\theta}'^{\dot{\alpha}},\theta''^{\alpha},\bar{\theta}''^{\dot{\alpha}}) &= \exp(\omega^{\mu\nu}L_{\mu\nu}+\tilde{\omega}^{\mu\nu}\tilde{L}_{\mu\nu}+x^{\mu}P_{\mu}+\tilde{x}^{\mu}\tilde{P}_{\mu} \\ &\quad +\theta'^{\alpha}Q'_{\alpha}+\theta'^{\alpha}Q'_{\alpha}+\bar{Q}''_{\dot{\alpha}}\bar{\theta}''^{\dot{\alpha}}+\bar{Q}''_{\dot{\alpha}}\bar{\theta}''^{\dot{\alpha}}). \end{aligned}$$

The grading of the exponent is zero (00) therefore all parameters are graded and we have permutation relations for the coordinates x^{μ}, $\tilde{x}^{\mu}$ and the Grassmann variables θ'^{α}, $\bar{\theta}'^{\dot{\alpha}}$, θ''^{α}, $\bar{\theta}''^{\dot{\alpha}}$:

(i) standard relations

$$\begin{aligned} [x_{\mu},\theta'_{\alpha}] &= [x_{\mu},\bar{\theta}'_{\dot{\alpha}}] = [x_{\mu},\theta''_{\alpha}] = [x_{\mu},\bar{\theta}''_{\dot{\alpha}}] = 0, \\ \{\theta'_{\alpha},\bar{\theta}'_{\dot{\beta}}\} &= \{\theta'_{\alpha},\theta'_{\beta}\} = \{\bar{\theta}'_{\dot{\alpha}},\bar{\theta}'_{\dot{\beta}}\} = 0, \\ \{\theta''_{\alpha},\bar{\theta}''_{\dot{\beta}}\} &= \{\theta''_{\alpha},\theta''_{\beta}\} = \{\bar{\theta}''_{\dot{\alpha}},\bar{\theta}''_{\dot{\beta}}\} = 0; \end{aligned}$$

(ii) alternative relations

$$\begin{aligned} \{\tilde{x}_{\mu},\theta'_{\alpha}\} &= \{\tilde{x}_{\mu},\bar{\theta}'_{\dot{\alpha}}\} = \{\tilde{x}_{\mu},\theta''_{\alpha}\}; = \{\tilde{x}_{\mu},\bar{\theta}''_{\dot{\alpha}}\} = 0, \\ [\theta'_{\alpha},\bar{\theta}''_{\dot{\beta}}] &= [\theta''_{\alpha},\bar{\theta}'_{\dot{\beta}}] = [\theta'_{\alpha},\theta''_{\beta}] = [\bar{\theta}'_{\dot{\alpha}},\bar{\theta}''_{\dot{\beta}}] = 0, \end{aligned}$$

Some Cosmological Speculations ("Na Zakusku")

So we constructed the complex double orthosymplectic superalgebra $\mathfrak{dosp}(1|4;1|2,2)$. It has 8 supercharges and it contains the de Sitter and anti-de Sitter algebras simultaneously and as well as their superextansions. The Lorentz algebra is a common part of these de Sitter and anti-de Sitter algebras. Such real form of $\mathfrak{dosp}(1|4;1|2,2)$ services two geometries with metrics $g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,1)$ (anti-de Sitter), and $g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,-1)$ (de Sitter) where $a,b=0,1,2,3,4$.

Modern models of our Univers say that there are two substational components: matter and dark energy.

The dark energy is associated with the cosmological constant Λ and moreover $\Lambda>0$ that is corresponding to the de Sitter metric $g_{ab}=\mathrm{diag}\,(1,-1,-1,-1,-1)$.

Moreover our space-time manifold does simultaneously consist of the standard coordinates x_μ ($\deg(x_\mu)=(00)$) and alternative ones $\tilde{x}_\mu$ ($\deg(\tilde{x}_\mu)=(11)$). Corresponding four-momenta are p_μ and $\tilde{p}_\mu$ with the grading $\deg(p_\mu)=(00)$ and $\deg(\tilde{p}_\mu)=(11)$. A Casimir element of our double super-Poincare is given by $p_\mu p^\mu+\tilde{p}_\mu\tilde{p}^\mu$. We can assume that the standard part $p_\mu p^\mu$ coreponds to matter and $\tilde{p}_\mu\tilde{p}^\mu$ is associated with the dark energy. We can show that $\tilde{p}_\mu\tilde{p}^\mu = -\, p_\mu p^\mu$, it means that

$$p_\mu p^\mu+\tilde{p}_\mu\tilde{p}^\mu=0.$$

This result is not contradict to modern models of our Universe.

THANK YOU FOR YOUR ATTENTION