Supersymmetric membrane in D=7

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Plan

- Introduction
- Preliminary considerations
    - Algebra and its consequences
    - Cartan forms
    - Covariant derivatives
    - Irreducibility conditions
- Broken supersymmetry and anzatz for action
    - S-SUSY transformations
    - New covariant derivative
    - Wess-Zumino term
- Unbroken supersymmetry invariance
    - $\nabla\psi$ and others
    - Anzatz for transformations
    - Variation of lagrangian
- Conclusion

Introduction

Partial Spontaneous Breaking of Global Supersymmetry is an approach, dedicated to study of superbranes and Born-Infeld theories. Such breaking of supersymmetry necessarily accompanies breaking of target space Poincaré symmetry, which is result of embedding of extended object (membrane, particle worldline) in the (flat) spacetime. Partial spontaneous breaking of global supersymmetry also implies appearance of Goldstone fermions; then supersymmetry is partially realized as nonlinear transformation of these fermions and related bosonic coordinates and bosonic fields.

In this talk we consider PBGS with $d = 3,\ N = 4$ hypermultiplet as Goldstone superfield, which describes membrane in $D = 7$. Originally we considered this system just as a simplified version of 3-brane in $D = 8$, described by $d = 4,\ N = 2$ hypermultiplet. However, it possibly leads to important insights about such systems in general.

Introduction

Then constructing component action for the system with spontaneous $N = 8 \rightarrow N = 4$ breaking of supersymmetry in $d = 3$, we use coset approach (previously outlined by Stefano Bellucci). It allows us to construct systematically

- Transformation laws for superfields;
- Cartan forms, invariant under these transformations;
- Covariant derivatives;
- Irreducibility conditions and bosonic action from bosonic forms;
- Conditions, that put system on-shell.

Then standard program is to check invariance of generalized bosonic action and Wess-Zumino term under broken supersymmetry, derive unbroken symmetry transformation laws for components, fix remaining arbitrary constants by invariance at low orders in fields and then check invariance of the action under full transformation laws.

Algebra and its consequences

Our starting point is $N = 8$ super-Poincaré algebra in $d = 3$. Its bosonic part consists of generators

- Momenta generators P_{ab} and cental charges $Z^{i\alpha}$ (together - $7D$ translations)
- M_{ab} - $3d$ Lorentz, two $su(2)$ algebras T^{ij}, $R^{\alpha\beta}$
- Generators $K^{i\alpha}_{ab}$ from coset $SO(1,6)/SO(1,2) \times SU(2) \times SU(2)$, mix together P_{ab} and $Z^{i\alpha}$

They are accompanied by supercharges Q^i_a, $\overline{Q}_{ia}$, S^α_a, $\overline{S}_{\alpha a}$ with anticommutators

$$\left\{Q^i_a, \overline{Q}_{jb}\right\} = 2\delta^i_j P_{ab}, \; \left\{S^\alpha_a, \overline{S}_{\beta b}\right\} = 2\delta^\alpha_\beta P_{ab}, \; \left\{Q^i_a, S^\alpha_b\right\} = 2\epsilon_{ab} Z^{i\alpha}, \; \left\{\overline{Q}_{ia}, \overline{S}_{\alpha b}\right\} = 2\epsilon_{ab} Z_{i\alpha}.$$

They have standard commutation relations with M_{ab}, T^{ij}, $R^{\alpha\beta}$. With $K^{i\alpha}_{ab}$

$$\mathrm{i}\left[K^{i\alpha}_{ab}, Q^j_c\right] = \epsilon^{ij}\left(\epsilon_{ac}\overline{S}^\alpha_b + \epsilon_{bc}\overline{S}^\alpha_a\right), \; \mathrm{i}\left[K^{i\alpha}_{ab}, S^\beta_c\right] = -\epsilon^{\alpha\beta}\left(\epsilon_{ac}\overline{Q}^i_b + \epsilon_{bc}\overline{Q}^i_a\right).$$

Cartan forms

Bosonic forms can be constructed with use of $ISO(1,6)/SO(1,2)\times SU(2)\times SU(2)$ coset element

$$g_{bos} = e^{\mathrm{i}x^{ab}P_{ab}}e^{\mathrm{i}q(x)^{i\alpha}Z_{i\alpha}}e^{\mathrm{i}\Lambda(x)^{ab}_{i\alpha}K^{i\alpha}_{ab}}.$$

Interesting bosonic forms from $g^{-1}_{bos}dg_{bos}$ then are

$$\Omega_P = \left[\left(\cosh 2\sqrt{Y}\right)_{ab}^{cd} dx^{ab} + 2\left(\frac{\sinh 2\sqrt{y}}{2\sqrt{y}}\right)_{i\alpha}^{j\beta} dq^{i\alpha}\Lambda^{cd}_{j\beta}\right] P_{cd},$$

$$\Omega_Z = \left[(\cosh 2\sqrt{y})^{j\beta}_{i\alpha}\, dq_{j\beta} + 2\Lambda^{ab}_{i\alpha}dx_{cd}\left(\frac{\sinh 2\sqrt{Y}}{2\sqrt{Y}}\right)^{cd}_{ab}\right] Z^{i\alpha}.$$

Here $Y^{cd}_{ab} = \Lambda^{i\alpha}_{ab}\Lambda^{cd}_{i\alpha}$, $y^{j\beta}_{i\alpha} = \Lambda^{ab}_{i\alpha}\Lambda^{j\beta}_{ab}$. Applying condition $\Omega_Z = 0$ (Inverse Higgs Phenomenon - Ivanov, Ogievetsky) leads to

$$dq_{i\alpha} = -2dx_{ab}\Lambda^{cd}_{i\alpha}\left(\frac{\tanh 2\sqrt{Y}}{2\sqrt{Y}}\right)^{cd}_{ab}, \qquad \Omega_P = dx^{ab}P_{cd}e^{cd}_{ab} \Rightarrow$$

$$e^{cd}_{ab} = \left(\frac{1}{\cosh 2\sqrt{Y}}\right)^{cd}_{ab}, \qquad g^{cd}_{ab} = \left(e^2\right)^{cd}_{ab} = \delta^c_a\delta^d_b - \partial_{ab}q^{i\alpha}\partial^{cd}q_{i\alpha}, \qquad \mathcal{L}_{bos} = \sqrt{\det g}.$$

Cartan forms

Overall coset element, that takes into account fermions, reads

$$g = e^{\mathrm{i}x^{ab}P_{ab}}\, e^{\theta^a_i Q^i_a + \bar\theta^{ia}\overline{Q}_{ia}}\, e^{\mathrm{i}\mathbf{q}_{i\alpha}Z^{i\alpha}}\, e^{\psi^a_\alpha S^\alpha_a + \bar\psi^{a\alpha}\overline{S}_{a\alpha}}\, e^{\mathrm{i}\boldsymbol{\Lambda}^{ab}_{i\alpha}K^{i\alpha}_{ab}}.$$

Just one fermionic form is of interest:

$$\Omega_S = \left[\left(\cosh 2\sqrt{W}\right)_{\alpha b}^{\gamma a} d\psi^b_\gamma + 2d\bar\theta^{jb}\left(\frac{\sinh 2\sqrt{\overline{T}}}{2\sqrt{\overline{T}}}\right)_{jb}^{kc}\boldsymbol{\Lambda}^a_{kc\alpha}\right] S^\alpha_a,$$

$$\overline{T}^{ia}_{jb} = \boldsymbol{\Lambda}^{ia}_{c\gamma}\boldsymbol{\Lambda}^{c\gamma}_{jb},\; W^{\beta b}_{\alpha a} = \boldsymbol{\Lambda}^{\beta j}_{ac}\boldsymbol{\Lambda}^{cb}_{j\alpha}.$$

Bosonic forms, modified by fermionic generators, can be found by replacing

$$dx^{ab} \rightarrow \triangle x^{ab} = dx^{ab} - \mathrm{i}\left(\theta^{(a}_i d\bar\theta^{b)i} + \bar\theta^{i(a}d\theta^{b)}_i + \psi^{(a}_\alpha d\bar\psi^{b)\alpha} + \bar\psi^{\alpha(a}d\psi^{b)}_\alpha\right),$$

$$dq_{i\alpha} \rightarrow \triangle\mathbf{q}_{i\alpha} = d\mathbf{q}_{i\alpha} - 2\mathrm{i}\left(\psi_{a\alpha}d\theta^a_i + \bar\psi_{a\alpha}d\bar\theta^a_i\right).$$

New subforms are invariant under both supersymmetries:

$$\delta_Q x^{ab} = \mathrm{i}\left(\epsilon^{(a}_i\bar\theta^{b)i} + \bar\epsilon^{i(a}\theta^{b)}_i\right),\; \delta_Q\theta^a_i = \epsilon^a_i,\; \delta_Q\bar\theta^{ia} = \bar\epsilon^{ai},$$

$$\delta_S\psi^a_\alpha = \varepsilon^a_\alpha,\; \delta_S\theta^{a\alpha} = 0,\; \delta_S x^{ab} = \mathrm{i}\left(\varepsilon^{(a}_\alpha\bar\psi^{b)\alpha} + \bar\varepsilon^{\alpha(a}\psi^{b)}_\alpha\right), \delta_S\mathbf{q}_{i\alpha} = 2\mathrm{i}\left(\varepsilon_{a\alpha}\theta^a_i + \bar\varepsilon_{a\alpha}\bar\theta^a_i\right).$$

Covariant derivatives

Subform $\triangle x^{ab}$, invariant under both supersymmetries, allows us to define derivatives, covariant with respect to both supersymmetries

$$\begin{aligned}
\nabla_{ab} &= \left(E^{-1}\right)_{ab}^{cd}\partial_{cd},\; E_{ab}^{cd} = \frac{1}{2}\left(\delta_a^c\delta_b^d + \delta_a^d\delta_b^c\right) - \mathrm{i}\left(\psi_\alpha^{(c}\partial_{ab}\bar{\psi}^{d)\alpha} + \bar{\psi}^{\alpha(c}\partial_{ab}\psi_\alpha^{d)}\right);\\
\nabla_a^i &= D_a^i - \mathrm{i}\left(\psi_\alpha^c\nabla_a^i\bar{\psi}^{d\alpha} + \bar{\psi}^{c\alpha}\nabla_a^i\psi_\alpha^d\right)\partial_{cd},\\
\overline{\nabla}_{ia} &= \overline{D}_{ia} - \mathrm{i}\left(\psi_\alpha^c\overline{\nabla}_{ia}\bar{\psi}^{d\alpha} + \bar{\psi}^{c\alpha}\overline{\nabla}_{ia}\psi_\alpha^d\right)\partial_{cd}.
\end{aligned}$$

“Ordinary” covariant derivatives read

$$D_a^i = \frac{\partial}{\partial\theta_i^a} - \mathrm{i}\bar{\theta}^{ib}\partial_{ab},\; \overline{D}_{ia} = \frac{\partial}{\partial\bar{\theta}^{ia}} - \mathrm{i}\theta_i^b\partial_{ab},\; \left\{D_a^i, \overline{D}_{jb}\right\} = -2\mathrm{i}\delta_j^i\partial_{ab}.$$

Irreducibility conditions

In the supersymmetric case, $d\theta$-part of condition $\Omega_Z = 0$ implies the relation between ψ^a_α and $\nabla^i_a \mathbf{q}_{j\alpha}$ (IHP):

$$\nabla^j_a \mathbf{q}_{i\alpha} + 2\mathrm{i}\psi_{a\alpha}\delta^j_i = 0, \quad \overline{\nabla}_{ja}\mathbf{q}_{i\alpha} + 2\mathrm{i}\bar{\psi}_{a\alpha}\epsilon_{ij} = 0 \Rightarrow$$
$$\nabla^{(j}_a \mathbf{q}^{i)}_\alpha = 0, \quad \overline{\nabla}_{a(j}\mathbf{q}_{i)\alpha} = 0, \quad \psi_{a\alpha} = \tfrac{\mathrm{i}}{4}\nabla^k_a \mathbf{q}_{k\alpha}, \quad \bar{\psi}_{a\alpha} = \tfrac{\mathrm{i}}{4}\overline{\nabla}_{ka}\mathbf{q}^k_\alpha.$$

Thus, not only the relation between ψ^a_α and $\nabla^i_a \mathbf{q}_{j\alpha}$ appears, but also the generalized conditions that define $d = 3$, $N = 4$ hypermultiplet. This is on-shell multiplet, with no auxiliary fields, and odd derivatives of ψ^a_α, $\bar{\psi}^{a\alpha}$, can be expressed in terms of $\nabla_{ab}\mathbf{q}^{i\alpha}$ with help of relations

$$\overline{\nabla}_{jb}\psi_{a\alpha} \;=\; \frac{\mathrm{i}}{3}\left[\left\{\nabla^k_a, \overline{\nabla}_{jb}\right\}\mathbf{q}_{k\alpha} + \frac{1}{2}\left\{\nabla_{ja}, \overline{\nabla}_{kb}\right\}\mathbf{q}^k_\alpha\right],$$

and so on.

S-SUSY transformations

Let us now construct the anzatz for the component action and check its invariance under broken supersymmetry. As ψ^a_α can be expressed in terms of $\nabla^i_a \mathbf{q}_{j\alpha}$, we may define

$$q_{i\alpha} = \mathbf{q}_{i\alpha}|_{\theta\to 0},\ \psi^a_\alpha = \boldsymbol{\psi}^a_\alpha|_{\theta\to 0},\ \bar{\psi}^{a\alpha} = \bar{\boldsymbol{\psi}}^{a\alpha}|_{\theta\to 0}.$$

Then broken SUSY transformations of these components, defined at point, read

$$\delta^\star_S \psi^a_\alpha = \varepsilon^a_\alpha - U^M \partial_M \psi^a_\alpha\ \delta^\star_S q_{i\alpha} = -U^M \partial_M q_{i\alpha},\ U^M = \mathrm{i}\left(\varepsilon^{(a}_\alpha \bar{\psi}^{b)\alpha} + \bar{\varepsilon}^{\alpha(a}\psi^{b)}_\alpha\right)\left(\sigma^M\right)_{ab}.$$

(with vector notations with proper σ^A matrices.) One may define derivative, covariant with respect to broken supersymmetry, that acts on components

$$\mathcal{D}_A = \left(\mathcal{E}^{-1}\right)_A^B \partial_B,\ \mathcal{E}^B_A = \delta^B_A - \mathrm{i}\left(\psi^c_\alpha \partial_A \bar{\psi}^{d\alpha} + \bar{\psi}^{d\alpha}\partial_A \psi^c_\alpha\right)\left(\sigma^B\right)_{cd}.$$

Clear advantage of using this derivative is that $\delta^\star_S \mathcal{D}_A q_{i\alpha} = -U^M \partial_M \mathcal{D}_A q_{i\alpha}$. Also $\delta^\star_S \det \mathcal{E}^B_A = -\partial_M \left(U^M \det \mathcal{E}\right)$. Bosonic action then can be made invariant with respect to S-supersymmetry ($\partial_A \to \mathcal{D}_A, \quad \times \det \mathcal{E}$)

$$\mathcal{L}_0 = \det \mathcal{E}\left(A + \sqrt{\det g^D_C}\right),\ g_{AB} = \eta_{AB} - 2d_{AB},\ d_{AB} = \mathcal{D}_A q^{i\alpha}\mathcal{D}_B q_{i\alpha}.$$

Wess-Zumino term

Another important ingredient of component Lagrangian is Wess-Zumino term. The way exists to systematically construct such term, developed by Mezincescu. At first, one should construct 4-form Ω_4 (in $d=3$ case), invariant under broken supersymmetry, and with property $d\Omega_4 = 0$. As $\Lambda^{i\alpha}_A$, and θ^a_i, $\bar\theta^{ia}$, are invariant under broken supersymmetry, good candidates are $(\Omega_Z)^{i\alpha}|_{\Lambda,\theta\to 0} = dq^{i\alpha}$, $(\Omega_S)^a_\alpha|_{\Lambda,\theta\to 0} = d\psi^a_\alpha$, $(\bar\Omega_S)_{a\alpha}|_{\Lambda,\theta\to 0} = d\bar\psi^{a\alpha}$.

As i-index exists only on $dq^{i\alpha}$, when these forms can only be interleaved as $dq^{i\alpha}\wedge dq^\beta_i$; then, obviously, $\Omega_4 \sim \mathrm{i} dq^{i\alpha}\wedge dq^\beta_i \wedge d\psi_{a\alpha}\wedge d\bar\psi^a_\beta$. Ω_4 can be represented as $d\Omega_3$, and $\int \Omega_3$ is the right Wess-Zumino term. It reads

$$\Omega_3 \sim \mathrm{i} dq^{i\alpha}\wedge dq^\beta_i \wedge \left(\psi_{a\alpha}\wedge d\bar\psi^a_\beta + \bar\psi^a_\beta \wedge d\psi_{a\alpha}\right),$$

$$S_{WZ} = \mathrm{i}\int d^3x \epsilon^{ABC}\partial_A q^{i\alpha}\partial_B q^\beta_i \left(\psi_{a\alpha}\partial_C\bar\psi^a_\beta + \bar\psi^a_\beta\partial_C\psi_{a\alpha}\right).$$

Other possible terms (with different amounts of q's and fermions) can be excluded on dimensional grounds.

Wess-Zumino term

One may choose another representation for Wess-Zumino term, in which it looks more covariant:

$$
\begin{aligned}
\mathcal{L}_{WZ} &= \mathrm{i}\epsilon^{ABC}\partial_A q^{i\alpha}\partial_B q_i^{\beta}\left(\psi_{a\alpha}\partial_C\bar{\psi}^a_\beta + \bar{\psi}^a_\beta\partial_C\psi_{a\alpha}\right), \quad \partial_A = \mathcal{E}_A^B\mathcal{D}_B \Rightarrow \\
\mathcal{L}_{WZ} &= \mathrm{i}\det\mathcal{E}\,\epsilon^{ABC}\mathcal{D}_A q^{i\alpha}\mathcal{D}_B q_i^{\beta}\left(\psi_{a\alpha}\mathcal{D}_C\bar{\psi}^a_\beta + \bar{\psi}^a_\beta\mathcal{D}_C\psi_{a\alpha}\right)
\end{aligned}
$$

Then we vary the second equivalent form, all terms with U^M (come from shift of coordinates) combine into total divergence; the rest is

$$
\mathrm{i}\det\mathcal{E}\,\epsilon^{ABC}\mathcal{D}_A q^{i\alpha}\mathcal{D}_B q_i^{\beta}\left(\varepsilon_{a\alpha}\mathcal{D}_C\bar{\psi}^a_\beta + \bar{\varepsilon}^a_\beta\mathcal{D}_C\psi_{a\alpha}\right)
$$

and becomes full divergence after removing determinant and $\mathcal{E}^{-1}$.

Complete anzatz for the action is, therefore,

$$
\begin{aligned}
&\mathcal{L} = \mathcal{L}_0 + B\mathcal{L}_{WZ} = \\
&= \det\mathcal{E}\left(A + \sqrt{\det g_A^B}\right) + \mathrm{i}B\det\mathcal{E}\,\epsilon^{ABC}\mathcal{D}_A q^{i\alpha}\mathcal{D}_B q_i^{\beta}\left(\psi_{a\alpha}\mathcal{D}_C\bar{\psi}^a_\beta + \bar{\psi}^a_\beta\mathcal{D}_C\psi_{a\alpha}\right).
\end{aligned}
$$

with two constants A, B, which have to be fixed by the unbroken supersymmetry. Moreover, one may also add constant term to Lagrangian, if necessary.

$\nabla\psi$ and others

Next natural step is to define transformation laws for components under unbroken supersymmetry. This can be done with help of formulaes

$$\delta^{\star}_{Q} f \quad = \quad -\left(\epsilon^{a}_{i} D^{i}_{a} + \bar{\epsilon}^{ai}\overline{D}_{ai}\right) \mathbf{f}|_{\theta\to 0},$$

where D^i_a can be expressed in terms of ∇^i_a. To know the complete set of laws, we therefore, have to define $\nabla^i_a \psi^b_\alpha$, $\overline{\nabla}_{ia}\bar{\psi}^{a\alpha}$, $\overline{\nabla}_{ia}\psi_{b\alpha}$, $\nabla_{ib}\bar{\psi}_{a\alpha}$.

As happened in the systems with auxiliary fields, we may use condition $\Omega_S|_{d\theta} = 0$ to extract information about such relations:

$$\Omega_S \quad = \quad \left[\left(\cosh 2\sqrt{W}\right)^{\gamma a}_{\alpha b} d\psi^b_\gamma + 2 d\bar{\theta}^{jb} \left(\frac{\sinh 2\sqrt{\overline{\overline{T}}}}{2\sqrt{\overline{\overline{T}}}}\right)^{kc}_{jb} \mathbf{\Lambda}^{a}_{kc\alpha} \right] S^{\alpha}_{a}.$$

Therefore, $\nabla^i_a \psi^b_\alpha$, $\overline{\nabla}_{ia}\bar{\psi}^{b\alpha}$ are zero. Also, comparing Ω_S with its conjugate, one may note that $\overline{\nabla}_{ia}\psi_{b\alpha} = \nabla_{ia}\bar{\psi}_{b\alpha}$. It is difficult to achieve more this way, however, as there is no obvious way to resum power series in $W^{\beta b}_{\alpha a} = \mathbf{\Lambda}^{\beta j}_{ac}$ in terms of $Y^{cd}_{ab} = \mathbf{\Lambda}^{i\alpha}_{ab}\mathbf{\Lambda}^{cd}_{i\alpha}$.

$\nabla\psi$ and others

$\overline{\nabla}_{ia}\psi_{b\alpha},\ \nabla_{ib}\bar{\psi}_{a\alpha}$ also can be found with help of anticommutators of covariant derivatives. Let us split them into symmetric and antisymmetric parts in $\{a, b\}$:

$$\overline{\nabla}_{ia}\psi_{b\alpha}|_{\theta\to 0} = \nabla_{ia}\bar{\psi}_{b\alpha}|_{\theta\to 0} = \left(\sigma^A\right)_{ab} J_{Ai\alpha} + \epsilon_{ab}X_{i\alpha}.$$

This together with $\nabla^i_a\psi^b_\alpha = 0$ $\overline{\nabla}_{ia}\bar{\psi}^{b\alpha} = 0$ has to be substituted into equations

$$\overline{\nabla}_{ia}\psi_{b\alpha} + \nabla_{ib}\bar{\psi}_{a\alpha} = \frac{\mathrm{i}}{2}\left\{\nabla^k_b, \overline{\nabla}_{ka}\right\}\mathbf{q}_{i\alpha},$$

$$\overline{\nabla}_{ia}\psi_{b\alpha} - \nabla_{ib}\bar{\psi}_{a\alpha} = -\frac{\mathrm{i}}{6}\left[\left\{\nabla_{kb}, \overline{\nabla}_{ia}\right\}\mathbf{q}^k_\alpha + \left\{\nabla_{ib}, \overline{\nabla}_{ka}\right\}\mathbf{q}^k_\alpha\right].$$

This results in system of matrix equations

$$2J^A_{i\alpha} = \left(2 - J^B_{k\beta}J^{k\beta}_B + X_{k\beta}X^{k\beta}\right)\mathcal{D}_A q_{i\alpha} + 2\left(J^B_{k\beta}\mathcal{D}_B q_{i\alpha}\right)J^{k\beta}_A,$$

$$X_{i\alpha} = \tfrac{1}{3}\epsilon^{ABC}J_{Ai\beta}J^\beta_{Bk}\mathcal{D}_C q^k_\alpha - \tfrac{1}{3}\left(X_{i\beta}J^\beta_{Bk} + X_{k\beta}J^\beta_{Bi}\right)\mathcal{D}^B q^k_\alpha.$$

But this system is also difficult to solve.

Another approach

As it is difficult to learn directly from equations, what $J^A_{i\alpha}$, $X_{i\alpha}$ are, let us adopt another strategy. There are no substantial doubts that action is finally invariant, with properly adjusted constants. We, therefore, may use the invariance of action as condition on $J^A_{i\alpha}$ and $X_{i\alpha}$ and, once they are found, ensure that nonlinear equations are satisfied. Clear advantage of this approach is that $J^A_{i\alpha}$ and $X_{i\alpha}$ enter not action, but transformation laws of ψ^a_α and $\bar{\psi}^{a\alpha}$, and therefore, resulting conditions are linear. Let us fix constants. In first approximation in fields, $\delta q_{i\alpha} = 2\mathrm{i}\epsilon^a_i \psi_{a\alpha}$,

$$\delta_Q q_{i\alpha} \sim 2\mathrm{i}\epsilon^a_i \psi_{a\alpha},\ \delta_Q \psi^a_\alpha \sim 0,\ \delta_Q \bar{\psi}^{a\alpha} \sim -\epsilon^b_j \left(\sigma^A\right)^a_b \partial_A q^{j\alpha},$$

$$\mathcal{L} \sim \mathrm{i}(1+A)\left(\psi^c_\alpha \partial_A \bar{\psi}^{d\alpha} + \bar{\psi}^{d\alpha}\partial_A \psi^c_\alpha\right)\left(\sigma^A\right)_{cd} + \partial_A q^{i\alpha}\partial^A q_{i\alpha}.$$

With help of integration by parts, variation reads

$$\delta_Q \mathcal{L} \sim 2\mathrm{i}\epsilon_{bj}\partial_A \psi^c_\alpha \partial_B q^{j\alpha}\left((A-1)\eta^{AB}\delta^b_c + (1+A)\epsilon^{ABC}\left(\sigma_C\right)^b_c\right) \Rightarrow A = 1.$$

Coefficent near Wess-Zumino term can be fixed by considering variation in first order in fermions and third in bosons. Finally,

$$\mathcal{L} = 2 - \det\mathcal{E}\left(1 + \sqrt{\det g^B_A}\right) + 2\mathrm{i}\det\mathcal{E}\epsilon^{ABC}\mathcal{D}_A q^{i\alpha}\mathcal{D}_B q^\beta_i \left(\psi_{a\alpha}\mathcal{D}_C \bar{\psi}^a_\beta + \bar{\psi}^a_\beta \mathcal{D}_C \psi_{a\alpha}\right).$$

Analysis of variation

Then one should vary this action with respect to unbroken supersymmetry with arbitrary $J^A_{i\alpha}$, $X_{i\alpha}$. Careful analysis (to complicated to present there) shows that action is invariant if

$$J^A_{i\alpha} = \frac{F}{2}\left(\eta^{AB} + \sqrt{\det g}(g^{-1})^{AB}\right)\mathcal{D}_B q_{i\alpha},\; X_{i\alpha} = \frac{F}{3}\epsilon^{ABC}\mathcal{D}_A q_{i\beta}\mathcal{D}_B q^\beta_k \mathcal{D}_C q^k_\alpha,$$

$$F = \frac{2}{1-\frac{1}{2}\sqrt{\det g}+\frac{1}{2}\sqrt{\det g}\,\mathrm{tr}(g^{-1})}.$$

As expected, they satisfy system of nonlinear equations

$$2J^A_{i\alpha} = \left(2 - J^B_{k\beta}J^{k\beta}_B + X_{k\beta}X^{k\beta}\right)\mathcal{D}_A q_{i\alpha} + 2\left(J^B_{k\beta}\mathcal{D}_B q_{i\alpha}\right)J^{k\beta}_A,$$

$$X_{i\alpha} = \frac{1}{3}\epsilon^{ABC}J_{Ai\beta}J^\beta_{Bk}\mathcal{D}_C q^k_\alpha - \frac{1}{3}\left(X_{i\beta}J^\beta_{Bk} + X_{k\beta}J^\beta_{Bi}\right)\mathcal{D}^B q^k_\alpha.$$

Expressions for $J(\mathcal{D}q)$ can be considered significant result on their own, as one may find that $\overline{\nabla}\psi$ can be expressed in terms of $\mathcal{D}q$ in all currently studied systems by the formulaes with the same structure, with proper metric and possibly adjusted coefficients in F_3. In particular, this happens in $N=2 \to N=1$ and $N=4 \to N=2$ PBGS in $d=3$ and $N=2 \to N=1$ in $d=4$. As in previously studied systems equations $\overline{\nabla}\psi(\mathcal{D}q)$ were solved before proving invariance of action, it was not noticed.

Conclusion

Let us present summary of the main results of this talk.

- On the way of understanding of component actions for P-branes, invariance of action with maximal amount of supersymmetries has been proven:

$$
\begin{aligned}
\mathcal{L} &= 2 - \det \mathcal{E}\left(1 + \sqrt{\det g_A^B}\right) + \\
&\quad + 2\mathrm{i} \det \mathcal{E} \epsilon^{ABC} \mathcal{D}_A q^{i\alpha} \mathcal{D}_B q_i^{\beta} \left(\psi_{a\alpha} \mathcal{D}_C \bar{\psi}^a_\beta + \bar{\psi}^a_\beta \mathcal{D}_C \psi_{a\alpha}\right), \\
&\quad g_{AB} = \eta_{AB} - 2\mathcal{D}_A q_{i\alpha} \mathcal{D}_B q^{i\alpha}, \ \mathcal{E}_A^B = \delta_A^B - \mathrm{i}\left(\psi^c_\alpha \partial_A \bar{\psi}^{d\alpha} + \bar{\psi}^{d\alpha} \partial_A \psi^c_\alpha\right)\left(\sigma^B\right)_{cd}.
\end{aligned}
$$

- Applied method of proving invariance of action appeared to be effective and is likely useful in analysis of system with hypermultiplet in $d = 4$.
- Expression of second components of fermions, found on the way of proving, seems to be rather general and applicable for all previously studied systems. Geometric meaning of expression $J(\mathcal{D}q)$ and exact degree of generality of this result are not clear at present time. However, it is supposed to be helpful in search of action for higher dimensional systems.