Semiclassical Calculation of the Chiral Magnetic Effect in Even Spacetime Dimensions

Ö. F. DAYI

Istanbul Technical University, Physics Department, Turkey

Supersymmetry in Integrable Systems-SIS'14,
Dubna, 11–13 September 2014

with M. Elbistan .

Outline

Motivation

Outline

Motivation

Semiclassical formulation for the Dirac-like systems

Outline

Motivation

Semiclassical formulation for the Dirac-like systems

Weyl Hamiltonian and the Berry Gauge Field

Outline

Motivation

Semiclassical formulation for the Dirac-like systems

Weyl Hamiltonian and the Berry Gauge Field

Chiral Kinetic Theory in d+1=2n+2 Dimensions

- Chiral Anomaly
- Chiral Magnetic Effect

Outline

Motivation

Semiclassical formulation for the Dirac-like systems

Weyl Hamiltonian and the Berry Gauge Field

Chiral Kinetic Theory in d+1=2n+2 Dimensions

- Chiral Anomaly
- Chiral Magnetic Effect

Conclusions

Motivation

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.
- In $3+1$ dimensions a part of the chiral particle current is parallel to the magnetic field $\rightarrow$ Chiral Magnetic Effect. This was generalized to higher dimensions.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.
- In $3+1$ dimensions a part of the chiral particle current is parallel to the magnetic field $\rightarrow$ Chiral Magnetic Effect. This was generalized to higher dimensions.
- Chiral anomaly and chiral magnetic effect are embodied in classic kinetic theory in $3+1$ dimensions. The Berry curvature plays the main role.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.
- In $3+1$ dimensions a part of the chiral particle current is parallel to the magnetic field $\rightarrow$ Chiral Magnetic Effect. This was generalized to higher dimensions.
- Chiral anomaly and chiral magnetic effect are embodied in classic kinetic theory in $3+1$ dimensions. The Berry curvature plays the main role.
- Chiral anomaly in higher dimensions was obtained.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.
- In $3+1$ dimensions a part of the chiral particle current is parallel to the magnetic field $\rightarrow$ Chiral Magnetic Effect. This was generalized to higher dimensions.
- Chiral anomaly and chiral magnetic effect are embodied in classic kinetic theory in $3+1$ dimensions. The Berry curvature plays the main role.
- Chiral anomaly in higher dimensions was obtained.
- Chiral magnetic effect in higher dimensions was only conjectured which is supported by computations.

Motivation

- We deal with the kinetic theory of massless Dirac fermions in the external electromagnetic fields in even space-time dimensions.
- Non-conservation of the chiral particle current $\rightarrow$ Chiral Anomaly.
- In $3+1$ dimensions a part of the chiral particle current is parallel to the magnetic field $\rightarrow$ Chiral Magnetic Effect. This was generalized to higher dimensions.
- Chiral anomaly and chiral magnetic effect are embodied in classic kinetic theory in $3+1$ dimensions. The Berry curvature plays the main role.
- Chiral anomaly in higher dimensions was obtained.
- Chiral magnetic effect in higher dimensions was only conjectured which is supported by computations.
- We formulated semiclassical theory by keeping spin dependence explicit. We obtain anomalies as well as the chiral magnetic effect within the same formulation straightforwardly.

Semiclassical formulation for Dirac-like systems

Wave packet composed of the Positive energy solutions

$$H_0(\boldsymbol{p})u^{(\alpha)}(\boldsymbol{p},\boldsymbol{x}) = E_\alpha u^{(\alpha)}(\boldsymbol{p},\boldsymbol{x});\ \ E_\alpha > 0.$$

The normalization is $u^{\dagger(\alpha)}(\boldsymbol{p},\boldsymbol{x})u^{(\beta)}(\boldsymbol{p},\boldsymbol{x}) = \delta_{\alpha\beta}$.
Center of the wave packet $(\boldsymbol{x}_c,\boldsymbol{p}_c)$

$$\Psi_{\boldsymbol{x}}(\boldsymbol{p}_c,\boldsymbol{x}_c) = \int [dp]|\delta(\boldsymbol{p}_c - \boldsymbol{p}) \sum_\alpha \xi_\alpha u^{(\alpha)}(\boldsymbol{p},\boldsymbol{x}_c))e^{i\boldsymbol{p}\cdot\boldsymbol{x}},$$

The one-form η, defined through

$$d\mathcal{S} \equiv \int [dx]\delta(\boldsymbol{x}_c - \boldsymbol{x})\Psi^\dagger_{\boldsymbol{x}}\left(id - H_0 dt\right)\Psi_{\boldsymbol{x}} = \sum_{\alpha\beta} \xi^*_\alpha \eta^{\alpha\beta}\xi_\beta.$$

By renaming $\boldsymbol{x}_c, \boldsymbol{p}_c \to \boldsymbol{x}, \boldsymbol{p}$ we obtain

$$\eta^{\alpha\beta} = \delta^{\alpha\beta}\boldsymbol{x} \cdot d\boldsymbol{p} + \boldsymbol{a}^{\alpha\beta} \cdot d\boldsymbol{x} + \boldsymbol{\mathcal{A}}^{\alpha\beta} \cdot d\boldsymbol{p} - H_0^{\alpha\beta} dt.$$

The matrix valued Berry gauge fields

$$\begin{aligned} \boldsymbol{a}^{\alpha\beta} &= u^{\dagger(\alpha)}(\boldsymbol{p}, \boldsymbol{x}) \frac{\partial}{\partial \boldsymbol{x}} u^{(\beta)}(\boldsymbol{p}, \boldsymbol{x}), \\ \boldsymbol{\mathcal{A}}^{\alpha\beta} &= u^{\dagger(\alpha)}(\boldsymbol{p}, \boldsymbol{x}) \frac{\partial}{\partial \boldsymbol{p}} u^{(\beta)}(\boldsymbol{p}, \boldsymbol{x}). \end{aligned}$$

The symplectic two-form

$$w = d\eta - i\eta \wedge \eta,$$

can be employed to define the volume form

$$\Omega_{d+1} = \frac{(-1)^{[d/2]}}{d} w^d \wedge dt,$$

and obtain the Liouville eq. etc.

Weyl Hamiltonian and the Berry Gauge Field

The massless, free Dirac electron can be separated into the right and left handed states with the Weyl Hamiltonian

$$\mathcal{H}_{\mathrm{W}} = \boldsymbol{\Sigma} \cdot \boldsymbol{p},$$

where Σ_{A} are $2^{[(d-1)/2]} \times 2^{[(d-1)/2]}$ matrices.

$$\mathcal{H}_{\mathrm{W}} \psi_E(\boldsymbol{p}) = E \psi_E(\boldsymbol{p}).$$

$E = (p, -p)$ where $p = |\boldsymbol{p}|$. Focusing on the positive energy solutions $|\psi^\alpha\rangle;\ \alpha = 1, ..., 2^{[\frac{d-3}{2}]}$, one can define the Berry gauge field

$$\mathcal{A}_{\mathrm{A}}^{\alpha\beta} = i \langle \psi^\alpha | \frac{\partial}{\partial p_{\mathrm{A}}} | \psi^\beta \rangle.$$

$\mathcal{A}$ is Abelian for $d = 3$, it becomes to be non-Abelian for higher dimensions. Thus, in general the Berry field strength

$$\mathcal{G}_{\mathrm{AB}}^{\alpha\beta} = \frac{\partial \mathcal{A}_{\mathrm{B}}^{\alpha\beta}}{\partial p_{\mathrm{A}}} - \frac{\partial \mathcal{A}_{\mathrm{A}}^{\alpha\beta}}{\partial p_{\mathrm{B}}} - i [\mathcal{A}_{\mathrm{A}}, \mathcal{A}_{\mathrm{B}}]^{\alpha\beta}.$$

5+1 dimensional Berry fields

The Hamiltonian is

$$\mathcal{H}_{\mathrm{W}} = \begin{pmatrix} \sigma_a p_a & i(p_4 + ip_5) \\ -i(p_4 - ip_5) & -\sigma_a p_a \end{pmatrix}.$$

The Berry fields are 2×2 matrices. e.g.

$$\begin{aligned} \mathcal{A}_1 &= \frac{1}{2p(p-p_3)} \begin{pmatrix} -p_2 & -i\sqrt{p_4^2 + p_5^2} \\ i\sqrt{p_4^2 + p_5^2} & p_2 \end{pmatrix}, \\ \mathcal{G}_{45} &= \tfrac{1}{2p^3(p-p_3)} \begin{pmatrix} (p_1^2+p_2^2+p_3^2)-pp_3 & (p_1+ip_2)\sqrt{(p_4^2+p_5^2)} \\ (p_1-ip_2)\sqrt{(p_4^2+p_5^2)} & -(p_1^2+p_2^2+p_3^2)+pp_3 \end{pmatrix}. \end{aligned}$$

$d+1=2n+2$ dimensional Chiral Kinetic Theory

Switch on the external electromagnetic fields: The magnetic field $\mathcal{F}=\frac{1}{2}\mathcal{F}_{\mathrm{AB}}dx_{\mathrm{A}}\wedge dx_{\mathrm{B}}$, and the electric field pointing towards the $\hat{x}_{\mathrm{A}}$ direction $\mathcal{E}_{\mathrm{A}}$.

Define the symplectic two-form as the $2^{n-1}\times 2^{n-1}$ matrix

$$\tilde{W}_H \equiv dp_{\mathrm{A}}\wedge dx_{\mathrm{A}}+\mathcal{F}-\mathcal{G}-\hat{p}_{\mathrm{A}}dp_{\mathrm{A}}\wedge dt+\mathcal{E}_{\mathrm{A}}dx_{\mathrm{A}}\wedge dt.$$

$\mathcal{G}=\frac{1}{2}\mathcal{G}_{\mathrm{AB}}dp_{\mathrm{A}}\wedge dp_{\mathrm{B}}$. Introduce the matrix valued vector field

$$\tilde{V}=\frac{\partial}{\partial t}+\dot{X}_{\mathrm{A}}\frac{\partial}{\partial x_{\mathrm{A}}}+\dot{P}_{\mathrm{A}}\frac{\partial}{\partial p_{\mathrm{A}}},$$

in terms of the matrix valued time evolutions $\dot{X}_{\mathrm{A}},\dot{P}_{\mathrm{A}}$.

The equations of motion are obtained in terms of the interior product as

$$i_{\tilde{V}}\tilde{W}_H=0,$$

yielding

$$\dot{P}_{\mathrm{A}}=\dot{X}_{\mathrm{B}}\mathcal{F}_{\mathrm{AB}}+\mathcal{E}_{\mathrm{A}},\ \dot{X}_{\mathrm{A}}=\mathcal{G}_{\mathrm{AB}}\dot{P}_{\mathrm{B}}+\hat{p}_{\mathrm{A}}.$$

Liouville Equation

We do not treat spin degrees of freedom as dynamical variables. Classical dynamics is asserted through the phase space variables, so that we define the volume form as

$$\tilde{\Omega} \equiv \frac{(-1)^{n+1}}{(2n+1)!} \tilde{W}_H^{2n+1} \wedge dt.$$

Express it in terms of the $2d$ dimensional Liouville measure dV as

$$\tilde{\Omega} = \tilde{W}_{1/2} dV \wedge dt.$$

$\tilde{W}_H$ is a two-form in the phase space variables,so that $\tilde{W}_{1/2}$ is the Pfaffian of the $(4n+2) \times (4n+2)$ matrix

$$\begin{pmatrix} \mathcal{F}_{\mathrm{AB}} & -\delta_{\mathrm{AB}} \\ \delta_{\mathrm{AB}} & -\mathcal{G}_{\mathrm{AB}} \end{pmatrix}.$$

Definition: For M, a skew-symmetric matrix, $[Pf(M)]^2 = \det M$, a polynomial in the elements of M.

The explicit form of $\tilde{W}_{1/2}$ can be provided within our formalism.

Liouville equation can be expressed formally as

$$\begin{aligned} L_{\tilde{V}}\tilde{\Omega} &= (i_{\tilde{V}}d + di_{\tilde{V}})\tilde{W}_{1/2}\; dV \wedge dt \\ &= \left(\frac{\partial}{\partial t}\tilde{W}_{1/2} + \frac{\partial}{\partial x_{\mathrm{A}}}(\dot{X}_{\mathrm{A}}\tilde{W}_{1/2}) + \frac{\partial}{\partial p_{\mathrm{A}}}(\tilde{W}_{1/2}\dot{P}_{\mathrm{A}})\right) dV \wedge dt. \end{aligned}$$

To find it one needs to calculate

$$L_{\tilde{V}}\tilde{\Omega} = \frac{(-1)^{n+1}}{(2n+1)!} d\tilde{W}_H^{2n+1}.$$

We introduced matrix valued quantities. However, measure of the related path integral should be a scalar.
An appropriate definition of the path integral measure is

$$\sqrt{W} \equiv \mathrm{Tr}\,[\tilde{W}_{1/2}].$$

To obtain the corresponding classical chiral kinetic theory we let $\dot{x}_{\mathrm{A}}, \dot{p}_{\mathrm{A}}$, denote the ordinary velocities and define

$$\mathrm{Tr}\,[(\dot{X}_{\mathrm{A}}\tilde{W}_{1/2}] \equiv \sqrt{W}\dot{x}_{\mathrm{A}},\; \mathrm{Tr}\,[\tilde{W}_{1/2}\dot{P}_{\mathrm{A}})] \equiv \sqrt{W}\dot{p}_{\mathrm{A}}.$$

Hence, let us discuss the trace properties:

Properties of the Berry Curvature and the Σ Matrices

When we take the trace, a generic term which we should calculate

$$\epsilon_{\mathrm{A}_1\mathrm{A}_2\ldots\mathrm{A}_{2m+1}\ldots\mathrm{A}_{2n+1}}\mathrm{Tr}\,[\mathcal{G}_{\mathrm{A}_2\mathrm{A}_3}\ldots\mathcal{G}_{\mathrm{A}_{2m}\mathrm{A}_{2m+1}}].$$

For the Weyl Hamiltonian $\mathcal{H}_{\mathrm{W}} = \boldsymbol{\Sigma}\cdot\boldsymbol{p}$, the Berry fields can be written in terms of

$$P^+ = \frac{1}{2}(\frac{\mathcal{H}_{\mathrm{W}}}{p} + 1).$$

Thus the generic term

$$= (2i)^m\epsilon_{\mathrm{A}_1\mathrm{A}_2\ldots\mathrm{A}_{2m+1}\ldots\mathrm{A}_{2n+1}}\mathrm{Tr}\,[P^+(\partial_{\mathrm{A}_2}P^+)\ldots(\partial_{\mathrm{A}_{2m+1}}P^+)],$$

which can be written as

$$= (2i)^m\frac{p_{\mathrm{A}}}{(2p)^{2m+1}}\epsilon_{\mathrm{A}_1\mathrm{A}_2\ldots\mathrm{A}_{2m+1}\ldots\mathrm{A}_{2n+1}}\mathrm{Tr}\,[\Sigma_{\mathrm{A}}\Sigma_{\mathrm{A}_2}\ldots\Sigma_{\mathrm{A}_{2m+1}}].$$

$\Sigma_{\rm A}$ obey the Clifford algebra,

$$\{\Sigma_{\rm A}, \Sigma_{\rm B}\} = 2\delta_{\rm AB}.$$

Moreover, they are traceless,

$$\mathrm{Tr}\left[\Sigma_{\rm A}\right] = 0,$$

and in $2n+2$ dimensional spacetime they satisfy the identity,

$$\Sigma_1 ... \Sigma_{2n+1} = i^{n+2} \mathbf{1}_{2^n \times 2^n}.$$

Thus the trace of $2n+1$ antisymmetric product of the Σ matrices yields

$$\frac{1}{(2n+1)!} \epsilon_{\rm A_1 ... A_{2n+1}} \mathrm{Tr}\left[\Sigma_{\rm A_1} \ldots \Sigma_{\rm A_{2n+1}}\right] = i^{n+2} 2^n.$$

Actually one can observe that the trace of the product of even number of different Σ matrices always vanishes because of satisfying the Clifford algebra:

$$\mathrm{Tr}\left[\Sigma_{\mathrm{A}_1}\ldots\Sigma_{\mathrm{A}_{2m}}\right]=0.$$

Moreover, it can be easily shown that the trace of the product of $2m+1$ different Σ matrices is equal to the trace of the product of the remaining $2(n-m)$, Σ matrices which is equal to zero. Therefore, the trace of the antisymmetrized product of the Berry field strength vanishes

$$\epsilon^{\mathrm{A}_1\mathrm{A}_2\ldots\mathrm{A}_{2m-1}\mathrm{A}_{2m}\ldots\mathrm{A}_{2n+1}}\mathrm{Tr}\left[\mathcal{G}_{\mathrm{A}_1\mathrm{A}_2}\ldots\mathcal{G}_{\mathrm{A}_{2m-1}\mathrm{A}_{2m}}\right]=0,$$

for the case $m<n$. When $m=n$ one finds

$$\epsilon_{\mathrm{A}_1\mathrm{A}_2\mathrm{A}_3\ldots\mathrm{A}_{2n}\mathrm{A}_{2n+1}}\mathrm{Tr}\left[\mathcal{G}_{\mathrm{A}_2\mathrm{A}_3}\ldots\mathcal{G}_{\mathrm{A}_{2n}\mathrm{A}_{2n+1}}\right]=(-1)^{n+1}(2n)!\frac{p_{\mathrm{A}_1}}{2p^{2n+1}},$$

which is the Dirac monopole field: $\boldsymbol{b}=\frac{\boldsymbol{p}}{2p^{2n+1}}$.

The Chiral Anomaly

The semiclassical anomalous Liouville equation

$$\left(\frac{\partial}{\partial t}\sqrt{W}+\frac{\partial}{\partial x_{\mathrm{A}}}(\sqrt{W}\dot{x}_{\mathrm{A}})+\frac{\partial}{\partial p_{\mathrm{A}}}(\sqrt{W}\dot{p}_{\mathrm{A}})\right)=$$

$$\frac{(-1)^{n+1}(2n)!}{(n!)^2\ 2^{2n+1}}\mathrm{Vol}(S^{2n})\delta^{2n+1}(p)\epsilon_{\mathrm{ABC\dots DE}}\mathcal{E}_{\mathrm{A}}\overbrace{\mathcal{F}_{\mathrm{BC}}\dots\mathcal{F}_{\mathrm{DE}}}^{n\ \ times}.$$

Introduce the phase space distribution $f(x,p,t)$ satisfying the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial x_{\mathrm{A}}}\dot{x}_{\mathrm{A}}+\frac{\partial f}{\partial p_{\mathrm{A}}}\dot{p}_{\mathrm{A}}=0.$$

Define the probability density $\rho(x, p, t) = \sqrt{W} f$;
The chiral particle density $n(x, t) = \int \frac{d^{2n+1}}{(2\pi)^{2n+1}} \rho$;
The chiral current density $j_{\mathrm{A}} = \int \frac{d^{2n+1}}{(2\pi)^{2n+1}} \rho \dot{x}_{\mathrm{A}}$.
Non-conservation of the chiral current

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = \frac{(-1)^{n+1}}{n!\ 2^n\ (2\pi)^{n+1}} f(x, p = 0, t) \mathcal{E}_{\mathrm{A}} \overbrace{\mathcal{F}_{\mathrm{BC}} \ldots \mathcal{F}_{\mathrm{DE}}}^{n \ times}.$$

is the semiclassical manifestation of the chiral anomaly.

The Chiral Magnetic Effect

Comparison of the formal and the explicit expressions for $L_{\tilde{V}}\tilde{\Omega}$ provides the solutions of the equations of motion for $\tilde{W}_{1/2}, (\dot{X}_{\mathrm{A}}\tilde{W}_{1/2}, \tilde{W}_{1/2}\dot{P}_{\mathrm{A}})$ in terms of $(x_{\mathrm{A}}, p_{\mathrm{A}})$. Write only

$$\begin{aligned}
\dot{X}_{\mathrm{A}}\tilde{W}_{1/2} = \quad & \frac{(-1)^n}{(2n)!} dx_{\mathrm{A}} \wedge \hat{p}_{\mathrm{B}} dp_{\mathrm{B}} \wedge dt \wedge \overbrace{dp_{\mathrm{C}} \wedge dx_{\mathrm{C}} \ldots dp_{\mathrm{D}} \wedge dx_{\mathrm{D}}}^{2n\ \ times} \\
& + \frac{(-1)^n}{(2n-1)!} dx_{\mathrm{A}} \wedge \mathcal{E}_{\mathrm{B}} dx_{\mathrm{B}} \wedge dt \wedge \mathcal{G}\, \overbrace{dp_{\mathrm{C}} \wedge dx_{\mathrm{C}} \ldots dp_{\mathrm{D}} \wedge dx_{\mathrm{D}}}^{2n-1\ \ times} \\
& + \frac{(-1)^{n+1}}{(2n-2)!} dx_{\mathrm{A}} \wedge \hat{p}_{\mathrm{B}} dp_{\mathrm{B}} \wedge dt \wedge \mathcal{G}\mathcal{F}\, \overbrace{dp_{\mathrm{C}} \wedge dx_{\mathrm{C}} \ldots dp_{\mathrm{D}} \wedge dx_{\mathrm{D}}}^{2n-2\ \ times} \\
& + \ \ldots \ + \frac{1}{(n!)^2 \ 2^{2n}} \epsilon_{\mathrm{ABC}\ldots\mathrm{DE}} \overbrace{\mathcal{F}_{\mathrm{BC}} \ldots \mathcal{F}_{\mathrm{DE}}}^{n\ \ times} \epsilon_{\mathrm{IJK}\ldots\mathrm{LM}} \hat{p}_{\mathrm{I}} \overbrace{\mathcal{G}_{\mathrm{JK}} \ldots \mathcal{F}_{\mathrm{LM}}}^{n\ \ times} .
\end{aligned}$$

In the semiclassical approximation we define the current as

$$j_{\mathrm{A}} = \int \frac{d^{2n+1}p}{(2\pi)^{2n+1}} \mathrm{Tr}\,[\dot{X}_{\mathrm{A}} \tilde{W}_{1/2}] f(x,p,t).$$

$f(x,p,t)$ is the probability function.
CME is generated by the terms depending on the external magnetic field $\mathcal{F}_{\mathrm{AB}}$.

Once we take the trace over the spin indices there remains only one term depending on the external magnetic field $\mathcal{F}_{\rm AB}$. Therefore the chiral magnetic current is calculated

$$
\begin{aligned}
j_{\rm A}^{\rm CME} &= \frac{1}{2^{2n}(n!)^2}\int \frac{d^{2n+1}p}{(2\pi)^{2n+1}}\epsilon_{\rm ABC\ldots DE}\overbrace{\mathcal{F}_{\rm BC}\ldots\mathcal{F}_{\rm DE}}^{n\ \ times} \\
&\quad {\rm Tr}\,[\epsilon_{\rm IJK\ldots LM}\hat{p}_{\rm I}\overbrace{\mathcal{G}_{\rm JK}\ldots\mathcal{G}_{\rm LM}}^{n\ \ times}]f(x,p,t) \\
&= \frac{(-1)^{n+1}(2n)!}{2^{2n}(n!)^2}\int \frac{d^{2n+1}p}{(2\pi)^{2n+1}}\epsilon_{\rm ABC\ldots DE}\overbrace{\mathcal{F}_{\rm BC}\ldots\mathcal{F}_{\rm DE}}^{n\ \ times}(\hat{\boldsymbol{p}}\cdot\boldsymbol{b})f(x,p,t).
\end{aligned}
$$

When we deal with an "isotropic momentum distribution $f = f(E)$, the angular part of it can be computed, so that we establish the chiral magnetic current as

$$
\begin{aligned}
j_{\mathrm{A}}^{\mathrm{CME}} &= \frac{(-1)^{n+1}(2n)!}{2^{2n}(n!)^2}\frac{\mathrm{Vol}(S^{2n})}{2(2\pi)^{2n+1}}\epsilon_{\mathrm{ABC\ldots DE}}\overbrace{\mathcal{F}_{\mathrm{BC}}\ldots\mathcal{F}_{\mathrm{DE}}}^{n\ \ times}\int dEf(E), \\
&= \frac{(-1)^{n+1}}{2^n(2\pi)^{n+1}n!}\epsilon_{\mathrm{ABC\ldots DE}}\overbrace{\mathcal{F}_{\mathrm{BC}}\ldots\mathcal{F}_{\mathrm{DE}}}^{n\ \ times}\int dEf(E).
\end{aligned}
$$

This is the chiral magnetic current conjectured in *(R.Loganayagam and P.Surowka, JHEP, 2012). and supported by computations in (R.Loganayagam, JHEP, 2013).*

Conclusions

Conclusions

- We calculated the CME and chiral anomaly in any even dimensions within the same formulation.

Conclusions

- We calculated the CME and chiral anomaly in any even dimensions within the same formulation.
- A semiclassical study of the massive Dirac particle within the formalism presented here can be performed in the ordinary classical phase space without enlarging it with some new dynamical variables corresponding to spin.

Conclusions

- We calculated the CME and chiral anomaly in any even dimensions within the same formulation.
- A semiclassical study of the massive Dirac particle within the formalism presented here can be performed in the ordinary classical phase space without enlarging it with some new dynamical variables corresponding to spin.
- The formalism based on the matrix valued symplectic form is not restricted to even dimensions. It can be employed to establish solution of the equations of motion in terms of phase space variables in any dimensions. These solutions which exhibit the spin dependence explicitly can be useful to formulate some interesting physical phenomena like the spin Hall effect.

Thank You!