Supersymmetric bag as source of the Kerr spinning particle, structure of the Dirac equation

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Based on:

A.B., Regularized Kerr-Newman solution as Gravitating Soliton, J.Phys.A, 43, 392001 (2010).

A.B., Emergence of the Dirac Equation in the Solitonic Source of the Kerr Spinning Particle, [arXiv:1404.5947].

KERR-NEWMAN SPINNING PARTICLE AS A DRESSED ELECTRON.

Black holes as elementary particles[G.'t Hooft (1990), A. Sen (1995), C.F.E. Holzhey and F. Wilczek (1992), A.Salam and J. Strathdee (1976)].

Gravitational and electromagnetic fields of the electron are described by the Kerr-Newman (KN) black hole solution!

(B. Carter, 1989) Gyromagnetic ratio for KN solution $g = 2$, $\Rightarrow$ The experimentally observable parameters of the electron, mass m, spin J, charge e and magnetic moment μ, correspond to KN solution.

Spin of electron is extremely high, $a = J/m >> m$ $(a/m \approx 10^{44}$), and the black hole horizons disappear, which corresponds to OVER-EXTREMAL KN solution containing a topological defect - naked singular ring.
NAKED SINGULAR RING HAS THE COMPTON RADIUS, and represents branch line of space-time into two sheets.

It forms a "door" to a mirror world. $\Rightarrow$ *TWO-SHEETED space-time!*

So, THE CONFLICT BETWEEN GRAVITY AND QUANTUM THEORY APPEARS ON THE COMPTON SCALE – much before the Planck scale!

Quantum theory requires a normal FLAT space and to remove this conflict, the Kerr space should be REGULARIZED:

1) space-time should be set FLAT INSIDE the source !

2) space-time should be taken the Kerr-Newman solution OUTSIDE the source!

These two requirements determine UNAMBIGUOUSLY structure of a soliton source of the KN solution.

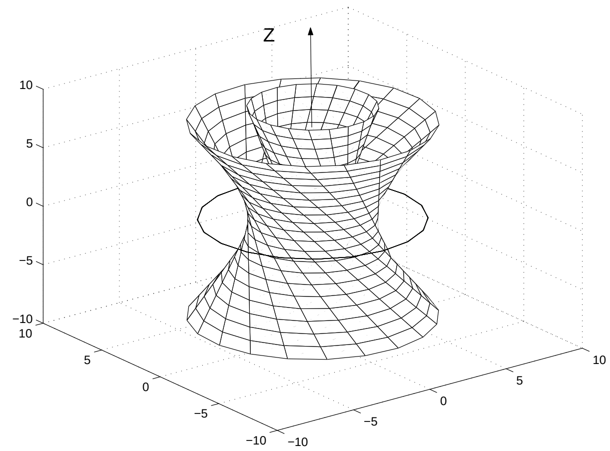
The SOURCE takes the shape of a BAG (false vacuum bubble), similar to MIT-bag and SLAC-bag models of the extended hadrons, but with reversed inside out vacuum states.

This bag takes the Compton size, and we should consider it as a bag model for a *dressed* electron.

Where is there the Dirac equation in the bag-like source of the KN solution?

REAL structure of the Kerr-Newman solution:

Metric $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}$, **and**
electromagnetic vector potential $A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu$ **are collinear with** k^μ.



The Kerr singular ring is a branch line of the TWOSHEETED Kerr space!
The congruence of null directions $k^\pm_\mu$ **creates two metrics** $g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2Hk^\pm_\mu k^\pm_\nu$.

BAG-LIKE SOURCE covers the "door" to negative sheet!

The Kerr congruences are controlled by KERR THEOREM:
Geodesic and Shear-free congruences are obtained as analytic solutions of the equation $F(T^a) = 0$, ***where*** F ***is a holomorphic function of the projective twistor coordinates in CP***^{3}, $\quad T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar\zeta\}$.
TWISTOR ⇔ SPINOR relation is origin of the Dirac equation.

PECULIARITIES OF MIT-BAG AND KN-BAG MODELS.

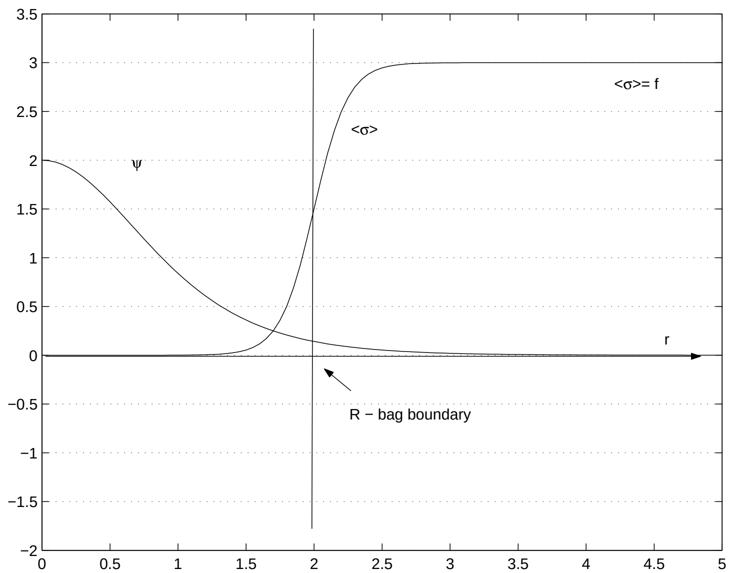


Figure 1: Illustration of the quark confinement in the bag models. Vacuum field σ is determined by quartic potential.

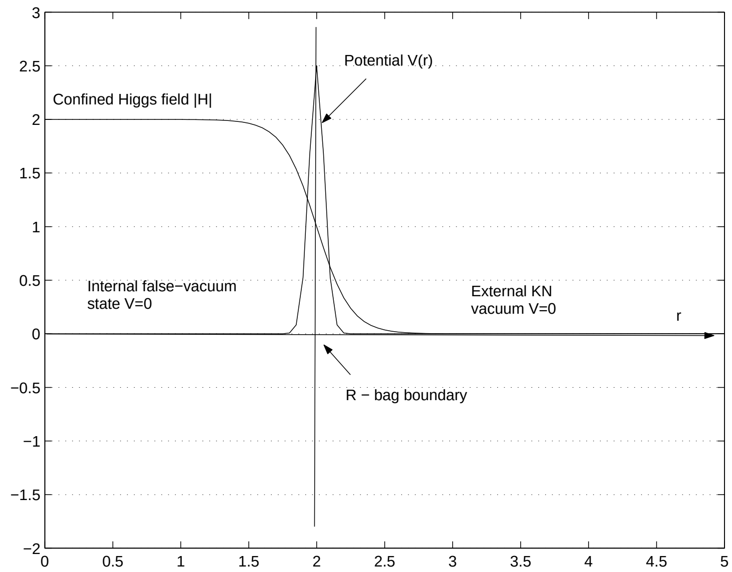


Figure 2: The KN soliton bag model (Q-ball). Potential $V(R)$ forms a narrow spike at the bag boundary. The Higgs field H is confined inside the bag forming a false-vacuum state.

In MIT- and SLAC-bag models the gauge symmetry is broken outside the bag: the standard quartic potential $V(r) = \lambda(|\phi|^2 - \Phi^2)^2$ is unacceptable for solitons with external gravitational and electromagnetic field!
KN-bag should have $V_{int} = V_{ext} = 0$, and a narrow spike at the bag-boundary.
Formation of such a potential requires *a few chiral fields* $\Phi^i(r), \ i = 1, 2, 3.$

Supersymmetric field model of phase transition.

Triplet of the chiral fields $\Phi^{(i)} = \{H, Z, \Sigma\}$, where H is Higgs field.

Lagrangian

$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{3} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \frac{1}{2}\sum_{i=1}^{3} (\mathcal{D}_\mu^{(i)}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^* + V, \tag{1}$$

covariant derivatives $\mathcal{D}_\mu^{(i)} = \nabla_\mu + ieA_\mu^{(i)}$.

Superpotential

$$W = \Phi^{(2)}(\Phi^{(1)}\bar{\Phi}^{(1)} - \eta^2) + (\Phi^{(2)} + \mu)\Phi^{(3)}\bar{\Phi}^{(3)}, \tag{2}$$

determines the potential

$$V(r) = \sum_i |\partial_i W|^2, \tag{3}$$

$\mathcal{H} \equiv \Phi^{(1)}$ **is taken as Higgs field.**

Vacuum states $V_{(vac)} = 0$ **are determined by the conditions** $\partial_i W = 0$. **The model yields two vacuum solutions:**

(I) vacuum state inside the bag: $|\mathcal{H}| = \eta\sqrt{\lambda};\ Z = -\mu;\ \Sigma = 0,$

(II) external vacuum state: $|\mathcal{H}| = 0;\ Z = 0;\ \Sigma = \eta.$

The Higgs field $\mathcal{H}$ is confined inside the bag. Gauge symmetry is broken $\Rightarrow$ supersymmetric false vacuum state.

Basic equations for interaction of the electromagnetic and the Higgs field $\mathcal{H}(x) = |\mathcal{H}|e^{i\chi(x)}$ confined inside the bubble:

$$\mathcal{D}^{(1)}_\nu \mathcal{D}^{(1)\nu} \mathcal{H} = \partial_{\mathcal{H}^*} V, \tag{4}$$

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e|\mathcal{H}|^2 (\chi,_\mu + eA_\mu). \tag{5}$$

Peculiarities of the KN soliton model:

(i) the Kerr singular ring is regularized, and forms a circular string of the Compton radius $r_c \approx a$ along the sharp border of the disklike bag,

(ii) closed flux of the KN electromagnetic potential forms a *quantum Wilson loop* $\oint eA_\varphi d\varphi = -4\pi ma$, which results in quantization of the soliton spin, $J = ma = n\hbar/2, \ n = 1, 2, 3, ...,$

(iii) the Higgs condensate forms a *coherent vacuum state* oscillating with the frequency $\omega = 2m$ − oscillons, Q-balls (G.Rosen 1968, Coleman 1985).

Shape of the bag is unambiguously determined from the form of KN metric $g^{(KN)}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)} k_\mu k_\nu$ by the condition

$$H_{(KN)} = mr - e^2/2 = 0 \Rightarrow r = e^2/2m \tag{6}$$

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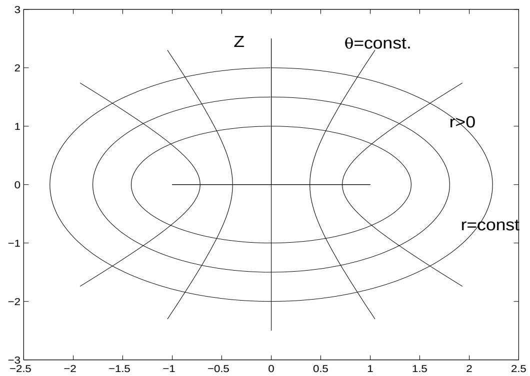

Figure 3: **Kerr's oblate spheroidal coordinates cover space-time twice.**

The bag forms an oblate disk of the Compton radius $r_c \approx a = \frac{1}{2m}$ with the thickness of classical EM radius of electron $r_e = \frac{e^2}{2m}$, so that $r_e/r_c = \alpha \approx 137^{-1}$. The soliton bag closes the door to negative sheet of the Kerr space-time.

DIRAC EQUATION splits in the Weyl representation into two equations

$$\sigma^{\mu}_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \tag{7}$$

the "left-handed" and "right-handed" electron fields.

One of them, say "left" field can be associated with spinor structure of the Kerr congruence.

The Kerr theorem determines all the geodesic and *shear free* congruences as analytical solutions of the equation

$$F(T^A) = 0, \tag{8}$$

where F is an arbitrary holomorphic function of the projective twistor variables

$$T^A = \{Y,\ \zeta - Yv,\ u + Y\bar{\zeta}\}, \qquad A = 1, 2, 3, \tag{9}$$

and $\zeta = (x + iy)/\sqrt{2}, \quad \bar{\zeta} = (x - iy)/\sqrt{2}, \quad u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}$ are the null Cartesian coordinates of the Minkowski space $x^\mu = (t, x, y, z) \in M^4$.

Projective spinor coordinate

$$Y = \phi_1/\phi_0, \tag{10}$$

is equivalent to the Weyl two-component spinor ϕ_α.

Function F for the Kerr and KN solutions may be represented in the quadratic in Y form, $F(Y,x^\mu) = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu)$. In this case, the equation (8) is explicitly solved, leading to two solutions

$$Y^\pm(x^\mu) = (-B \mp \tilde{r})/2A, \tag{11}$$

where $\tilde{r} = (B^2 - 4AC)^{1/2}$. These solutions are antipodally conjugate

$$Y^+ = -1/\bar{Y}^-, \tag{12}$$

and (11) determine two Weyl spinor fields ϕ^α and $\bar{\chi}_{\dot{\alpha}}$, corresponding to antipodal congruences

$$Y^+ = \phi_1/\phi_0 \quad , \quad Y^- = \bar{\chi}^{\dot{1}}/\bar{\chi}^{\dot{0}}. \tag{13}$$

Since Y is also a projective angular coordinate $Y^+ = e^{i\phi}\tan\frac{\theta}{2}$, the created spinor fields ϕ_α and $\bar{\chi}^{\dot{\alpha}}$ acquire explicit dependence dependence on ϕ and θ. For the congruence Y^+ this dependence takes the form

$$\phi_\alpha = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \tag{14}$$

and for $Y^- = -1/\bar{Y}^+$, we have

$$\bar{\chi}^{\dot{\alpha}} = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \tag{15}$$

Only one of the fields, say "left", $k_\mu^{(+)}(x)$ is "retarded" and corresponds to the external KN solution. The field $k_\mu^{(-)}(x) = (1, -\mathbf{k})$, retains the time-like direction and reflects the space orientation.

The spinor fields created by the Kerr theorem ϕ_α and $\bar{\chi}^{\dot{\alpha}}$ correspond to the left out-field and right-in fields, i.e. the retarded and advanced fields correspondingly.

Removing twosheetedness by the bag-source, we meet it again from the external side!

The null vector fields $k^{\mu\pm}(x)$ differ on the retarded and advanced sheets, and generate different metrics

$$g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)} k^{\pm}_\mu k^{\pm}_\nu . \tag{16}$$

The "left" and "right" Dirac fields should be positioned on SEPARATE SHEETS.

Following to Dirac and Feynman, the retarded potentials A_{ret} are represented in the form

$$A_{ret} = \frac{1}{2}[A_{ret} + A_{adv}] + \frac{1}{2}[A_{ret} - A_{adv}], \tag{17}$$

where half-difference corresponds to radiation, and half-sum – to a self-interaction of the source.

This problem disappears inside the bag, where the space is flat, and the both null congruences $k^{\pm}_{\mu}(x)$ are null with respect to the flat Minkowski space.

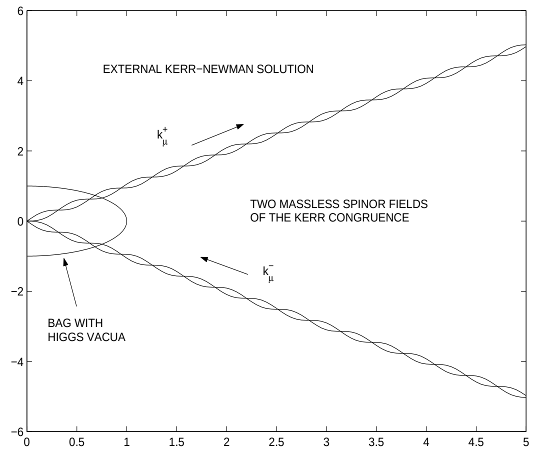

Figure 4: Dirac equation is built of two massless spinor fields $\phi_\alpha \quad \bar{\chi}^{\dot{\alpha}}$ positioned on two different sheets of the Kerr geometry covered by antipodal Kerr congruences (solutions $Y^+(x)$ and $Y^-(x) = -1/\bar{Y}^+$). Penetrating inside the bag these fields acquire Yukawa coupling which yields mass term to the Dirac equation.

Extending the left and right spinor fields inside the solitonic bag, we obtain that they transfer into the flat Minkowski space, where the both null congruences turn out to be compatible, so far as these congruences are null with respect to the same flat sheet of the common internal Minkowski space. Inside the soliton they are united into Dirac bispinor $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$, **corresponding to the massless Dirac equation.**

$$(\gamma^\mu \partial_\mu)\Psi(x) = 0. \tag{18}$$

The confined inside the bag Higgs field $\mathcal{H}$ **adds the mass term, generating the Dirac equation with VARIABLE mass term**

$$m \equiv g\mathcal{H}. \tag{19}$$

The variable mass term is a typical feature of the bag models!

The Dirac wave function is strongly deformed. In the conception of the MIT- and SLAC- bag-models is used a variational principle for the states of minimal energy.

For the KN bag model, the correlations of the wave function with the Kerr congruence is retained by deformations of the bag.

Solutions are localized in the narrow boundary of bubble – effectively similar to the SLAC-bag model.

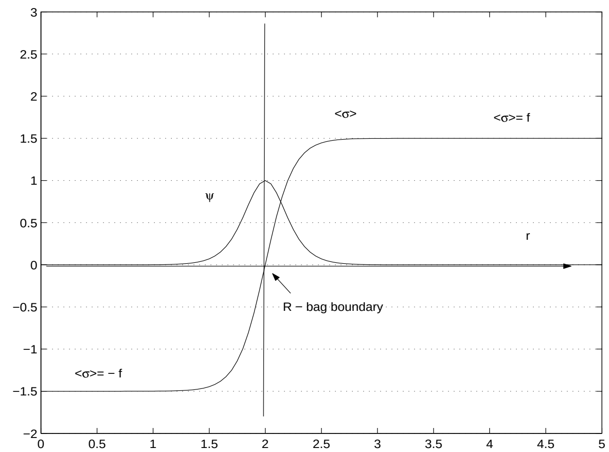

Figure 5: **Potential and the quark wave-function in the SLAC-bag model.**

Bags are deformed by excitations and take the stringy forms (Vinciarelli 1972-1975, Bardeen at al 1975, Chodos at.al 1974, C. Thorn 1976).

Typical deformations are open flux-tube strings with radial and rotational excitations. **The Kerr-Newman rotating disk-like bag may be considered as a deformed spherical bag of the *Dirac "extensible" electron* (*first prototype of the bag models was suggested by Dirac, Ann. of Phys. 1962*). The KN bag stringy structures (AB, 1975- 2014).**

Under vanishing rotation, $a = 0$, the KN disk-like bag turns into the spherical Dirac "extensible" electron model. The non-rotating spherical KN bag has just the Dirac radius R corresponding to classical radius of the electron, $R = r_e = e^2/2m$. **The disk-like bag of the KN rotating source may be considered as a bag obtained by the rotational stretch from the Dirac "extensible" spherical bag.** Kerr's parameter of rotation $a = J/m$ stretches the spherical bag to the disk of the Compton radius $a = \hbar/2mc$. It corresponds to the zone of vacuum polarization of a "dressed" electron. The degree of oblateness is $\alpha = 137^{-1}$. The fine structure constant acquires a geometrical interpretation.

For the KN bag, concentration of the wave function at the border of the KN disk results in the appearance of the circular light-like string, similar to obtained by Sen fundamental string to low energy heterotic string theory. Traveling wave deforms the bag creating an oscillating singular point, D0-brane.

RELATIVISTICALLY ROTATING OBLATE BUBBLE, BOUNDED BY CIRCULAR STRING + point-like QUARKS, complex of D0-D1-D2-D3-branes!

The lowest excitation of the Kerr closed string creates a circulating singular point which may be interpreted as a confined quark in the conception of the bag models, or as a point-like bare electron with zitterbewegung of the Dirac theory, either as an end point of an open circular string (D0-brane) in the conception of string theory.

COMPTON SIZE of the KN BAG indicates its relation to a DRESSED electron - of an electroweak sector of the (Beyond) Standard Model.

The singular end points (quarks) of the KN CIRCULAR STRING, similar to confined quarks of the SLAC-bag model correspond to the point-like BARE electron.
String excitations (traveling waves) create "zitterbewegung".

CONCLUSION:

- **the KN gravitating bag model does not contradict to quantum theory,**
- **Higgs field is confined inside the KN gravitating bag in an oscillating false-vacuum state,**
- **consistency of the KN gravity determines unambiguously shape of the bag as an oblate and relativistically rotating disk,**
- **a circular string is formed on the perimeter of the oblate bag, and circulating end points of the string (poles) correspond to zitterbewegung of the BARE electron.**
- **association with a DRESSED electron may be extended for electroweak sector Beyond Standard Model.**

THANK YOU FOR ATTENTION!