Component on-shell actions of supersymmetric 3-brane in D=6

Stefano Bellucci

INFN LNF, Frascati, Italy

in collaboration with S.Krivonos, N.Kozyrev and A.Sutulin

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Introduction

The usual treatment of the superbranes as extended objects arising as solitons of supersymmetric field theories, leads to the interpretation of the dynamics of branes fluctuations as the field theories on the worldvolume of the superbrane. Thus, it seems to be important to have the explicit form of brane actions, preferably written in terms of covariant objects.

Unfortunately, the gauge fixing procedure in the standard Green-Schwartz-type formulation is not unique. Moreover, the existing possibility to choose different worldvolume fermions makes the task to write the full component superbrane actions with all fermions included rather complicated.

The main problem in this way is to decide which type of covariant objects has to be taken to write the actions. Clearly, even if we would have at hands the proper superfield actions, after passing to the physical components the beauty of the superspace formulation will disappear leaving us with a long tail of fermionic terms having no explicit geometric meaning.

The standard superspace description of the superbranes is based on the fact that usually the single branes preserve half of the target-space supersymmetries. Thus, it seems to be a quite rational choice to introduce the superfields under unbroken supersymmetry, imposing by hands the invariance of the action under broken supersymmetry. One of the methods, heavily used to describe the superbranes, is the method of nonlinear realization of spontaneously broken symmetries. In this approach, from the beginning we are dealing with the theory in the static gauge in which the worldvolume fields, forming the proper supermultiplet of unbroken supersymmetry, correspond to the physical modes of the branes. Within the nonlinear realization approach one may easily construct the objects, the Cartan forms and covariant derivatives, which are covariant with respect to unbroken and broken supersymmetries. Moreover, by imposing the proper constraints on the Cartan forms, one may find the irreducibility conditions on the superfields involved and the covariant superfield equations of motion.

Nevertheless, one notices that the superfield action cannot be constructed within the nonlinear realization approach. The reason is simple - in all known cases the superfield Lagrangian is not an invariant object. Instead, it is shifted by the total space-time derivative under supersymmetry transformations. Thus, the superfield action cannot be constructed from the Cartan forms which are the natural ingredients of this approach. Just due to this reason, another approach in which the superfield Lagrangian is a component of the linear, with respect to both supersymmetries, supermultiplet has been elaborated.

Coming back to the nonlinear realization method, one may wonder why the broken supersymmetry plays such a small role. Indeed, it is well known since the pioneering papers by D. Volkov and V. Akulov that the Goldstone fermions, accompanying the partial breaking of supersymmetry, cannot enter the component action in an arbitrary way. They may appear in the action either through the covariant derivatives or vielbeins, only. Therefore, one may try to shift the attention to the broken supersymmetry while dealing with the component on-shell actions.

Indeed, it was demonstrated in our paper that with a suitable choice of the parametrization of the coset, the θ coordinates of the superspace together with the physical bosonic components do not transform under broken supersymmetry, while the physical fermions transform as the Goldstino of the Volkov-Akulov model. In such a situation, one should expect that all fermionic terms in the components can be "hidden" inside the covariant derivatives and vielbeins, covariant with respect to broken supersymmetry. Therefore, one may consider the reduced coset, which does not contain the θ coordinates at all. The corresponding reduced Cartan forms will be the building blocks for the component actions.

In the series of our papers we explicitly demonstrated that this is indeed the case and the component on-shell actions for low dimensional branes ($D < 4$) can be written through proper covariant derivatives of the bosonic fields and the fermionic vielbeins. In the present talk we analyze the more interesting cases with the 3-brane moving in $D = 6$. In the case of 3-brane the worldvolume effective actions will be the actions for standard four-dimensional field theories, while the corresponding superfields will be the most interesting one: the chiral superfields.

Three-brane in $D = 6$

It is well known that the action of the $N = 1$ supersymmetric 3-brane in $D = 6$ is just a minimal action for the Goldstone chiral $N = 1$ superfields accompanying the spontaneous breaking of $N = 1$, $D = 6$ supersymmetry down to $N = 1$, $d = 4$ one, or, in other words, the breaking of $N = 2$ supersymmetry down to $N = 1$ in four dimensions. In principle, this information is enough to construct the superfield action of the 3-brane in terms of chiral $N = 1$ superfields. The existence of hidden $N = 1$ supersymmetry completely fixes the action.

However, passing to the on-shell component action almost completely destroys its very nice superfields form. Moreover, it seems there is no way to combine the long tail of fermionic terms in some more readable form which would reflect the presence of higher symmetry in the theory.

We will show in the beginning that one may reformulate the on-shell component action in such a way that all its ingredients will have the clear geometric properties.

Our procedure includes three steps:

- construction of the superfield equations of motion,

- checking the bosonic action,

- construction of the full component action.

Superalgebra, Cartan forms and covariant derivatives

In $d = 4$ notation the $N = 1, D = 6$ Poincaré superalgebra is a two central charges extended $N = 2$ Poincaré superalgebra containing the following set of generators

$$\text{N=2, d=4 SUSY} \quad \propto \quad \left\{ P_A, Q_\alpha, \overline{Q}_{\dot\alpha}, S_\alpha, \overline{S}_{\dot\alpha}, Z, \overline{Z}, L_{AB}, K_A, \overline{K}_A, U \right\}.$$

Here, P_A, Z and $\overline{Z}$ are $D = 6$ translation generators, $Q_\alpha, \overline{Q}_{\dot\alpha}$ and $S_\alpha, \overline{S}_{\dot\alpha}$ are the generators of supertranslations, the generators L_{AB} form $d = 4$ Lorentz algebra $so(1,3)$, the generators K_A and $\overline{K}_A$ belong to the coset $SO(1,5)/SO(1,3) \times U(1)$, while U span $u(1)$. The most important anticommutators read

$$\left\{ Q_\alpha, \overline{Q}_{\dot\alpha} \right\} = 2 \left(\sigma^A \right)_{\alpha\dot\alpha} P_A, \quad \left\{ S_\alpha, \overline{S}_{\dot\alpha} \right\} = 2 \left(\sigma^A \right)_{\alpha\dot\alpha} P_A,$$

$$\left\{ Q_\alpha, S_\beta \right\} = 2\epsilon_{\alpha\beta} Z, \quad \left\{ \overline{Q}_{\dot\alpha}, \overline{S}_{\dot\beta} \right\} = 2\epsilon_{\dot\alpha\dot\beta} \overline{Z}.$$

Let us define the coset element as follows

$$g = e^{ix^A P_A} e^{\theta^\alpha Q_\alpha + \bar\theta^{\dot\alpha} \overline{Q}_{\dot\alpha}} e^{\psi^\alpha S_\alpha + \bar\psi^{\dot\alpha} \overline{S}_{\dot\alpha}} e^{i(\varphi Z + \bar\varphi \overline{Z})} e^{i(\Lambda^A K_A + \overline{\Lambda}^A \overline{K}_A)}.$$

Here, $(x^A, \theta^\alpha, \bar\theta^{\dot\alpha})$ are $N = 1, d = 4$ superspace coordinates, while the remaining coset parameters are Goldstone superfields.

The transformation properties of the coordinates and superfields with respect to all symmetries can be found by acting from the left on the coset element g by the different elements of $N = 2$, $d = 4$ Poincaré supergroup. In particular, for the unbroken $(Q, \overline{Q})$ and broken $(S, \overline{S})$ supersymmetries we have

- Unbroken supersymmetry:

$$\delta_Q x^A = i \left(\epsilon^\alpha \bar{\theta}^{\dot{\alpha}} + \bar{\epsilon}^{\dot{\alpha}} \theta^\alpha \right) \left(\sigma^A \right)_{\alpha \dot{\alpha}}, \quad \delta_Q \theta^\alpha = \epsilon^\alpha, \quad \delta_Q \bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}}.$$

- Spontaneously broken supersymmetry:

$$\delta_S x^A = i \left(\varepsilon^\alpha \bar{\psi}^{\dot{\alpha}} + \bar{\varepsilon}^{\dot{\alpha}} \psi^\alpha \right) \left(\sigma^A \right)_{\alpha \dot{\alpha}},$$

$$\delta_S \psi^\alpha = \varepsilon^\alpha, \quad \delta_S \bar{\psi}_{\dot{\alpha}} = \bar{\varepsilon}_{\dot{\alpha}}, \quad \delta_S \varphi = 2i\varepsilon_\alpha \theta^\alpha, \quad \delta_S \bar{\varphi} = 2i\bar{\varepsilon}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}.$$

The local geometric properties of the system are specified by the Cartan forms.
The most convenient structure of them can be written after passing to new superfields $(\lambda^A, \bar{\lambda}^A)$ which are connected with the superfields $(\Lambda^A, \overline{\Lambda}^A)$ as

$$\lambda^A = \left(\frac{\tanh\sqrt{\frac{Y}{2}}}{\sqrt{\frac{Y}{2}}}\right)^A_{B} \Lambda^B, \quad \bar{\lambda}^A = \left(\frac{\tanh\sqrt{\frac{Y}{2}}}{\sqrt{\frac{Y}{2}}}\right)^A_{B} \overline{\Lambda}^B,$$

where

$$Y_A^{B} = \Lambda_A \overline{\Lambda}^B + \overline{\Lambda}_A \Lambda^B, \quad y_A^{B} = \lambda_A \bar{\lambda}^B + \bar{\lambda}_A \lambda^B.$$

Thus, for example, for the bosonic forms one gets

$$(\omega_P)^A = \triangle x^B \left(\frac{1 + y/2}{1 - y/2}\right)^A_{B} - 2\left(\triangle\varphi\,\bar{\lambda}^B + \triangle\bar{\varphi}\,\lambda^B\right)\left(\frac{1}{1 - y/2}\right)^A_{B},$$

$$\omega_Z = \triangle\varphi + \left(\triangle\varphi\,\bar{\lambda}^A + \triangle\bar{\varphi}\,\lambda^A\right)\left(\frac{1}{1 - y/2}\right)^B_{A}\lambda_B - \triangle x^A \left(\frac{1}{1 - y/2}\right)^B_{A}\lambda_B,$$

$$\bar{\omega}_Z = \triangle\bar{\varphi} + \left(\triangle\varphi\,\bar{\lambda}^A + \triangle\bar{\varphi}\,\lambda^A\right)\left(\frac{1}{1 - y/2}\right)^B_{A}\bar{\lambda}_B - \triangle x^A \left(\frac{1}{1 - y/2}\right)^B_{A}\bar{\lambda}_B,$$

where

$$\triangle x^A = dx^A - i\left(\theta^\alpha d\bar{\theta}^{\dot{\alpha}} + \bar{\theta}^{\dot{\alpha}} d\theta^\alpha + \psi^\alpha d\bar{\psi}^{\dot{\alpha}} + \bar{\psi}^{\dot{\alpha}} d\psi^\alpha\right)\left(\sigma^A\right)_{\alpha\dot{\alpha}},$$

$$\triangle\varphi = d\varphi - 2i\psi_\alpha d\theta^\alpha, \quad \triangle\bar{\varphi} = d\bar{\varphi} - 2i\bar{\psi}_{\dot{\alpha}} d\bar{\theta}^{\dot{\alpha}}.$$

Taking into account that the quantities $\triangle x^A$, $d\theta^\alpha$ and $d\bar{\theta}^{\dot\alpha}$ are invariant with respect to both broken and unbroken supersymmetries, one may define the semi-covariant derivatives $\nabla_A, \nabla_\alpha, \overline{\nabla}_{\dot\alpha}$

$$\nabla_A = (E^{-1})_A{}^B \quad \partial_B, \ E_A{}^B = \delta_A{}^B - \mathrm{i} \left(\psi^\alpha \partial_A \bar{\psi}^{\dot\alpha} + \bar{\psi}^{\dot\alpha} \partial_A \psi^\alpha \right) \left(\sigma^B \right)_{\alpha\dot\alpha},$$

$$\nabla_\beta = D_\beta - \mathrm{i} \left(\psi^\alpha \nabla_\beta \bar{\psi}^{\dot\alpha} + \bar{\psi}^{\dot\alpha} \nabla_\beta \psi^\alpha \right) \left(\sigma^B \right)_{\alpha\dot\alpha} \partial_B,$$

$$\overline{\nabla}_{\dot\beta} = \overline{D}_{\dot\beta} - \mathrm{i} \left(\psi^\alpha \overline{\nabla}_{\dot\beta} \bar{\psi}^{\dot\alpha} + \bar{\psi}^{\dot\alpha} \overline{\nabla}_{\dot\beta} \psi^\alpha \right) \left(\sigma^B \right)_{\alpha\dot\alpha} \partial_B.$$

These derivatives satisfy the following (anti)commutation relations

$$\{\nabla_\alpha, \nabla_\beta\} = -2\mathrm{i} \left(\nabla_\alpha \psi^\gamma \nabla_\beta \bar{\psi}^{\dot\gamma} + \nabla_\beta \psi^\gamma \nabla_\alpha \bar{\psi}^{\dot\gamma} \right) \left(\sigma^C \right)_{\gamma\dot\gamma} \nabla_C,$$

$$[\nabla_A, \nabla_B] = 2\mathrm{i} \left(\nabla_A \psi^\gamma \nabla_B \bar{\psi}^{\dot\gamma} - \nabla_B \psi^\gamma \nabla_A \bar{\psi}^{\dot\gamma} \right) \left(\sigma^C \right)_{\gamma\dot\gamma} \nabla_C,$$

$$\{\nabla_\alpha, \overline{\nabla}_{\dot\beta}\} = -2\mathrm{i} \left(\delta_\alpha^\gamma \delta_{\dot\beta}^{\dot\gamma} + \nabla_\alpha \psi^\gamma \overline{\nabla}_{\dot\beta} \bar{\psi}^{\dot\gamma} + \overline{\nabla}_{\dot\beta} \psi^\gamma \nabla_\alpha \bar{\psi}^{\dot\gamma} \right) \left(\sigma^C \right)_{\gamma\dot\gamma} \nabla_C,$$

$$[\nabla_A, \nabla_\alpha] = -2\mathrm{i} \left(\nabla_A \psi^\gamma \nabla_\alpha \bar{\psi}^{\dot\gamma} + \nabla_\alpha \psi^\gamma \nabla_A \bar{\psi}^{\dot\gamma} \right) \left(\sigma^C \right)_{\gamma\dot\gamma} \nabla_C.$$

The flat covariant spinor derivatives obey the standard relations

$$\left\{ D_\alpha, \overline{D}_{\dot\alpha} \right\} = -2\mathrm{i} \left(\sigma^A \right)_{\alpha\dot\alpha} \partial_A, \quad \left\{ D_\alpha, D_\beta \right\} = 0, \quad \left\{ \overline{D}_{\dot\alpha}, \overline{D}_{\dot\beta} \right\} = 0.$$

Superfield equations of motion

The next tasks, that we are going to perform within the superfield approach, include

- reduction of the number of independent superfields
- imposing on the superfields the covariant irreducibility constraints
- finding the covariant equations of motion.

As we already demonstrated in our previous paper, all these tasks can be solved simultaneously by imposing the following constraints on the Cartan forms

$$\omega_Z = 0, \quad \bar{\omega}_Z = 0, \qquad\qquad (kinematical)$$

$$\omega_S| = 0 \quad \bar{\omega}_S| = 0, \qquad\qquad (dynamical)$$

where $|$ means the $d\theta$ and $d\bar{\theta}$-projections.

These constraints are similar to superembedding conditions.

The first set of constraints are purely kinematical ones, and they result in the following equations

$$\overline{\nabla}_{\dot\alpha}\varphi = 0, \quad \nabla_\alpha\varphi = -2i\psi_\alpha, \qquad \nabla_A\varphi = 2\frac{(2+\lambda\bar\lambda)\lambda_A - \lambda^2\bar\lambda_A}{(2+\lambda\bar\lambda)^2 - \lambda^2\bar\lambda^2},$$

$$\nabla_\alpha\bar\varphi = 0, \quad \overline{\nabla}_{\dot\alpha}\bar\varphi = -2i\bar\psi_{\dot\alpha}, \qquad \nabla_A\bar\varphi = 2\frac{(2+\lambda\bar\lambda)\bar\lambda_A - \bar\lambda^2\lambda_A}{(2+\lambda\bar\lambda)^2 - \lambda^2\bar\lambda^2}.$$

These equations allow us to express the superfields $\psi_\alpha, \bar\psi_{\dot\alpha}$ and $\lambda_A, \bar\lambda_A$ through the covariant derivatives of φ and $\bar\varphi$ (this is the so called Inverse Higgs Phenomenon). In addition, the superfields φ and $\bar\varphi$ are subjected to the covariant (anti)chirality conditions.

Thus, the bosonic, covariantly (anti)chiral Goldstone superfields φ and $\bar\varphi$ are the only essential superfields needed for this case of partial breaking of the global supersymmetry.

The situation with the second set constraints is more interesting. First, the $d\bar{\theta}$ ($d\theta$) projection of the form ω_S ($\bar{\omega}_S$) relates the spinor derivative of the superfield ψ_α ($\bar{\psi}_{\dot\alpha}$) and $x-$derivative of the superfield φ ($\bar{\varphi}$)

$$J_{\alpha\dot\alpha} \equiv \overline{\nabla}_{\dot\alpha}\psi_\alpha = \partial_{\alpha\dot\alpha}\varphi + \dots, \qquad \overline{J}_{\alpha\dot\alpha} \equiv \nabla_\alpha\bar{\psi}_{\dot\alpha} = \partial_{\alpha\dot\alpha}\bar{\varphi} + \dots,$$

where we explicitly write only the leading, linear in $\partial_{\alpha\dot\alpha}\varphi$ and $\partial_{\alpha\dot\alpha}\bar{\varphi}$ terms.
At the same time, the $d\theta$ ($d\bar{\theta}$) projection of the form ω_S ($\bar{\omega}_S$) gives the equations

$$\nabla_\alpha\psi_\beta = 0, \qquad \overline{\nabla}_{\dot\alpha}\bar{\psi}_{\dot\beta} = 0.$$

To see that these equations are really equations of motion, note that from $\omega_S| = \bar{\omega}_S| = 0$ and the algebra of covariant derivatives it follows that

$$\{\nabla_\alpha, \nabla_\beta\} = 0, \qquad \{\overline{\nabla}_{\dot\alpha}, \overline{\nabla}_{\dot\beta}\} = 0,$$

and, therefore, we conclude that

$$\nabla^\alpha\nabla_\alpha\varphi = 0, \qquad \overline{\nabla}^{\dot\alpha}\overline{\nabla}_{\dot\alpha}\bar{\varphi} = 0.$$

This is the covariant form of the superfield equations of motion, which is a proper covariantization of the free equations of motion.

Component action

As was already mentioned, it is not clear how to construct the superfield action within the nonlinear realization approach. For the supersymmetric 3-brane in $D = 6$ the corresponding superfield actions were constructed within different frameworks (*Bagger, Galperin; Rocek, Tseytlin*).

Nevertheless, the component action, being constructed from the superfield one, is very complicated. It contains a lot of fermionic terms with completely unclear geometric structure.

Alternatively, as we demonstrated in our previous papers, the component actions can be constructed within the nonlinear realization approach in such way that the invariance with respect to broken supersymmetry becomes almost evident.

The useful ingredients for this construction include the reduced Cartan forms and reduced covariant derivatives, covariant with respect to broken supersymmetry only.

The basic steps of our approach are

- construction of the bosonic action
- covariantization of the bosonic action with respect to broken supersymmetry
- construction of the Wess-Zumino terms
- imposing the invariance with respect to unbroken supersymmetry.

Let us perform all these steps for the supersymmetric 3-brane in $D = 6$.

Bosonic action

The bosonic equations of motion can be extracted from the superfield equations. But the calculations are rather involved. Instead, one can construct the corresponding action directly, using the fact that such an action should possess invariance with respect to $D = 6$ Poincaré symmetry spontaneously broken to $d = 4$.
One of the key ingredients of such a construction is the bosonic limit of the Cartan forms which explicitly reads (here we use again $y_A^B = \lambda_A \bar{\lambda}^B + \bar{\lambda}_A \lambda^B$)

$$(\omega_P)_{bos}^A = dx^B \Big(\frac{1 + y/2}{1 - y/2} \Big)_B^A - 2 \Big(d\varphi \bar{\lambda}^B + d\bar{\varphi} \lambda^B \Big) \Big(\frac{1}{1 - y/2} \Big)_B^A,$$

$$(\omega_Z)_{bos} = d\varphi + \Big(d\varphi \bar{\lambda}^A + d\bar{\varphi} \lambda^A \Big) \Big(\frac{1}{1 - y/2} \Big)_A^B \lambda_B - dx^A \Big(\frac{1}{1 - y/2} \Big)_A^B \lambda_B,$$

$$(\bar{\omega}_Z)_{bos} = d\bar{\varphi} + \Big(d\varphi \bar{\lambda}^A + d\bar{\varphi} \lambda^A \Big) \Big(\frac{1}{1 - y/2} \Big)_A^B \bar{\lambda}_B - dx^A \Big(\frac{1}{1 - y/2} \Big)_A^B \bar{\lambda}_B.$$

Imposing now the same constraint $\omega_Z = \bar{\omega}_Z = 0$, we will get the bosonic analog of the supersymmetric relations which were given before

$$\partial_A \varphi = 2 \frac{(2 + \lambda \bar{\lambda}) \lambda_A - \lambda^2 \bar{\lambda}_A}{(2 + \lambda \bar{\lambda})^2 - \lambda^2 \bar{\lambda}^2}, \qquad \partial_A \bar{\varphi} = 2 \frac{(2 + \lambda \bar{\lambda}) \bar{\lambda}_A - \bar{\lambda}^2 \lambda_A}{(2 + \lambda \bar{\lambda})^2 - \lambda^2 \bar{\lambda}^2}.$$

Plugging these expressions in the form $(\omega_P)^A_{bos}$ and using the explicit expression for the matrix $\left(\frac{1}{1-y/2}\right)^B_A$

$$\left(\frac{1}{1-y/2}\right)^B_A = \delta^B_A + \frac{(2-\lambda\bar{\lambda})\left(\lambda_A\bar{\lambda}^B + \bar{\lambda}_A\lambda^B\right) + \left(\bar{\lambda}^2\lambda_A\lambda^B + \lambda^2\bar{\lambda}_A\bar{\lambda}^B\right)}{(2-\lambda\bar{\lambda})^2 - \lambda^2\bar{\lambda}^2},$$

one may obtain

$$(\omega_P)^A_{bos} = dx^B\, e_B{}^A = dx^B\left(\delta^A_B - 2\frac{(2+\lambda\bar{\lambda})\left(\lambda_B\bar{\lambda}^A + \bar{\lambda}_B\lambda^A\right) - \left(\bar{\lambda}^2\lambda_B\lambda^A + \lambda^2\bar{\lambda}_B\bar{\lambda}^A\right)}{(2+\lambda\bar{\lambda})^2 - \lambda^2\bar{\lambda}^2}\right).$$

Now, the unique invariant which can be constructed from the Cartan form $(\omega_P)^A_{bos}$ is the volume form which explicitly reads

$$\epsilon_{ABCD}\,(\omega_P)^A_{bos} \wedge (\omega_P)^B_{bos} \wedge (\omega_P)^C_{bos} \wedge (\omega_P)^D_{bos} \sim$$

$$\sim d^4x\,\det(e) = d^4x\frac{(2-\lambda\bar{\lambda})^2 - \lambda^2\bar{\lambda}^2}{(2+\lambda\bar{\lambda})^2 - \lambda^2\bar{\lambda}^2}.$$

Finally, the bosonic action, being rewritten in terms of $\partial_A \varphi$ and $\partial_A \bar\varphi$ acquires the form

$$S_{bos} = \int d^4 x \, \sqrt{\left(1 - 2\left(\partial\varphi\partial\bar\varphi\right)\right)^2 - 4\left(\partial\varphi\partial\varphi\right)\left(\partial\bar\varphi\partial\bar\varphi\right)}.$$

This is the static gauge Nambu-Goto action for the 3-brane in $D = 6$.
One may explicitly check that this action is invariant with respect to $K_A, \overline{K}_A$
transformations from the coset $SO(1,5)/SO(1,3) \times U(1)$ realized as

$$\delta x^A = 2\bar\alpha^A \varphi + 2\alpha^A \bar\varphi, \quad \delta\varphi = \alpha_A x^A, \quad \delta\bar\varphi = \bar\alpha_A x^A,$$

and therefore, it is invariant with respect to the whole $D = 6$ Poincaré group.

Covariantization with respect to broken supersymmetry

In contrast with the standard approach, in which the superfields with respect to unbroken $(Q, \overline{Q})$ supersymmetry play the main role and are the building blocks for the superfield actions, in the component approach we are prefer to concentrate on the broken $(S, \overline{S})$ supersymmetry. Thus, the first task is to modify the bosonic action in such a way to achieve invariance with respect to broken supersymmetry.

Due to the transformations laws undre the broken supersymmetry, the coordinates x^A and the first components of the superfields $\varphi, \bar{\varphi}, \psi^\alpha, \bar{\psi}^{\dot\alpha}$ transform under broken supersymmetry as follows

$$\delta_S x^A = i \left(\varepsilon^\alpha \bar{\psi}^{\dot\alpha} + \bar{\varepsilon}^{\dot\alpha} \psi^\alpha \right) \left(\sigma^A \right)_{\alpha\dot\alpha},$$

$$\delta_S \psi^\alpha = \varepsilon^\alpha, \quad \delta_S \bar{\psi}^{\dot\alpha} = \bar{\varepsilon}^{\dot\alpha}, \quad \delta_S \varphi = 0, \quad \delta_S \bar{\varphi} = 0.$$

Thus, the volume d^4x and the derivatives $\partial_A \varphi, \partial_A \bar{\varphi}$ are not the covariant objects.

To find the proper objects, let us consider the reduced coset element

$$g_{red} = e^{\mathrm{i} x^A P_A} e^{\psi^\alpha S_\alpha + \bar\psi^{\dot\alpha} \bar S_{\dot\alpha}} e^{\mathrm{i}(\varphi Z + \bar\varphi \bar Z)},$$

where the fields $\varphi, \bar\varphi, \psi^\alpha, \bar\psi^{\dot\alpha}$ depend on the coordinates x^A only.
The corresponding reduced Cartan forms read

$$(\omega_P)^A_{red} = \mathcal{E}^A{}_B dx^B, \quad \mathcal{E}^A{}_B \equiv \delta^A_B - \mathrm{i}\left(\psi^\alpha \partial_B \bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha} \partial_B \psi^\alpha\right)\left(\sigma^A\right)_{\alpha\dot\alpha}$$

$$(\omega_Z)_{red} = d\varphi, \quad (\bar\omega_Z)_{red} = d\bar\varphi, \quad (\omega_S)^\alpha_{red} = d\psi^\alpha, \quad (\bar\omega_S)^{\dot\alpha}_{red} = d\bar\psi^{\dot\alpha}.$$

These forms are invariant with respect to transformations of broken supersymmetry.
Therefore, the covariant x-derivative will be

$$\mathcal{D}_A = \left(\mathcal{E}^{-1}\right)_A{}^B \partial_B,$$

while the invariant volume can be constructed now from the forms $(\omega_P)^A_{red}$. Thus, the proper covariantization of the bosonic action, having the right bosonic limit, reads

$$S_1 = \int d^4 x \, \det(\mathcal{E}) \, \sqrt{\left(1 - 2\left(\mathcal{D}\varphi\mathcal{D}\bar\varphi\right)\right)^2 - 4\left(\mathcal{D}\varphi\mathcal{D}\varphi\right)\left(\mathcal{D}\bar\varphi\mathcal{D}\bar\varphi\right)}.$$

The action S_1 reproduces the fixed kinetic terms for bosons and fermions

$$(S_1)_{lin} = \int d^4 x \left(2\,\partial^A\varphi\partial_A\bar\varphi - \mathrm{i}\left(\psi^\alpha \partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha}\partial_{\alpha\dot\alpha}\psi^\alpha\right)\right).$$

This would be too strong to maintain unbroken supersymmetry.
Therefore, we have to introduce one more, evidently invariant, action

$$S_2 = \alpha \int d^4x \, \det(\mathcal{E}).$$

Consequently, our Ansatz for the invariant supersymmetric action of the 3-brane acquires the form

$$S = S_0 + S_1 + S_2 =$$

$$= (1 + \alpha) \int d^4x - \int d^4x \, \det(\mathcal{E}) \left(\alpha + \sqrt{(1 - 2\,(\mathcal{D}\varphi\mathcal{D}\bar{\varphi}))^2 - 4\,(\mathcal{D}\varphi\mathcal{D}\varphi)\,(\mathcal{D}\bar{\varphi}\mathcal{D}\bar{\varphi})} \right),$$

where α is a constant that has to be defined, and we have added the trivial invariant action

$$S_0 = \int d^4x$$

to have a proper limit

$$S_{\varphi, \psi \to 0} = 0.$$

Wess-Zumino term

The construction of the Wess-Zumino term, which is not strictly invariant, but which is shifted by a total derivative under broken supersymmetry, goes in a standard way. First, one has to determine the close five form Ω_5, which is invariant under $d = 4$ Lorentz and broken supersymmetry transformations. Moreover, in the present case this form has to disappear in the bosonic limit, because our Ansatz for the action already reproduces the proper bosonic action of the 3-brane.

Such a form can be easily constructed in terms of the Cartan forms

$$\Omega_5 = \omega_Z \wedge \bar\omega_Z \wedge \omega_S^\alpha \wedge \bar\omega_S^{\dot\alpha} \wedge \omega_P^A \, (\sigma_A)_{\alpha\dot\alpha} = d\varphi \wedge d\bar\varphi \wedge d\psi^\alpha \wedge d\bar\psi^{\dot\alpha} \wedge \omega_P^A \, (\sigma_A)_{\alpha\dot\alpha}.$$

To see that Ω_5 is indeed closed, one should take into account that the exterior derivative of $(\omega_P)_{\alpha\dot\alpha}$ is given by the expression

$$d\,(\omega_P)_{\alpha\dot\alpha} \sim (\omega_S)_\alpha \wedge (\bar\omega_S)_{\dot\alpha} = d\psi_\alpha \wedge d\bar\psi_{\dot\alpha},$$

and, therefore, $d\Omega_5 = 0$, because

$$d\psi^\alpha \wedge d\psi_\alpha = d\bar\psi^{\dot\alpha} \wedge d\bar\psi_{\dot\alpha} = 0.$$

Next, one has to write Ω_5 as the exterior derivative of a 4-form Ω_4

$$\Omega_5 = d\Omega_4 \;\Rightarrow\; \Omega_4 = d\varphi \wedge d\bar\varphi \wedge \left(\psi^\alpha d\bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha} d\psi^\alpha\right) \wedge \omega_P^A \;(\sigma_A)_{\alpha\dot\alpha}\,.$$

Finally, the Wess-Zumino term is given by

$$S_{WZ} = \int \Omega_4 = \int d^4x \det(\mathcal{E})\, \epsilon^{ABCD} \nabla_A \varphi \nabla_B \bar\varphi \left(\psi^\alpha \nabla_C \bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha} \nabla_C \psi^\alpha\right)\,(\sigma_D)_{\alpha\dot\alpha}\,.$$

Before going further, let us note that the Wess-Zumino term could be equivalently represented as

$$S_{WZ} = \int d^4x \det(\mathcal{E})\, \epsilon^{ABCD} \nabla_A \varphi \nabla_B \bar\varphi \left(\psi^\alpha \nabla_C \bar\psi^{\dot\alpha} + \bar\psi^{\dot\alpha} \nabla_C \psi^\alpha\right)\left(\mathcal{E}^{-1}\right)_D^{\;E}\,(\sigma_E)_{\alpha\dot\alpha}\,.$$

One can check that the difference between these two expressions is equal to zero because

$$\epsilon^{ABCD}\left(\psi^2 \nabla_C \bar\psi^{\dot\alpha} \nabla_D \bar\psi_{\dot\alpha}\right) = \epsilon^{ABCD}\left(\bar\psi^2 \nabla_C \psi^\alpha \nabla_D \psi_\alpha\right) = 0,$$

$$\epsilon^{ABCD}\left(\nabla_C(\psi^2)\nabla_D(\bar\psi^2) + \nabla_D(\psi^2)\nabla_C(\bar\psi^2)\right) = 0.$$

In this form, the invariance of WZ-action under broken supersymmetry is easily to check.

As the last step, one has to prove that the Wess-Zumino term is invariant with respect to broken supersymmetry transformations. Omitting the technical details of the proof of its invariance, we present the final expression of the full component action of the 3-brane in $D = 6$

$$S = (1 + \alpha) \int d^4x - \int d^4x \det(\mathcal{E}) \left(\alpha + \sqrt{(1 - 2(\mathcal{D}\varphi\mathcal{D}\bar{\varphi}))^2 - 4(\mathcal{D}\varphi\mathcal{D}\varphi)(\mathcal{D}\bar{\varphi}\mathcal{D}\bar{\varphi})} \right)$$

$$+ \beta \int d^4x \det(\mathcal{E}) \, \epsilon^{ABCD} \mathcal{D}_A \varphi \mathcal{D}_B \bar{\varphi} \left(\psi^\alpha \mathcal{D}_C \bar{\psi}^{\dot\alpha} + \bar{\psi}^{\dot\alpha} \mathcal{D}_C \psi^\alpha \right) \left(\mathcal{E}^{-1} \right)_D^{\ E} (\sigma_E)_{\alpha\dot\alpha}.$$

Here α and β are two constants that have to be defined by invariance with respect to unbroken supersymmetry.

Thus, after imposing broken supersymmetry, the component action is fixed up to two constants α and β. No other terms in the action are admissible.

The role of the unbroken supersymmetry is just to fix this constants.

Unbroken supersymmetry

In contrast with the standard superfield approach, the most technically complicated part of our approach is to maintain the unbroken supersymmetry, despite the fact that all we need is to fix two constants in the action.

The first step in this task is to find the transformations of the needed ingredients in the action under unbroken supersymmetry.

We will explicitly present here only the ϵ-part of the transformations

$$\delta_Q \mathcal{D}_A \varphi = 2\mathrm{i}\epsilon^\alpha \mathcal{D}_A \psi_\alpha - 2\mathrm{i}\epsilon^\beta \mathcal{D}_A \psi^\alpha \overline{J}_\beta{}^{\dot\alpha} \mathcal{D}_{\alpha\dot\alpha} \varphi - 2\mathrm{i}H^C \partial_C \mathcal{D}_A \varphi,$$

$$\delta_Q \mathcal{D}_A \bar\varphi = -2\mathrm{i}\epsilon^\beta \mathcal{D}_A \psi^\alpha \overline{J}_\beta{}^{\dot\alpha} \mathcal{D}_{\alpha\dot\alpha} \bar\varphi - 2\mathrm{i}H^C \partial_C \mathcal{D}_A \bar\varphi,$$

$$\delta_Q \psi_\alpha = -2\mathrm{i}H^A \partial_A \psi_\alpha,$$

$$\delta_Q \bar\psi_{\dot\alpha} = -\epsilon^\alpha \overline{J}_{\alpha\dot\alpha} - 2\mathrm{i}H^A \partial_A \bar\psi_{\dot\alpha},$$

$$\delta_Q \mathcal{D}_A \psi_\alpha = -2\mathrm{i}\epsilon^\beta \mathcal{D}_A \psi^\gamma \overline{J}_\beta{}^{\dot\gamma} \mathcal{D}_{\gamma\dot\gamma} \psi_\alpha - 2\mathrm{i}H^C \partial_C \mathcal{D}_A \psi_\alpha,$$

$$\delta_Q \mathcal{D}_A \bar\psi_{\dot\alpha} = -\epsilon^\alpha \mathcal{D}_A \overline{J}_{\alpha\dot\alpha} - 2\mathrm{i}\epsilon^\beta \mathcal{D}_A \psi^\gamma \overline{J}_\beta{}^{\dot\gamma} \mathcal{D}_{\gamma\dot\gamma} \bar\psi_{\dot\alpha} - 2\mathrm{i}H^C \partial_C \mathcal{D}_A \bar\psi_{\dot\alpha},$$

$$\delta_Q \left(\mathcal{E}^{-1}\right)_A{}^B = 2\mathrm{i}\mathcal{D}_A H^B - 2\mathrm{i}\epsilon^\beta \mathcal{D}_A \psi^\gamma \overline{J}_\beta{}^{\dot\gamma} \left(\mathcal{E}^{-1}\right)_D{}^B \left(\sigma^D\right)_{\gamma\dot\gamma} - 2\mathrm{i}H^C \partial_C \left(\mathcal{E}^{-1}\right)_A{}^B$$

Here we use the convenient for further analysis variables H^A

$$H^{\alpha\dot\alpha} \equiv \epsilon^\beta \psi^\alpha \overline{J}_\beta{}^{\dot\alpha}, \quad H^A = \frac{1}{2} H^{\alpha\dot\alpha} \left(\sigma^A\right)_{\alpha\dot\alpha}.$$

As a consequence of the transformation properties, we will have

$$\delta_Q \det(\mathcal{E}) = 2i\epsilon^\beta \mathcal{D}_{\alpha\dot\alpha}\psi^\alpha \overline{J}_\beta{}^{\dot\alpha} \det(\mathcal{E}) - 2i\partial_A\left(H^A \det(\mathcal{E})\right).$$

Therefore, in any action of the form

$$S \sim \int d^4x \det(\mathcal{E})\mathcal{F}\left[\mathcal{D}\varphi, \mathcal{D}\bar\varphi, \psi, \bar\psi, \mathcal{D}\psi, \mathcal{D}\bar\psi, \mathcal{E}^{-1}\right]$$

with arbitrary function $\mathcal{F}\left[\mathcal{D}\varphi, \mathcal{D}\bar\varphi, \psi, \bar\psi, \mathcal{D}\psi, \mathcal{D}\bar\psi, \mathcal{E}^{-1}\right]$, the H-dependent terms will be converted into the full derivative $-2i\partial_A\left(H^A \det(\mathcal{E})\,\mathcal{F}\right)$.

Thus, one may ignore these parts of the given transformations (except the $2i\mathcal{D}_A H^B$ term in the transformations of $\left(\mathcal{E}^{-1}\right)_A{}^B$), because our action just belongs to that class of actions.

The parameter α can be defined if we will consider the kinetic terms for bosonic $(\varphi, \bar{\varphi})$ and fermionic $(\psi_\alpha, \bar{\psi}_{\dot{\alpha}})$ components in the action

$$S_{kin} = \int d^4x \left(2i(1+\alpha)\bar{\psi}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\psi^\alpha + 2\partial^A\varphi\partial_A\bar{\varphi} \right).$$

The action has to be invariant with respect to linearized transformations of unbroken supersymmetry

$$\delta_Q\partial_A\varphi = 2i\epsilon^\alpha\partial_A\psi_\alpha, \quad \delta_Q\partial_A\bar{\varphi} = 0, \quad \delta_Q\psi_\alpha = 0, \quad \delta_Q\bar{\psi}_{\dot{\alpha}} = -\epsilon^\alpha\partial_{\alpha\dot{\alpha}}\bar{\varphi}.$$

Varying the integrand in S_{kin} and integrating by parts, we will get

$$\delta_Q S_{kin} = \int d^4x \left(2i(1+\alpha)\epsilon^\beta\bar{\varphi}\,\partial_\beta{}^{\dot{\alpha}}\,\partial_{\alpha\dot{\alpha}}\psi^\alpha - 4i\epsilon^\alpha\bar{\varphi}\,\partial^A\partial_A\psi_\alpha \right).$$

Therefore, we have to fix

$$\alpha = 1.$$

The fixing of the parameter β is more involved. Ignoring the H-dependent terms in the transformation laws, the variation of the actions without the Wess-Zumino term with $\alpha = 1$ under unbroken supersymmetry can be represented as

$$\delta_Q S = \int d^4 x \det(\mathcal{E}) \, \frac{4\epsilon_\beta \mathcal{D}_A \psi^\alpha}{1 - J^2 \overline{J}^2} \left((1 - J \cdot \overline{J}) \overline{J}_B \left(\sigma^{AB} \right)_\alpha^\beta + \overline{J}^A J_C \overline{J}_B \left(\sigma^{CB} \right)_\alpha^\beta \right).$$

The full variation of the Wess-Zumino term is more complicated. But if we limit ourselves to the terms which are of the first order in fermions, this variation reads

$$\delta_Q S_{WZ} \sim -2\beta \int d^4 x \frac{\epsilon_\alpha \partial_A \psi^\beta}{1 - J^2 \overline{J}^2} \left(-J \cdot \overline{J} \, \overline{J}_B \left(\sigma^{AB} \right)_\alpha^\beta + \overline{J}^2 J_B \left(\sigma^{AB} \right)_\alpha^\beta + \overline{J}^A J_B \overline{J}_C \left(\sigma^{BC} \right)_\alpha^\beta \right).$$

If we choose now $\beta = 2$, then the variation of the full action acquires the form

$$\delta_Q S \sim \int d^4 x \frac{4\epsilon_\beta \partial_A \psi^\alpha}{1 - J^2 \overline{J}^2} \left(\overline{J}_B - \overline{J}^2 J_B \right) \left(\sigma^{AB} \right)_\alpha^\beta = 4 \int d^4 x \, \epsilon_\beta \, \partial_A \psi^\alpha \, \partial_B \overline{\varphi} \left(\sigma^{AB} \right)_\alpha^\beta.$$

Thus, the integrand in the last expression is a full derivative and, therefore, the full action including the Wess-Zumino term is invariant in this approximation.

The careful analysis of the terms with higher order in the fermions shows that they also cancel out.

To conclude, let us write the full component action of supersymmetric 3-brane in $D = 6$

$$S = 2 \int d^4x - \int d^4x \det(\mathcal{E}) \left(1 + \sqrt{(1 - 2\,(\mathcal{D}\varphi\mathcal{D}\bar{\varphi}))^2 - 4\,(\mathcal{D}\varphi\mathcal{D}\varphi)\,(\mathcal{D}\bar{\varphi}\mathcal{D}\bar{\varphi})} \right)$$

$$+ 2 \int d^4x \det(\mathcal{E})\, \epsilon^{ABCD} \mathcal{D}_A\varphi \mathcal{D}_B\bar{\varphi} \left(\psi^\alpha \mathcal{D}_C \bar{\psi}^{\dot\alpha} + \bar{\psi}^{\dot\alpha} \mathcal{D}_C \psi^\alpha \right) \left(\mathcal{E}^{-1} \right)_D{}^E \, (\sigma_E)_{\alpha\dot\alpha} \, ,$$

This action is invariant with respect to both broken and unbroken supersymmetries and, therefore, it is the supersymmetric 3-brane action in $D = 6$.

Conclusion

In this talk we have demonstrated how to construct the on-shell component action for $N = 1, D = 6$ supersymmetric 3-brane within the nonlinear realization approach. We treated the worldvolume action of this 3-brane as the action of four-dimensional field theory which realized the partial breaking of $N = 2, d = 4$ global supersymmetry down to the $N = 1, d = 4$ one. The $N = 2, d = 4$ Poincaré superalgebra we considered, contains two central charges which are just two translation generators from the $D = 6$ point of view. These two translations, as well as half of the $N = 2, d = 4$ supersymmetry, are supposed to be spontaneously broken. As the result of this spontaneous breakdown we have one complex bosonic and one fermionic Goldstone superfields in the theory. The first components of these superfields constitute the physical fields of the corresponding 3-brane.

The main result presented in the talk, is the construction of the component action of the 3-brane. By shifting attention to the broken supersymmetry and considering the reduced coset, we introduced the covariant derivatives and the fermionic vielbeins (covariant with respect to broken supersymmetry) which are the building blocks of the action.

Moreover, the main part of the component action just mimics its bosonic cousin in which the ordinary space-time derivatives and the bosonic world volume are replaced by their covariant supersymmetric analogs.

It is funny, that the Wess-Zumino term in the action, which does not exist in the bosonic case, can be also easily constructed in terms of reduced Cartan forms.

Keeping the broken supersymmetry almost explicit, one may write the Ansatz for the component action fully defined up to two constant parameters.

The role of the unbroken supersymmetry is to fix these constants.

Of course, the component action we have explicitly constructed, can be in principle obtained from the superfield action of the papers (*Bagger, Galperin; Rocek, Tseytlin*).

Nevertheless, using the introduced covariant derivatives and fermionic vielbeins makes the action quite simple.