Course 345: INTRODUCTION TO SOLITONS

Problem Set 2

Date Issued: February 18, 2008
Date due: February 25, 2008

1. (10 points) It has been shown (cf. Set 1, Problem 2) that the transformation of the KdV equation $u_t - 6uu_x + u_{xxx} = 0$ by substituting $u(x,t) = -(3t)^{-2/3} f(\eta)$; $\eta = x/(3t)^{1/3}$ yields
$$f''' + (6f - \eta)f' - 2f = 0$$
Verify that the following transformation of the Miura's type $f = v' - v^2$, $v = v(\eta)$ yields the Painleve equation of the second kind for v:
$$v'' - \eta v - 2v^3 = 0$$

2. (5 points) Show that $xu + 3tu^2$ is a conserved density for the KdV equation $u_t - 6uu_x + x_{xxx} = 0$.

3. (5 points) Find first three conservation laws for the modified KdV equation $u_t - 6u^2u_x + x_{xxx} = 0$.

4. (5 points) Use the Bäcklund transformation of the Bürgers equation $u_t + uu_x = \alpha u_{xx}$ where α is a positive constant,
$$v_x = -\frac{uv}{2\alpha}; \qquad v_t = \frac{v}{4\alpha}\left(u^2 - 2\alpha u_x\right)$$
to show that
$$u_t + uu_x = \alpha u_{xx} \qquad \text{and} \qquad v_t = \alpha v_{xx}$$

5. (5 points) Hirota's method. Using parametrization $u(x,t) = -2\frac{\partial}{\partial x}\left(\frac{f_x}{f}\right)$ transform the Boussinesq equation
$$u_{tt} - u_{xx} + 3(u^2)_{xx} - u_{xxxx} = 0$$
into the bilinear form.