AdS Holography and dyonic black holes

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The AdS/CFT correspondence: a brief summary

- **Originally, the AdS/CFT correspondence provides a relation between string theory and/or (super)gravity in asymptotically anti-de Sitter (AdS) space-time *(the "bulk")* and Conformal Field Theory *(CFT)* on the boundary. (Example: Type IIB string theory on $AdS_5 \times S^5 \rightarrow$ N=4 strongly coupled SUSY Yang-Mills gauge theory on AdS_5 boundary - *Maldacena, Witten...*)**

- **Such correspondence allows us to compute Quantum Field Theory observables on the boundary by expanding around classical solutions of the gravity (+ other fields) theory in the bulk (large N expansion using saddle point approximation).**

- **Correlation functions of the QFT on the boundary in the strong coupling regime can be computed from classical gravity solutions in the bulk: the bulk quantum corrections are 1/N quantum corrections.**

- **The boundary/bulk connection provides an explicit realization of the so called *holographic principle*.**

AdS/CFT correspondence: another view

- The dual gravitational system has at least one extra dimension **z**; the field theory properties can be extracted by working on the boundary.

- The extra dimension **z** should be interpreted as an energy scale.

- It represents the renormalisation group flow of the quantum field theory defined on the boundary.

- The AdS/CFT correspondence "geometrises" the field theory energy scale.

- Geometrisation: in the dual bulk gravitational description the energy scale is treated geometrically on an equal footing to the spatial directions of the boundary field theory

Large N gauge theory Strong coupled field theory at finite timperature d-dim space-time	Classical Einstein gravity d+1 - dim space-time

Einstein-Hilbert action in AdS space

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}(R-2\Lambda); \qquad \Lambda = -\frac{d(d-1)}{2L^2} < 0$$

Equations of motion and highly symmetrical vacuum solution:

$$R_{\mu\nu} = -\frac{d}{L^2}g_{\mu\nu} \Rightarrow ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \left(1+\frac{r^2}{L^2}\right)^{-1}dr^2 + r^2 d\Omega_{d-1}^2$$

- The limit ***r << L*** reduces AdS to Minkowski space.
- In the opposite limit ***L << r*** the total metric becomes the **Poincaré metric**

$$ds^2 = \frac{r^2}{L^2}\left(-dt^2 + dx_1^2 + dx_2^2 + \cdots + dx_{d-1}^2\right) + \frac{L^2}{r^2}dr^2 \qquad \boxed{\sum_{i=1}^{d} x_i^2 = L^2}$$

Change of variables:
(Poincaré coordinates)

$$z = \frac{L^2}{r} \Rightarrow \boxed{ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx_i dx_i + dz^2\right)}$$

- Coordinates on the boundary: (t, x_i)
- Extra radial coordinate z running from z=0 (the boundary) to z = ∞ (the origin)

- Each radial slice is a d-dim ***Minkowski space***

Conformal symmetry and AdS isometries

$$x \to x'; \qquad g'_{\mu\nu}(x') = M(x) g_{\mu\nu}(x)$$

- **Generators:**

$$\begin{cases} M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu) \\ P_\mu = -i\partial_\mu \\ D = -ix_\mu \partial^\mu \\ K_\mu = i(x^2 \partial_\mu - 2x_\mu x_\nu \partial^\nu) \end{cases}$$

Rotations

Translations

Dilatations

Special conformal transformations

$$\boxed{ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx_i dx_i + dz^2\right)}$$

The symmetry group of the AdS$_{d+1}$ metric is the conformal group in *d*-dim Minkowski spacetime: SO(2,d).
A subgroup is a scaling x Poincaré transformations

The symmetries of the bulk action act on the boundary quantum field theory as conformal transformations → **QFT on the AdS boundary is CFT**

Deformations of the geometry away from AdS can be thought of as deformations of the field theory away from conformal fixed point

Planar AdS_{2+1}: Renormalisation group flow

$$ds^2 = \frac{r^2}{L^2}\left(-dt^2 + dx^2 + dy^2\right) + \frac{L^2}{r^2}dr^2$$
$$= \frac{L^2}{z^2}\left(dz^2 - dt^2 + dx^2 + dy^2\right)$$

$$\boxed{z = \frac{L^2}{r}}$$

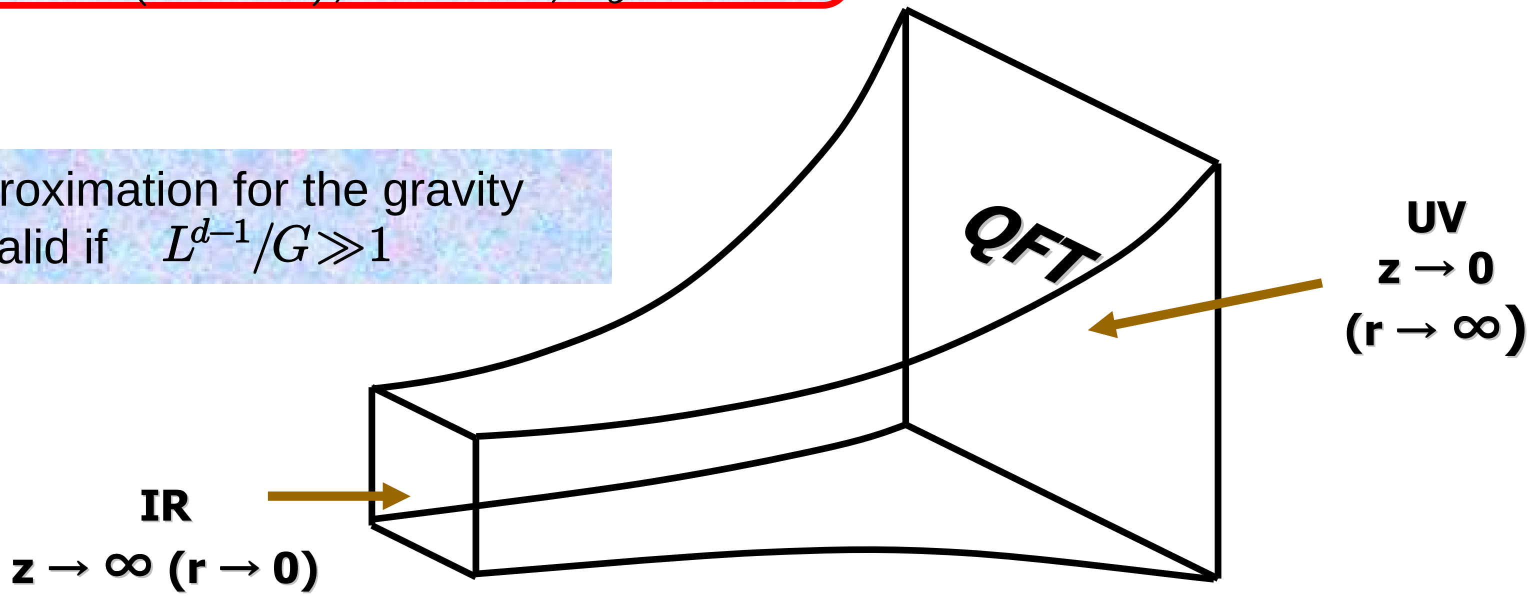
- **Scale invariance:**

$$\{z, t, x, y\} \to \{\lambda z, \lambda t, \lambda x, \lambda y\}$$

IR: scale $z \to \infty \quad (r \to 0), \quad x \to \infty, \quad y \to \infty$

UV: scale $z \to 0 \quad (r \to \infty), \quad x \to 0, \quad y \to 0$

A classical approximation for the gravity theory will be valid if $L^{d-1}/G \gg 1$



Holographic correspondence

- **QFT generating functional:**

$$Z_{QFT}[\Phi_0] = \int \mathcal{D}A e^{iS_{QFT} + \int \Phi_0 \mathcal{Q}[A]}$$

$$\langle \mathcal{O}(x) \rangle = \frac{1}{Z}\frac{\delta Z}{\delta \Phi_0}$$

Fundamental fields of QFT

A source (non-dynamical)

Gauge invariant operator of the QFT

The idea of holography: to promote the source Φ_0 to a dynamical field in the bulk

(Gubser, Klebanov, Polyakov and Witten, 1998)

The QFT operators defined on the z=0 boundary are given by the theory in the bulk

The boundary value of the bulk field $\Phi \to \Phi_0$ gives a backround source for the field theory operator

Holographic correspondence:

$$Z_{QFT}[\Phi_0] \approx e^{iS_{bulk}}\Big|_{\Phi \to \Phi_0}$$

AdS bulk

dynamical field Φ

⟷

QFT on the boundary

Operator $\mathcal{O}_{field}$

Holographic correspondence

The catch: suppose we have S_{QFT}- which theory of gravity in the bulk it corresponds to?

- **Approach:** **Start from the bulk gravity and identify the boundary theory**

$$\boxed{Z_{bulk}\,\big|_{\Phi(z\sim 0,x)} = \langle \exp\int_{boundary} d^3x\sqrt{-g_0}\,\Phi_0(x)\mathcal{O}(x)\rangle} \quad \Longleftrightarrow \quad \boxed{Z_{QFT}[\Phi_0] \approx e^{iS_{bulk}}\big|_{\Phi\to\Phi_0}}$$

$$\langle\mathcal{O}(x)\rangle = \frac{1}{Z_{QFT}}\frac{\delta Z_{QFT}}{\delta\Phi_0} = \frac{\delta \ln Z_{QFT}}{\delta\Phi_0} = \frac{\delta S_{bulk}}{\delta\Phi_0}$$

Simple analogy: mechanical action as a function of boundary conditions (on-shell action)

$$S = \int_0^T L(\dot{x}, x, t)\,dt \;\Rightarrow\; \delta S = \int_0^T dt\left[\cancel{\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)}\right]\delta x + \left[\frac{\partial L}{\partial \dot{x}}\delta x\right]_0^T \;\Rightarrow\; \frac{\partial S}{\partial x} = \frac{\partial L}{\partial \dot{x}}\bigg|_T = p(T)$$

How it works: massive scalar field in AdS

$$S[\phi] = \frac{1}{2}\int d^{d+1}x\sqrt{|g|}\,\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right]$$

$$\Rightarrow \quad \partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi) = m^2\sqrt{|g|}\phi$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{ij}dx^i dx^i + dz^2\right)$$

Ansatz: $\phi = \phi(z)e^{ik\cdot x}$

(Translation invariance parallel to the boundary)

Equation of motion:

$$\left(g^{ij}k_i k_j - \frac{1}{\sqrt{g}}\partial_z(\sqrt{g}g^{zz}\partial_z) + m^2\right)\phi(z) = 0$$

$$\sqrt{g} \equiv \left(\frac{L}{z}\right)^{d+1}$$

$$\Downarrow$$

$$-z^{d+1}\partial_z(z^{-d+1}\partial_z)\phi(z) + (k^2z^2 + m^2L^2)\phi(z) = 0$$

$$\nu = \sqrt{d^2 + 4m^2L^2}$$

$$\boxed{\phi(z) = c_1 z^{\frac{d}{2}} J_{\frac{\nu}{2}}(kz) + c_2 z^{\frac{d}{2}} Y_{\frac{\nu}{2}}(kz)}$$

IR Asymptotic at $z \to \infty$: $J_{\frac{\nu}{2}}(kz) \sim e^{-kz}$

Bessel function of the first kind $J_n(z)$

Bessel function of the second kind $Y_n(z)$

How it works: massive scalar field in AdS

UV Asymptotic at $z \to 0$: $\phi(z) \to z^{\Delta}$

$$-z^{d+1}\partial_z(z^{-d+1}\partial_z)\phi(z) + (k^2z^2 + m^2L^2)\phi(z) = 0$$

$$(k^2z^2 - \Delta(\Delta - d) + m^2L^2)z^{\Delta} = 0; \Rightarrow \text{as} \quad z \to 0$$

$$\Delta(\Delta - d) = m^2L^2$$

The roots $\Delta_{\pm} = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2L^2}$

Breitenlohner-Freedman bound:

$$m^2_{BF} = -\left(\frac{d}{2}\right)^2 > m^2L^2$$

- **The solution near the boundary:**

$$\phi(z,x) \approx \left(\frac{z}{L}\right)^{\Delta_-}\phi_0(x) + \left(\frac{z}{L}\right)^{\Delta_+}\phi_1(x) + \ldots$$

The boundary condition:

$$\phi \to \left(\frac{z}{L}\right)^{\Delta_-}\phi_0(x)$$

$$\frac{\delta S}{\delta x} = \frac{\partial L}{\partial \dot{x}}\Big|_0 = -p(0) \Rightarrow \frac{\partial L}{\partial(\partial_z\phi)}\Big|_{z=0} = \Pi(0,x) \Rightarrow \frac{\delta S}{\delta\phi(0,x)} = -\Pi(0,x)$$

Canonical momentum

$$\frac{\delta S}{\delta\phi_0} = \frac{\delta S}{\delta\phi}\frac{\delta\phi}{\delta\phi_0} = \left(\frac{z}{L}\right)^{\Delta_-}\frac{\delta S}{\delta\phi} = -\left(\frac{z}{L}\right)^{\Delta_-}\Pi(0,x)$$

$$\Pi(z,x) = \sqrt{|g|}g^{zz}\partial_z\phi \approx \left(\frac{L}{z}\right)^{d+1}\frac{z^2}{L^2}\left[\left(\frac{z}{L}\right)^{\Delta_-}\phi_0 + \left(\frac{z}{L}\right)^{\Delta_+}\phi_1\right]$$

$$\boxed{\langle\mathcal{Q}\rangle = \frac{\delta S}{\delta\phi_0} \approx -\frac{\Delta_+}{L}\phi_1}$$

How it works: a beautiful/perverse way to derive Ohm's law

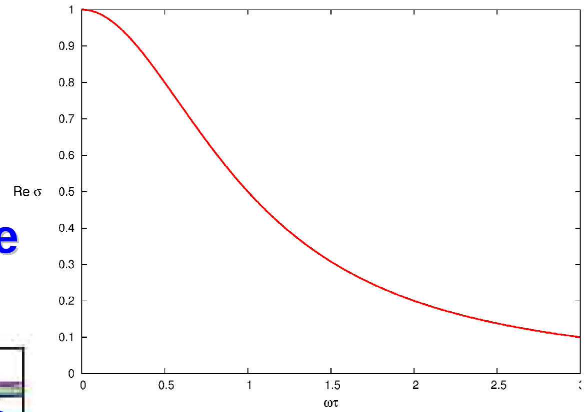
Ohm's law: relation between the induced current density $\vec{j}(t)$ and external electric field $\vec{E}(t)$

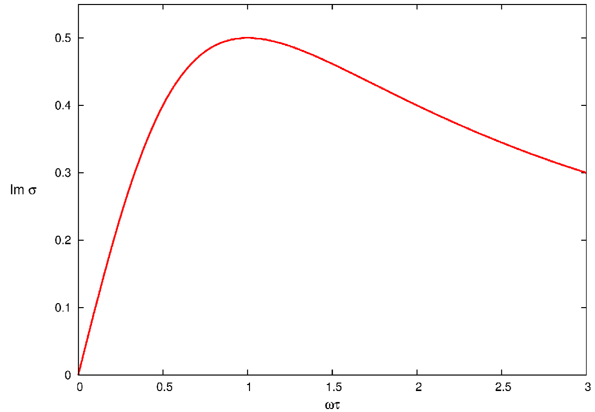
Fourier transform: $\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$

Drude model: $\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$

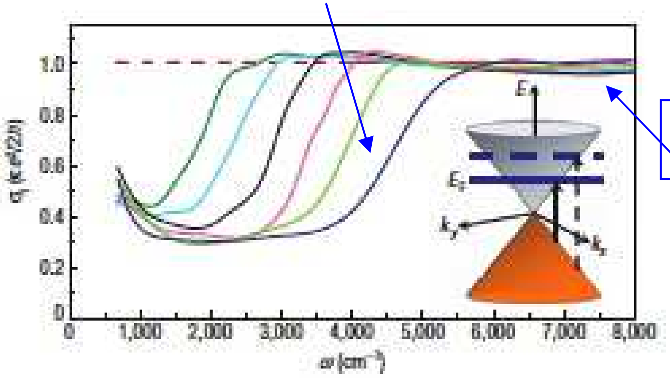
High frequencies:

- **Drude model: $\sigma \to -1/i\omega\tau$**





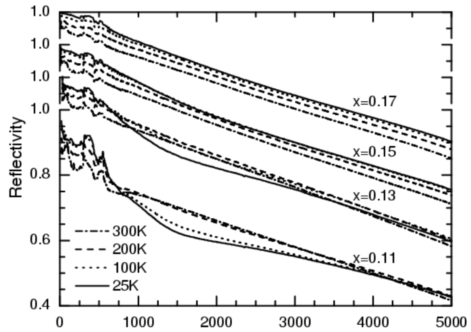
- **Field theory: particle-hole production (graphene)**



(Z. Q. Li, et al, Nature Physics 4, 532 (2008))

- **in d=2+1 σ is dimensionless, the model manifests its scale invariance**

- **Problems with understanding of strongly interacting systems e.g, unconventional superconductors (cuprates)**



$\boxed{?}$

$$|\sigma| = \frac{C}{\omega^{2/3}}$$

Holographic conductivity

Bulk: 3+1 dim
Einstein-Maxwell model

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}(R-2\Lambda) - F_{\mu\nu}F^{\mu\nu}\right\}$$

Probe limit (fixed AdS): $ds^2 = \frac{L^2}{z^2}\left(\eta_{ij}dx^i dx^i + dz^2\right)\,; \quad \Lambda = -3$

Holographic correspondence:

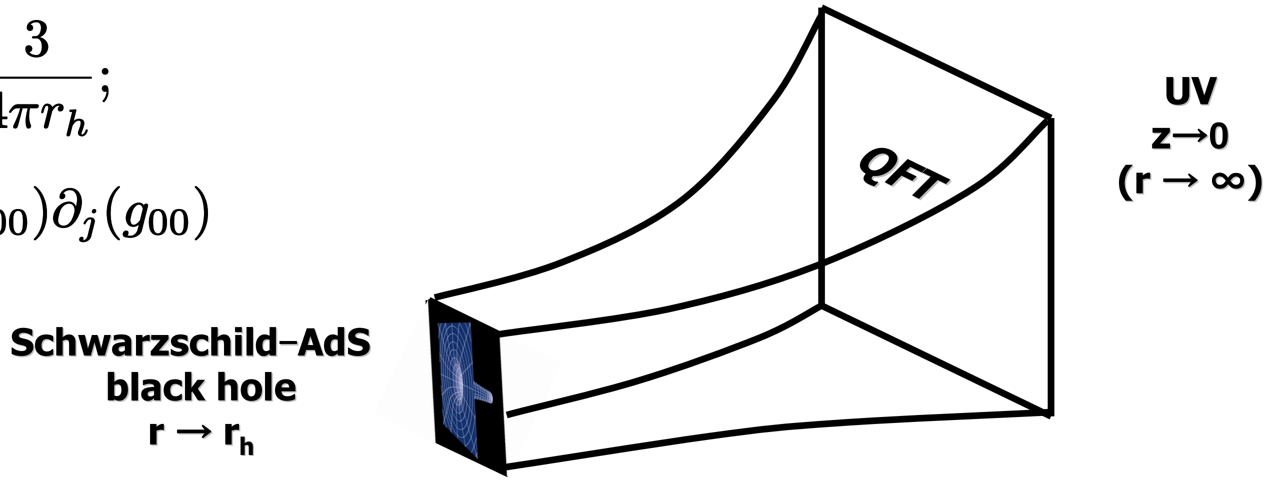
- Gauge field A_μ $\Leftrightarrow$ U(1) current J_μ
- Graviton $g_{\mu\nu}$ $\Leftrightarrow$ energy-momentum tensor $T_{\mu\nu}$

Challenging question: how to study the boundary operators at finite temperature?

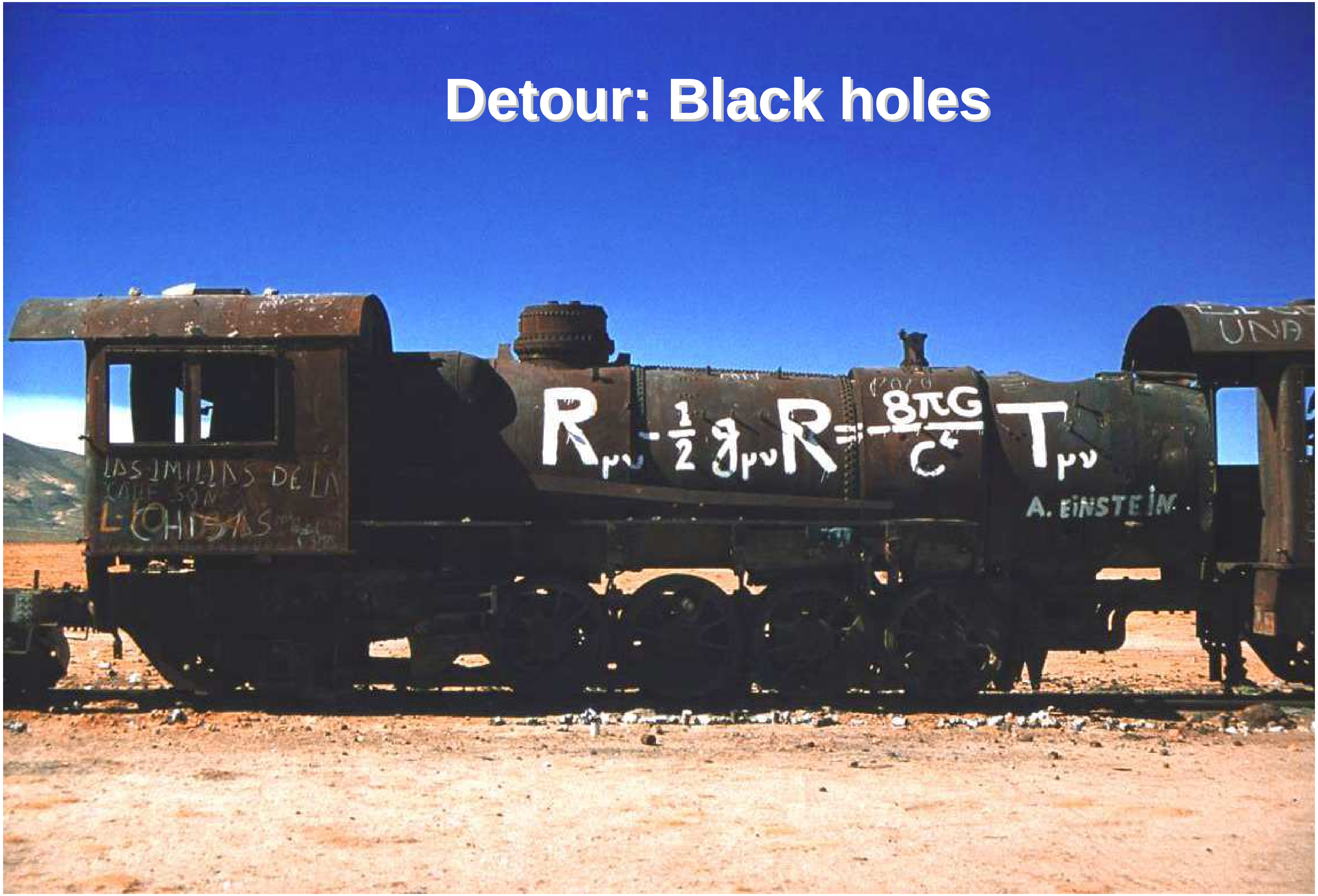
<u>The idea: consider AdS black holes instead of the regular AdS gravity</u>

$$T = \frac{\hbar c^3}{8\pi M G k_B} = \frac{3}{4\pi r_h};$$

$$\kappa^2 = -\frac{1}{4}g^{00}g^{ij}\partial_i(g_{00})\partial_j(g_{00})$$



Detour: Black holes



Black holes

Schwarzschild, 1916

Static spherically symmetric metric, solution of the Einstein equations with $T_{\mu\nu} = 0$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

where $$f(r) = 1 - \frac{2MG}{r} = 1 - \frac{r_H}{r}$$

Karl Schwarzschild
1873 - 1916

- Coordinate singularity at the event horizon $r = r_H$
- True singularity at $r = 0$
- Scwarzshild black holes are non-rotating and uncharged

Rotating black holes

Kerr, 1963

Stationary metric, a solution of the Einstein equations with non-zero angular momentum (rotating black hole)

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a \ \sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}(a \ dt - \rho_0^2 d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

where $\rho^2 = r^2 + a^2 \ \cos^2\theta,$

$$\rho_0^2 = r^2 + a^2, \quad \Delta = r^2 - 2Mr + a^2$$

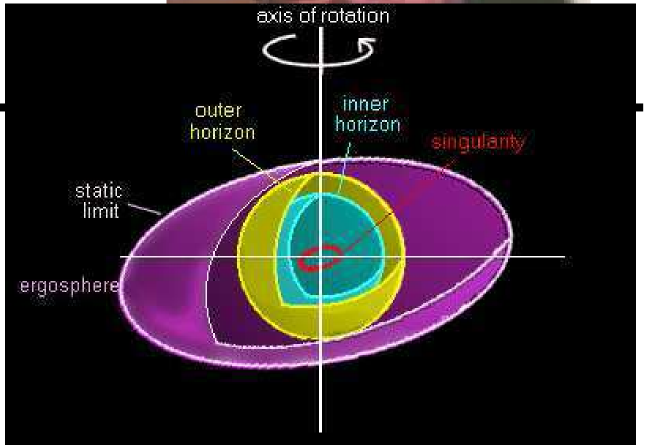
and $a = J/M$

Roy Patrik Kerr

Newman, 1965

Electrically charged generalization of the Kerr solution

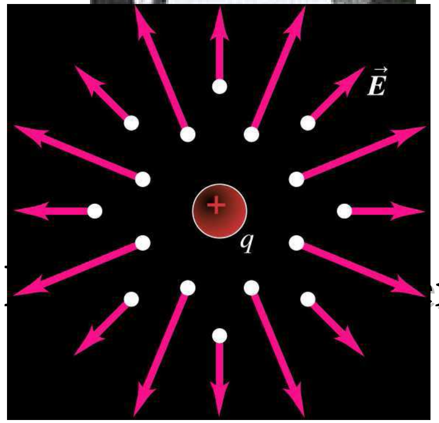
$$\Delta = r^2 - 2Mr + a^2 + Q^2$$



Reissner–Nordström Black Holes

H. Reissner, 1916; G.Nordström, 1918

Charged static spherically symmetric black holes, the Schwarzschild metric with



$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2 + g^2}{r^2}$$

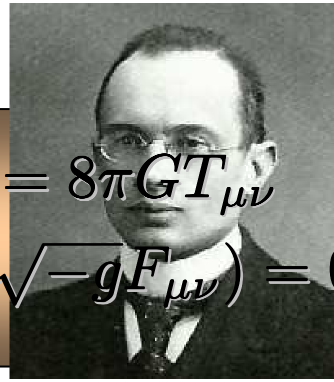
- Electrically (***Q***) and magnetically (***g***) charged black holes;
- The event horizons are located at

$$r_\pm = M \pm \sqrt{M^2 - Q^2 - g^2}$$

- $T_{\mu\nu} \neq 0$. Energy density outside the horizon is due to the Coulomb fields of the charges ***Q*** and ***g***

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} F_{\mu\nu} \right) = 0$$



Gunnar Nordström
1881 - 1923

Einstein equations are solved together with Maxwell field equations

Introducing temperature: AdS black holes

- *Hawking temperature is dual to the temperature of the system on the boundary in d=3*
- *Temperature of the black hole is proportional to the surface gravity, T=κ/2π*
- *Entropy of a black hole is proportional to surface area of event horizon*
- *Dynamics in the bulk yields the boundary thermal field theory including non-equilibrium processes (dissipation)*

Planar AdS Schwarzschield:

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

$$f(z) = 1 - \left(\frac{z}{r_h}\right)^3; \quad \Lambda = -3$$

Planar AdS Reissner-Nordström

$$f(z) = 1 - \left(1 + \frac{r_h^2\mu^2}{\gamma^2}\right)\left(\frac{z}{r_h}\right)^3 + \frac{r_h^2\mu^2}{\gamma^2}\left(\frac{z}{r_h}\right)^4$$

Chemical potential μ = Q

$$\boxed{\gamma = \frac{e^2L^2}{8\pi G}}$$

Classical solution

$$\boxed{A_0 = Q\left(1 - \frac{z}{r_h}\right)} \qquad T = \frac{1}{4\pi r_h}\left(3 - \frac{r_h^2 Q^2}{\gamma^2}\right)$$

On the boundary: A_0 is a source for J_0 (charge density)

$$\boxed{\langle J_0\rangle \sim \mu/r_h}$$

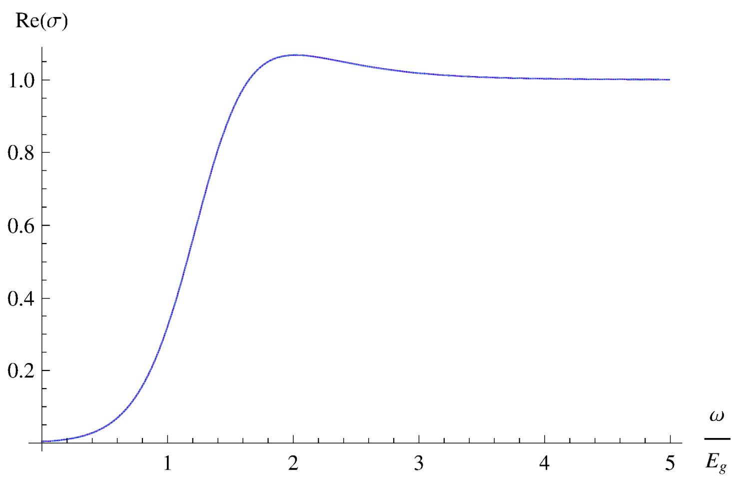
Holographic conductivity

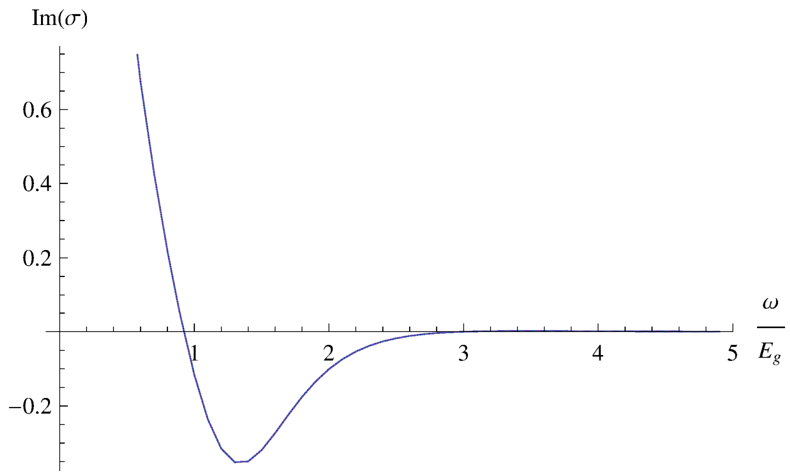
An electric field on the boundary: $A_x = (E/i\omega)e^{i\omega t}; \quad \frac{\partial A_x}{\partial t} = Ee^{i\omega t}$

In the bulk: $A_x = (E/i\omega)e^{i\omega t} + \langle J_x \rangle z + \dots$

Field equation in the fixed AdS space: $$\boxed{(fA_x')' + \frac{\omega^2}{f}A_x = \frac{4\mu^2}{\gamma^2 r_h^2}z^2 A_x}$$

Conductivity: $\sigma(\omega) = \frac{1}{e^2}\frac{A_x'}{i\omega A_x}\bigg|_{z=0}$





Holographic superconductivity

Bulk: 3+1 dim
Abeian Higgs model

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |\nabla_\mu - iA_\mu\phi|^2 + m^2|\phi|^2\right\}$$

Probe limit (fixed planar AdS_{3+1} Schwarzschield):

$$ds^2 = -f(z)dt^2 + \frac{dz^2}{f(z)} + z^2 dx_i dx^i$$

$$f(z) = z^2\left(1 - \frac{r_h^3}{z^3}\right); \quad L = 1$$

- **Ansatz:** $\phi = |\phi| = \phi(z); \quad A_\mu = \omega(z)\delta_{0\mu}$

- **Field equations** $\phi'' + \left(\frac{f'}{f} + \frac{2}{z}\right)\phi' + \left(\cancel{\frac{\omega^2}{f^2}} - \frac{m^2}{f}\right)\phi = 0$ $\qquad \omega'' + \frac{2}{z}\omega' - \frac{2\phi^2}{f}\omega = 0$

Low T *(small r_h)*
Effective mass is negative,
there is a long-range scalar field (the hair)

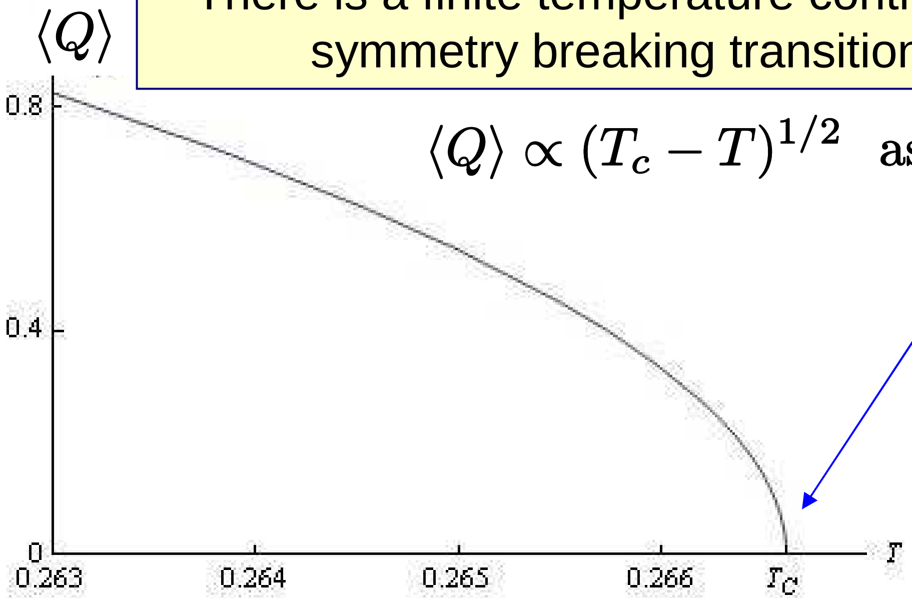
Hight T *(large r_h)*
Effective mass is positive,
Solution for scalar field is trivial (no hair)

- **Holographic correspondence:** $\phi \propto \langle Q\rangle \Rightarrow$

$$T < T_c: \quad \langle Q\rangle \neq 0;$$
$$T > T_c: \quad \langle Q\rangle = 0$$

Holographic phase transition

There is a finite temperature continuous symmetry breaking transition



$$\langle Q \rangle \propto (T_c - T)^{1/2} \quad \text{as } T \to T_c$$

Second order phase transition with critical exponent 1/2

Note: Semiclassical approximation to the gravity (weak curvature) means:

$$\frac{L}{G} \sim \frac{L}{l_{pl}} \gg 1$$

On the boundary it should match the large N limit:

$$\boxed{\frac{L}{G} \sim N \gg 1}$$

- As N decreases, the quantum gravitational effects become important;
- Holographic superconductors are described not in terms of electrons and Cooper pairs but in the framework of the order parameter of strongly coupled field theory defined on the boundary of the AdS space; this charged operator condenses below a critical temperature.
- In the AdS Abelian Higgs model the order parameter $\langle Q \rangle$ is a scalar (s-wave)

Holographic correspondence

Next step: extend the model by inclusion of the non-Abelian matter fields in AdS$_4$ space:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}(R-2\Lambda) - \frac{1}{e^2}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)(D^\mu\Phi) - V(\Phi)\right\}$$

In the AdS$_4$ EYMH model the correspondence is as follows:

- **Scalar field Φ $\Longleftrightarrow$ triplet of scalar operators $\mathcal{O}$**
- **Gauge field A$_\mu$ $\Longleftrightarrow$ global SU(2) current J$_\mu$**

  - gauge symmetry in the bulk ⇆ conservation of the current J$_\mu$
- **Graviton g$_{\mu\nu}$ $\Longleftrightarrow$ energy-momentum tensor T$_{\mu\nu}$**

  - classical scale invariance in the bulk ⇆ conservation of stress-energy tensor
- **Fermion field $\Longleftrightarrow$ fermion operator $\mathcal{O}_\psi$**

Asymptotic matters! $\Phi^a \to v^a + \frac{\langle\mathcal{O}^a\rangle}{r^3} + \ldots \quad \Longleftarrow \quad \Phi_0^a = v^a; \quad \langle\mathcal{O}(x)\rangle = \frac{\delta S}{\delta\Phi_0}$

A very interesting case: **SU(2) symmetry on the boundary is explicitly broken to Abelian group via asymptotic behavior of the Higgs field:**

$$\partial^\mu J_\mu^a = \varepsilon_{abc} v^b \mathcal{O}^c$$

U(1) symmetry remains unbroken

Detour: Einstein vs Yang-Mills

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichenrowitz: there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Pure Yang-Mills (attraction/repulsion)

$$L = \frac{1}{2}\mathrm{Tr}\ F_{\mu\nu}^2$$

Deser, Coleman: Classical Yang-Mills theory in 3+1 dim is scale invariant - there is no soliton solutions

Israel's theorem: Static Einstein-Maxwell black holes are spherically symmetric

'No-hair' theorem: Stationary black holes are completely characterized by their mass **M**, charge **Q** and angular momentum **J**

Einstein-Yang-Mills model

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{(R-2\Lambda) - \mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\}$$

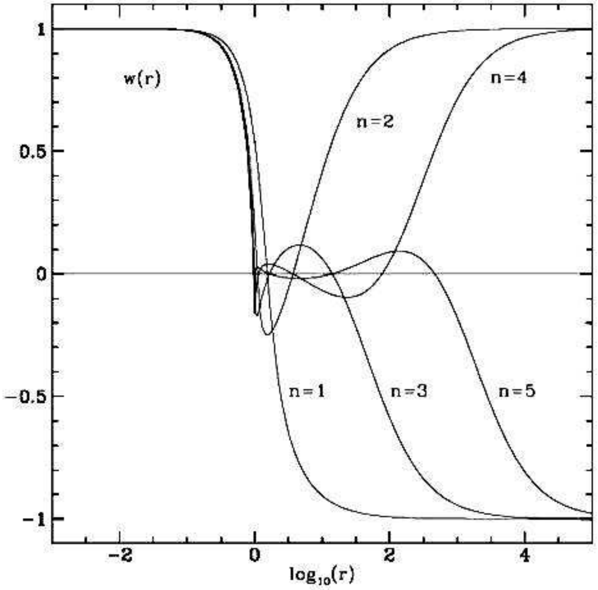
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}; \qquad D_\mu F^\mu_\nu = \nabla_\mu F^\mu_\nu + [A_\mu, F^\mu_\nu] = 0$$

Spherical symmetry:

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Static asymptotically flat solution

$$A^a_k = \varepsilon_{iak}\frac{x^k}{r^2}(w(r) - 1)$$



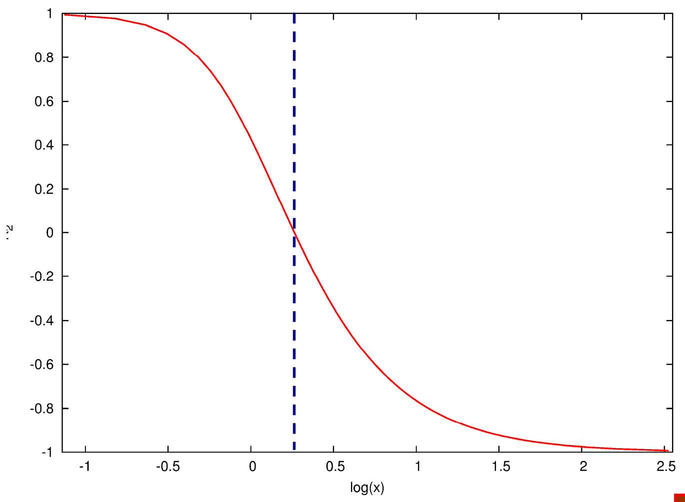
The Bartnik-McKinnon solitons

- Found numerically by the shooting method;
- The solution is globally regular;
- Analytic proof of existence of solutions of the differential equation;
- Gauge function $\boldsymbol{\omega}(\mathbf{r})$ has at least one zero, the solutions are characterized by the number of nodes of the $\boldsymbol{\omega}(\mathbf{r})$

Properties of the solutions

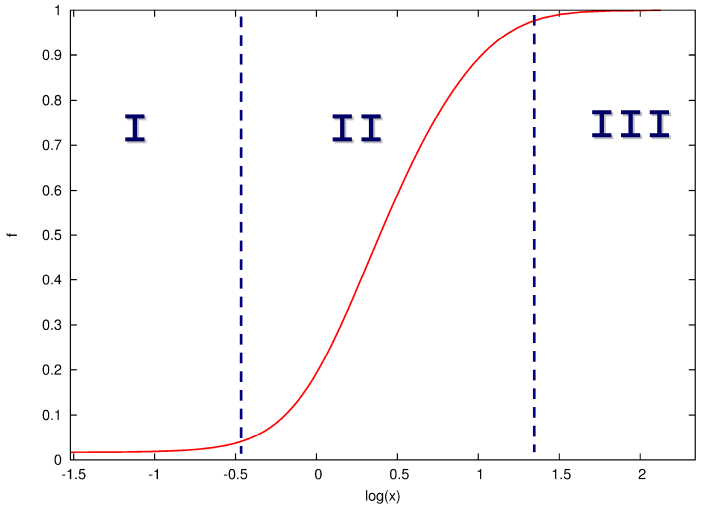
Dimensionless variables: $x = \frac{e}{\sqrt{4\pi G}} r \sim \frac{r}{l_{Pl}}; \quad \tilde{M} = eM\sqrt{\frac{G}{4\pi}} \sim \frac{M}{M_{Pl}}$

$$\boxed{M_{Pl} \sim 1/\sqrt{G}; \;\; l_{Pl} \sim \sqrt{G}}$$



- **Region I:** Yang-Mills field is almost trivial, the metric is close to Schwarzchild
- **Region II:** Yang-Mills field corresponds to monopole the metric is almost Reissner–Nordström
- **Region III:** Yang-Mills field is almost trivial, the metric is asymptotically Schwarzchild

All Bartnik-McKinnon configurations are sphalerons



Galtsov, Volkov: **There are EYM black hole solutions with long-range non-abelian fields (hairy black holes)**

BM solutions are static asymptotically flat gravitationally bound EYM sphaleron solutions; the exterior of the limiting solution approaches RN black hole

Generalised Bartnik-McKinnon solitons

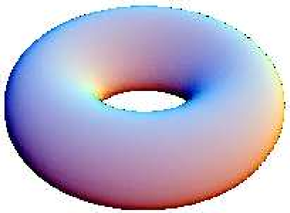
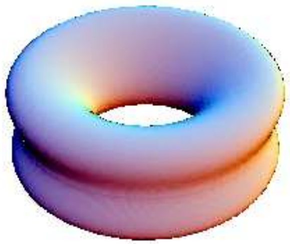
Axial symmetry:

$$ds^2 = -f\,dt + \frac{m}{f}\left(dr^2 + r^2 d\theta^2\right) + \frac{l}{f} r^2 \sin^2\theta\, d\varphi$$

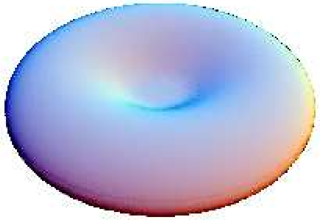
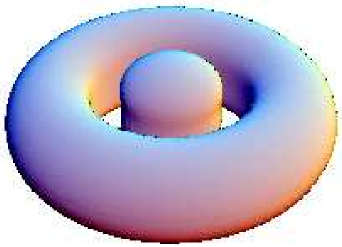
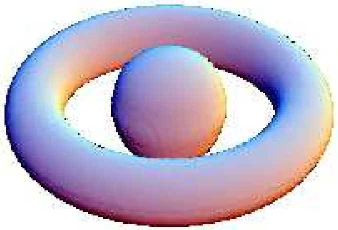
$$A_\mu dx^\mu = \left(\frac{K_1}{r}dr + (1-K_2)d\theta\right)\frac{\tau_\varphi^{(n)}}{2e} - n\sin\theta\left(K_3\frac{\tau_r^{(n,k)}}{2e} + (1-K_4)\frac{\tau_\theta^{(n,k)}}{2e}\right)d\varphi$$

Bartnik-McKinnon solutions are composite states

k=1 k=2 k=3

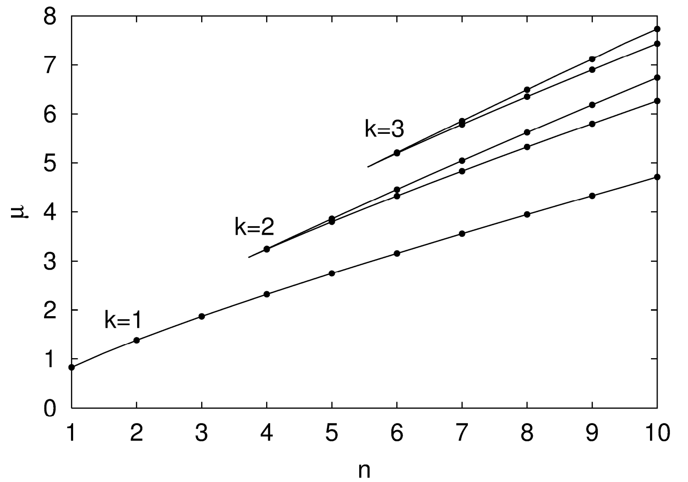




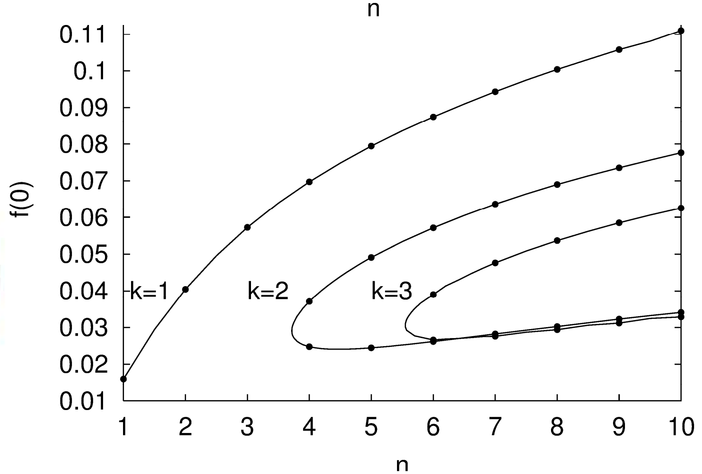
n=1 n=7 n=7

n=4

(Kleihaus,Kunz, Shnir et al)

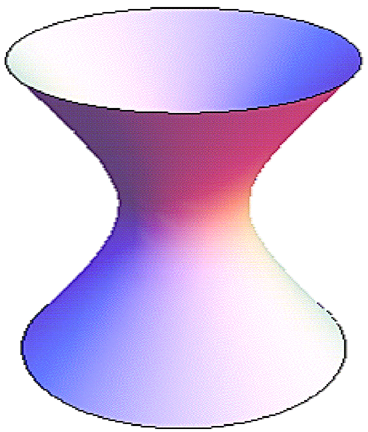




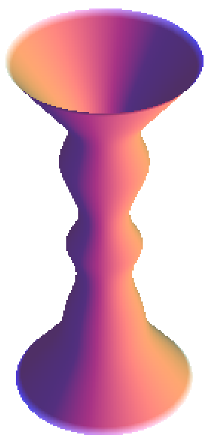
AdS Bartnik-McKinnon solitons

(Maison, Winstanley, Radu, Bjoraker, Hosotani et al)

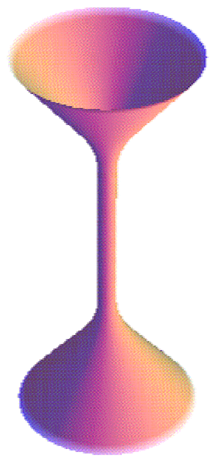
- Found numerically: There are continous families of solutions;
- Boundary conditions on the gauge function $\omega(\mathbf{r})$ can be relaxed
- Gauge function $\omega(\mathbf{r})$ may have no zero, the solutions possess a non-integer magnetic charge
- There are rotating and electrically charged BM solitons
- There are stable configurations, both colored black holes and self-gravitating lumps



Fixed AdS space



Asymptotically AdS space



Limiting AdS

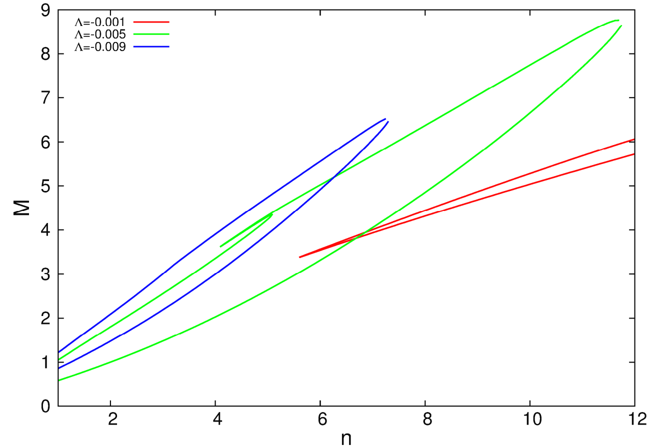
Bartnik-McKinnon solitons in asympotically AdS space

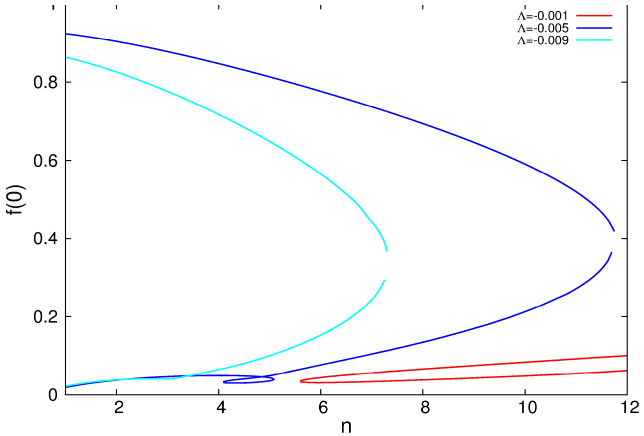
Axial symmetry in the bulk:

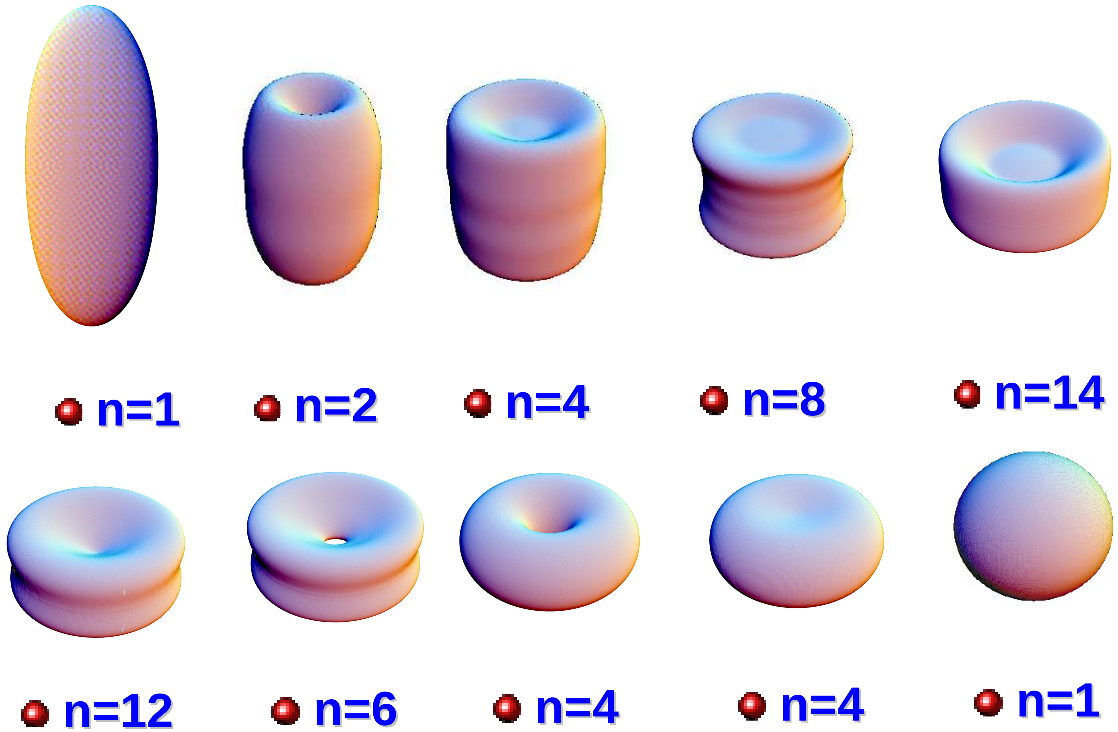
$$ds^2 = -f\left(1-\frac{\Lambda}{3}r^2\right)dt^2 + \frac{m}{f}\left(\frac{dr^2}{1-\frac{\Lambda}{3}r^2} + r^2 d\theta^2\right) + \frac{l}{f} r^2 \sin^2\theta d\varphi^2$$

Dimensionless variables: $\Lambda \to \dfrac{e^2}{4\pi G}\Lambda$

Two branches of the solutions

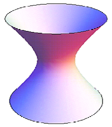




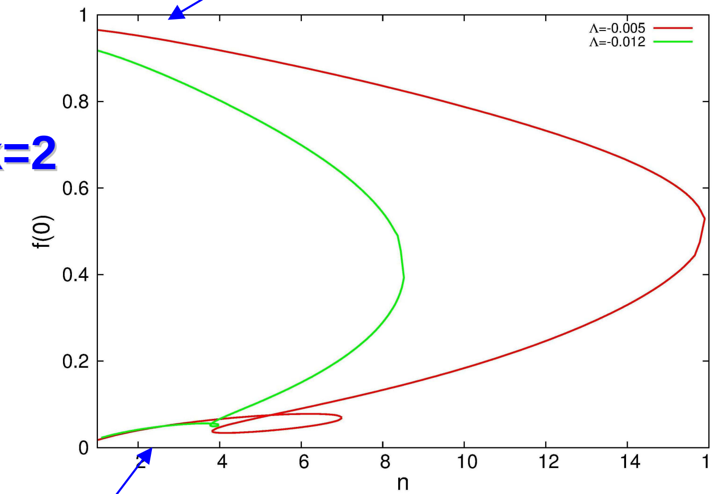


Bartnik-McKinnon solitons: composite structure

Fixed AdS



k=2



Throat + AdS

Note: Yang-Mills interaction can be both *repulsive* and *attractive*

- **Simplest k=1 spherically symmetric solution has 2 components on top of each other: MAP**

- **k=1 solution possesses a non-vanishing magnetic dipole moment**
- **The constitutents of the k=1 solution are aligned:** $\uparrow\uparrow$
- **k solution consists of k MAPs**

- **Lower branch: it has no flat space limit; gauge interaction is repulsive** $\Longrightarrow$ as $g\uparrow,\ G\uparrow$
- **Critical point: configuration decays into components**
- **Upper branch: the components are anti-aligned:** $\uparrow\downarrow$ **gauge interaction is attractive** $\Longrightarrow$ as $g\downarrow,\ G\uparrow$
- **Along the loop constituents are separated**
- **Limit of very strong gravity: the space is split into the internal region and the outer region**

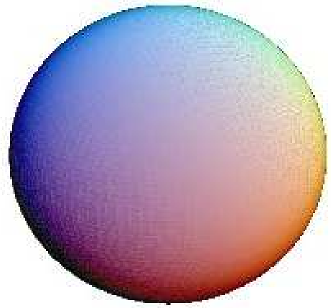
Yang-Mills-Higgs Theory

$$S=\frac{1}{2}\int d^4x\,\{\mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}+\mathrm{Tr}\ (\mathrm{D}_\mu\Phi)(\mathrm{D}^\mu\Phi)-\mathrm{V}(\Phi)\}$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+ie[A_\mu,A_\nu]$$

$$D_\mu\Phi=\partial_\mu\Phi+ie[A_\mu,\Phi]$$

$$V(\Phi)=\lambda\ (\Phi^2-\eta^2)^2$$



´t Hooft-Polyakov static spherically symmetric solution: monopole

$$\phi^a=\frac{r^a}{er^2}H(e\eta r)$$

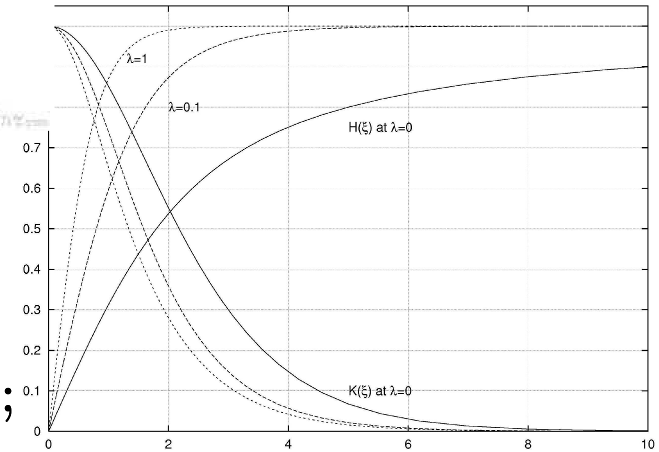
$$A_n^a=\varepsilon_{amn}\frac{r^m}{er^2}(1-K(e\eta r))$$

$$M=-\int d^3x\sqrt{-g}T_0^0;$$

$$g=\int d^3x\sqrt{-g}\ \mathrm{Tr}({}^*F^{0n}D_n\Phi);$$

$$Q=\int d^3x\sqrt{-g}\ \mathrm{Tr}(F^{0n}D_n\Phi);$$

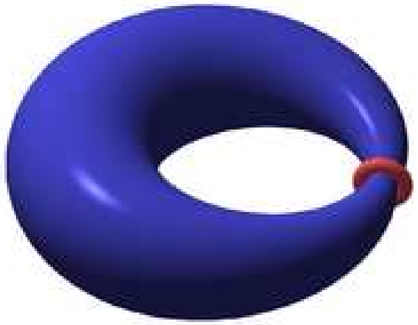
$$J=2\int d^3x\sqrt{-g}\ \mathrm{Tr}(F_{r\varphi}F^{r0}+F_{\theta\varphi}F^{\theta 0}+D_\varphi\Phi D^0\Phi);$$

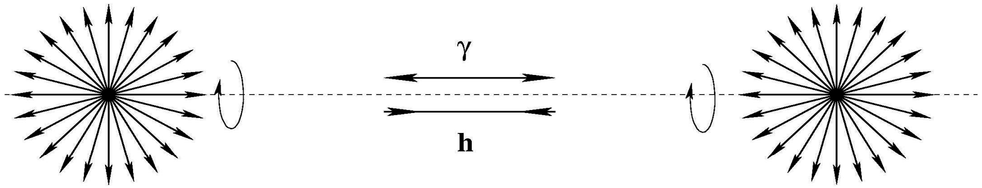


Monopole core: $R_c \sim m_V^{-1}$

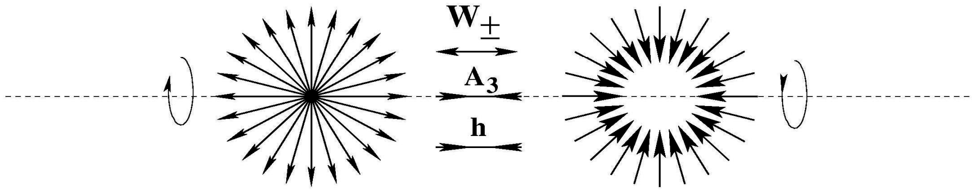
Non-BPS axially symmetric MAP

MA pair: magnetic dipole (Taubes, Nahm, Rüber, Kleihaus,Kunz & Shnir)





$$U = e^{im\theta\tau_\phi^{(n)}}$$

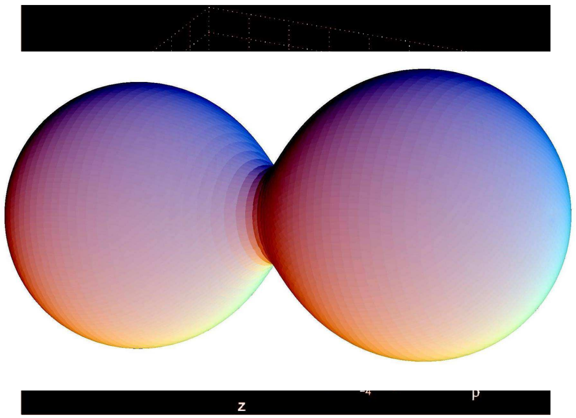


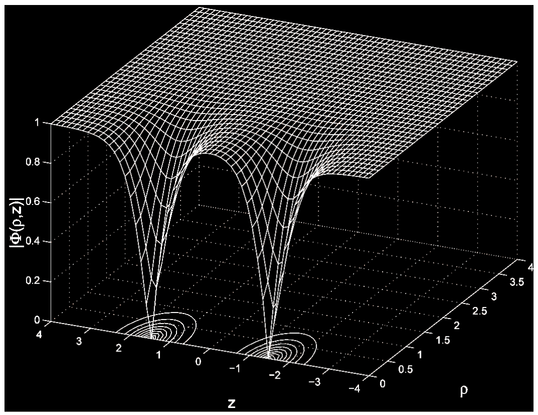
$$A_\mu dx^\mu = \left(\frac{K_1}{r}dr + (1-K_2)d\theta\right)\frac{\tau_\varphi^{(n)}}{2e} - n\sin\theta\left(K_3\frac{\tau_r^{(n,m)}}{2e} + (1-K_4)\frac{\tau_\theta^{(n,m)}}{2e}\right)d\varphi;$$

$$\Phi = \Phi^a\frac{\tau^a}{2} = a\left(H_1\frac{\tau_r^{(n,m)}}{2} + H_2\frac{\tau_\theta^{(n,m)}}{2}\right) .$$

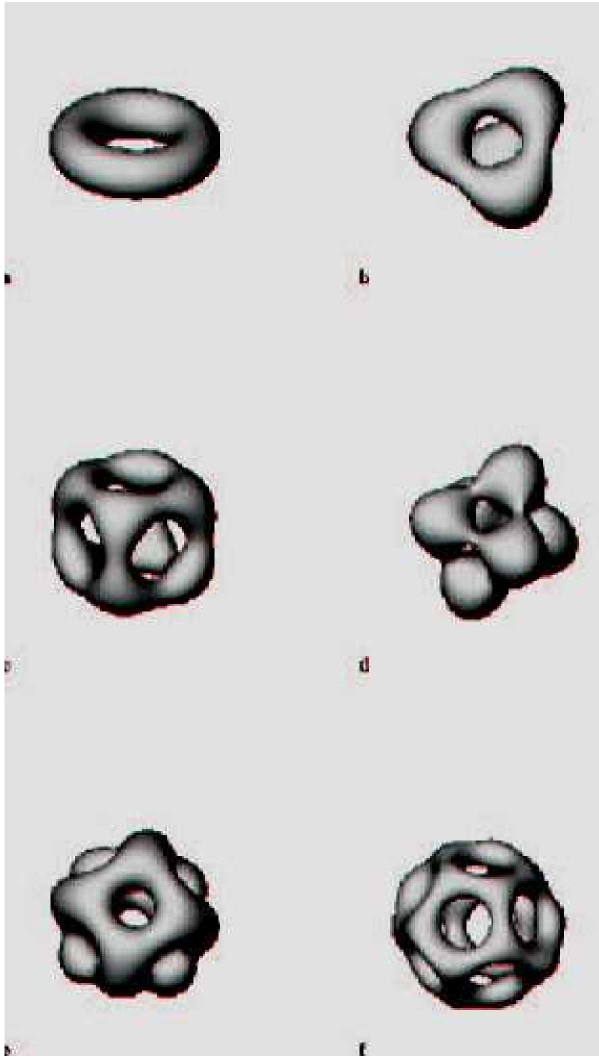
Magnetic charge:

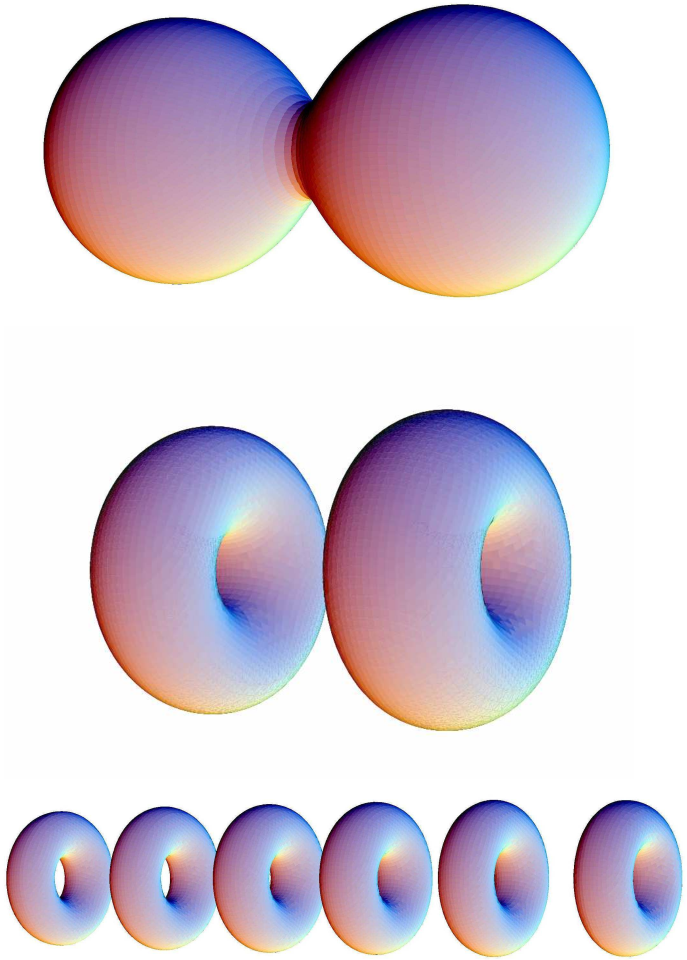
$$Q = \frac{1}{2\pi}\int_{S_2}(\Phi d\Phi \wedge d\Phi)$$
$$= \frac{1}{2}n\left[1-(-1)^m\right]$$





Multimonopoles, monopole chains and vortices





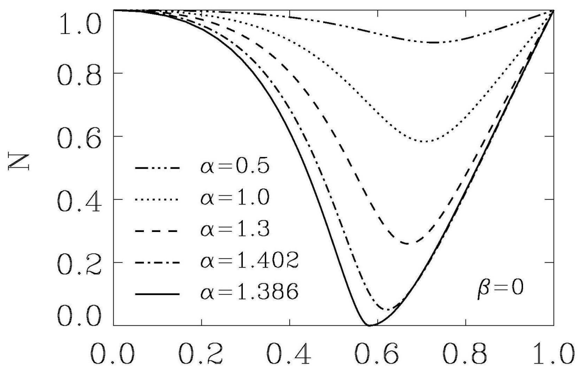
(P. Sutcliffe, N.Manton et al)

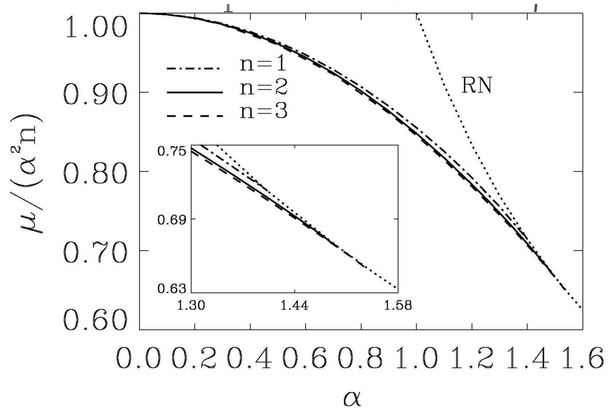
(Kleihaus,Kunz, Shnir et al)

Self-gravitating Dyons

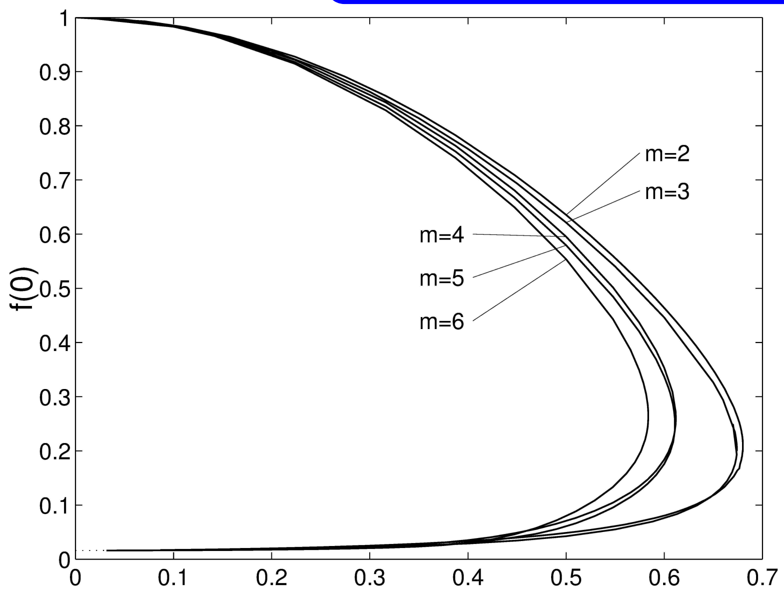
- **n=1, m=1** — Branch of gravitating solutions links the monopole to the RN black hole

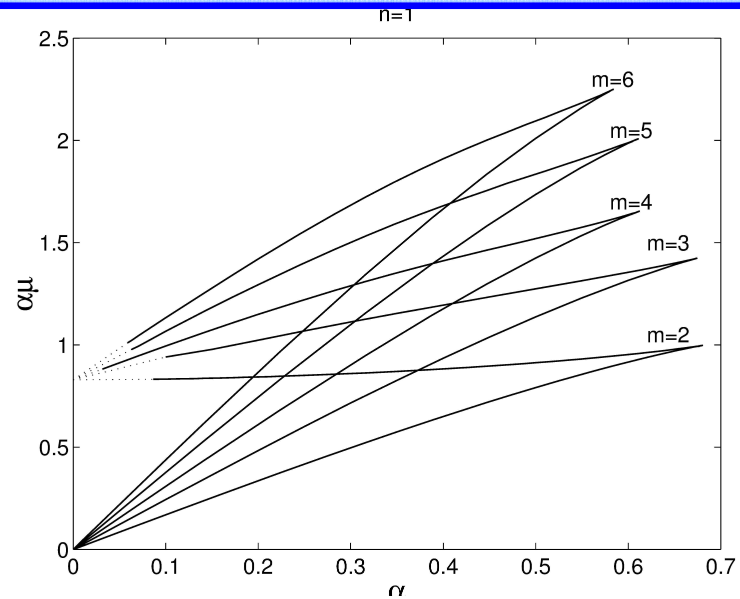
Dimensionless parameters of the model: $\alpha^2 = 4\pi^2 \, G\eta^2, \;\; \beta^2 = e^2/\eta$





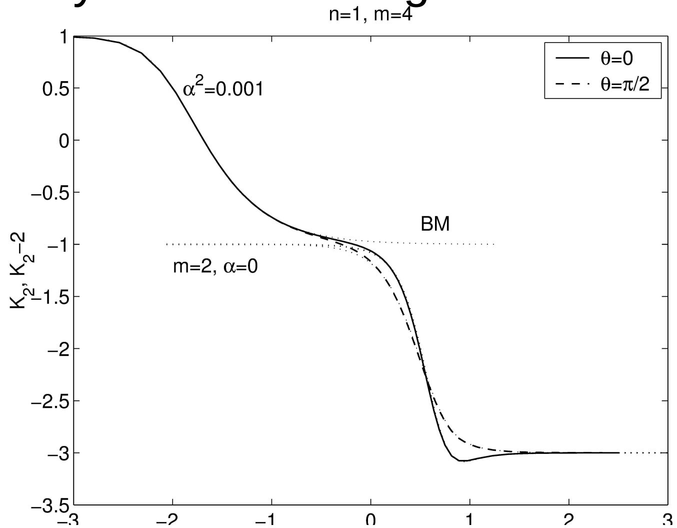
- **n=1, m=2,3..** — Branch of gravitating MA-chains is linked to the BM solutions

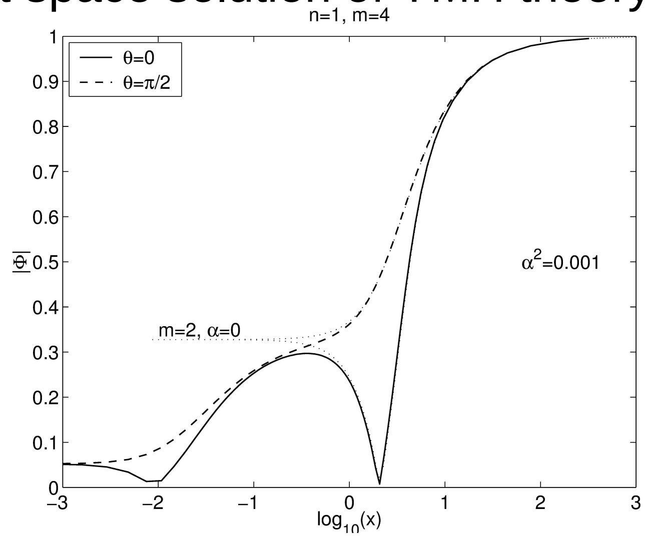


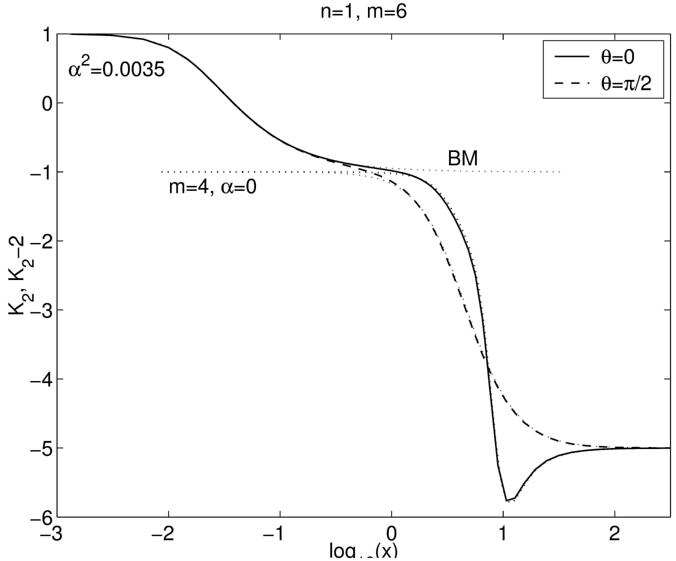


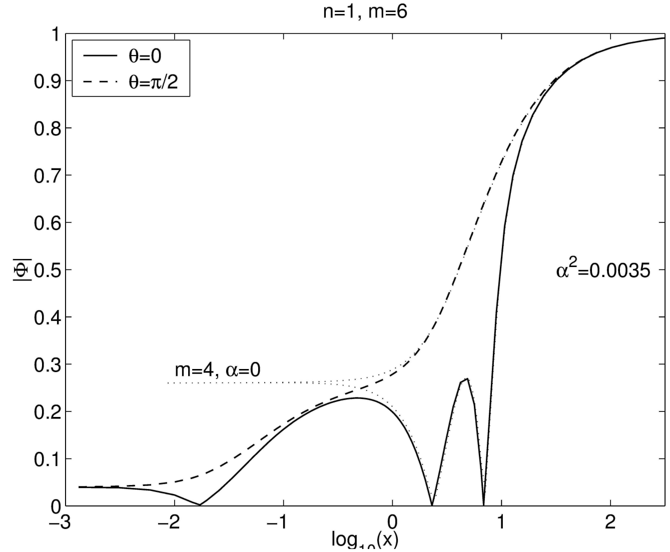
From gravitating Dyons to Bartnik-MacKinnon solutions

As gravity increases, the second branch of graviting axially symmetric **n**-MA chain evolve toward composite system of a Bartnik-McKinnon solution of EYM theory in the inner region and an outer **n − 2** flat space solution of YMH theory.



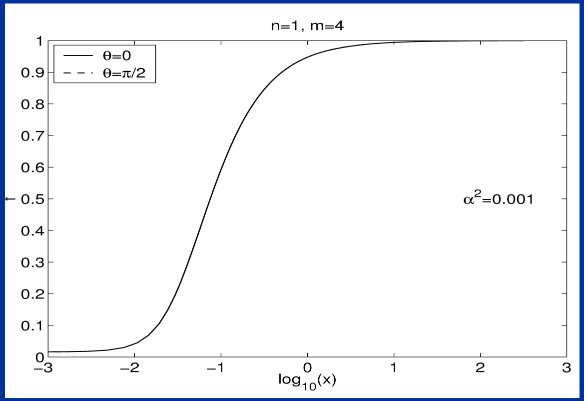


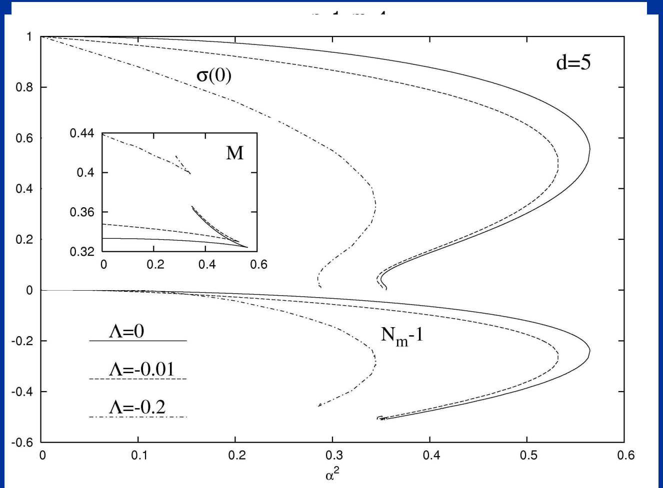


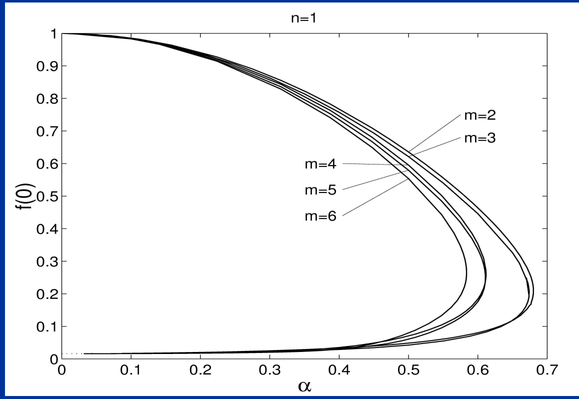


Gravitating EYMH solitons: strong gravity limit

- **Branch structure: Two distincts limits $\alpha \to 0$, $(\alpha^2 = 4\pi^2 G\eta^2)$**
- **Lower branch is linked to the flat space solution** $\Longrightarrow$ as $G\uparrow,\ \alpha\uparrow$
- **Critical point: configuration decays into components**
- **Upper branch:** $\Longrightarrow$ as $G\uparrow,\ \eta\downarrow$
- **Limiting behavior at semi-Planck scales: splitting of the space into an interior and an outer regions: BM sphaleron is confined (intermediate β);**
- **Change of the geometry: a long throat is formed;**
- **Conical singularity occurs in the limit of strong gravity**



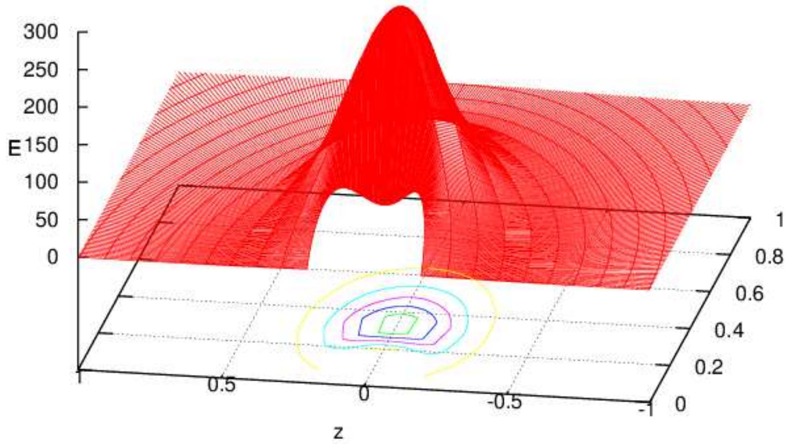


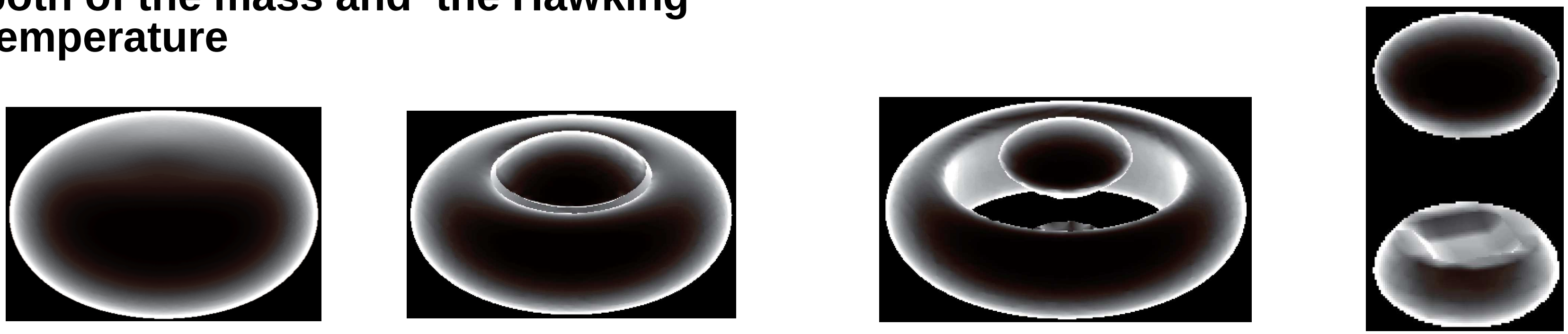


Non-Abelian Hairy Black Holes

(Lee, Weinberg, Breitenlohner, Forgacs, Maison, Hartmann, Kleihaus Kunz, Shnir…)

- **Black hole solutions are linked to gravitating monopoles; there are m=1,n>1 axially symmetric black hole solutions with a regular deformed S_2 horizon and non-trivial non-Abelian fields outside**
- **Neither Israel theorem nor the *"no-hair"* theorem cannot be generalized to theories with nonabelian fields**
- **Regular horizon ($f(r_h) = 0$)**
- **No uniqueness**
- **A globally regular multimonopole solution corresponds to a family of black holes for range of values of r_h**
- **Increasement of r_h yields increasement both of the mass and the Hawking temperature**





AdS SU(2) EYMH theory

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{(R-2\Lambda) - \mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} - \mathrm{Tr}\ (\mathrm{D}_\mu\Phi)(\mathrm{D}^\mu\Phi) - \mathrm{V}(\Phi)\}$$

(Maison, Breitenlocher, Shaposhnik, Moreno, Tong, Bolognesi, Kunz, Radu, Shnir..)

Boundary CFT

V = 0: **Gauge field A_μ $\Leftrightarrow$ triplet of conserved currents J_μ^a**

Scalar field Φ^a $\Leftrightarrow$ scalar operators $\mathcal{Q}^a$

BPS limit:

$$\Phi^a(z) \to \eta^a + \frac{C^a}{z^3} + \dots$$

SU(2) global symmetry on the boundary is broken:

$$\partial^\mu J_\mu^a = \varepsilon^{abc}\eta^b\mathcal{Q}^c \qquad \eta^b = \mathrm{const} \Longrightarrow U(1)$$

- **V < 0:** **Abelian symmetry in the bulk $\Leftrightarrow$ U(1) conserved boundary current J_μ, massive gauge boson $\Leftrightarrow$ charged spin-1 operator**

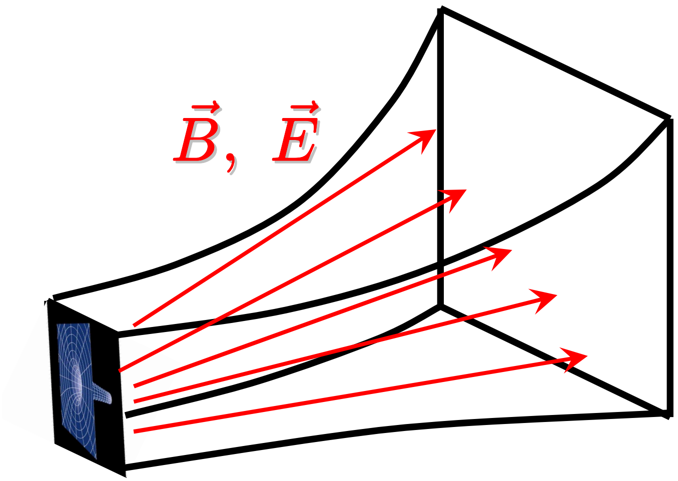
  Scalar field Φ $\Leftrightarrow$ relevant scalar operator

$$\Phi^a(z) \to n^a\left(\eta + \frac{C_0}{z^{\Delta_-}} + \frac{C_1}{z^{\Delta_+}} + \dots\right)$$

- **V > 0:** **Scalar field is irrelevant**

Holographic AdS dyon

BPS limit: V = 0



$$A_k = B\varepsilon_{ki}x_i + \ldots \Longrightarrow L_{QFT}(A) = A_i J_i$$

$$\Phi^a = \eta^a + \frac{C^a}{r^3} + \ldots \Longrightarrow L_{QFT}(\Phi) = \Phi \cdot \mathcal{O}(x)$$

$$A_0^a = e\eta\hat{n}^a \left(\mu + \frac{Q}{r} + \ldots\right)$$

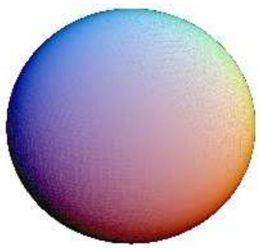
On the boundary: d=2+1 Abelian Quantum Field Theory which undergoes a phase transition exhibiting condensation below a critical temperature.

Abelian Higgs model at finite temperature

In the bulk we have:

- d=3+1 Yang-Mills-Higgs theory;
- Schwarzschild-AdS black hole

Holographic p-wave superconductors



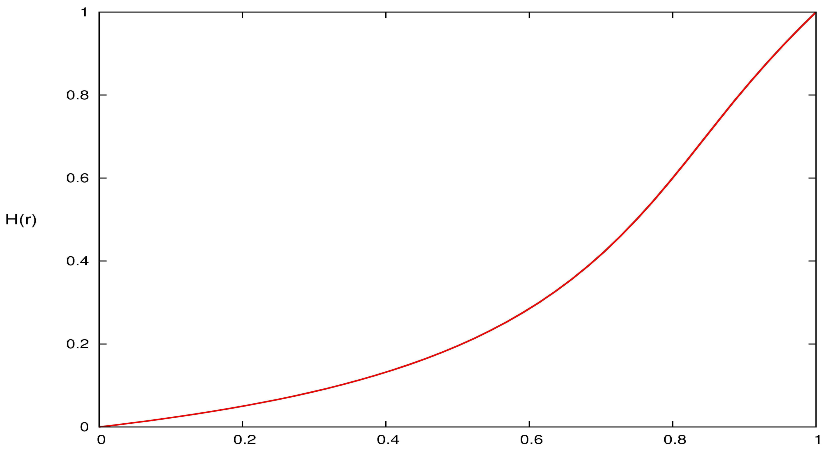
$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - N(r)dt^2$$

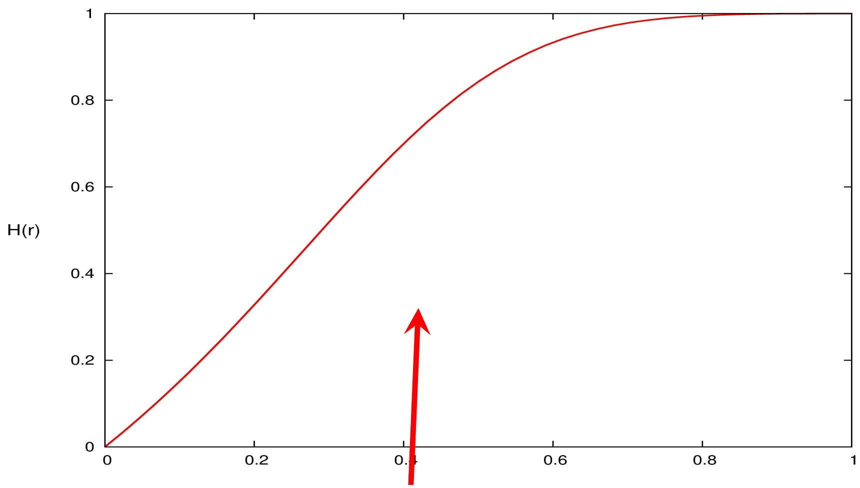
$$N(r) = 1 - \tfrac{\Lambda}{3}r^2 - \tfrac{2M}{r}$$

$$N(r) = (1 - \tfrac{r_h}{r})\left[1 - \tfrac{\Lambda}{3}\left(r^2 + rr_h + r_h^2\right)\right]$$

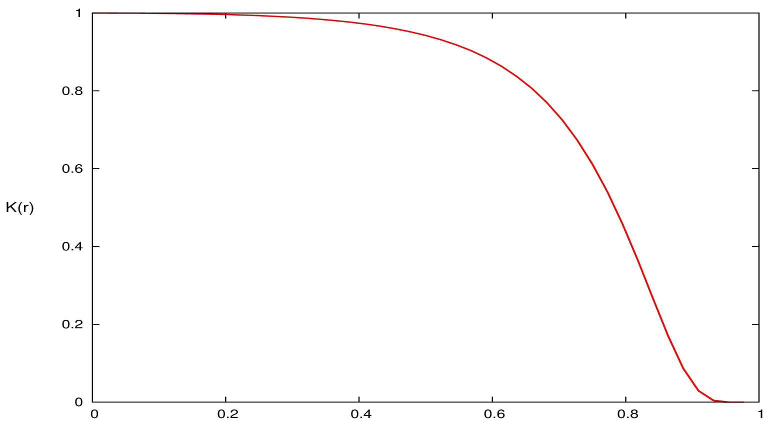
Dyons in AdS space

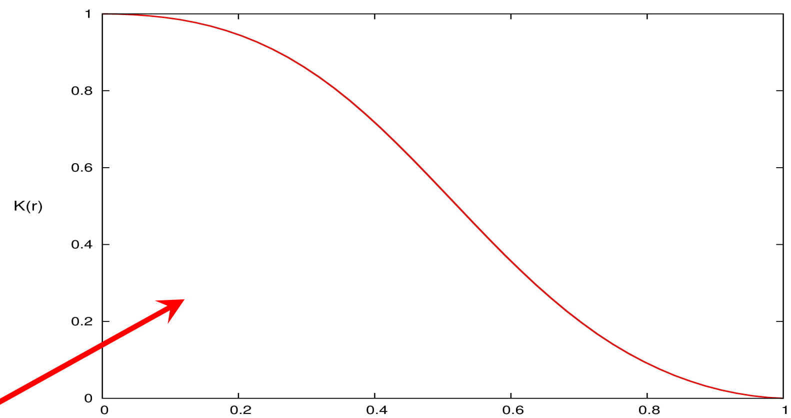
- Λ=0
- Λ=-3





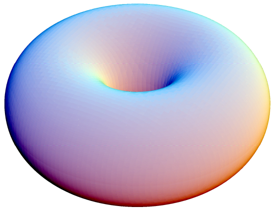
The Higgs field on the boundary becomes a constant
It does not qualify as a proper order parameter

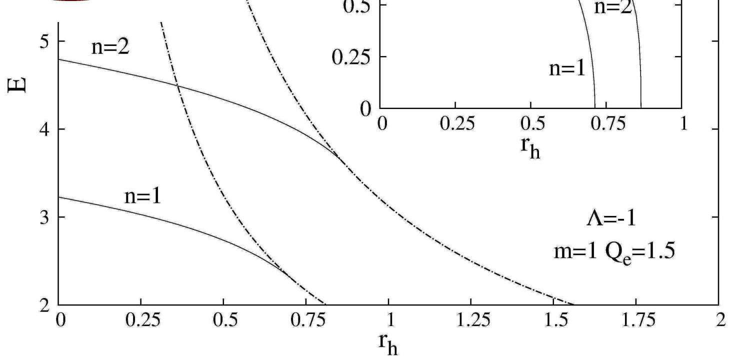


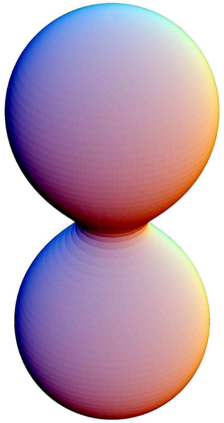


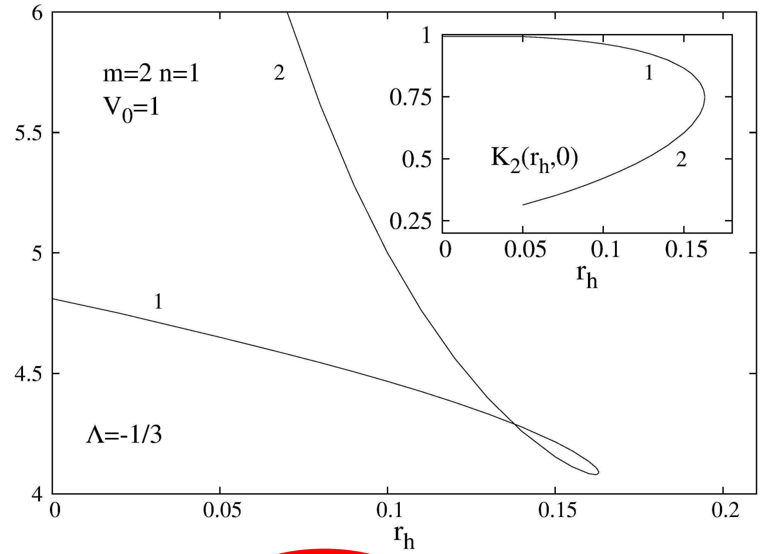
The order parameter is the gauge function K

Some more numerics..



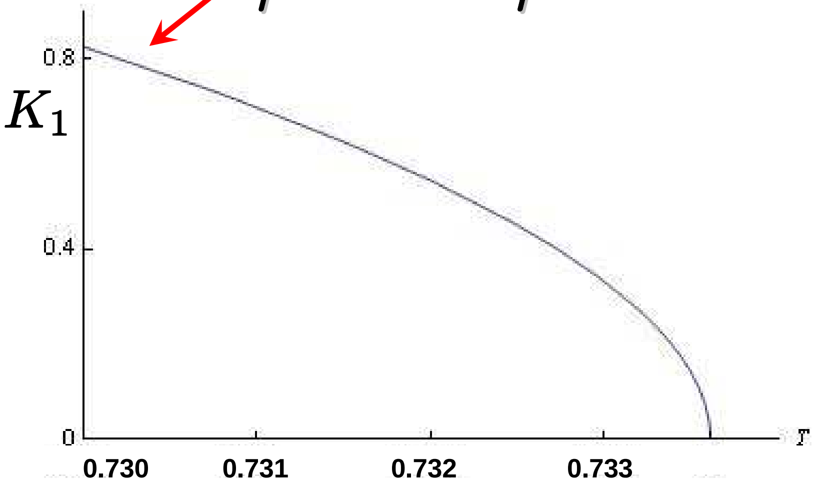


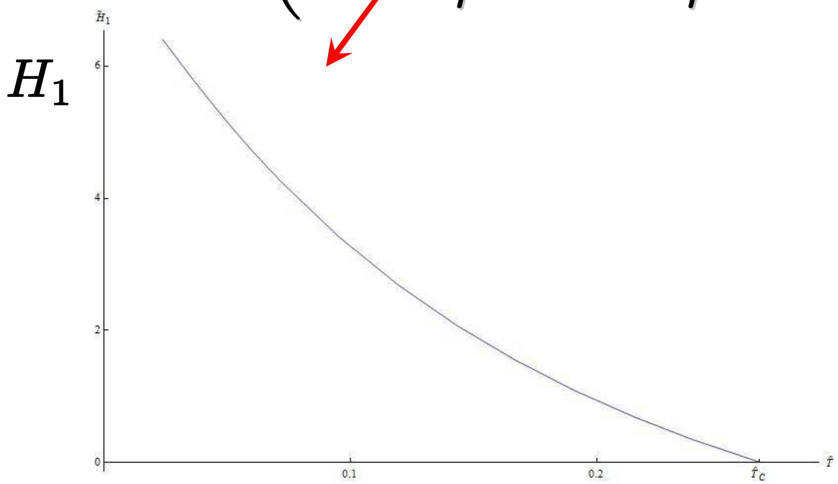




$$K(r) = \frac{K_1(T)}{r^{\nu+1}} + \frac{K_2(T)}{r^{\nu+3}} + \dots$$

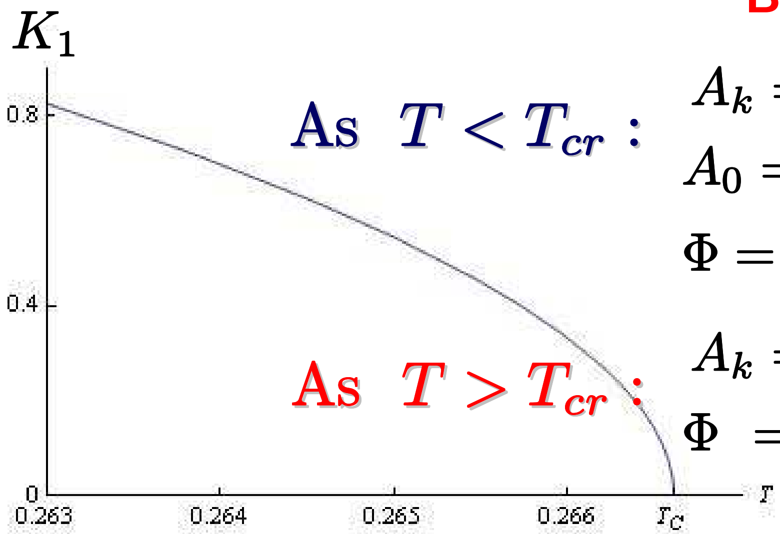
$$H(r) = \eta\left(1 - \frac{H_1(T)}{r^3} + \frac{H_2(T)}{r^5} + \dots\right)$$





Physics in the bulk/boundary

Interpretation: Phase transition in the bulk at T=T_{cr}



	Bulk:	**Boundary:**
As $T < T_{cr}$:	$A_k = B\varepsilon_{ki}x_i + \ldots$	$L_{QFT}(A) = A_i J_i$
	$A_0 = C_0 + \frac{C_1}{r^{\circ}} + \ldots$	$L_{QFT}(\Phi) = \Phi \cdot \mathcal{O}(x)$
	$\Phi = v + \frac{C}{r^{\alpha}} + \ldots$	
As $T > T_{cr}$:	$A_k = \quad const$	$J_k = 0; B = 0$
	$\Phi \quad = \quad const$	$\mathcal{O} = 0$

- To the right of T_{cr} the conguration becomes trivial, SU(2) global symmetry is restored.
- To the left of T_{cr} the congurations, which correspond to v.e.v.'s in the dual field theory are non-trivial.
- There is a finite temperature continuous symmetry breaking transition.
- The system condenses below a critical temperature $T_{\spadesuit}$
- Fitting the curves one confirms that this is a second order phase transition:

$$\boxed{K_1 \propto (T_{cr} - T)^{1/2}; \qquad H_1 \propto (T_{cr} - T)}$$

Summary and Outlook

- AdS/CFT may become AdS/CondMatterTheory (no strings attached)
- AdS/gauge duality is able to compute dynamical transport properties of strongly coupled systems at nonzero T.
- On the gravity side: a 3 + 1 EYMH model in AdS-Schwarzschild black hole background.
- On the QFT side: a 2 + 1 Abelian Higgs Model
- We constructed generalized BM AdS solutions
- We obtained static axially symmetric dyonic solutions in fixed AdS background
- Dyonic black hole in AdS yields phase transition on the boundary at critical temperature
- Vortex condensation?
- Is there is strongly coupled holographic superconductor? P-wave? D-wave?

Thank you for your attention!