Covariant relativistic separable kernel approach for the electrodisintegration of the deuteron at high momentum transfer

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Abstract. The electrodisintegration of the deuteron for the kinematic conditions of the JLab experiment E-94-019 is considered. The calculations are performed within the covariant Bethe-Salpeter approach with the separable kernel of interaction. The results are obtained within the relativistic plane wave impulse approximation and compared with the experimental data and other models. The influence of nucleon electromagnetic form factors is investigated.

Keywords: Bethe-Salpeter equation; Separable ansatz; Deuteron electrodisintegration
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FORMALISM

The deuteron electrodisintegraton is considered within the Bethe-Salpeter (BS) approach [1] with a separable kernel of NN interactions. It is based on the solution of the BS equation:

$$\Phi^{JM}(k;K) = \frac{i}{(2\pi)^4} S_2(k;K) \int d^4p V(p,k;K) \Phi^{JM}(p;K) \tag{1}$$

for the bound state of the neutron-proton (np) system with the total angular momentum J and its projection M which is described by the BS amplitude Φ^{JM}. Here the total $K = k_p + k_n$ and the relative $k = (k_p - k_n)/2$ momenta are used instead of the proton k_p and neutron k_n momenta. In general, the BS amplitude can be decomposed by the partial-wave states through the generalized spherical harmonic $\mathscr{Y}$ and the radial part ϕ [2] as:

$$\Phi^{JM}_{\alpha\beta}(k;K_{(0)}) = \sum_a (\mathscr{Y}_{aM}(\mathbf{k}) U_C)_{\alpha\beta}\ \phi_a(k_0, |\mathbf{k}|; K^2_{(0)}), \tag{2}$$

where $K_{(0)} = (M_d, 0)$ is the total momentum of the NN system in its rest frame (here it is the deuteron rest frame called the laboratory system, LS); M_d is a mass of the deuteron; U_C is the charge conjugation matrix; α, β denote matrix indices; a is a short notation of the partial-wave state ${}^{2S+1}L^{\rho}_J$ with spin S, orbital L and total J angular momenta, ρ means positive- or negative-energy partial-wave state. $S_2(k;K)$ is the free two-particle Green function:

$$S_2^{-1}(k;K) = \left(\tfrac{1}{2} K\cdot\gamma + k\cdot\gamma - m\right)^{(1)} \left(\tfrac{1}{2} K\cdot\gamma - k\cdot\gamma - m\right)^{(2)}.$$

In calculations, it is more convenient to use the BS vertex function Γ^{JM} which is connected with the BS amplitude by the following relation:

$$\Phi^{JM}(k;K) = S_2(k;K)\Gamma^{JM}(k;K). \tag{3}$$

After using the decomposition of type (2) for the vertex function the relation between Φ^{JM} and Γ^{JM} radial parts can be deduced:

$$\phi_a(k_0,|\mathbf{k}|) = \sum_b S_{ab}(k_0,|\mathbf{k}|;s)g_b(k_0,|\mathbf{k}|), \tag{4}$$

where S_{ab} is the one-nucleon propagator [2]. To solve the BS equation (1) we use the separable ansatz for the interaction kernel

$$V_{ab}(p_0,|\mathbf{p}|;k_0,|\mathbf{k}|;s) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[a]}(p_0,|\mathbf{p}|) g_j^{[b]}(k_0,|\mathbf{k}|), \tag{5}$$

where N is a rank of the kernel, g_i are model functions; λ is a parameter matrix satisfying the symmetry property $\lambda_{ij}(s) = \lambda_{ji}(s)$; k $[p]$ is the relative momentum of the initial [final] nucleons; $s = (p_p + p_n)^2$ where p_p is the outgoing proton and p_n is the neutron momentum, respectively. If the radial part of the vertex function Γ_{JM} is written in the following form:

$$g_a(p_0,|\mathbf{p}|) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[a]}(p_0,|\mathbf{p}|) c_j(s), \tag{6}$$

the initial integral BS equation (1) is transformed into a system of linear homogeneous equations for the coefficients $c_i(s)$:

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s)\lambda_{kj}(s)c_j(s) = 0, \tag{7}$$

where

$$h_{ij}(s) = -\frac{i}{4\pi^3}\sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0,|\mathbf{k}|) g_j^{[a]}(k_0,|\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\varepsilon)^2 - k_0^2} \tag{8}$$

and $E_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$. Using (4) and taking into account only positive-energy partial-wave states for the deuteron ${}^3S_1^+$, ${}^3D_1^+$ the radial part of the BS amplitude can be written as:

$$\phi_a(k_0,|\mathbf{k}|) = \frac{g_a(k_0,|\mathbf{k}|)}{(M_d/2 - E_{\mathbf{k}} + i\varepsilon)^2 - k_0^2}. \tag{9}$$

Thus, using the separable functions g we can calculate observables describing the np system.

CROSS SECTION

The exclusive $d(e,e'n)p$ process when all particles are unpolarized can be described by the cross section in LS:

$$\frac{d^3\sigma}{dQ^2 d|\mathbf{p}_n| d\Omega_n} = \frac{\sigma_{\rm Mott}\pi\mathbf{p}_n^2}{2(2\pi)^3 M_d E_e E_e'} \times$$
$$\times \left[l_{00}^0 W_{00} + l_{++}^0 (W_{++} + W_{--}) + l_{+-}^0 \cos 2\phi \; 2{\rm Re} W_{+-} - l_{+-}^0 \sin 2\phi \; 2{\rm Im} W_{+-} \right.$$
$$\left. - l_{0+}^0 \cos\phi \; 2{\rm Re}(W_{0+} - W_{0-}) - l_{0+}^0 \sin\phi \; 2{\rm Im}(W_{0+} + W_{0-}) \right], \qquad (10)$$

where $\sigma_{\rm Mott} = (\alpha \cos\frac{\theta}{2}/2E_e \sin^2\frac{\theta}{2})^2$ is the Mott cross section, $\alpha = e^2/4\pi$ is the fine structure constant; E_e $[E_e']$ is the energy of the initial [final] electron; Ω_e' is the outgoing electron solid angle; θ is the electron scattering angle; $Q^2 = -q^2 = -\omega^2 + \mathbf{q}^2$, where $q = (\omega, \mathbf{q})$ is the momentum transfer. The outgoing neutron is described by the momentum $\mathbf{p}_n$ and the solid angle $\Omega_n = (\theta_n, \phi)$ with the zenithal angle θ_n between $\mathbf{q}$ and $\mathbf{p}_n$ momenta and azimuthal angle ϕ between $(\mathbf{ee}')$ and $(\mathbf{qp}_n)$ planes. The photon density matrix elements have the following form:

$$l_{00}^0 = \frac{Q^2}{\mathbf{q}^2}, \quad l_{0+}^0 = \frac{Q}{|\mathbf{q}|\sqrt{2}} \sqrt{\frac{Q^2}{\mathbf{q}^2} + \tan^2\frac{\theta}{2}},$$
$$l_{++}^0 = \tan^2\frac{\theta}{2} + \frac{Q^2}{2\mathbf{q}^2}, \quad l_{+-}^0 = -\frac{Q^2}{2\mathbf{q}^2}. \qquad (11)$$

The hadron density matrix elements W can be calculated using Cartesian components of the hadron tensor

$$W_{\mu\nu} = \frac{1}{3} \sum_{s_d s_n s_p} \left| < np : SM_S | j_\mu | d : 1M > \right|^2, \qquad (12)$$

where S is a spin of the np pair and M_S is its projection, and the photon polarization vectors ε according to the relation

$$W_{\lambda\lambda'} = W_{\mu\nu} \varepsilon_\lambda^\mu \varepsilon_{\lambda'}{}^\nu, \qquad (13)$$

here λ, λ' are photon helicity components [3]. The hadron current j_μ in (12) can be written according to the Mandelstam technique [4] and, within the relativistic impulse approximation, has the following form:

$$< np : SM_S | j_\mu | d : 1M > =$$
$$i \sum_{r=1,2} \int \frac{d^4 p}{(2\pi)^4} {\rm Sp} \left\{ \Lambda(\mathscr{L}^{-1}) \bar{\psi}_{SM_S}(P^{\rm CM}, p^{\rm CM}) \Lambda(\mathscr{L}) \times \right.$$
$$\left. \Gamma_\mu^{(r)}(q) S^{(r)} \left(\frac{K_{(0)}}{2} - (-1)^r p - \frac{q}{2} \right) \Gamma^M \left(K_{(0)}, p + (-1)^r \frac{q}{2} \right) \right\}, \qquad (14)$$

the sum over $r = 1,2$ corresponds to the interaction of the virtual photon with the proton and with the neutron in the deuteron, respectively. The total P^{CM} and the relative p^{CM} momenta of the outgoing nucleons are considered in the final np pair rest frame (center-of-mass system, CM) and can be written in LS using the Lorenz-boost transformation along the $\mathbf{q}$ direction. The Lorenz transformation of the np pair wave function ψ_{SM_S} from CM to LS is:

$$\Lambda(\mathscr{L}) = \left(\frac{1+\sqrt{1+\eta}}{2}\right)^{\frac{1}{2}} \left(1+\frac{\sqrt{\eta}\gamma_0\gamma_3}{1+\sqrt{1+\eta}}\right). \tag{15}$$

where $\eta = \mathbf{q}^2/s$. The interaction vertex is chosen in the on-mass-shell form:

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{4m}\left(\gamma_\mu \not{q} - \not{q}\gamma_\mu\right) F_2(q^2), \tag{16}$$

here $F_1(q^2)$ is the Dirac form factor, $F_2(q^2)$ - Pauli form factor. The form factors are described by the dipole fit model [5] or modified dipole fit [6, 7]. If the outgoing nucleons are supposed to be non-interacting it is the so-called plane-wave approximation. In this case the np pair wave function can be written in the following form:

$$\bar{\psi}_{SM_S}(P,p;p^*) \;\rightarrow\; \bar{\psi}^{(0)}_{SM_S}(P,p;p^*) = (2\pi)^4 \bar{\chi}_{SM_S}\delta(p-p^*), \tag{17}$$

where $p^* = (0,\mathbf{p}^*)$ is the relative momentum of on-mass-shell nucleons, χ_{SM_S} describes spinor states of the pair. Taking into account the representation (17), the hadron current (14) can be transformed into a sum:

$$< np : SM_S|j_\mu|d : 1M >= i \sum_{r=1,2} \left\{\Lambda(\mathscr{L}^{-1})\bar{\chi}_{SM_S}\left(P^{\mathrm{CM}}, p^{\mathrm{CM}*}\right)\Lambda(\mathscr{L})\Gamma^{(r)}_\mu(q)\,\cdot\right.$$
$$\left.\cdot S^{(r)}\left(\frac{K_{(0)}}{2} - (-1)^r p^* - \frac{q}{2}\right)\Gamma^M\left(K_{(0)}, p^* + (-1)^r\frac{q}{2}\right)\right\}. \tag{18}$$

In the paper the cross section of the exclusive electrodisintegration of the deuteron $d^2\sigma/dQ^2d|\mathbf{p}_n|$ [8] is calculated. It can be obtained from (10) after integration over the neutron solid angle:

$$\frac{d^2\sigma}{dQ^2d|\mathbf{p}_n|} = \int\limits_{\Omega_n} \frac{d^3\sigma}{dQ^2d|\mathbf{p}_n|d\Omega_n} d\Omega_n. \tag{19}$$

According to [8] the integration is performed over Ω_n: $20^\circ \leqslant \theta_n \leqslant 160^\circ$, $0^\circ \leqslant \phi \leqslant 360^\circ$. Four different Q^2 are considered. The obtained results are discussed in the next section.

DISCUSSION AND CONCLUSION

In this paper the exclusive cross section of the electrodisintegration (19) for the kinematic conditions of the JLab experiment [8] is calculated within the Bethe-Salpeter

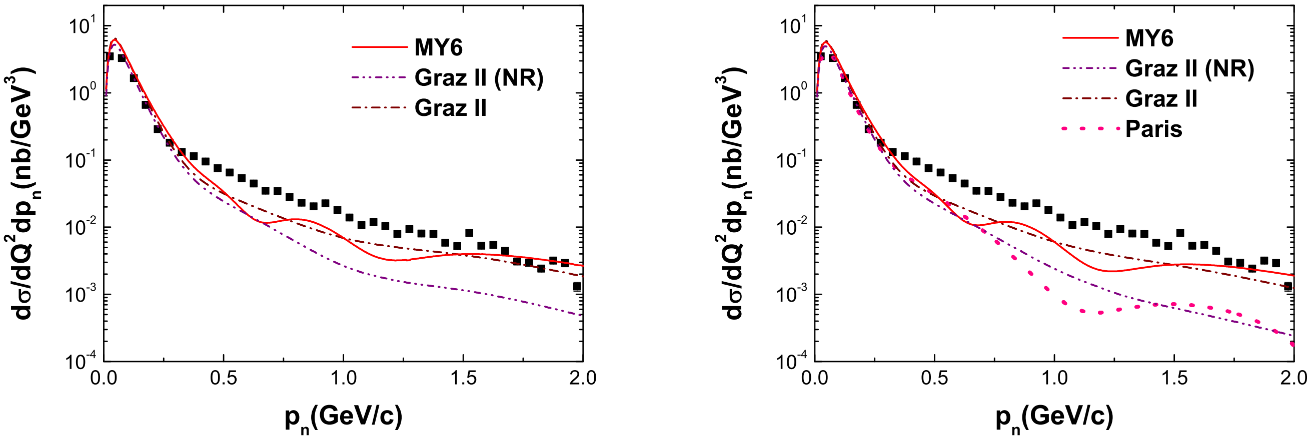

FIGURE 1. The cross section (19) for $Q^2 = 2 \pm 0.25\,\mathrm{GeV}^2$ depending on the neutron momentum $\mathbf{p}_n$ is considered. Calculations with Graz II (NR) [11] (purple dash-dot-dotted line), Graz II [12] (brown dash-dotted line), MY6 [9] (red solid line) and Paris [10] (pink dotted line) are present. The dipole fit [5] (on the left) and modified dipole fit [5, 6, 7] (on the right) models for nucleon electromagnetic form factors are considered.

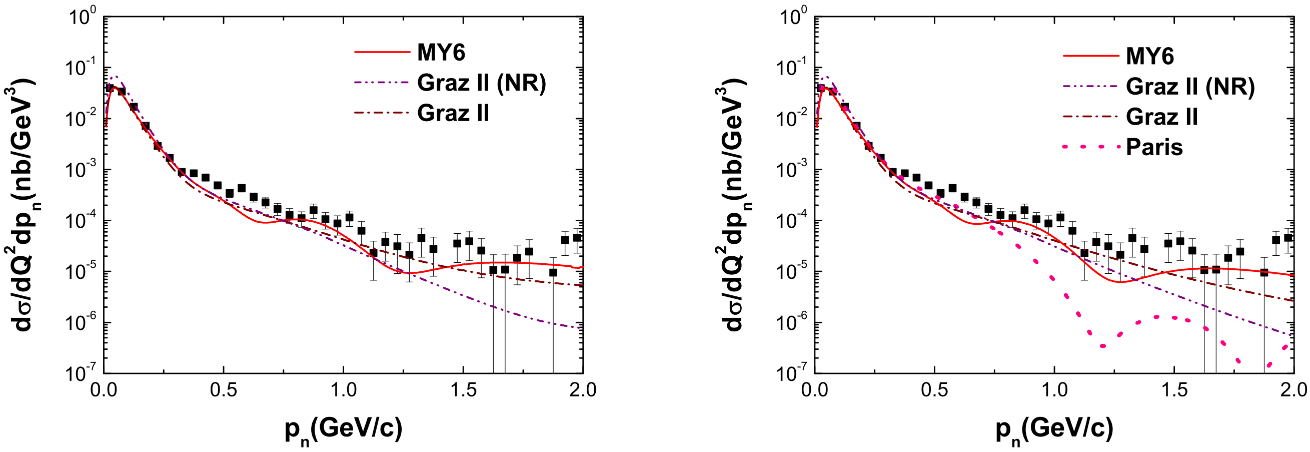

FIGURE 2. As in Fig.1, but for $Q^2 = 5 \pm 0.5\,\mathrm{GeV}^2$.

approach with the rank-six separable kernel MY6 [9]. The calculations are performed within the relativistic plane-wave impulse approximation. The obtained results are compared with the experimental data and other theoretical models, the Paris potential [10], the nonrelativistic Graz II (NR) [11] and relativistic Graz II [12] separable interaction kernels.

In Fig.1, the cross section depending on the outgoing neutron momentum $\mathbf{p}_n$ is present for Q^2=2 GeV^2. The dipole fit [5] (figure on the left) and modified dipole fit [6, 7] (figure on the right) models for the nucleon electromagnetic form factors are considered. From

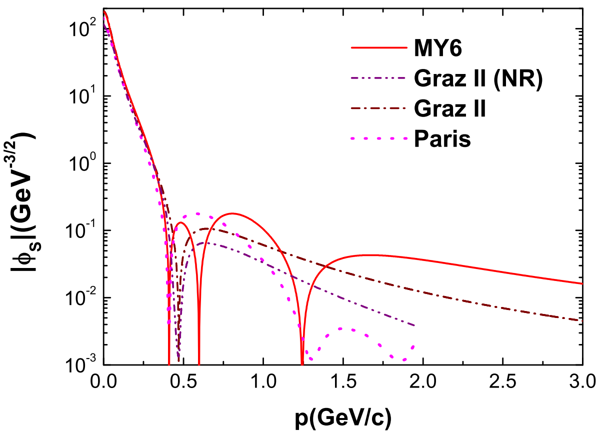

FIGURE 3. The wave function (9) at $k_0 = M_d/2 - E_{\mathbf{k}}$ for the ${}^3S_1^+$ partial-wave state in the deuteron rest frame for the MY6 model in comparison with those of Graz II (NR) [11], Graz II [12], and Paris [10].

the figure, a good agreement with the experimental data can be seen at low neutron momenta $|\mathbf{p}_n| \leqslant 0.25$ GeV/c. The discrepancy between the theoretical calculations and the experiment increases with the increase of $|\mathbf{p}_n|$ for the nonrelativistic separable Graz II (NR) [11] and Paris [10] potential models. For the relativistic MY6 [9] and Graz II [12] separable models, a good agreement with the experimental data can be seen not only at low $\mathbf{p}_n$ but also at high $|\mathbf{p}_n|$>1.5 GeV/c.

In Fig.2, the cross section for Q^2=5 GeV^2 is present. As in previous figure, two models of nucleon electromagnetic form factors are considered. The relativistic MY6 and Graz II models agree with the experimental data much better than for Q^2=2 GeV^2 whereas the nonrelativistic Graz II (NR) and Paris potentials deviate from the experimental points at $|\mathbf{p}_n|$>0.25 GeV/c increasingly more than in previous case (Fig.1). Therefore, it can be concluded that the influence of relativistic effects increases with the increase of the energy of the nucleons and the momentum transfer.

It should be noticed that the behavior of the calculated cross section is similar to the behavior of the corresponding wave function for the deuteron ${}^3S_1^+$ partial-wave state which is shown in Fig.3. From the comparison of Figs.1, 2 and Fig.3, it is seen that the cross section at high $|\mathbf{p}_n|$ is similar to the asymptotic form of the ${}^3S_1^+$ wave function.

From Figs.1,2, it is seen that results obtained within the dipole fit model [5] for nucleon electromagnetic form factors are similar to those obtained with modified form factors [6, 7]. Thus, we can summarize that the choice of nucleon electromagnetic form factors does not play an important role in the description of the cross section at high momentum transfer. It is interesting that the results calculated within the dipole fit model, which does not describe the behavior of the electric form factor of the proton at high Q^2, is virtually undistinguishable from those obtained with the modified proton electric form factor [6]. However, the final conclusion which model gives the best result can be made only when negative-energy partial-wave states (P waves) and final state interaction effects will be taken into account.

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