Properties of ^{26}Mg and ^{26}Si in the sd shell model and the determination of the ^{25}Al(p,γ)^{26}Si reaction rate[1]

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Abstract. We present results for levels in ^{26}Si (the mirror of nucleus ^{26}Mg). The calculated gamma-decay lifetimes and ^{25}Al to ^{26}Si spectroscopic factors together with experimental information on the levels of excited states are used to determined the ^{25}Al(p,γ)^{26}Si reaction rates together with a theoretical error on this rate based on the use of the USDA and USDB interactions.

Keywords: nuclear shell model, gamma-decay lifetime
PACS: 26.30.-k, 21.60.Cs, 21.10.Sf, 21.10.Tg

INTRODUCTION

The production mechanism and production site for the long-lived radioactive isotope ^{26}Al has been of interest since the first indications of ^{26}Al enrichment in meteoritic inclusions was observed [1]. Understanding its origin would serve as a unique signature for nucleosynthesis in novae and supernovae. The main reaction sequence leading to ^{26}Al is ^{24}Mg(p,γ)^{25}Al($\beta^+ + \nu$)^{25}Mg(p,γ)^{26}Al. At the high-temperature conditions expected for shell carbon burning and explosive neon burning the ^{25}Al(p,γ)^{26}Si reaction becomes faster than the ^{25}Al β decay. Since ^{26}Si β decays to the short-lived 0^+ state of ^{26}Al, the long-lived (5^+) state becomes depleted.

The properties of the states of ^{26}Si required for the calculation of the ^{25}Al(p,γ)^{26}Si reaction rate are the energies, J^π values, proton-decay widths and gamma-decay widths for levels above the proton decay threshold of 5.51 MeV. Experiments have established the energy of some levels [2]. But there is uncertainty in their J^π values and (based on the known levels of ^{26}Mg) many levels have not yet been observed. Theoretical input is needed for the unobserved levels as well as the gamma and proton decay widths for all of the levels.

Several advances are made in this paper. A new method is used to calculate the energies of levels in ^{26}Si based upon the observed energies of levels of the analogue states

[1] This work was supported by NSF grant PHY-0758099 and Joint Institute for Nuclear Astrophysics, NSF-PFC

in ^{26}Al and ^{26}Mg, together with a calculation of the c-coefficient of the isobaric-mass-multiplet equation (IMME). Also the gamma and proton decay widths are calculated with several Hamiltonians to find their values and to estimate their theoretical uncertainties.

This paper follows from recent work on the properties of $(0d_{5/2}, 0d_{3/2}, 1s_{1/2})$ sd-shell nuclei that include new Hamiltonians [3], a comprehensive study of electromagnetic and beta-decay observables [4] and a comprehensive study of the properties of states in ^{26}Mg [5]. For ^{26}Mg assignments between theory and experiment for about 50 levels in ^{26}Mg levels up to 10 MeV in excitation have been made, based on a comparison of the experimental and theoretical electron scattering data cross sections and electromagnetic transition strengths [5]. Because of the uncertainty in levels of ^{26}Si, conventionally levels are assigned on the basis of known levels in the mirror nucleus ^{26}Mg. In the next section we base these assignments on a new and improved method.

PROCEDURE FOR DETERMINING ^{26}SI ENERGY LEVELS.

In the present work we make use of a novel method of calculating energy levels in ^{26}Si by using the measured binding energies of the T=1 partners and a theoretical value of the c coefficient of the IMME [6]. Specifically

$$B_{th}(^{26}\mathrm{Si}) = 2B(^{26}\mathrm{Al}) - B(^{26}\mathrm{Mg}) + 2c_{th}. \quad (1)$$

In Fig. 1 values of c from experiment and theory are compared for states in ^{26}Si ordered according to increasing experimental energy. The calculated values of c are obtained from

$$c_{th} = [B_{th}(^{26}\mathrm{Si}) - 2B_{th}(^{26}\mathrm{Al}) + B_{th}(^{26}\mathrm{Mg})]/2. \quad (2)$$

The experimental values are obtained for states where all three members of the multiplet are known. In general a good correspondence can be seen, the largest deviations being less than 30 keV. There is considerable state dependence with c values ranging from 300 keV (for the 0^+ ground state) down to 180 keV. Thus where data is not available in ^{26}Si to determine the c coefficient from experiment, a fairly reliable value can be obtained from the theoretical calculation, and the binding energies for states in ^{26}Si can be then be obtained from Eq. 1, with experimental values of binding energy for corresponding states in ^{26}Al and ^{26}Mg (when they are known in both).

Testing calculated excitation energies against known values in ^{26}Si indicates that corresponding levels can be obtained very accurately. This is shown in Fig. 2. The calculated values can then be used as a guide to the correct spin/parity assignments for measured levels in ^{26}Si. Where no levels in ^{26}Si are known, levels can be predicted. Two such levels are indicated by crosses in Fig. 2.

The three levels that are just above the proton-decay separation energy of 5.51 MeV and of potential importance for the capture reaction at low temperatures are indicated by the arrows in Fig. 2. The J^π of levels 16 and 17 are from the recent analysis of Wrede [7] where arguments for the J^π are based on all available data for these states.

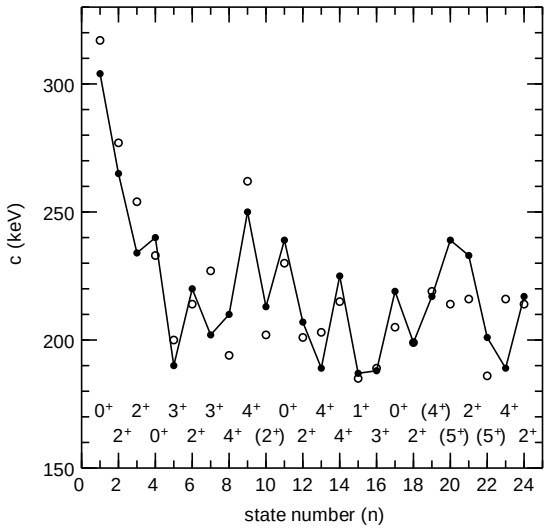

FIGURE 1. c coefficients from the isobaric mass multiplet equation (IMME: $E = a + bT_z + cT_z^2$) versus state number (in order of increasing energy) in ^{26}Si based on experimental energies (closed circles) and energies calculated from USDB (crosses).

RESULTS FOR THE REACTION RATE

The resonant reaction rate for capture on a nucleus in an initial state i, $N_A < \sigma v >_{\mathrm{res}\,i}$ for isolated narrow resonances is calculated as a sum over all relevant compound nucleus states f above the proton threshold [8]

$$N_A < \sigma v >_{\mathrm{res}\,i} = 1.540 \times 10^{11} (\mu T_9)^{-3/2} \times \sum_f \omega\gamma_{if}\ \mathrm{e}^{-\mathrm{E_{res}}/(kT)}\ \mathrm{cm^3\,s^{-1}mole^{-1}}. \quad (3)$$

Here T_9 is the temperature in GigaK, $\mathrm{E}_{res} = \mathrm{E}_f - \mathrm{E}_i$ is the resonance energy in the center of mass system, the resonance strengths in MeV for proton capture are

$$\omega\gamma_{if} = \frac{(2J_f+1)}{(2J_p+1)(2J_i+1)} \frac{\Gamma_{\mathrm{p}\,if}\Gamma_{\gamma f}}{\Gamma_{\mathrm{total}\,f}}. \quad (4)$$

$\Gamma_{\mathrm{total}\,f} = \Gamma_{\mathrm{p}\,if} + \Gamma_{\gamma f}$ is a total width of the resonance level and J_i, J_p and J_f are target (^{25}Al), the proton projectile ($J_p = 1/2$), and states in final nuclues (^{26}Si), respectively. The proton decay width depends exponentially on the resonance energy and can be calculated from the proton spectroscopic factor C^2S_{if} and the single-particle proton width $\Gamma_{\mathrm{sp}\,if}$ as $\Gamma_{\mathrm{p}\,if} = C^2S_{if}\Gamma_{\mathrm{sp}\,if}$. The single-particle proton widths were calculated from $\Gamma_{\mathrm{sp}} = 2\gamma^2 P(\ell, R_c)$ [9], with $\gamma^2 = \frac{\hbar^2 c^2}{2\mu R_c^2}$ and where the channel radius R_c was chosen to match the width obtained from an exact evaluation of the proton scattering cross section from a Woods-Saxon potential well and $Q = 0.1 - 0.5$ MeV. This simple model matches exact calculations in the sd-shell to within about 10%, and has the advantage that it

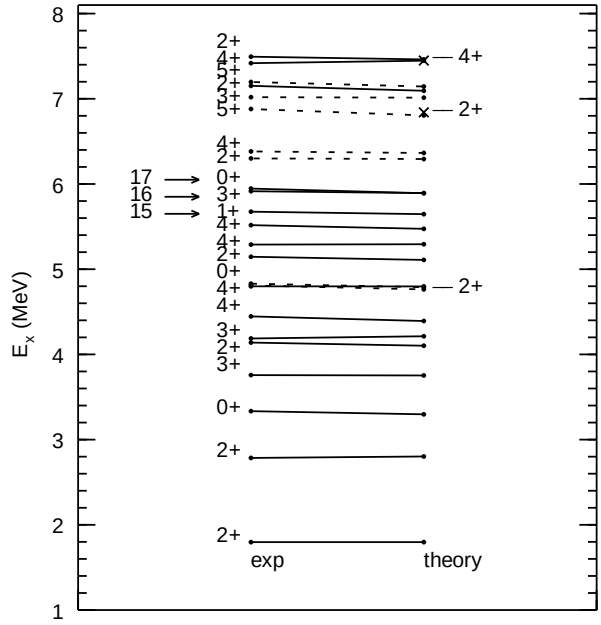

FIGURE 2. Adopted experimental excitation energies in ^{26}Si [2] versus predicted energies E_{th} based on experimental binding energies of ^{26}Mg and ^{26}Al and the theoretical c coefficient (USDB) (Eq. 1). The crosses correspond to predicted energies without experimental counterparts.

is fast and can be easily extrapolated to energies below 0.1 MeV where the scattering calculation becomes computationally difficult. We use a Coulomb penetration code from Barker [10].

The total rp reaction rates have been calculated for each of the interactions USD, USDA and USDB. The Q values required were based on measured energies in ^{26}Si, and where they were not known values calculated from Eq. 1 were used. In the cases with energies near 8 MeV and above where the energy of the T=1 state in ^{26}Al was not known, the energy of the state in ^{26}Si is based on the shift obtained from the average of five states in ^{26}Mg near 8 MeV. Above 8 MeV we use the energies obtained with USDB that includes the addition of about 170 states with $J^\pi \leq 5^+$ up to 14 MeV in excitation energy. The 0^+ state at 6.461 MeV [2] is much lower than the predicted energy of the fifth 0^+ state with USDB (at 8.040 MeV). Theory predicts a 1^+ state (at 6.620 MeV) which has no experimental counterpart. We have used the theoretical results of the 1^+ state for Γ_p and Γ_γ, instead of the 0^+ state.

Fig. 3 shows the results for the capture rate obtained using the properties of ^{26}Si. The Γ_p and Γ_γ in this case are all based on the USDB Hamiltonian. The contribution between log(T9)= -0.7 and 0.5 is dominated by the properties of the 3^+ state at 5.915 MeV (number 16). Since $\Gamma_\gamma < \Gamma_p$ the rate is determined by Γ_γ.

Above log(T9) of about 0.8 there will be contributions from negative parity states that should be taken from Hauser-Feshbach statistical model estimates for negative parity states [11].

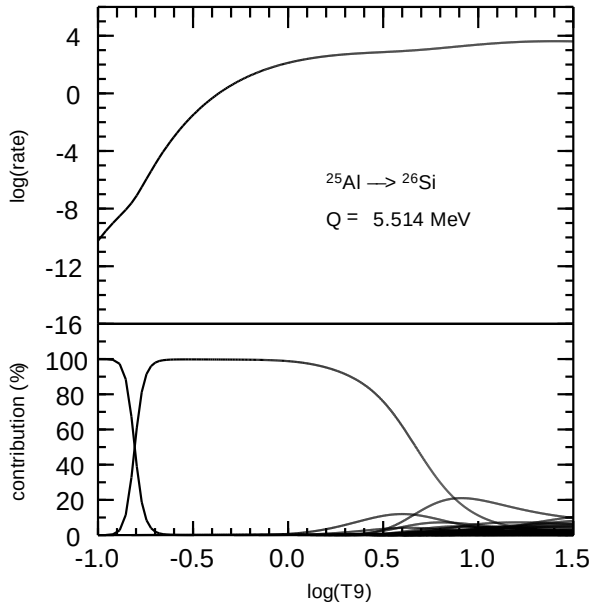

FIGURE 3. The total rp reaction rate versus temperature T9 (GigaK) (top panel) and the contribution of each of the final states (lower panel) with USDB. In the lower panel the dominant contribution below log(T9) = -0.8 is from state number 15, the 1^+ state at 5.675 MeV. Between log(T9) = -0.7 and 0.5 the dominant contribution is from state number 16, the 3^+ state at 5.915 MeV. Γ_γ calculated for ^{26}Si levels.

CONCLUSIONS

Because the calculation of the rp reaction rate for the ^{25}Al(p,γ)^{26}Si requires a knowledge of the energy levels in ^{26}Si, and many levels are uncertain, we have adopted a novel method of determining levels which is partly based on experiment and partly on theory. For the experimental part we used well-known binding energies of the T=1 analogue states of ^{26}Si. For the theoretical part we used calculated c coefficients of the isobaric mass multiplet equation. We have demonstrated that a good correspondence between theoretical and experimental values of the c coefficient for sd-shell nuclei exists. The method leads to a reliable prediction of energy levels in ^{26}Si. Using energy values in ^{26}Si constrained by our method for the Q values of the proton capture process on ^{25}Al, we obtained the required spectroscopic factors and gamma decay lifetimes for rate calculations from shell-model calculations using the new sd-shell interactions USDA and USDB. For comparison we also used the older USD interaction.

Reaction rates as well as contributions from individual states in ^{26}Si were then obtained for the different interactions. The variation in the rates give some indication of the theoretical error due to the use of different interactions and approximations for the gamma widths, and amounts to overall error band of $\pm$ 40%. It can also be concluded that using theoretical gamma widths from the mirror nucleus ^{26}Mg instead of ^{26}Si is an adequate approximation. The effect of negative parity states should also still be considered.

Acknowledgments This work is partly supported by NSF Grant PHY-0758099 and the National Research Foundation of South Africa.

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