Self consistent models of nuclear clustering

Collaboration

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Outline

- Motivation
- Formulation of binary clustering
- Core-Cluster Interaction
- Why Relativistic Mean Field Theory???
- Model Predictions
- Conclusions

Motivation

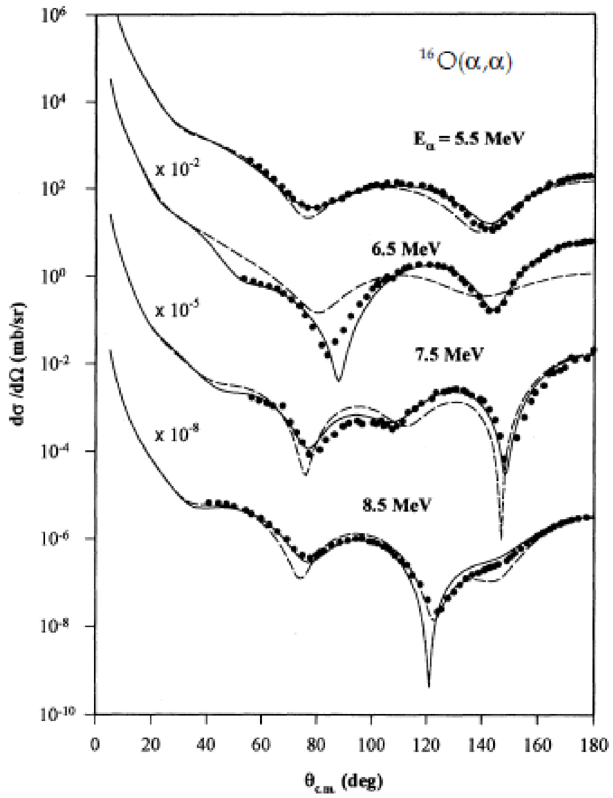
- We view nuclear cluster as the strong correlation between sub-system s of nucleons within a larger nuclear mass.

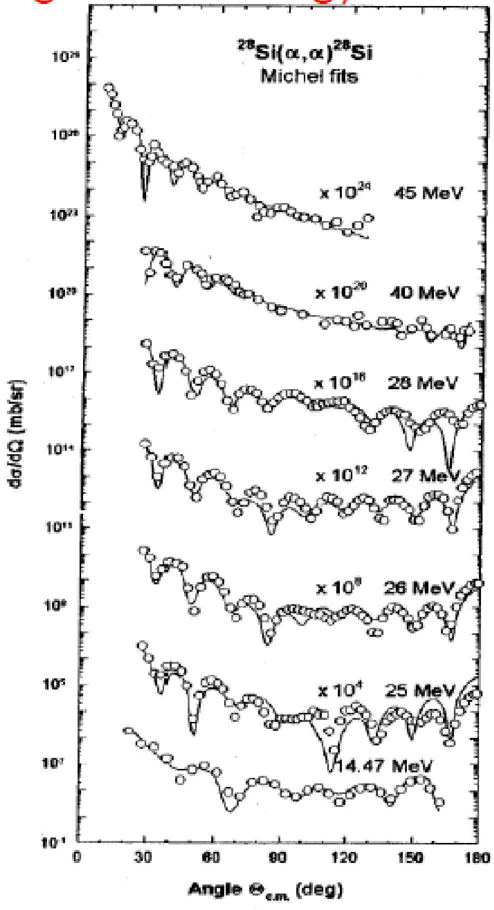
- Clustering in light nuclei has been studied in depth

- Focus has shifted to clustering phenomenon in heavy nuclei

- Semi-Classical Binary (SCB) model with Saxon-Woods core-cluster potential predicts most properties of nuclear clustering in light nuclei.

- **HOWEVER** SW model interaction fail to predict rainbow scattering (Anomalous Large Angle Scattering) observed in some nuclei!

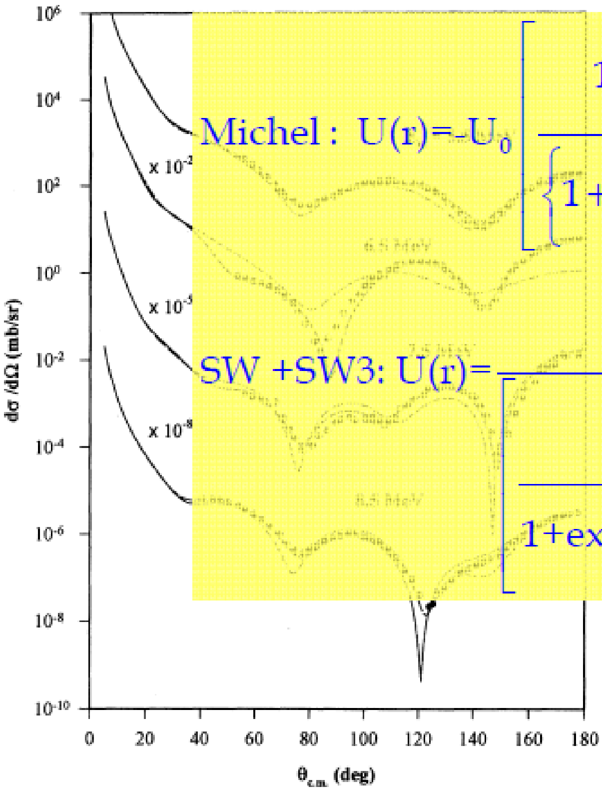


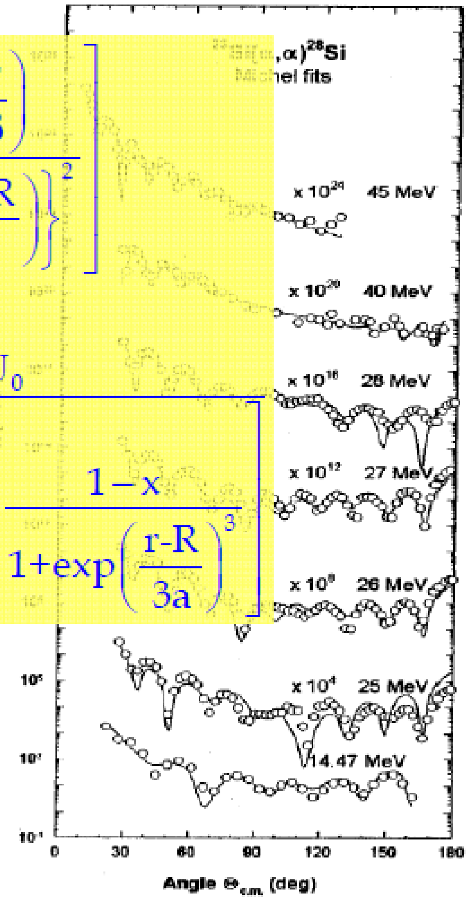


- **Modify SW:** Addition of high order SW term (Squared Michel) or (Cubic Buck, Merchant & Perez) fixes the problem.

$$\text{Michel}: \; U(r) = -U_0\left[\frac{1+\alpha\exp\left(\frac{r}{\beta}\right)}{\left\{1+\exp\left(\frac{r-R}{2a}\right)\right\}^2}\right]$$

$$\text{SW + SW3}: \; U(r) = \frac{-U_0}{\left[\frac{1}{1+\exp\left(\frac{r-R}{a}\right)} + \frac{1-x}{1+\exp\left(\frac{r-R}{3a}\right)^3}\right]}$$

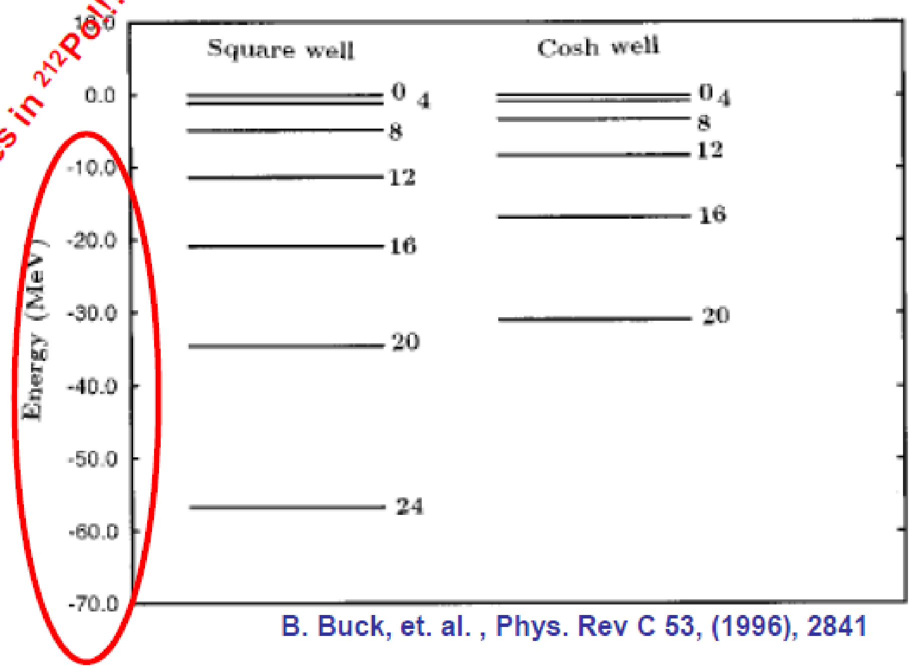




$$\text{SW}: \; U(r) = \frac{-U_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$$\text{Cosh}: \; U(r) = -U_0\left[\frac{1+\cosh\left(\frac{R}{a}\right)}{\cosh\left(\frac{r}{a}\right)+\cosh\left(\frac{R}{a}\right)}\right]$$

Inversion of + parity energy states in ^{212}Po!!!



B. Buck, et. al. , Phys. Rev C 53, (1996), 2841

SW + SW3 prediction (T.T. Ibrahim, PhD, 2009)

J^π	E_{expt}(MeV)	E_{cal}(MeV)
0^+	0.000	0.000
2^+	0.727	0.206
4^+	1.132	0.574
6^+	1.355	1.050
8^+	1.476	1.594
10^+	1.834	2.150
12^+	2.702	2.666
14^+	2.885	3.060
16^+	-	3.250
18^+	2.921	3.070

J^π	$T_{\frac{1}{2}}^{expt}$(ns)	$T_{\frac{1}{2}}^{cal}$(ns)
0^+	300	348
2^+	-	8.80×10^{-3}
4^+	-	1.11×10^{-1}
6^+	0.76	1.63
8^+	17.05	11.88
10^+	0.55	0.25
12^+	-	3.74×10^{-3}
14^+	-	7.10
16^+	-	-
18^+	4.5×10^{10}	4.0×10^{10}

SW + SW3 prediction (T.T. Ibrahim, PhD, 2009)

Lacks microscopic picture of clustering

J^π	E_{expt}(MeV)	E_{cal}(MeV)
0^+	0.000	0.000
2^+	0.727	0.206
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6^+	1.355	1.050
8^+	1.476	1.594
10^+	1.834	2.150
12^+	2.702	[illegible]
14^+	2.885	3.060
16^+	-	3.250
18^+	[illegible]	3.070

J^π	$T^{expt}_{\frac{1}{2}}$(ns)	$T^{cal}_{\frac{1}{2}}$(ns)
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- Microscopic double folded M3Y NN interaction predicts the α–decay half-life of the 212Po = 208Pb + α (~300 ns)

C. Xu and Z. Ren, Nucl. Phys. ***A753***, 174 (2005); Nucl. Phys. ***A760***, 303 (2005).

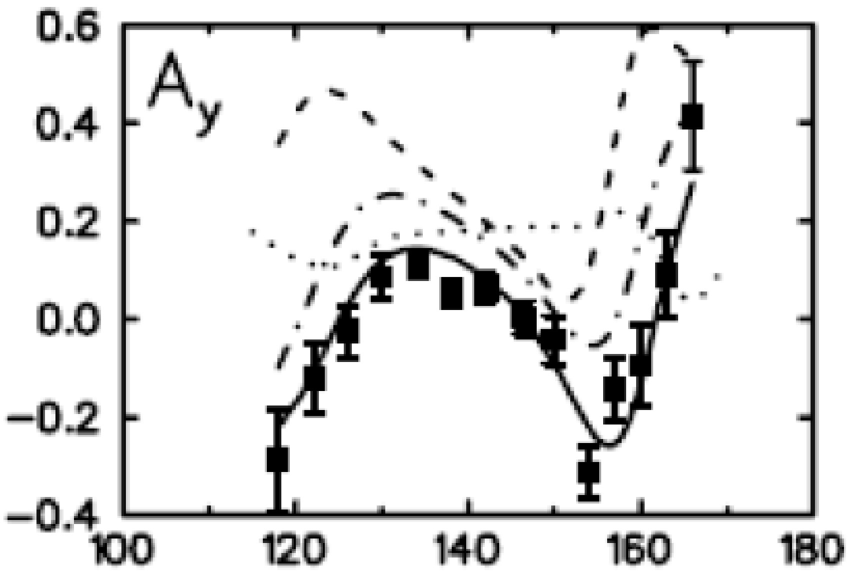
- Microscopic double folded M3Y NN interaction predicts an inverted energy spectrum for 212Po.

J^{π}	E_{expt}(MeV)	E_{cal}(MeV)
0^+	0.000	-0.004
2^+	0.727	-0.067
4^+	1.132	-0.229
6^+	1.355	-0.508
8^+	1.476	-0.930
10^+	1.834	-1.538
12^+	2.702	-2.358
14^+	2.885	-3.437
16^+	-	-4.800
18^+	2.921	-6.477

T. T. Ibrahim, PhD Thesis, (2009)

Why Relativistic Mean Field?

- A relativistic mean field theoretical approach has some attractive attributes
  - Lorentz convariance
  - Natural inclusion of spin phenomenon
  - Self consistance



Analysing for ^{208}Pb($\vec{p}$, 2p) from $2s_{1/2}$ state for incident proton lab energy of 202 MeV and scattering angles (28°, -54,6°).

[G. C. Hillhouse et. al., Phys. Rev. C 68, 034608 (2003)]

- Variations of RMF based models interaction exist, but they all use M3Y effective interaction with RMF theory constructed core & cluster densities.

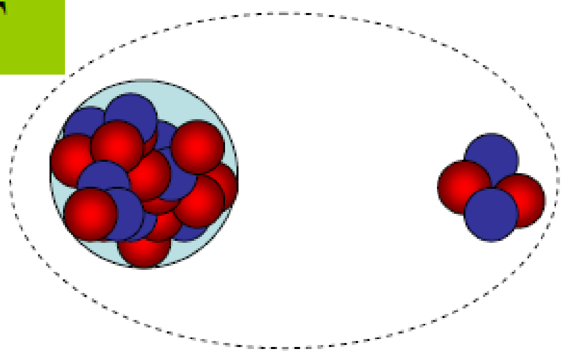
- We propose using Lorentz covariant form of the NN-interaction and fold it with the core and cluster densities obtained from RMF theory.

The Binary Cluster model

- Assume separated core + (preformed) cluster system.

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma}$$

V(r)=Total interaction potential



$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|E - V(r)|}$$

Energy of emitted alpha

$$\Gamma = P\frac{\hbar^2}{2\mu}\frac{exp(-2\int_{r_2}^{r_3} k(r)dr)}{\int_{r_1}^{r_2}[k^{-1}(r)]dr}$$

The Binary Cluster model

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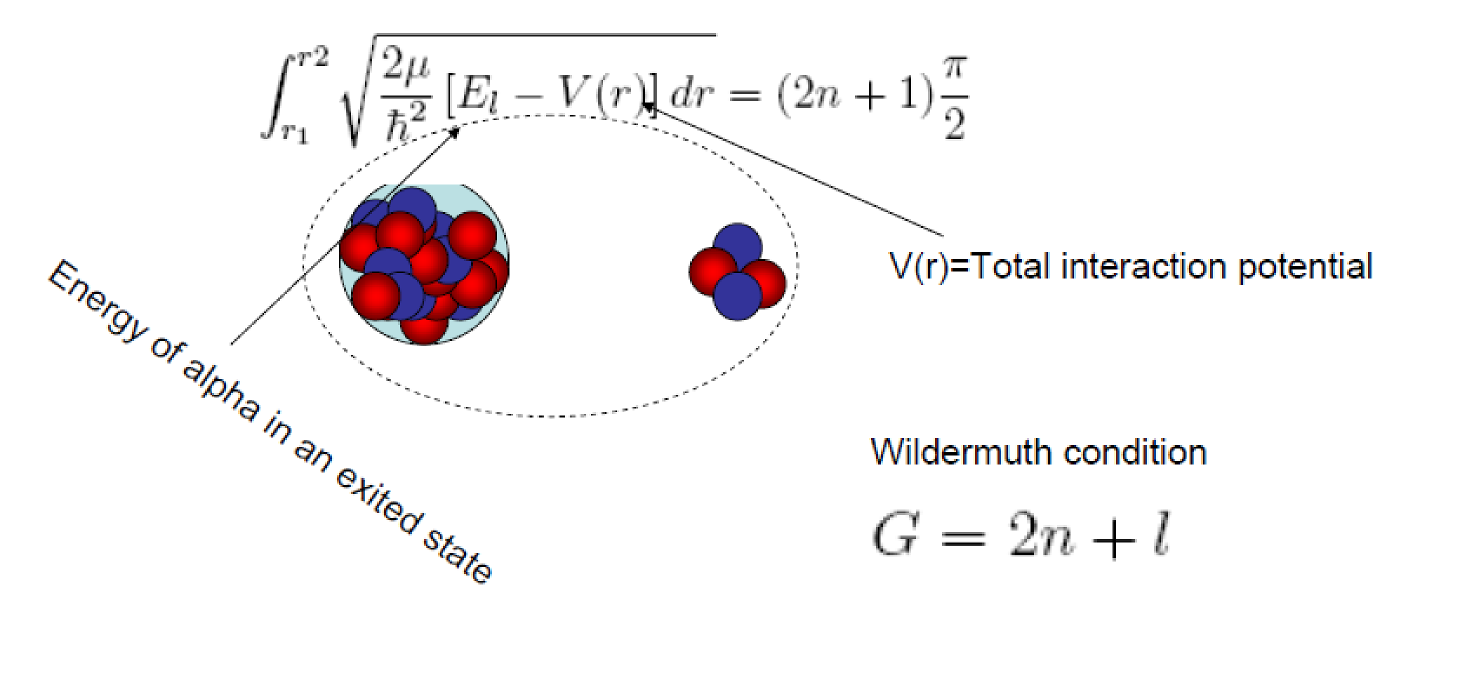

Core-Cluster Potential

- The total core-cluster potential is the sum of the attractive core-cluster term [U(r)], Coulomb interaction and angular momentum dependent term.

- The underlying attractive core-cluster term governs the dynamics of nuclear clustering.

- Saxon-Woods Plus cubic Saxon-Woods potentials contains an additional cubic term which is mixed with the usual Saxon-Woods potential functional form.

$$U(r) = U_0 \left[\frac{x}{1 + exp\left(\frac{r-R}{a}\right)} + \frac{1-x}{1 + exp\left(\frac{r-R}{3a}\right)^3} \right]$$

- From the McNeil, Ray & Wallace single folded we construct double folded “relativistic” core-cluster interaction

$$F = F^S I^a I_b + F^V \gamma^\mu_a \gamma_{\mu b} + F^{PS} \gamma^5_a \gamma_{5b} + F^T \sigma^{\mu\nu}_a \sigma_{\mu\nu b} + F^A \gamma^5_a \gamma^\mu_a \gamma^5_b \gamma_{\mu b}$$

↓

$$U^L(r,\epsilon) = -\frac{4\pi i p}{Mc^2} \int \frac{d^3q}{2\pi}^{3} e^{i\mathbf{q}\cdot\mathbf{r}} F^L(q,\epsilon) \int d^3r' e^{-i\mathbf{q}\cdot\,\mathbf{r}'} \rho^L_1(r') \int d^3r'' e^{-i\mathbf{q}\cdot\,\mathbf{r}''} \rho^L_2(r'')$$

Core & cluster densities

Relativistic Mean Field Approach

- Using the effective Lagrangian from the Walecka model one obtains the Dirac relationship as the dynamical equation.

$$\hat{H}\psi(\mathbf{r}) = \left(i\alpha\cdot\nabla - g_v\gamma^0 V^0(r) + \beta[M - g_s\phi(r)]\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Field operator

$$\hat{\psi}(\mathbf{r}) = \sum_{\Lambda}\left[A_\Lambda U_\Lambda(\mathbf{r}) + B_\Lambda^\dagger V_\Lambda(\mathbf{r})\right]$$

Positive energy solution

$$U_\Lambda \equiv U_{njlmt}(\mathbf{r}) = \begin{pmatrix} i\,[G_{njlt}(r)/r]\,\Phi_{jlm} \\ [F_{njlt}(r)/r]\,\Phi_{jl+1m} \end{pmatrix}\varsigma_t$$

- Baryon densities are constructed from the positive energy solution to the Dirac equation:

$$\left.\begin{matrix}\rho_B(\mathbf{r}) \\ \rho_s(\mathbf{r})\end{matrix}\right\} = \sum_{\Lambda} \bar{U}_{\Lambda}(\mathbf{r}) \begin{pmatrix}\gamma^0 \\ I\end{pmatrix} U_{\Lambda}(\mathbf{r})$$

Model Predictions

- We focused on clustering in 212Po.
- Compare the predictive power of SW3 & RMF core-cluster model potentials.
- Model $T_{1/2}$ of ground state and energy spectra of the positive parity states.

- For RMFT based model

m_ω	m_ρ	m_s	$g^2_{\ \omega}$	$g^2_{\ \rho}$	$g^2_{\ s}$
738 MeV	770 MeV	520 MeV	190.4	65.23	109.6

$T_{1/2}$ Prediction

$T_{1/2}$ (Exp)	$T_{1/2}$ (BMP)	$T_{1/2}$ (RMFT)
300 ns	348 ns	299.6 ns

Positive parity states

J^{π}	E (Exp) MeV	E (BMP) MeV	E (M3Y) MeV	E (RMFT) MeV
0^+	0.000	(0.495)	-0.004	0.203
2^+	0.727	0.659	-0.067	0.421
4^+	1.132	0.948	-0.229	0.699
6^+	1.355	1.318	-0.508	0.857
8^+	1.476	1.730	-0.930	1.085
10^+	1.834	2.145	-1.538	1.319
12^+	2.702	2.519	-2.358	1.553
14^+	2.885	2.805	-3.437	1.787
16^+	-------	2.941	-4.800	2.021
18^+	2.921	2.841	-6.477	2.255

Conclusions

- RMFT approach seems comparable to Experiment and BMP in the predictions of $T_{1/2}$ and E_L for 212Po

- Need to extend the test of other nuclei and observable quantities, ALAS, …

The way forward

- Model extensions to include:
  - Exited core + cluster system (Rel. Hartree Bogoliubov + BCS)
  - Replacing MRW representation of NN interaction with Rel. Love Franey representation with complete sets of NN parameters
  - Look at Heavier exotic cluster systems

English: **Thank you**
Afrikaans: Dankie
IsiNdebele: Ngiyathokoza
Sesotho: Ke a leboha
Northern Sotho: Ke a leboga
Setswana: Ke a leboga
SiSwati: Siyabonga
Xitsonga: Inkomu
Tshivenda: Ndo livhuwa / Ro livhuwa
IsiXhosa: Enkosi
IsiZulu: Ngiyabonga