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Models and Methods in Few- and Many-Body Systems
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Nuclear Structure and Double Beta Decay

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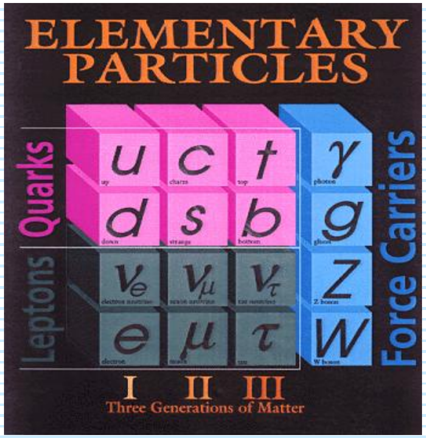
Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics

OUTLINE

- *Introduction*
- *$0\nu\beta\beta$-decay NMEs:* ***Current status***
- *Anatomy of the $0\nu\beta\beta$-decay*
- *$2\nu\beta\beta$-decay NMEs*
- *On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay NMEs*
- ***Co-existence*** *of few mechanisms of the $0\nu\beta\beta$-decay*
- *Conclusion*

Presented results obtained in collaboration with
Amand Faesler, V. Rodin, M. Saleh (Tuebingen U.),
P. Vogel (Caltech), J. Engel (North Caroline U.)
G. Pantis (U. Ioannina)
E. Moya de Guerra, P. Sarrigurren, O. Moreno (Madrid U.),
…



Standard Model Lepton Universality

Particle	Symbol	Anti − p.	mass [MeV]	L_e	L_μ	L_τ	life − time [s]
electron	e^-	e^+	0.511	1	0	0	stable
el.neutrino	ν_e	$\overline{\nu}_e$	$< 2.2\ 10^{-6}$	1	0	0	stable
muon	μ^-	μ^+	105.6	0	1	0	$2.2\ 10^{-6}$
muon neutr.	ν_μ	$\overline{\nu}_\mu$	< 0.19	0	1	0	stable
tau	τ^-	τ^+	1777.	0	0	1	$2.9\ 10^{-13}$
tau neutrino	ν_τ	$\overline{\nu}_\tau$	< 18.2	0	0	1	stable

Lepton Family Number Violation

NEW PHYSICS
massive neutrinos, SUSY...

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}$, $\overline{\nu}_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*observed*
$\mu^+ \to e^+ + \gamma$	$R \le 1.2 \times 10^{-11}$
$\mu^+ \to e^+ + e^- + e^+$	$R \le 1.0 \times 10^{-12}$
$K^+ \to \pi^+ + e^- + \mu^+$	$R \le 4.7 \times 10^{-12}$
$\tau^- \to e^- + \mu^+ + \mu^-$	$R \le 1.8 \times 10^{-6}$
$Z^0 \to e^\pm + \mu^\mp$	$R \le 1.7 \times 10^{-6}$
$\mu_b^- + (A,Z) \to (A,Z) + e^-$	$R \le 1.2 \times 10^{-11}$

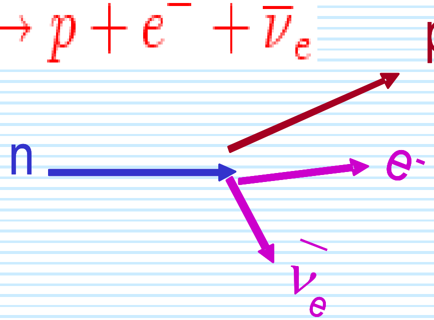
$\nu_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*not observed*
$K^+ \to \pi^- + e^+ + \mu^+$	$R \le 5 \times 10^{-10}$
$\tau^- \to \pi^- + \pi^+ + e^+$	$R \le 1.9 \times 10^{-6}$
$W^- + W^- \to e^- + e^-$	
$(A,Z) \to (A,Z+2) + e^- + e^-$	$T^{0\nu} \ge 1.9 \times 10^{-25}$
$\mu_b^- + (A,Z) \to (A,Z-2) + e^+$	$R \le 3.6 \times 10^{-11}$
$e^- + e^- \to \pi^- + \pi^-$	?

ν oscillations proposed by Bruno Pontecorvo in Dubna in 1957

1934 Fermi theory of beta decay

Fermi, Z. Physik 88 (1934) 161

$n \rightarrow p + e^- + \overline{\nu}_e$



Fermi 4-fermion contact interaction, Lagrangian of interaction (in analogy with electrodynamics):

$$\mathcal{L}(x) = -\frac{G_F}{\sqrt{2}}\left[\overline{\phi}_p(x)\gamma^\mu\phi_n(x)\right]\left[\overline{\phi}_e(x)\gamma^\mu\phi_\nu(x)\right]$$

G_F = Fermi coupling constant = (1.16637±0.000001) 10^{-5} GeV^{-2}

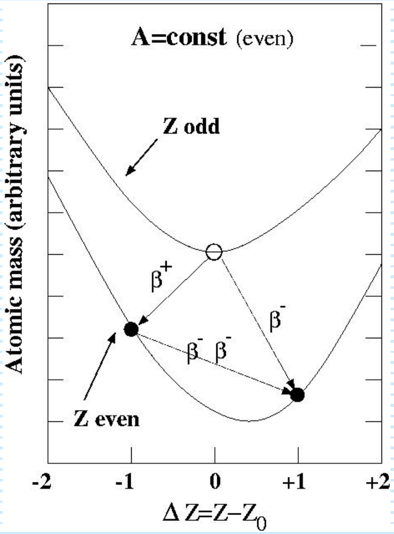
$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

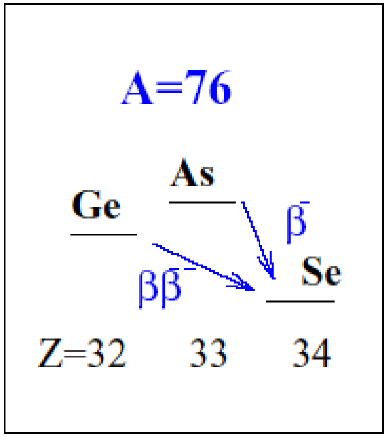
Eugene Wigner

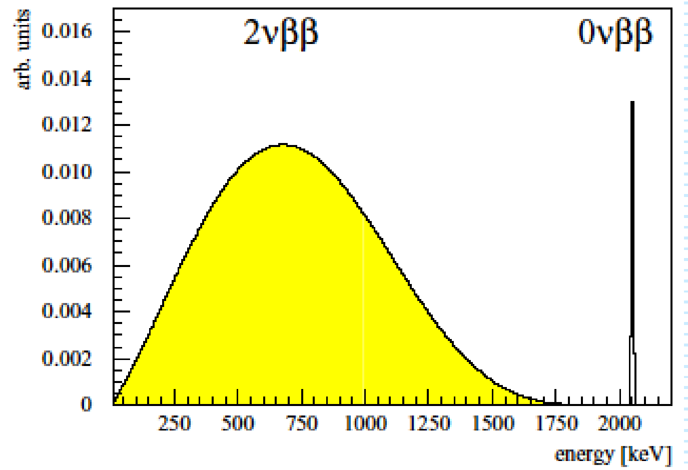
Maria-Goeppert Mayer

1935
Q-value about 10 MeV
$T_{1/2} \approx 10^{17}$ years

Double Beta Decay





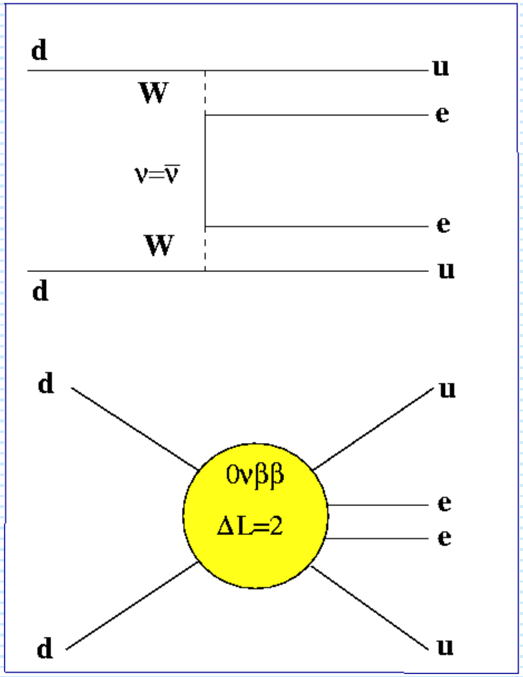


$$(A, Z) \to (A, Z+2) + 2e^- + 2\overline{\nu}_e$$

Observed for 10 isotopes: **^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo. ^{116}Cd. ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U,** **$T_{1/2} \approx 10^{18}$-10^{24} years**

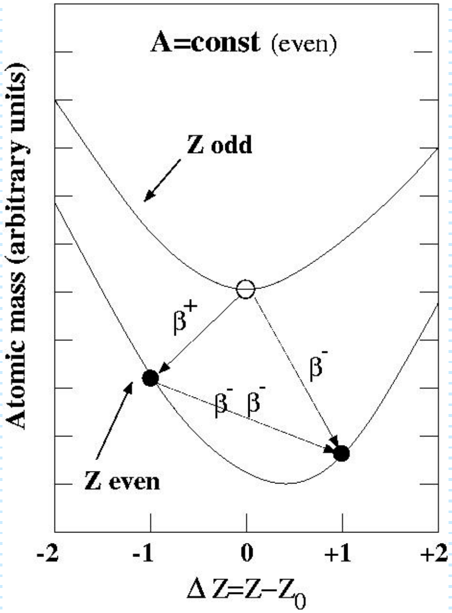
1967: ^{130}Te, Kirsten et al, Takaoka et al, (geochemical)
1987: ^{82}Se, Moe et al. (direct observation)
2006: ^{100}Mo, NEMO 3 coll. ~ 300 00 events

$$(A, Z) \to (A, Z+2) + 2e^-$$



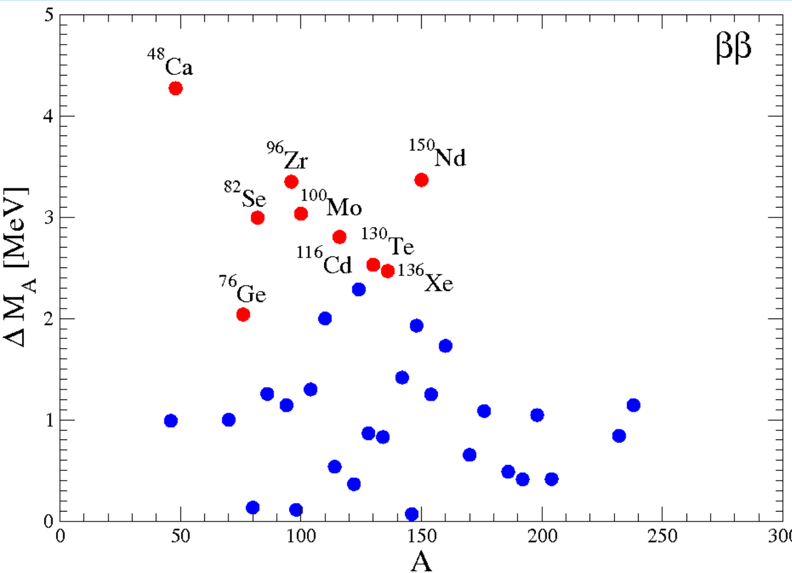
SM forbidden ,not observed yet: $T_{1/2}$ (^{76}Ge)>10^{25} years

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

transition	$G^{01}(E_0, Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\|M^{0\nu}\|^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?



The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \ ,$$

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \ m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

CP-violating phases

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

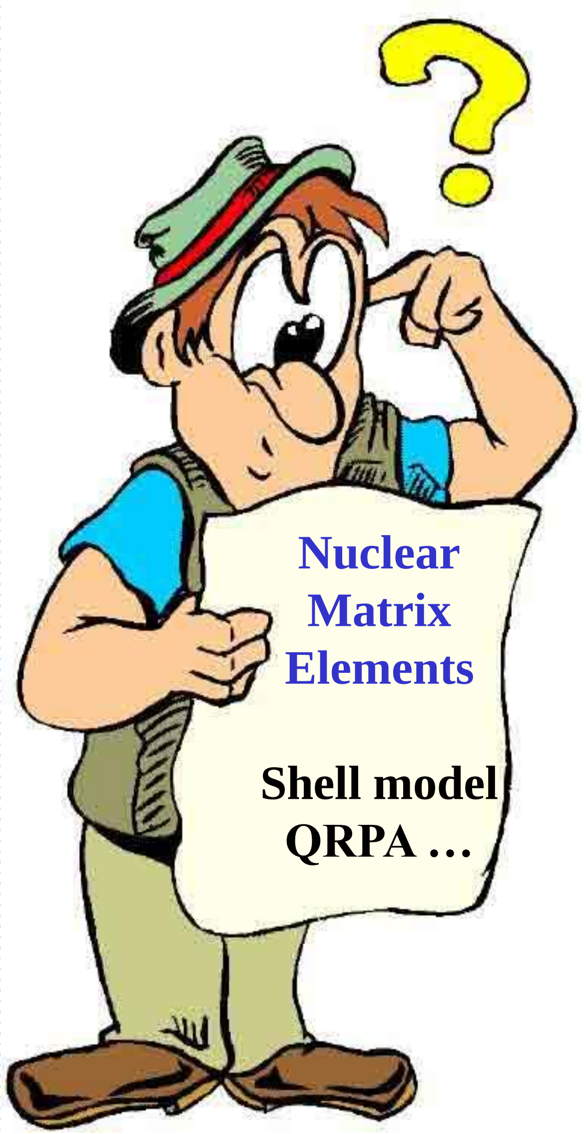
The $0\nu\beta\beta$-decay NMEs

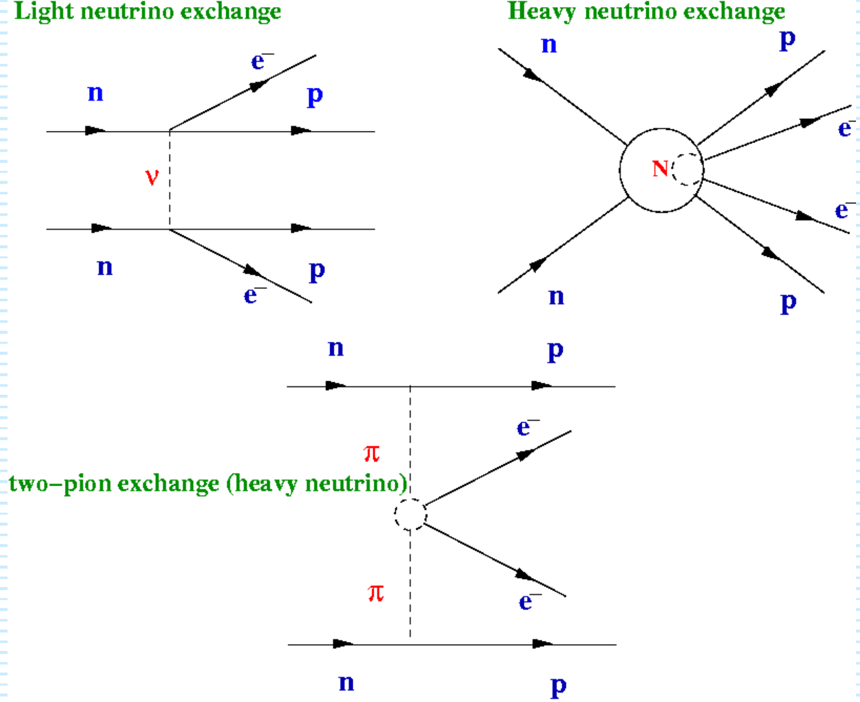
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge the quality of the result.

Calculation of NMEs is a complex task





$$\left(T^{0\nu}_{1/2}\right)^{-1} = \eta^{LNV}\, G^{0\nu}\, \left|M^{0\nu}\right|^2$$

NME´s: which mechanism, which transition?

- **Medium and heavy open shell nuclei with a complicated nuclear structure**
- **The construction of complete set of the states of the intermediate nucleus is needed**
- **Many-body problem ⇒ approximations needed**
- **Nuclear structure input has to be fixed**

The 0νββ-decay NME (light ν exchange mech.)

The 0νββ-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)qdq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors: finite nucleon size

$$\begin{aligned} h_F &= g_V^2(q^2) \\ h_{GT} &= g_A^2\left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \\ h_T &= g_A^2\left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \end{aligned}$$

Induced pseudoscalar coupling (pion exchange)

$$M_{K=F,GT,T} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$

$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$

$$\times \langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_J || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_J || 0_i^+\rangle$$

Jastrow f. s.r.c.

J^π = $0^+, 1^+, 2^+$... $0^-, 1^-, 2^-$...

Nuclear many-body calculations

Model space

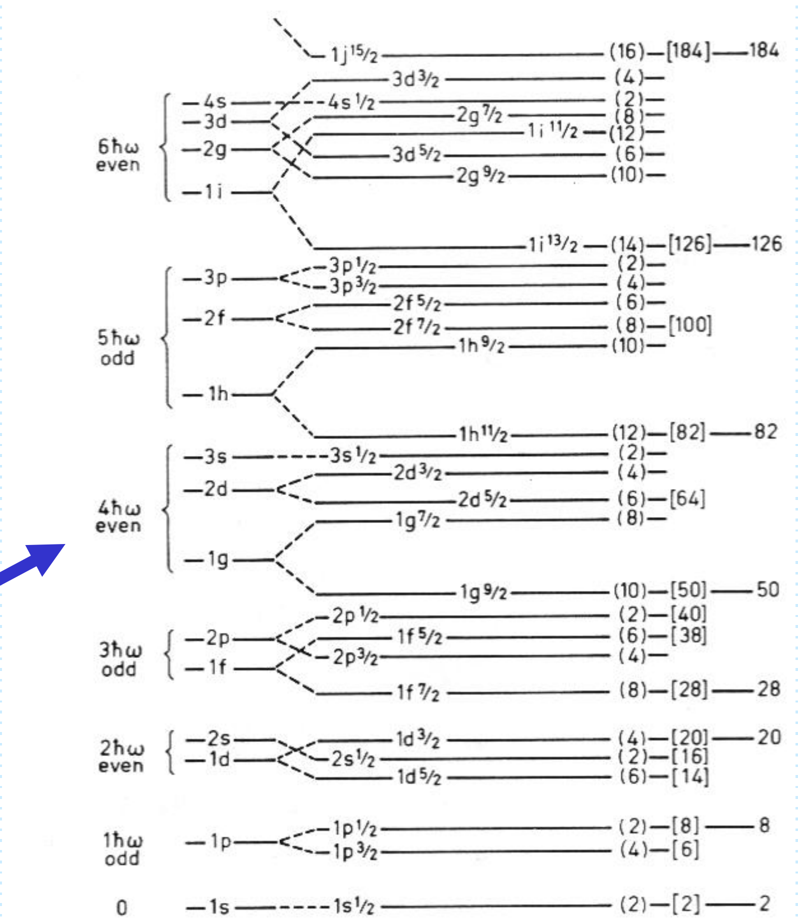
- *Start with the many-body Hamiltonian*

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V_{NN}(\vec{r}_i - \vec{r}_j)$$

- *Introduce a mean-field U to yield basis*

$$H = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + U(r_i) \right) + \underbrace{\sum_{i<j} V_{NN}(\vec{r}_i - \vec{r}_j) - \sum_i U(r_i)}_{\text{Residual interaction}}$$

Residual interaction



- *The mean field determines the shell structure*
- *In effect, nuclear-structure calculations rely on perturbation theory*

The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

Nuclear structure approaches

In **NSM** **(Madrid-Strassbourg group)** a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$-decay calculations

In **QRPA** **(Tuebingen-Caltech-Bratislava and Jyvaskula-La Plata groups)** a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more $0\nu\beta\beta$–decay calculations

In **IBM** **(Iachello, Barea)** the low lying states of the nucleus are modeled in terms of bosons. The bosons have either L=0 (s boson) or L=2 (d boson). The bosons can interact through one and to body forces giving rise to bosonic wave functions.

In **PHFB** **(India/Mexico groups)** w.f. of good angular momentum are obtained by making projection on the axially symmetric intrinsic HFB states. Nuclear Hamiltonian contains only quadrupole interaction.

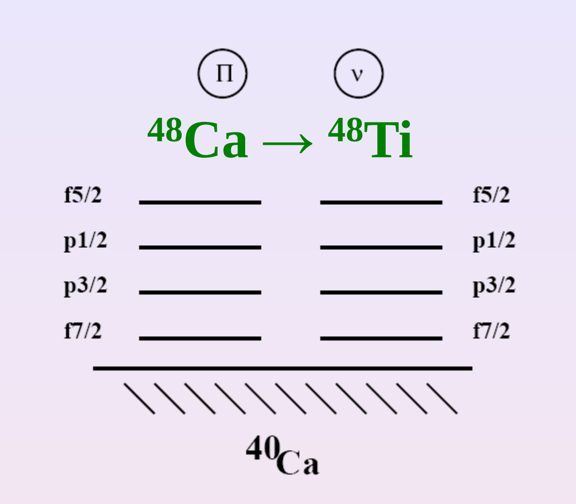
Differences: i) mean field; ii) residual interaction; iii) size of the model space
iv) many-body approximation

Nuclear Shell Model

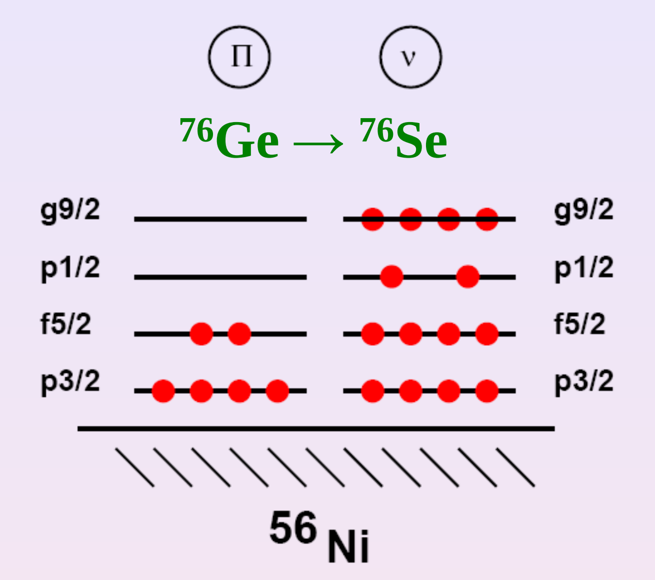
$$H = \sum_a \varepsilon_a a_a^+ a_a - \sum_{abcd} \frac{\langle j_a j_b; JT|V|j_c j_d; JT\rangle_A}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \left[\left[a_a^+ \otimes a_b^+\right]^{JT} \otimes \left[\tilde{a}_c \otimes \tilde{a}_d\right]^{JT} \right]^{00}_{00}$$

- Define a valence space
- Derive an effective interaction $H\,\Psi = E\,\Psi \rightarrow H_{eff}\,\Psi_{eff} = E\,\Psi_{eff}$
- Build and diagonalize Hamiltonian matrix (10^{10})
- Transition operator $< \Psi_{eff} \,|\, O_{eff} \,|\, \Psi_{eff} >$
- Phenomenological input:

  Energy of states, systematics of B(E2) and GT transitions (quenching f.)



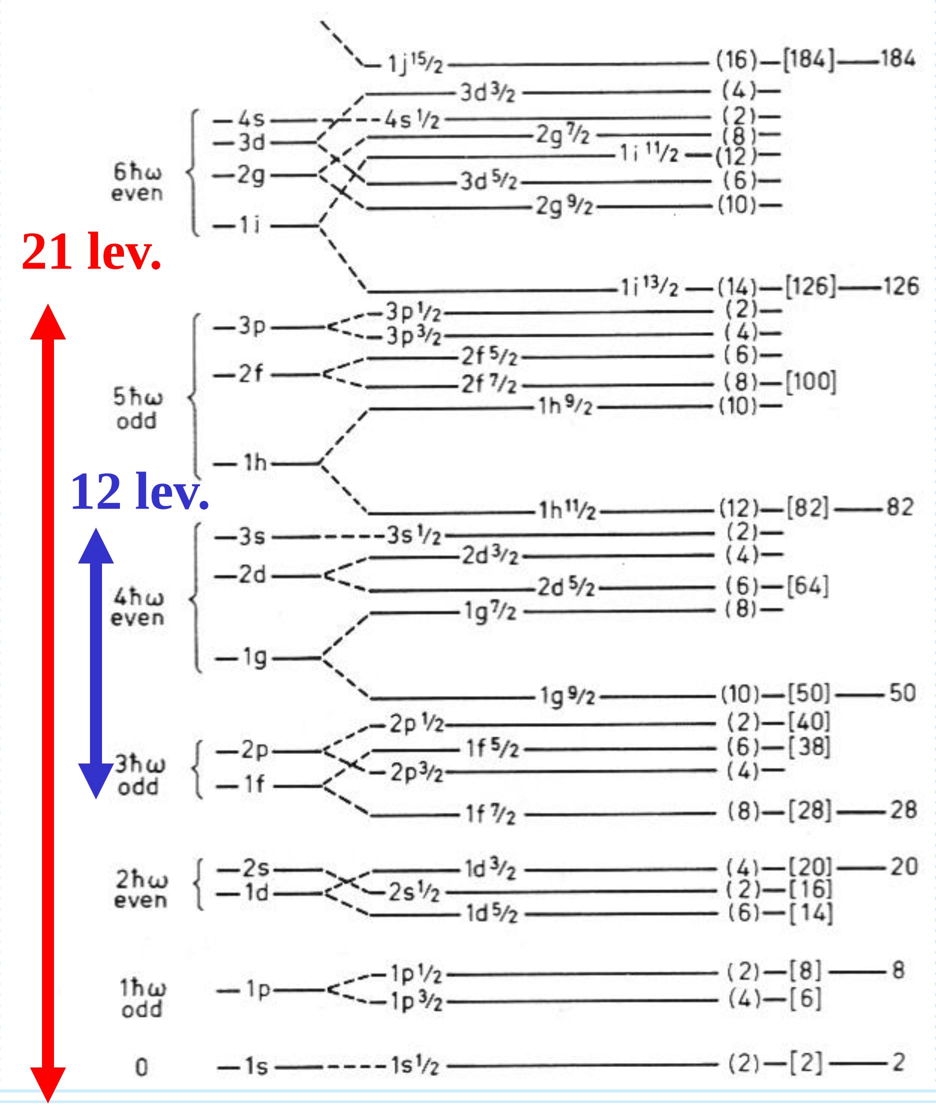
Small calculations



$^{76}Se_{42}$ in the valence

6 protons and 14 neutrons

Quasiparticle Random Phase Approximation (QRPA) and its variants



- **Large model space (up 23 s.p.l, ^{150}Nd – 60 active prot. and 90 neut.)**
- **Spin-orbit partners included**
- **Possibility to describe all multipolarities of the intermed. nucl. J^{π} ($\pi=\pm1$, J=0…9)**

$$H = H_0 + g_{ph} V_{ph} + g_{pp} V_{pp}$$

quasiparticle mean field | **Residual interaction**

Only Bratislava-Tuebingen group

Realistic NN-interactions used in the QRPA calculations

Modern *(phase-shift equivalent)* *NN* potentials

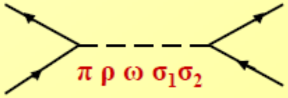
Nijmegen I - (P_D = 5.66%) - 41 parameters - χ^2/N_{data} = 1.03

Nijmegen II - (P_D = 5.64%) - 47 parameters - χ^2/N_{data} = 1.03

Argonne V_{18} - (P_D = 5.76%) - 40 parameters - χ^2/N_{data} = 1.09

CD Bonn - (P_D = 4.85%) - 43 parameters - χ^2/N_{data} = 1.02

based upon the OBE model



(1999 *NN* Database: 5990 *pp* and *np* scattering data)

Brueckner G-matrices from Tuebingen (H. Muether group)

Renormalization of the *NN* interaction

Difficulty in the derivation of V_{eff} from any modern *NN* potential: existence of a strong repulsive core which prevents its direct use in nuclear structure calculations.

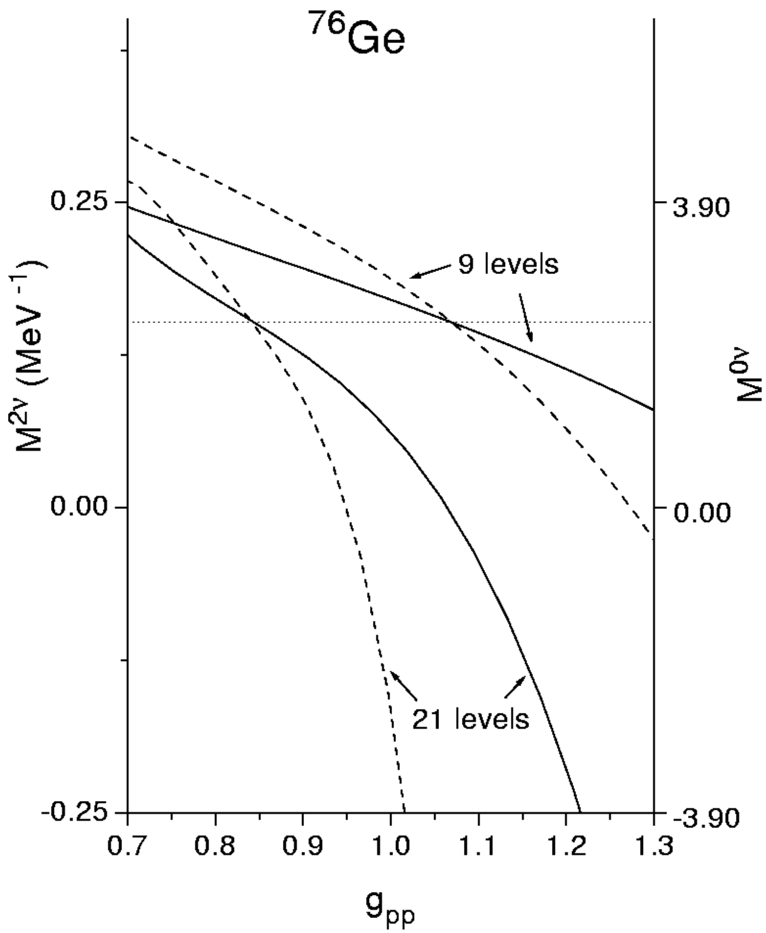
Traditional approach to this problem: Brueckner *G*-matrix method. The *G* matrix is model-space dependent as well as energy dependent.

Bethe-Goldstone equation

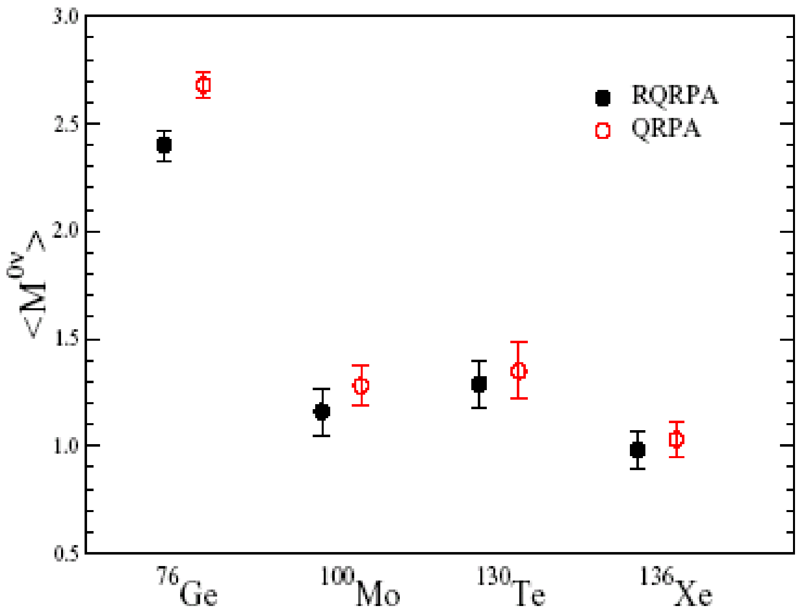
$$G = V + V \frac{Q}{W - H_0 + i\varepsilon} G$$

The 0νββ-decay NME: g_{pp} fixed to 2νββ-decay

Each point: (3 basis sets) x (3 forces) = 9 values



By adjusting of g_{pp} to 2νββ-decay half-life the dependence of the 0νββ-decay NME on other things that are not a priori fixed is essentially removed



Rodin, Faessler, Šimkovic, Vogel,
Phys. Rev. C 68, 044302 (2003)

The Interacting Boson Model¶

- The low-lying states of the nucleus, composed by n and z valence nucleons, are modeled in terms of (n+z)/2 bosons.
- The bosons have either L = 0 (*s* boson) or L = 2 (*d* boson).
- The bosons can interact through one-body and two-body forces giving rise to bosonic wave functions.
- Any observable can be calculated using these wave functions provided that the relevant operator is employed.

¶ F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987

Projected Hartree-Fock-Bogoliubov Model

PHFB Model

States of good angular momentum J

$$\left|\Psi_M^J\right\rangle = \frac{2J+1}{8\pi^2 a_J}\int d\Omega D_{MK}^J(\Omega)\hat{R}(\Omega)\left|\Phi_K\right\rangle$$

Axially symmetric HFB intrinsic state

$$\left|\Phi_0\right\rangle = \prod_{im}\left(u_{im} + v_{im} b_{im}^{+} b_{i\bar{m}}^{+}\right)$$

where

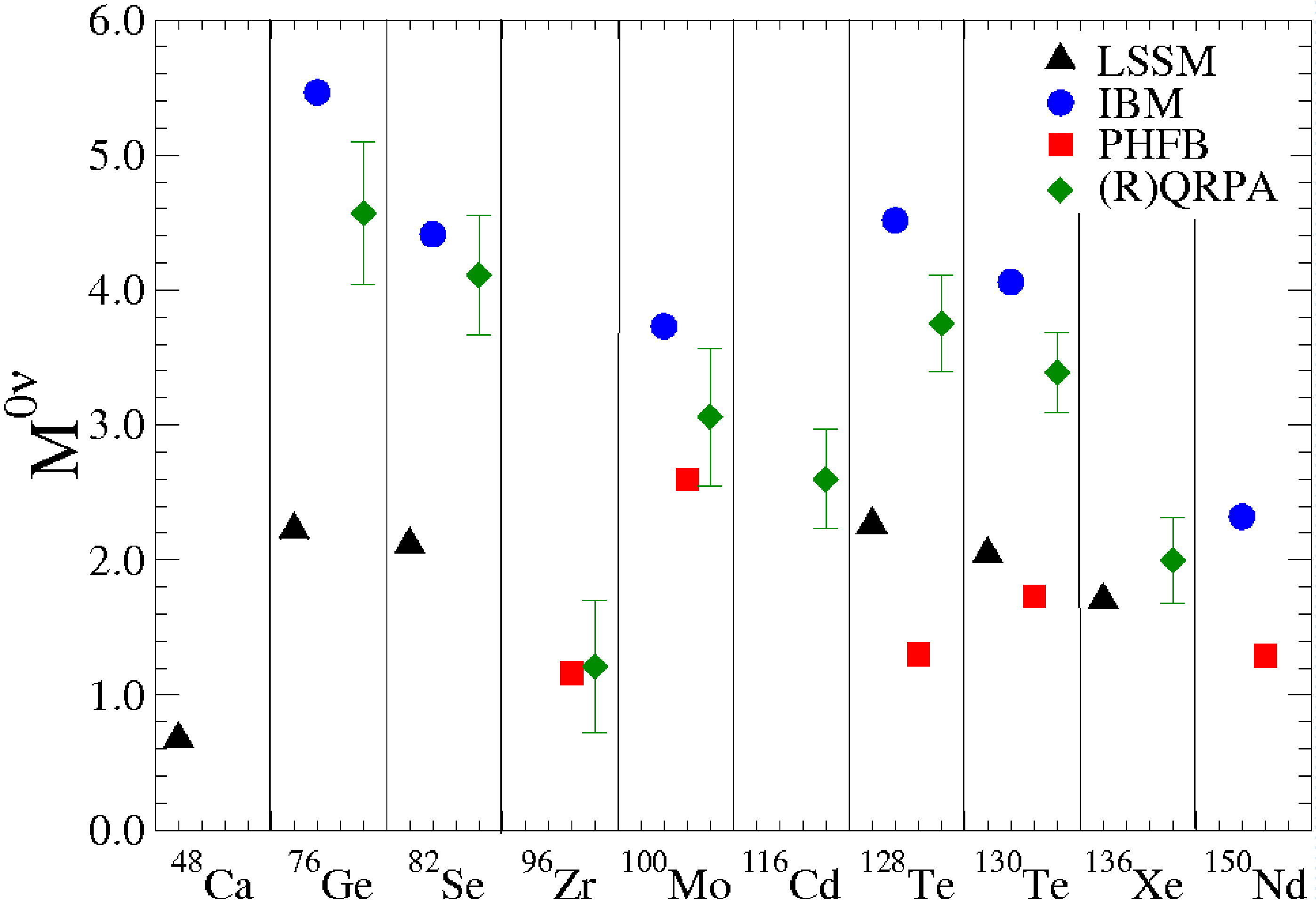
$$b_{im}^{+} = \sum_{m} C_{i\alpha m} a_{im}^{+} \qquad b_{i\bar{m}}^{+} = \sum_{m} (-1)^{l+j-m} C_{i\alpha m} a_{i-m}^{+}$$

Hamiltonian:

$$H = H_{sp} + V(P) + \zeta_{qq} V(QQ)$$

Only quadrupole interaction, GT interaction is missing

The 0νββ-decay NMEs (Status:2010)

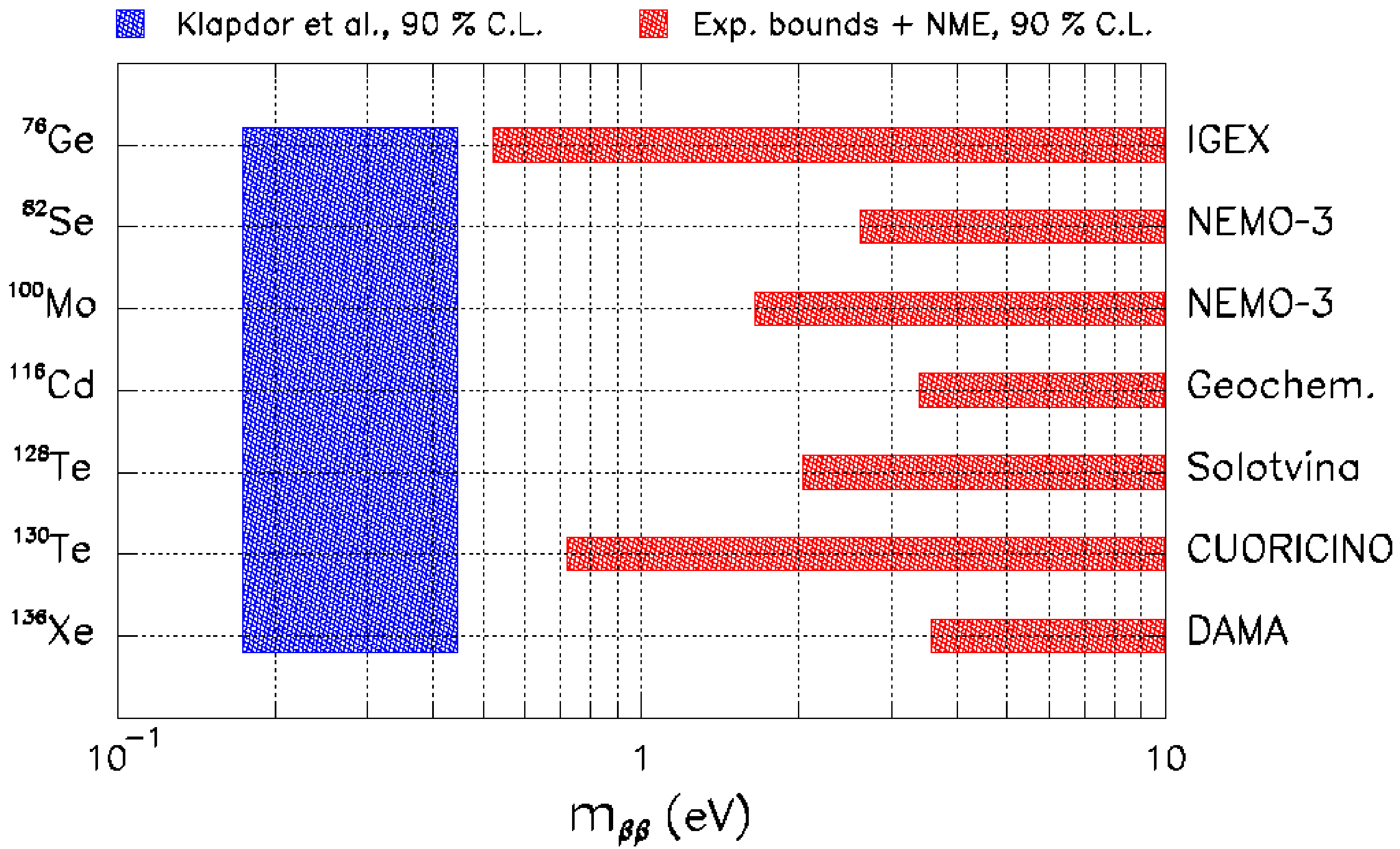


Nobody is perfect:

LSSM (small m.s., negative parity states)
PHFB (GT force neglected)
IBM (Hamiltonian truncated)
(R)QRPA (g.s. correlations not accurate enough)

Dominance of light ν mass mechanism of the 0νββ-decay

$$|m_{\beta\beta}| = \frac{m_e}{|M_1^{0\nu}|\sqrt{T_1\ G_1}} = \frac{m_e}{|M_2^{0\nu}|\sqrt{T_2\ G_2}} = \cdots$$



Faessler, Fogli, Lisi, Rodin, Rotunno, F.Š., PRD 79, 053001 (2009)

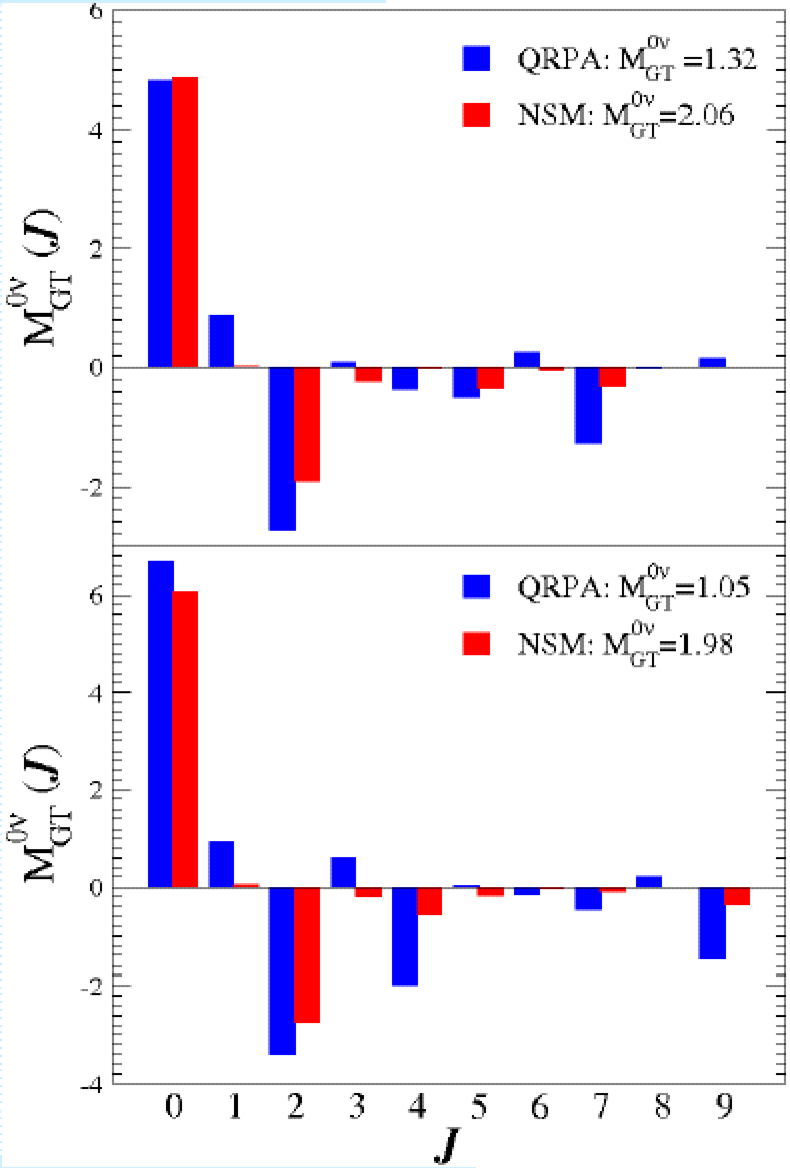
Anatomy of the 0νββ-decay NMEs

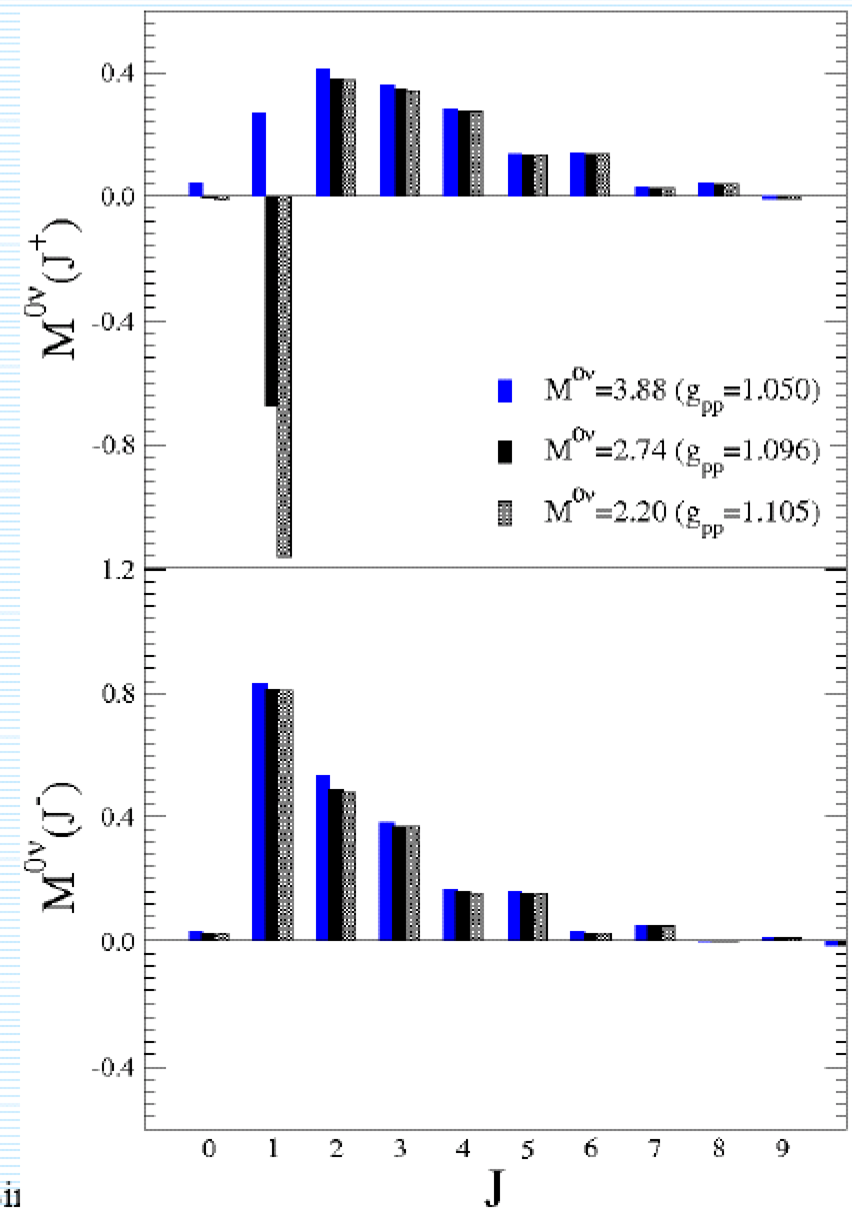
Dominance of Pairing mode (J=0)

Two types of decompositions (Particle-particle) and (particle-hole)

Sensitivity to g_{pp} of 1^+ state

$$\langle p(1), p'(2); \mathcal{J} \,||\, f(r_{12}) O_K f(r_{12}) \,||\, n(1), n'(2); \mathcal{J} \rangle$$



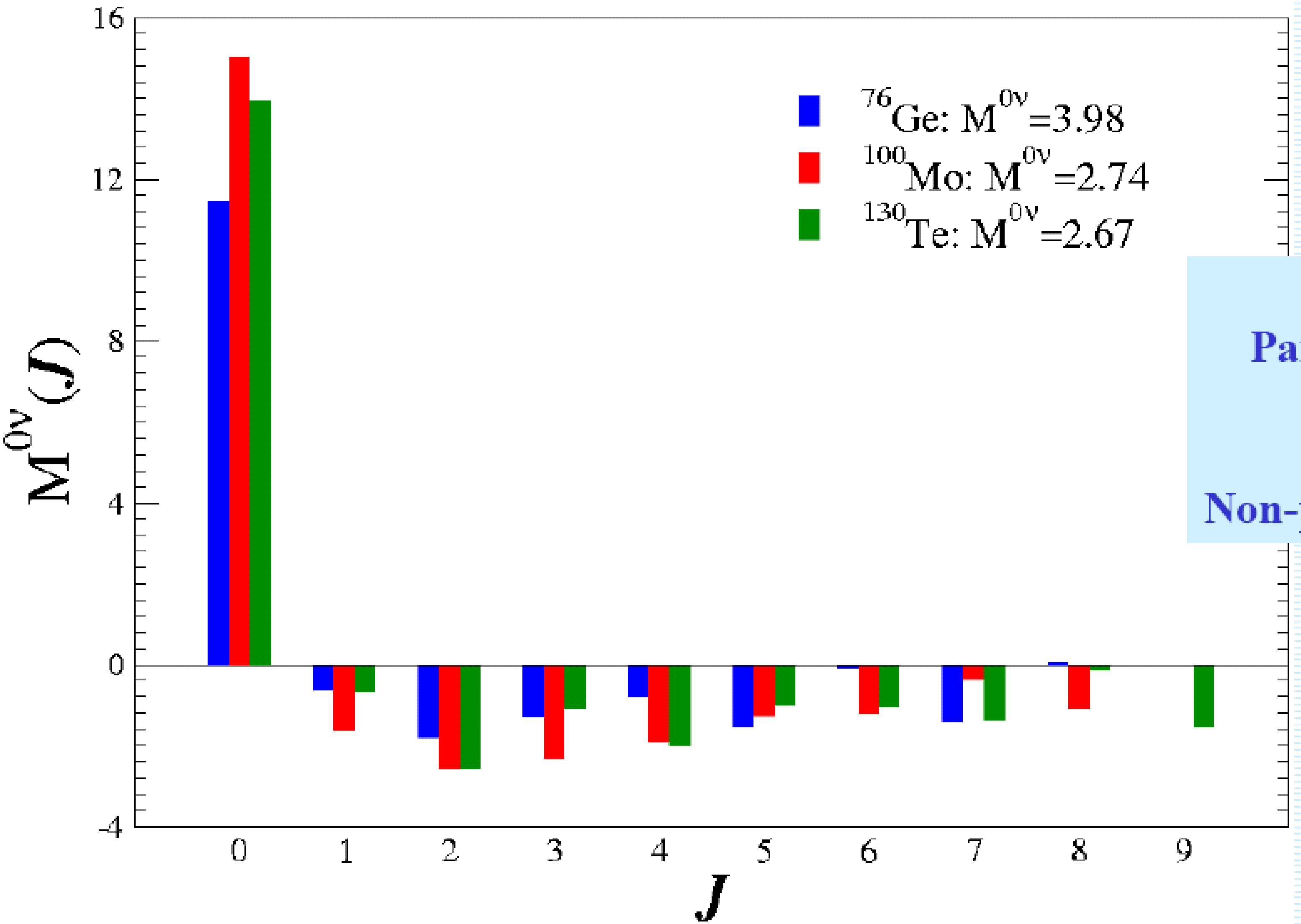


$$\langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_{\mathcal{J}} || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_{\mathcal{J}} || 0_i^+ \rangle$$

A comparison with NSM for the same model space

Decomposition in in pp and nn channels

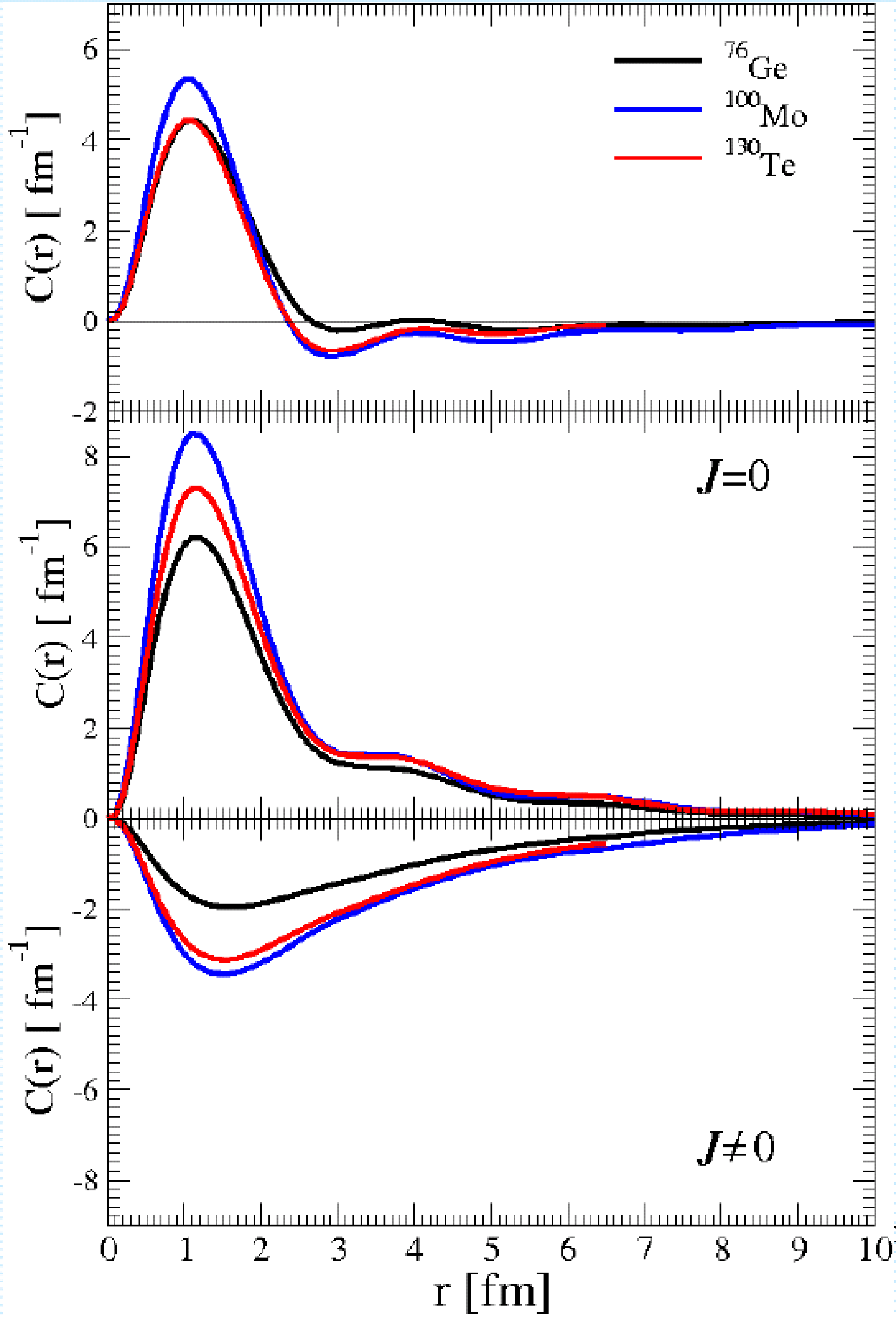
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle$$



J=0
Pairing mode

J≠0
Non-pairing mode

F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)



nkovic

r-dependence of the 0νββ-decay NME

The radial dependence of M^{0n} for the three indicated nuclei. The contributions summed over all components shown in the upper panel. The `pairing' *J* = 0 and `broken pairs' *J* ≠ 0 parts are shown separately below.

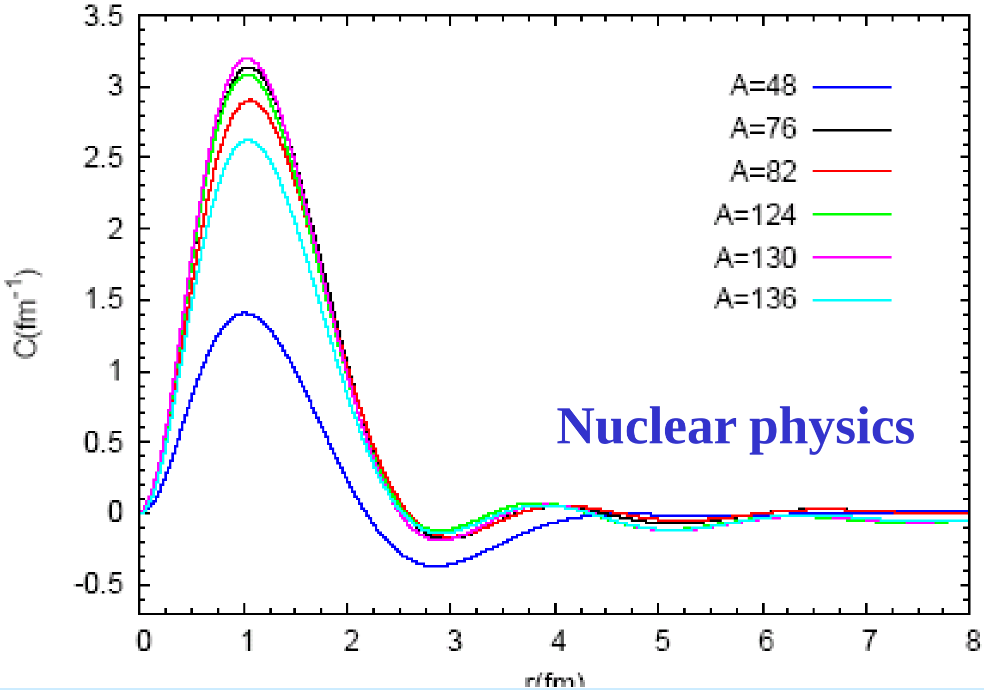
Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* and large values of *q* are quite important.

QRPA

F.Š, Faessler, Rodin, Vogel, Engel
PRC 77, 045503 (2008)

Large Scale Shell Model

Menendez, Poves, Caurier, Nowacki,
Arxive:0901.3760 [nucl-th]

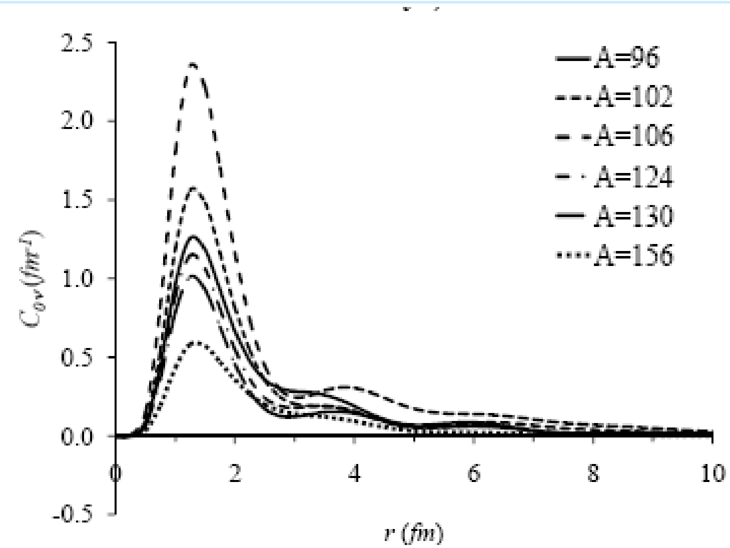


Nuclear physics

Nucleon physics

PHFB

P.Rath, R. Chandra, K. Chaturverdi, P.Raina, J.G. Hirsch, to be published in PRC



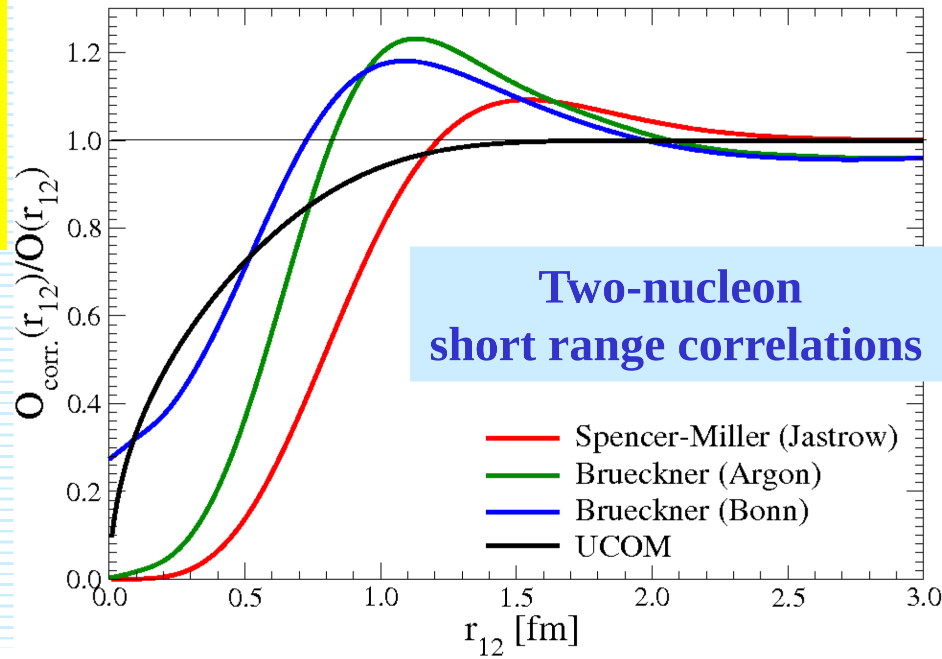
A consistent approach for the 0νββ-decay (pairing, s.r.c, g.s.c. calculated with the same NN potential- BonnCD, Argon)

Neutrino potential: I(r)/r

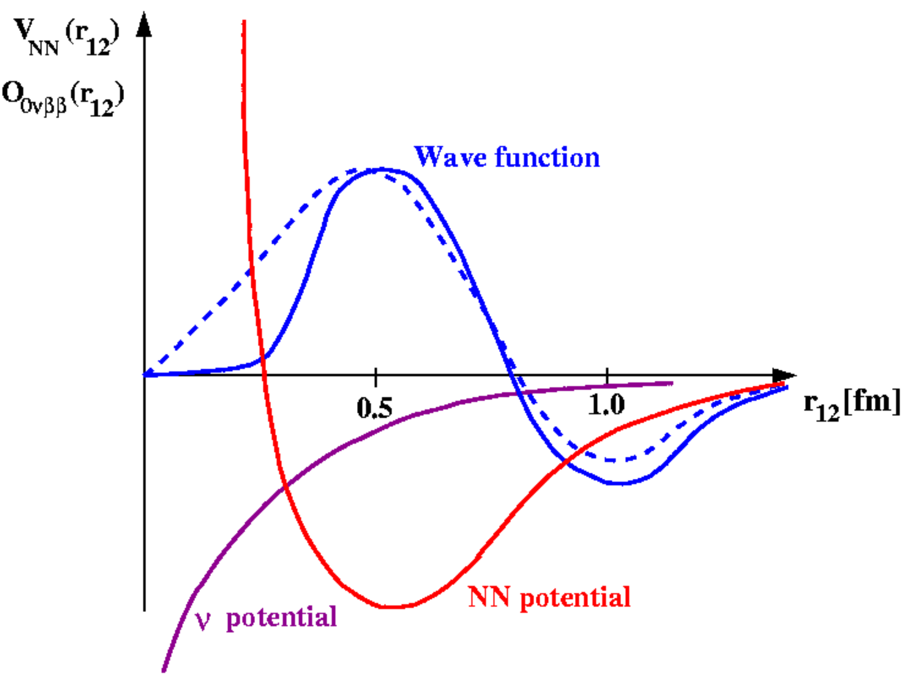
$$I(r) = \frac{2}{\pi}\int_0^\infty \frac{\sin(qr)}{(q+E_{aver})}\frac{dq}{(1+q^2/E_{cut}^2)^4}$$

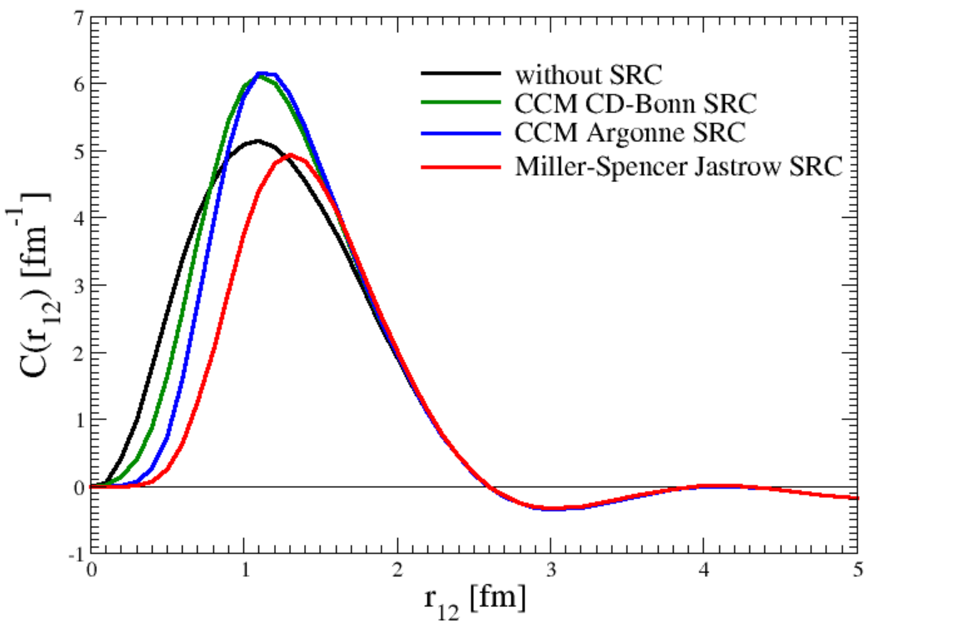
$$|\Psi>_{corr.} = f(r_{12})\,|\Psi>$$

$$O_{corr.}(r_{12}) = f(r_{12})O(r_{12})f(r_{12})$$

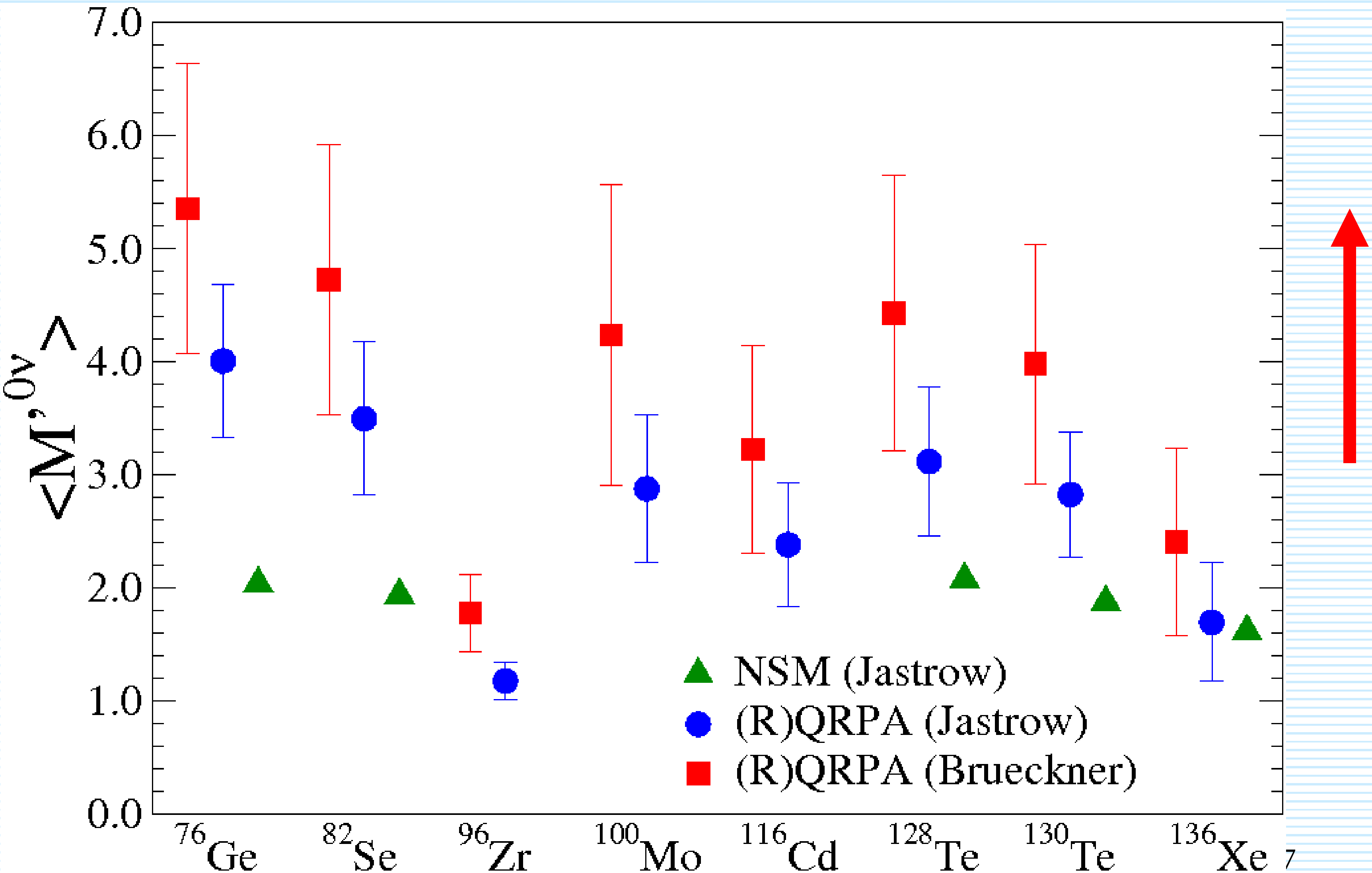


Nucleon–Nucleon Potential





The value of the 0νββ-decay NME calculated with consistent treatment of s.r.c. is increased



F.Š., Faessler, Muether, Rodin, Stauf, PRC 79, 055501 (2009)

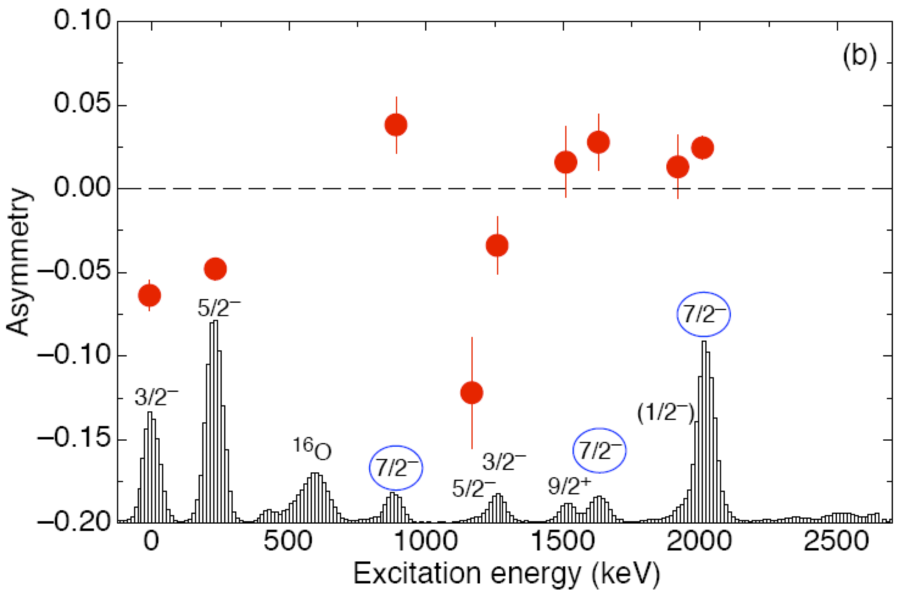
Constraining the $0\nu\beta\beta$-decay NMEs

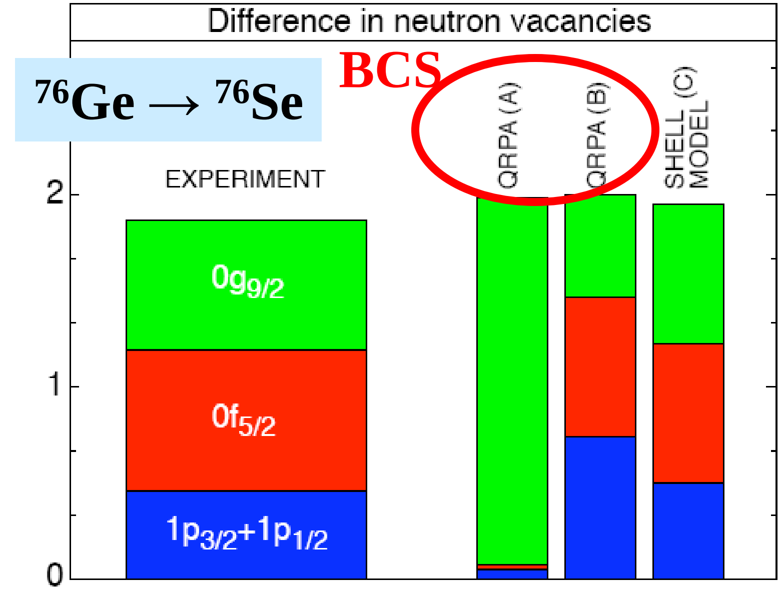
Nucleons that change from neutrons to protons are valence neutrons

Constraining the mean field with proton, neutron removing transfer reaction

Schiffer et al., PRL 100, 112501 (2008)

$$n_j^{exp} = \langle 0^+_{init} | \Sigma_m c^+_{j,m} c_{j,m} | 0^+_{init} \rangle$$





Adjusted WS mean field

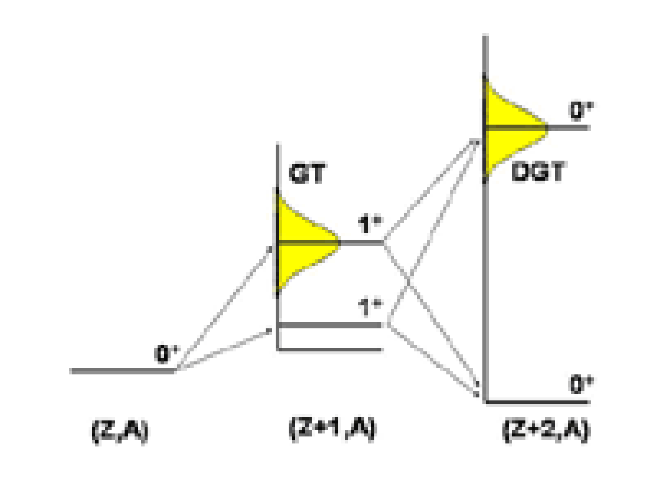
	^{76}Ge				^{76}Se			
neut.	BCS	Q	S	exp	BCS	Q	S	exp
p	5.65	5.27	4.64	4.9±0.2	5.57	5.05	4.12	4.4±0.2
$f_{5/2}$	5.54	5.12	4.34	4.6±0.4	5.53	5.00	3.63	3.8±0.4
$f_{7/2}$	7.91	7.67	7.62	-	7.90	7.54	7.37	-
$s_{1/2}$	0.01	0.05	0.07	-	0.01	0.04	0.08	-
$d_{3/2}$	0.03	0.14	0.15	-	0.02	0.14	0.16	-
$d_{5/2}$	0.09	0.30	0.36	-	0.07	0.27	0.39	-
$g_{7/2}$	0.14	0.53	0.48	-	0.12	0.56	0.58	-
$g_{9/2}$	4.63	4.78	6.35	6.5±0.3	2.78	3.55	5.66	5.8±0.3
prot.								
p	2.23	2.34	1.75	1.77±0.15	2.77	2.76	2.28	2.08±0.15
$f_{5/2}$	1.61	2.27	2.08	2.04±0.25	2.95	2.97	3.03	3.16±0.25
$f_{7/2}$	7.83	7.19	7.13	-	7.76	7.12	7.06	-
$s_{1/2}$	0.00	0.02	0.03	-	0.00	0.03	0.04	-
$d_{3/2}$	0.01	0.07	0.07	-	0.01	0.09	0.09	-
$d_{5/2}$	0.01	0.12	0.15	-	0.02	0.17	0.18	-
$g_{7/2}$	0.02	0.19	0.16	-	0.03	0.31	0.27	-
$g_{9/2}$	0.29	0.85	0.62	0.23±0.25	0.46	1.15	1.04	0.84±0.25

Reduction of NME within the SRQRPA

$^{76}Ge \rightarrow ^{76}Se$	prev.	new
Jastrow s.r.c.	4.24(0.44)	3.49(0.23)
UCOM s.r.c.	5.19(0.54)	4.60(0.39)

F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)

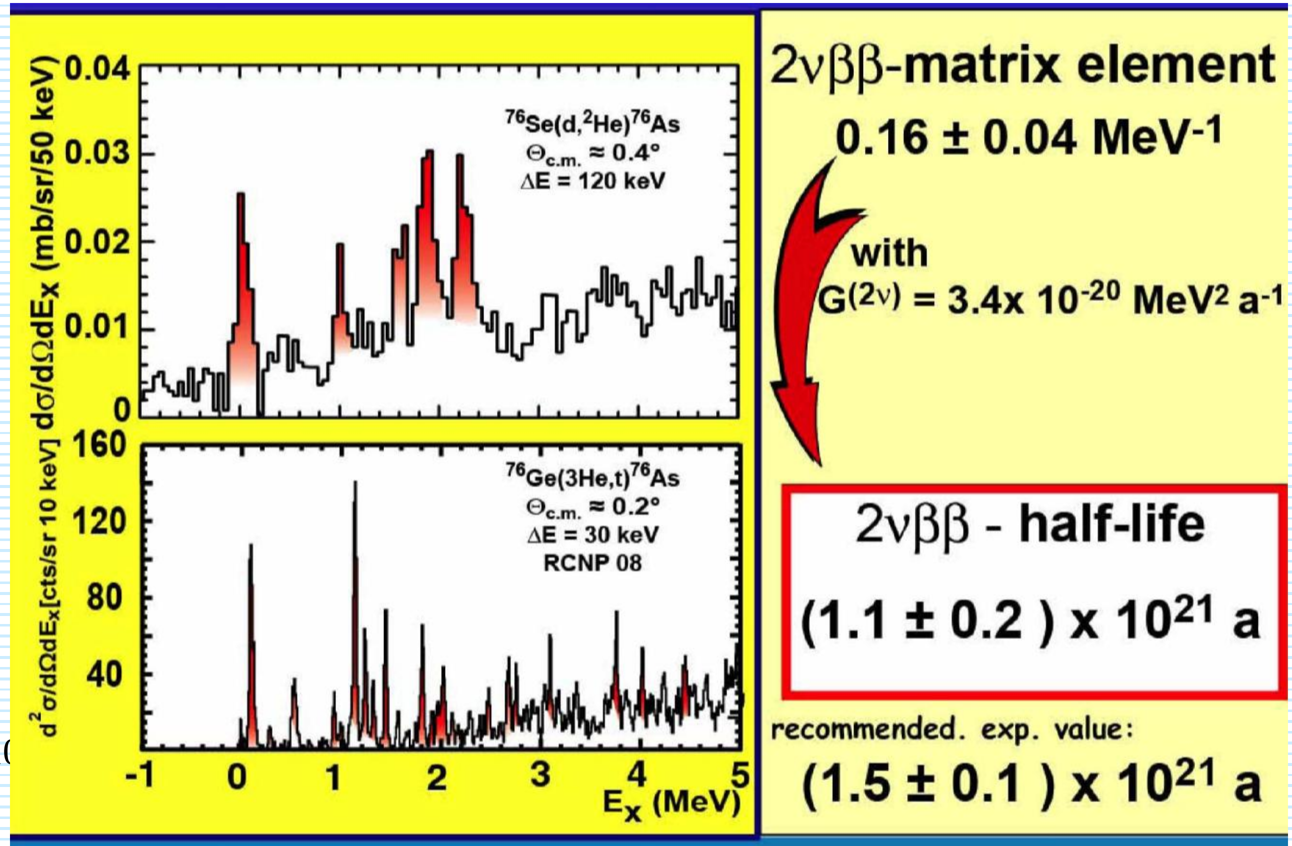
Constraining the 0νββ-decay NME



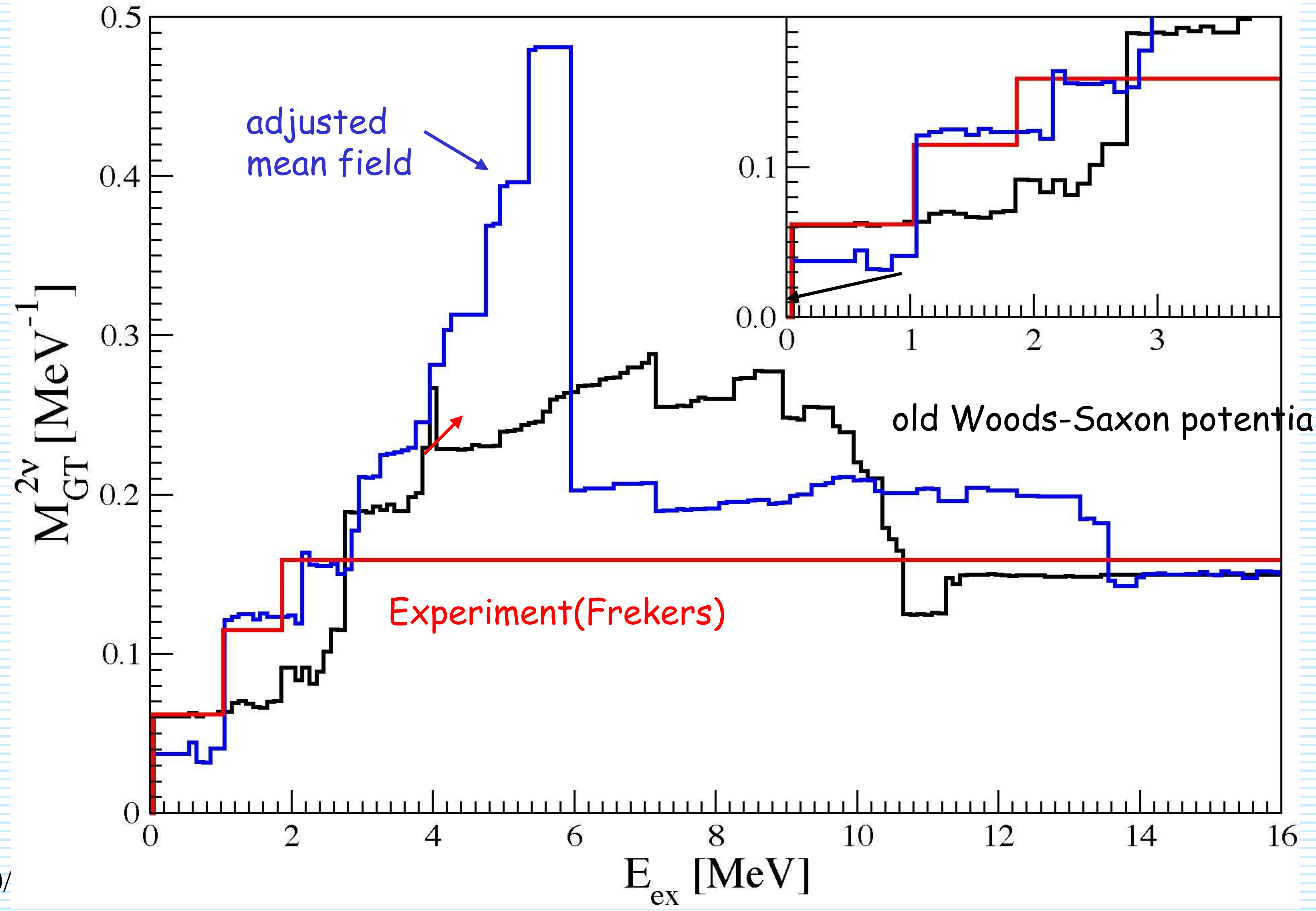
charge-exchange reactions

(t, ^{3}He)
(d, ^{2}He)

From D. Frekers, RIKEN 2008 lecture
The cross sections give B(GT) for β^+ and β^-, product of the amplitudes ($B(GT)^{1/2}$) gives the numerator of the $M^{2\nu}$ matrix element.



Staircase plot (running sum) of the contributions to the $2\nu\beta\beta$ decay ($^{76}Ge \rightarrow {}^{76}Se$)



Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

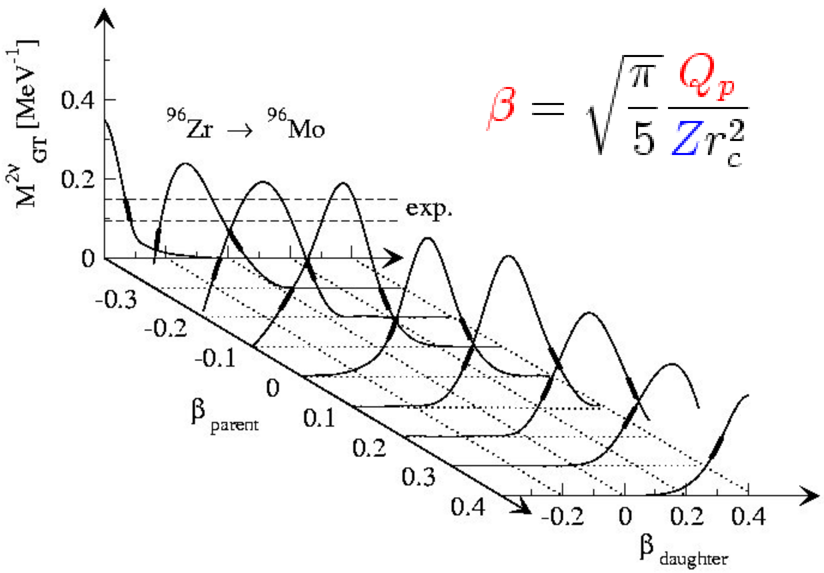
Exp. I (nuclear reorientation method)
Exp.II (based on measured E2 trans.)
Theor. I (Rel. mean field theory)
Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

Till now, in the QRPA-like calculations of the 0νββ-decay NME spherical symetry was assumed

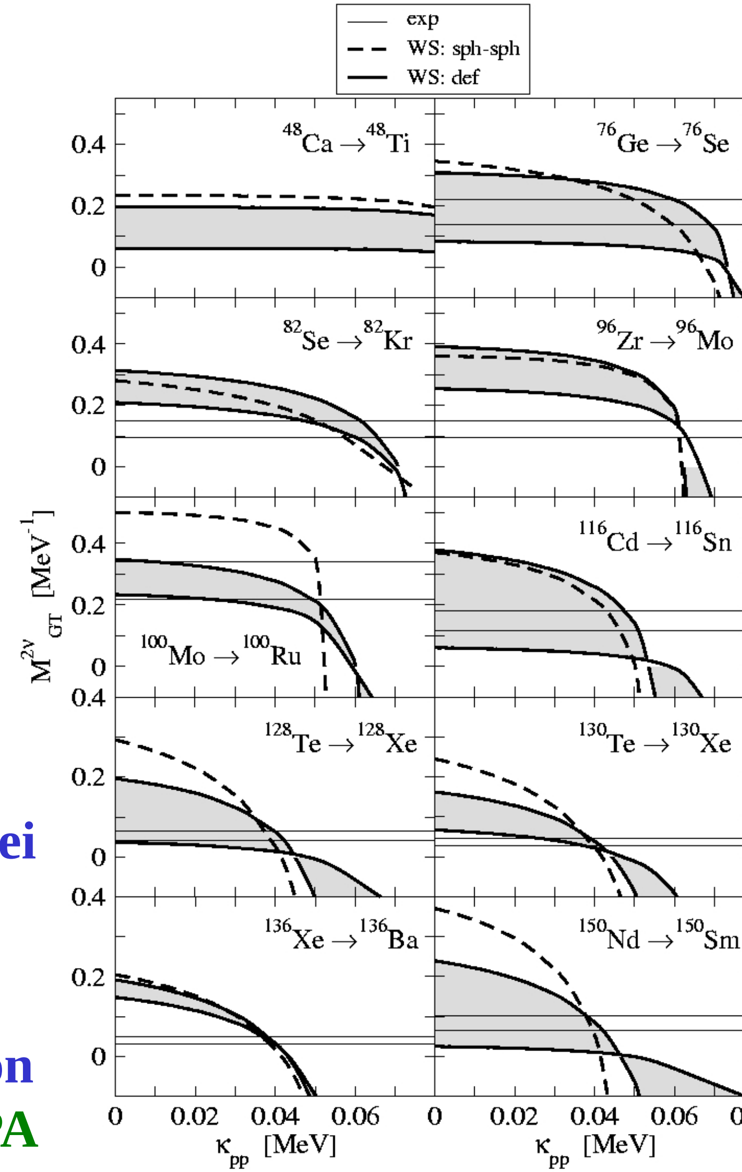
The effect of deformation on NME has to be considered

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
^{48}Ca	0.00	0.101	0.00	0.00
^{48}Ti	+0.17	0.269	-0.01	0.00
^{76}Ge	+0.09	0.26	0.16	0.14
^{76}Se	+0.16	0.31	-0.24	-0.24
^{82}Se	+0.10	0.19	0.13	0.15
^{82}Kr		0.20	0.12	0.07
^{96}Zr		0.081	0.22	0.22
^{96}Mo	+0.07	0.17	0.17	0.08
^{100}Mo	+0.14	0.23	0.25	0.24
^{100}Ru	+0.14	0.22	0.19	0.16
^{116}Cd	+0.11	0.19	-0.26	-0.24
^{116}Sn	+0.04	0.11	0.00	0.00
^{128}Te	+0.01	0.14	-0.00	0.00
^{128}Xe		0.18	0.16	0.14
^{130}Te	+0.03	0.12	0.03	0.00
^{130}Xe		0.17	0.13	-0.11
^{136}Xe		0.09	0.00	0.00
^{136}Ba		0.12	0.00	0.00
^{150}Nd	+0.37	0.28	0.22	0.24
^{150}Sm	+0.23	0.19	0.18	0.21

New Suppression Mechanism of the DBD NME



$$\beta = \sqrt{\frac{\pi}{5}}\,\frac{Q_p}{Z r_c^2}$$



The suppression of the NME depends on relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez,Sarriguren, Moya,Pacearescu, Faessler, F.Š.,
Phys. Rev. C 70 (2004) 321

There is a need for supporting experiments

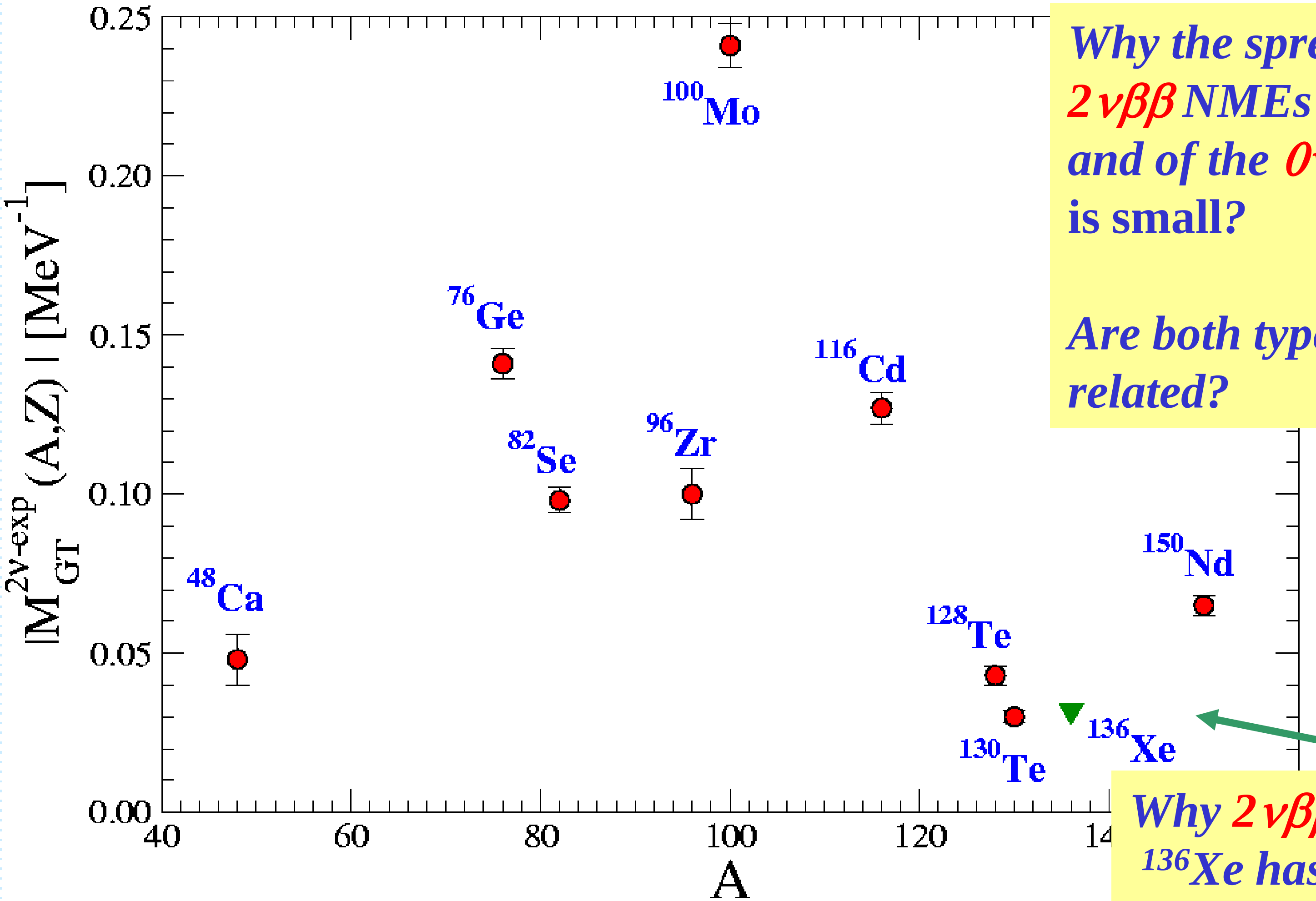
Nuclear matrix elements:

• *Mean field*	*p and n removing transfer reactions*
• β^- *and* β^+ *strengths*	*Charge-changing experiments*
• *deformation*	*Exp. to remeasure deformetion needed*
• $2\nu\beta\beta$*-decay*	*Double beta decay experiments*

2νββ-decay NMEs

2νββ-decay NMEs

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$



Why the spread of the 2νββ NMEs is large and of the 0νββ* NMEs *is small?

Are both type of NMEs related?

***Why 2νββ of ^{136}Xe has been not observed yet? Do this affect the value of 0νββ* NME**

Differencies among 2νββ-decay NMEs: up to factor 10

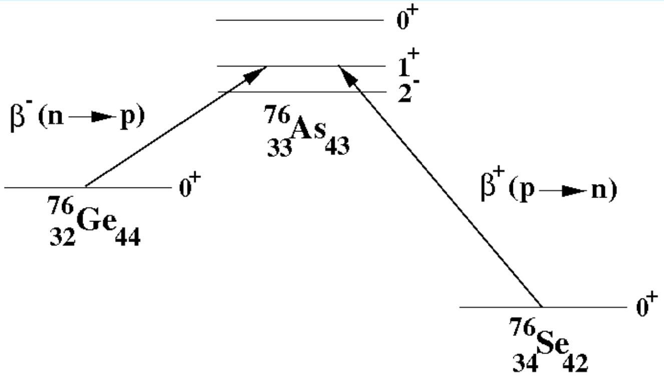
Low-lying states or GT resonance?

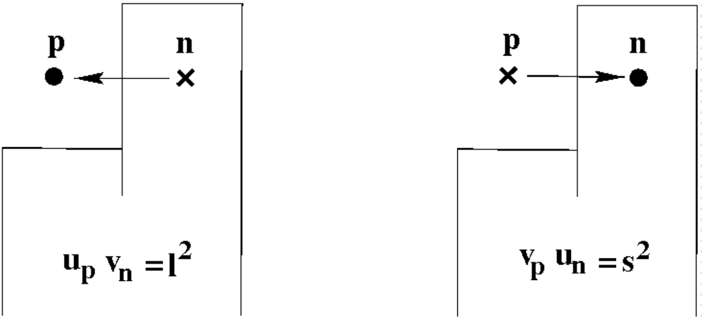
Isospin symmetry SU(2): $M_F=0$

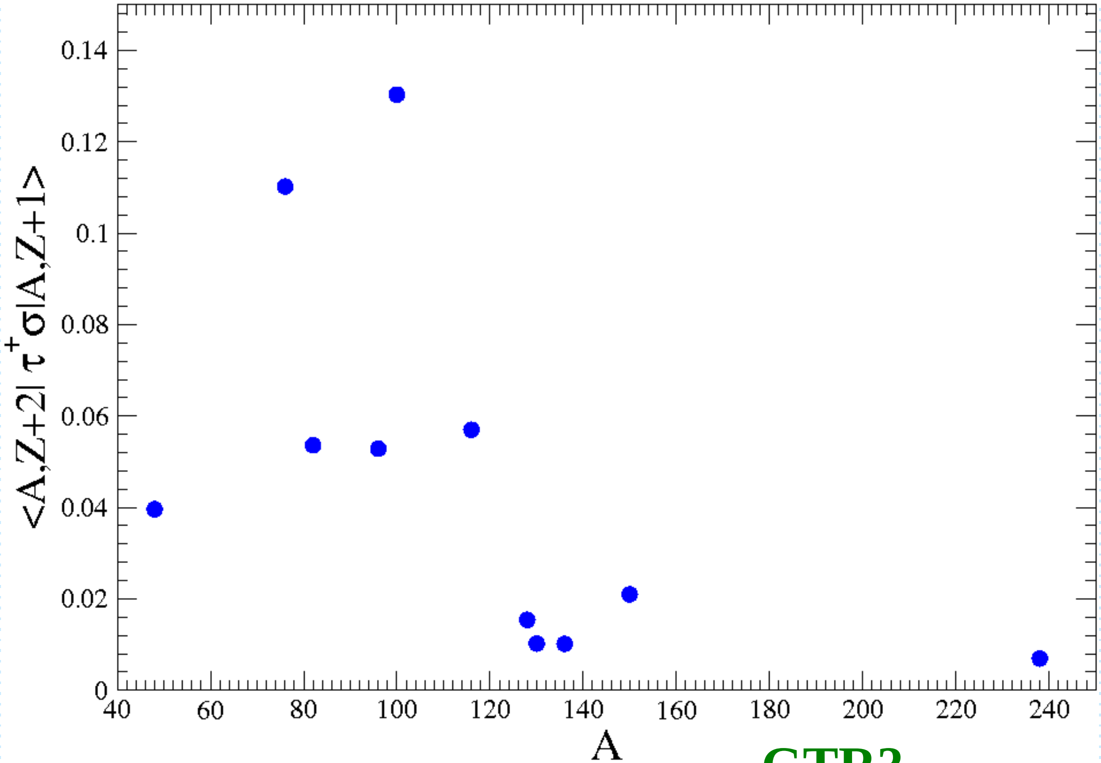
$M_F \neq 0$ (Coulomb)
$M_{GT} \neq 0$ (spin-orbit int.)

Isospin symmetry SU(4): $M_{GT}=0$

β^+-amplitude from M_{GT}^{exp} assuming single transition through GTR







low-lying states?

GTR?

2νββ-decay within the field theory

F.Š., G. Pantis, Phys. Atom. Nucl. 62 (1999) 585

Weak interaction Hamiltonian

$$\mathcal{H}^{\beta}(x) = \frac{G_F}{\sqrt{2}} 2 \left[\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x)\right] j_\alpha(x) + h.c.$$

2νββ-decay amplitude

$$< f|S^{(2)}|i > =$$
$$\frac{(-i)^2}{2}\left(\frac{G_F}{\sqrt{2}}\right)^2 L_{\mu\nu}(p_1, p_2, k_1, k_2) J_{\mu\nu}(p_1, p_2, k_1, k_2)$$
$$-(p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2)(k_1 \leftrightarrow k_2)$$

Hadron part of amplitude

$$J_{\mu\nu}(p_1, p_2, k_1, k_2) = \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2}$$
$$_{out}<p_f|T(J_\mu(x_1)J_\nu(x_2))|p_i>_{in}\, dx_1 dx_2$$

Integral representation of M_{GT}

$$M_{GT} = \frac{i}{2}\int_0^\infty (e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}) M_{AA}(t) dt$$

with

$$M_{AA}(t) = < 0_f^+ | \frac{1}{2}[A_k(t/2), A_k(-t/2)] | 0_i^+ >$$

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \qquad A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k, \;\; k = 1, 2, 3.$$

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

Completeness:
$\Sigma_n |n><n| = 1$

$$< A'|J_\alpha(x_1) J_\beta(x_2)|A > \;=\; \sum_n < A'|J_\alpha(0, \vec{x}_1)|n >< n|J_\beta(0, \vec{x}_2)|A > \times e^{-i(E'-E_n)x_{10}} e^{-i(E_n - E)x_{20}}$$

$$\int_0^\infty e^{-iat} dt \Rightarrow \lim_{\epsilon \to 0} \int_0^\infty e^{-i(a-i\epsilon)t} dt = \lim_{\epsilon\to 0} \frac{-i}{a - i\epsilon}$$

$$M_{GT} = \sum_n \frac{< 0_f^+|A(0)_k|1_n^+ >< 1_n^+|A(0)_k|0_i^+ >}{E_n - E_i + \Delta}$$

Double beta decay is a two-body process

$$\mathbf{H = one - body + two - body}, \qquad \mathrm{A}_k(0) = \text{one–body}$$

$$A_k(t) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

$$\begin{array}{llccccccc}
\text{n=0} & [H^{(0)}A] & [1] & & & & & & \\
 & & \downarrow & \searrow & & & & & \\
\text{n=1} & [H^{(1)}, A] & [1] & & [2] & & & & \\
 & & \downarrow & \searrow & \downarrow & \searrow & & & \\
\text{n=2} & [H^{(2)}, A] & [1] & & [2] & & [3] & & \\
 & & \downarrow & \searrow & \downarrow & \searrow & \downarrow & \searrow & \\
\text{n=3} & [H^{(3)}, A] & [1] & & [2] & & [3] & & [4]
\end{array}$$

$$If \quad \mathrm{H} \approx \text{one–body op.} \Longrightarrow \mathbf{A_k(t)} \text{ is one–body op.}$$

r_{12} -dependence of the 2νββ-decay NME

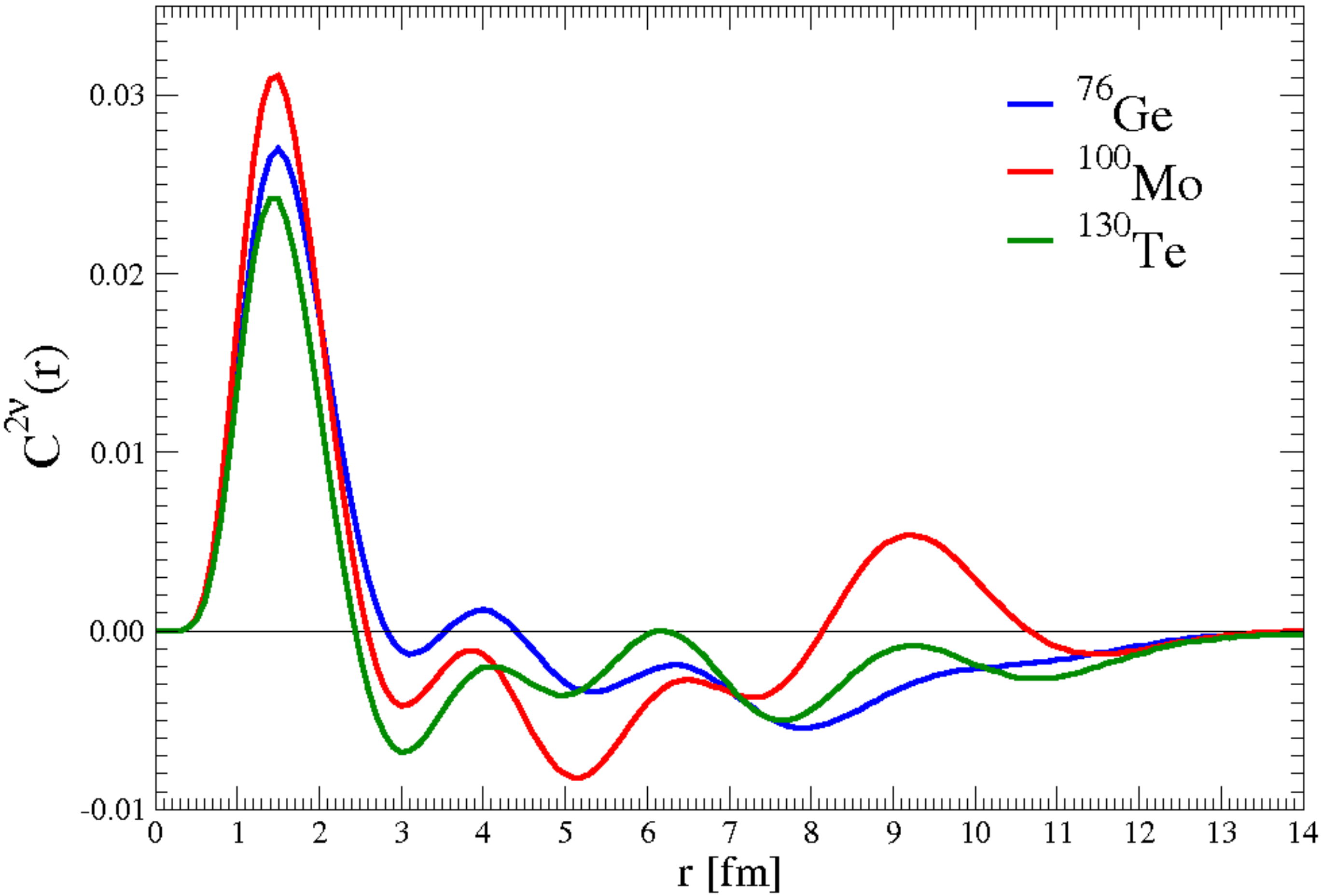
$$M^{2\nu}_{GT} = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel \sigma(1)\cdot\sigma(2) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0^+_f || [\widetilde{c^+_{p'} \tilde{c}_{n'}}]_J || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i || [c^+_p \tilde{c}_n]_J || 0^+_i \rangle$$

$$M^{2\nu}_{GT} = \int C^{2\nu}(r) dr$$



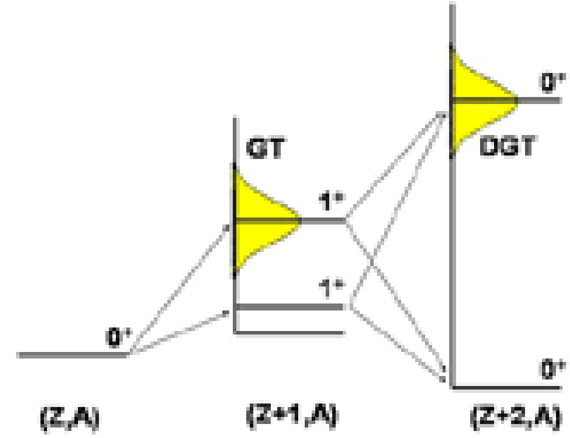
On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay (GT) NMEs

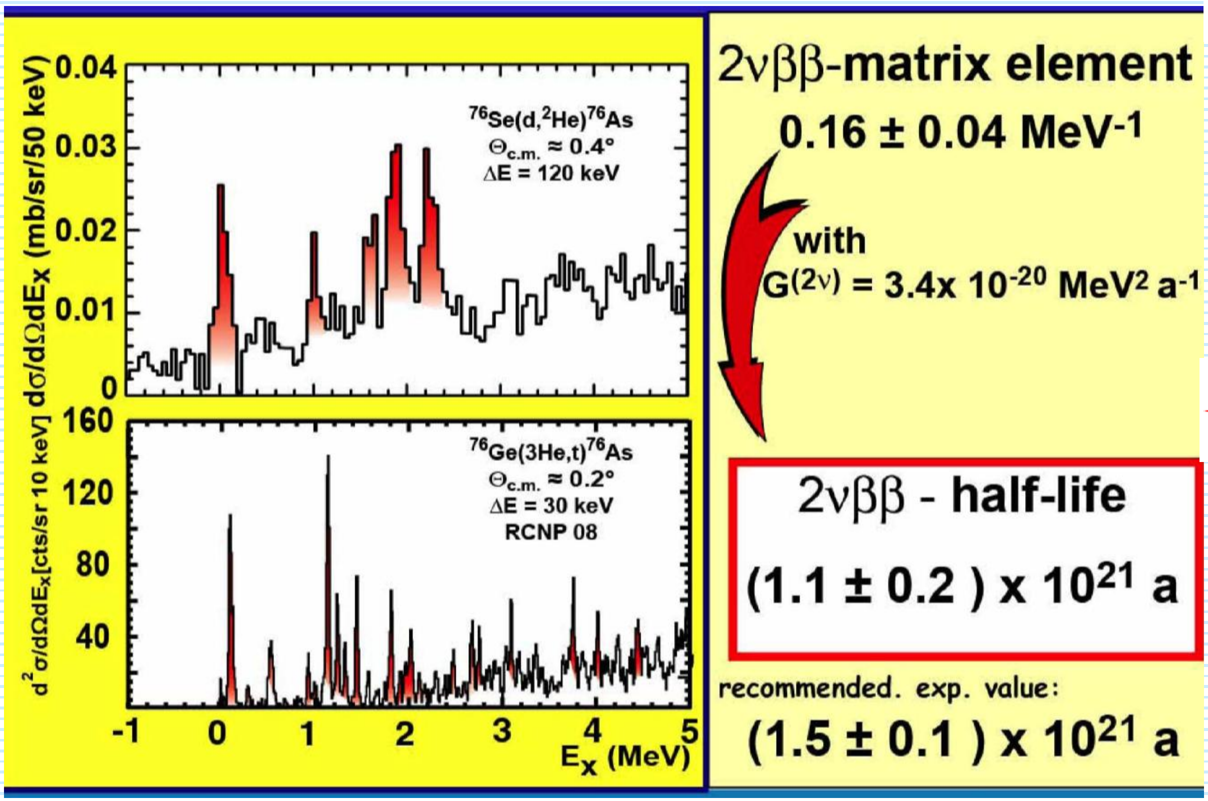
F.Š., Vogel, Hodak, Faessler, to be submitted

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

The cross sections of (t,^{3}He) and (d,^{2}He) reactions give B(GT$^{\pm}$) for β^+ and β^-, product of the amplitudes (B(GT)$^{1/2}$) entering the numerator of $M^{2\nu}_{GT}$

$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$





Grewe, …Frekers at al, PRC 78, 044301 (2008)

Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)$$

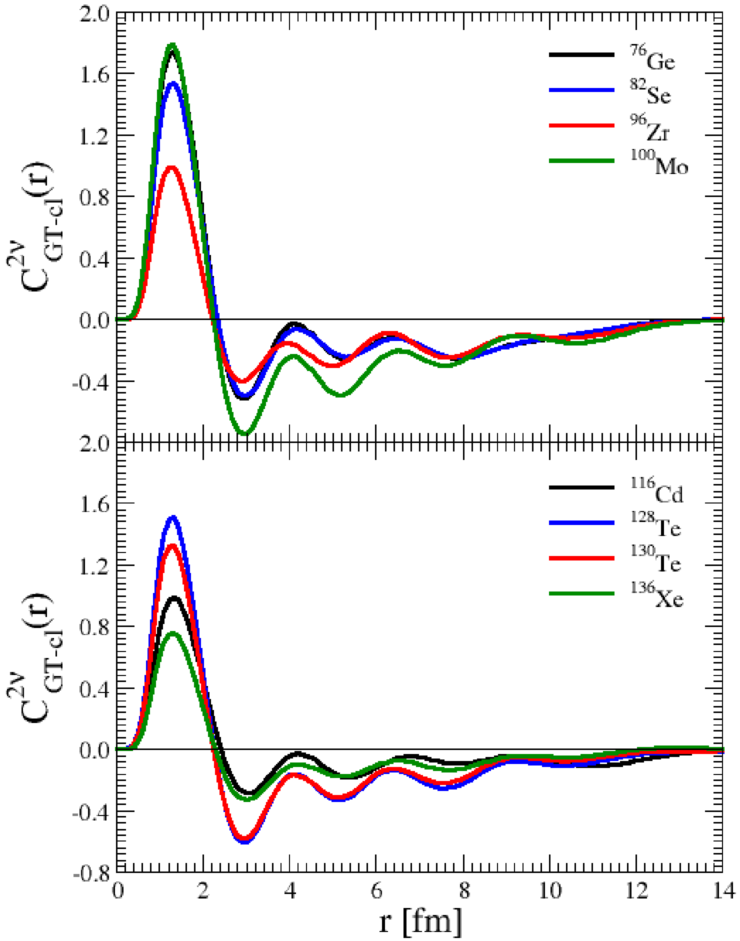
SSD hypothesis

$$g_A^2\ M^{2\nu}_{GT-cl} = \frac{3\ D}{\sqrt{ft_{EC}\, ft_{\beta^-}}}$$

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

***r**- relative distance of two nucleons*

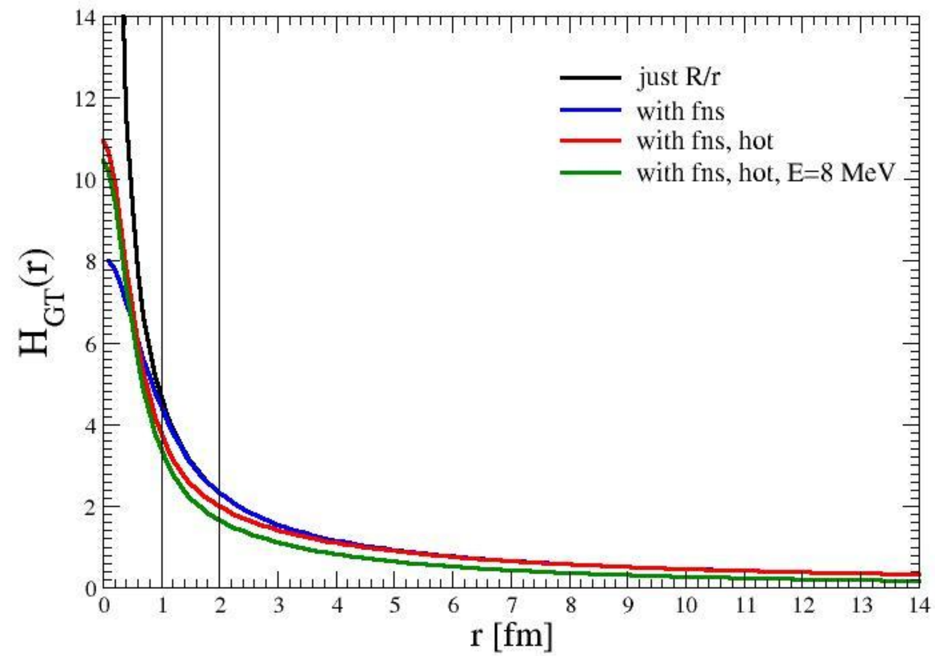


A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f^2_{FNS}(q^2) g_{HOT}(q^2)dr$$



Neutrino potential prefer short distances

Phenomenological estimation of $M^{0\nu}_{GT}$

Nucleus	$T^{2\nu-exp}_{1/2}$ [y] [years]	$\lvert g_A^2 M^{2\nu-exp}_{GT}\rvert$ $[MeV^-1]$	SSD $\lvert g_A^2 M^{2\nu}_{GT-cl}\rvert$	SSD $\lvert g_A^2 M^{0\nu-ph}\rvert$	ChER $\lvert M^{2\nu}_{GT}\rvert$ $[MeV^-1]$	ChER $\lvert M^{2\nu}_{GT-cl}\rvert$	ChER $\lvert M^{0\nu-ph}\rvert$
^{48}Ca	4.4×10^{19}	0.0735	-	-	0.083	0.355	3.19
^{76}Ge	1.5×10^{21}	0.219	-	-	0.159	0.840	8.80
^{96}Zr	2.3×10^{19}	0.145	-	-	-	0.357	4.04
^{100}Mo	7.1×10^{18}	0.373	0.564	6.47	-	-	-
^{116}Cd	2.8×10^{19}	0.203	0.562	6.78	0.064	0.491	5.92

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f^2_{FNS}(q^2)g_{HOT}(q^2)dr$$

$$\begin{aligned} M^{0\nu}_{GT} &= H_{GT}(r=0)\ M^{2\nu}_{GT-cl} \\ &\quad - \int_0^\infty \mathcal{F}(r)C^{2\nu}_{GT-cl}(r)dr \\ &= M^{0\nu-ph}_{GT} - M^{0\nu-rest}_{GT}\end{aligned}$$

with Taylor expansion

$$\begin{aligned} j_0(qr) &= 1 - \frac{1}{6}(qr)^2 + \frac{1}{120}(qr)^4 - \cdots \\ &= 1 - \mathcal{F}(r)\end{aligned}$$

A: Phenomen. prediction:
Too large
(~ factor 2)

B: Need to be calculated
Not negligable

There is no proportionality between $M^{0\nu}_{GT}$ and $M^{2\nu}_{GT-cl}$

Closure 2νββ GT NME

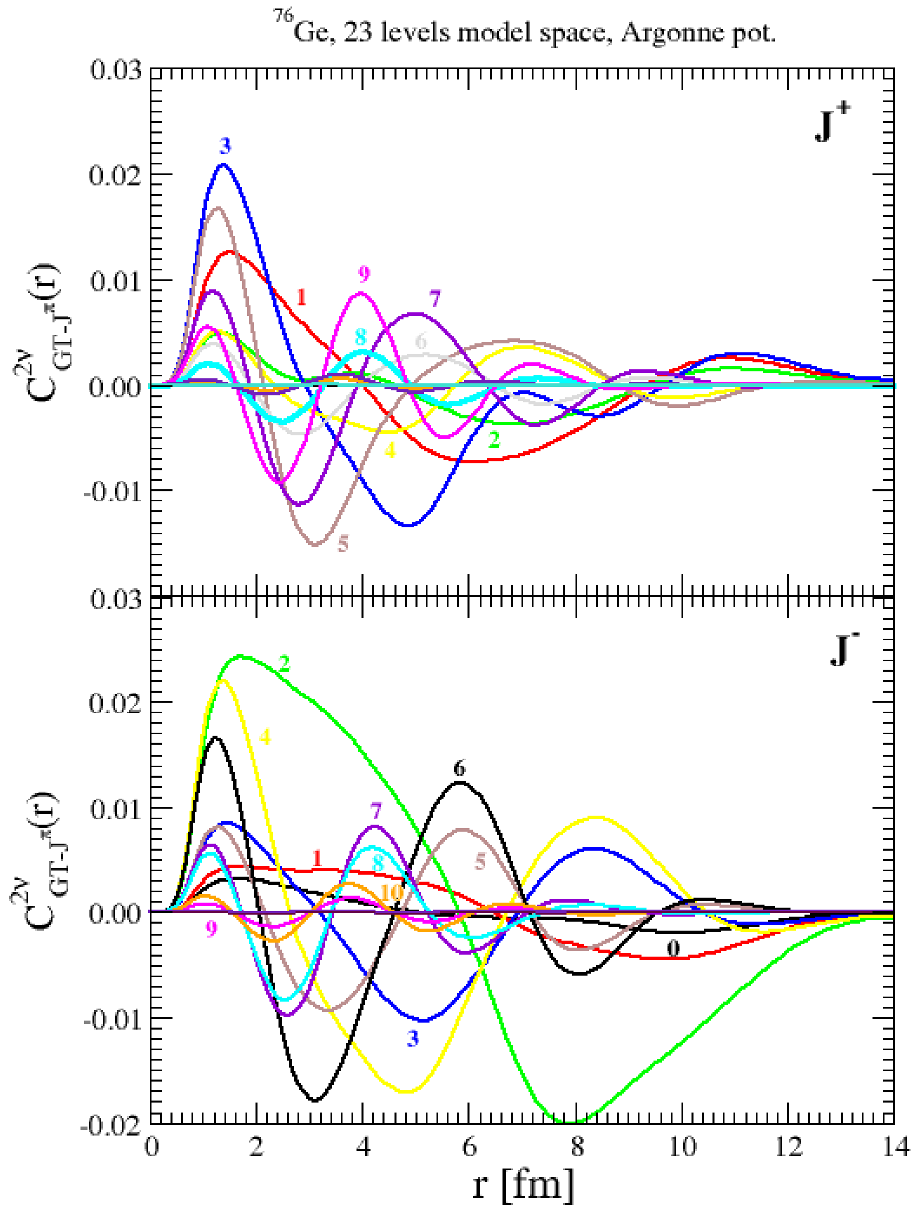
The only non-zero contribution from $J^{\pi}=1^{+}$

$$M^{2\nu}_{GT-cl} =$$

$$\sum_{J^\pi,m} < 0_f^+|\tau^+\vec{\sigma}|J^\pi,m> \cdot < J^\pi,m|\tau^+\vec{\sigma}|0_i^+>$$

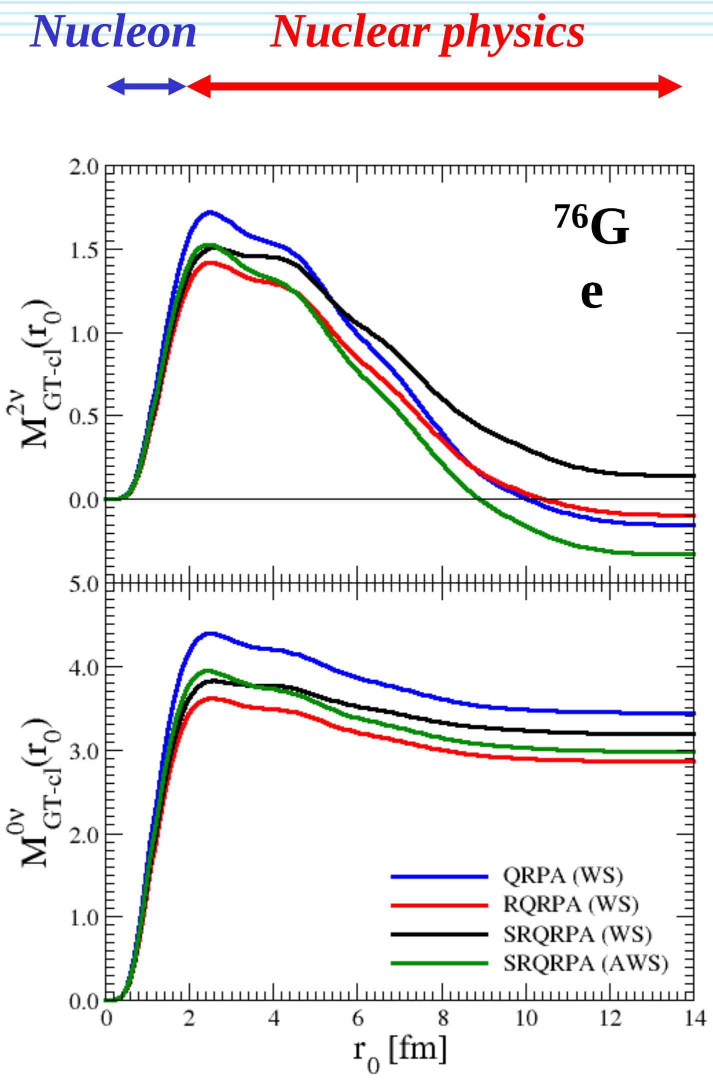
$$=\sum_{m} < 0_f^+|\tau^+\vec{\sigma}|1^+,m> \cdot < 1^+,m|\tau^+\vec{\sigma}|0_i^+>$$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi}\int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$



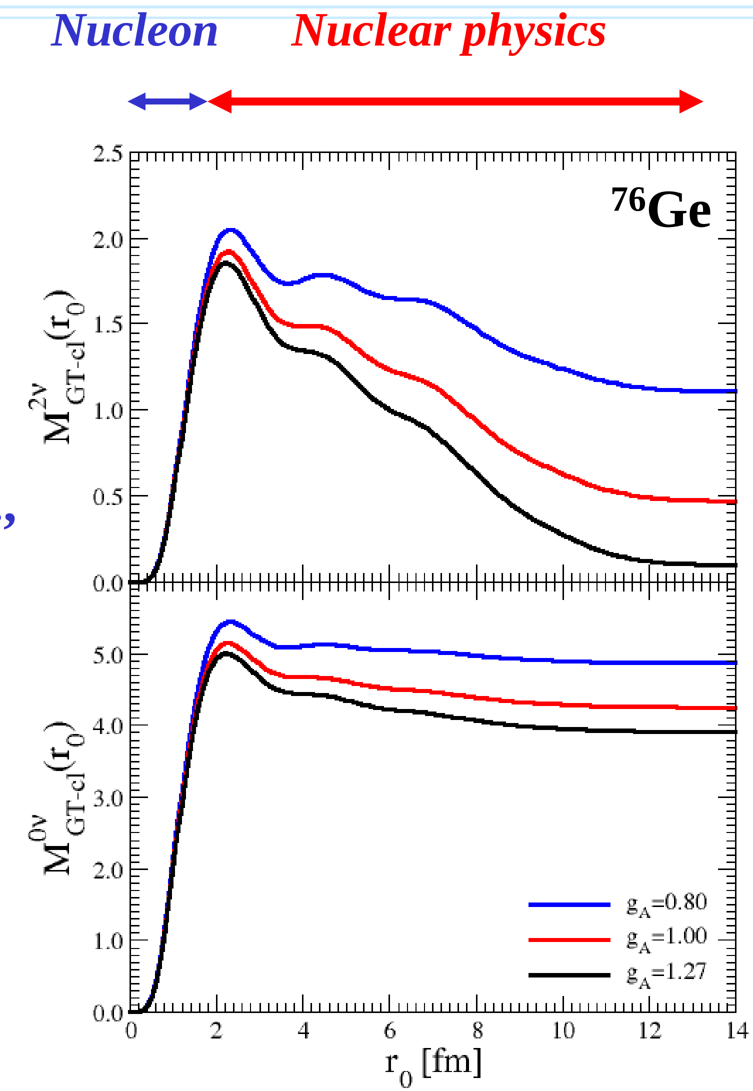
$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$



Different QRPA-like approches

F.Š.,



Dependence on axial-vector coupling

Co-existence of few mechanisms of the $0\nu\beta\beta$-decay

It may happen that in year 201? (or 2???) the $0\nu\beta\beta$-decay will be detected for 2-3 or more isotopes …

Co-existence of 2, 3 or more mechanisms of the 0νββ-decay

It is well-known that there exist many mechanisms that may contribute to the 0νββ. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

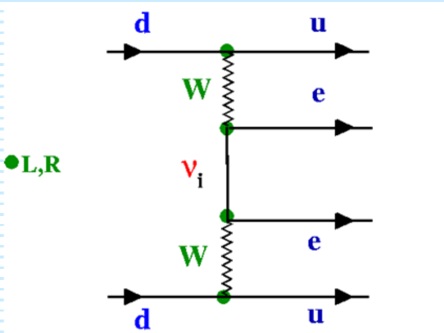
$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(E_0, Z)\left|\frac{m_{\beta\beta}}{m_e} M^{0\nu}_{\nu} + \eta^L_N M^{0\nu}_N + \eta_{\lambda'_{111}} M^{0\nu}_{\lambda'_{111}} \ldots\right|^2$$

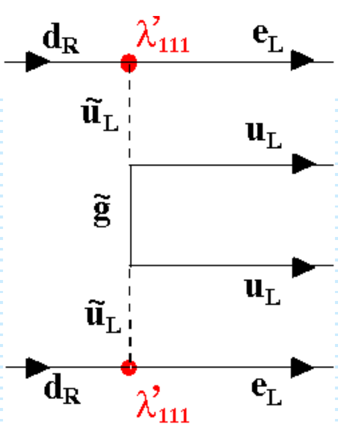
$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\eta^L_N = \sum_{k=4}^{6} |U^L{}_{ek}|^2\, \xi'_k\, \frac{m_p}{M_k},$$

$$\eta^R_N = \sum_{k=4}^{6} |U^R{}_{ek}|^2\, \xi'_k\, \frac{m_p}{M_k}.$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}}\frac{m_p}{m_{\tilde{g}}}\left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2.$$





Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina, Mod. Phys. A 21, 1547 (2009)

$$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$
$\xi_{Mo} < 2.6$

We introduce

$$\xi = \frac{|M^{\nu}_1|\sqrt{T_1\ G_1}}{|M^{\nu}_2|\sqrt{T_2\ G_2}}$$

ξ=0, non-observation ($T_2 \to \infty$)
ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq.

$$\frac{\pm 1}{\sqrt{T_1\ G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^\nu + \eta M_1^\eta$$

$$\frac{\pm 1}{\sqrt{T_2\ G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^\nu + \eta M_2^\eta$$

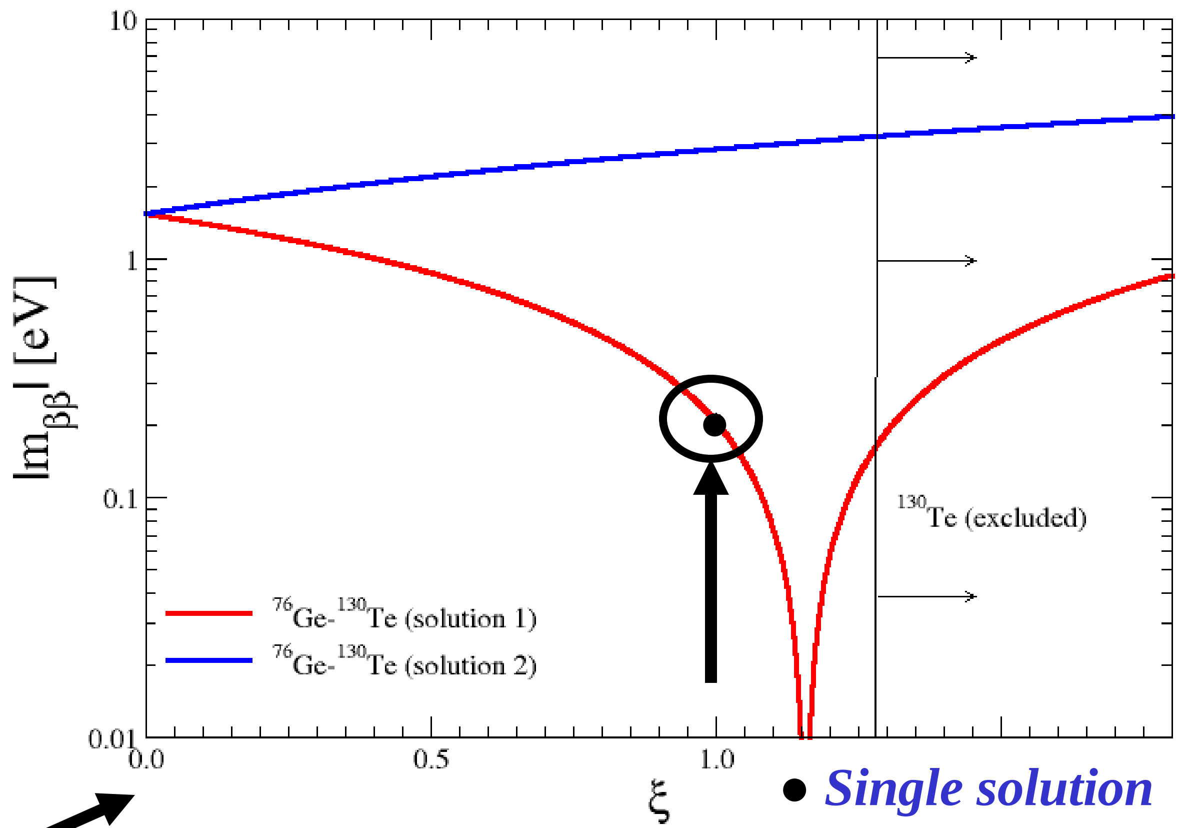
2 different solutions

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^\nu \sqrt{T_1\ G_1}} \frac{M_1^\nu\ M_2^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \pm \frac{m_e}{M_2^\nu \sqrt{T_2\ G_2}} \frac{M_2^\nu\ M_1^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \right|$$

2 active mechanisms of the 0νββ-decay: Light and heavy ν-mass mechanism

Non-observation of the 0νββ-decay for some isotopes might be in agreement with $M_{\beta\beta}$ in sub eV region

CP-conservation assumed



Non-observation for ^{130}Te

• ***Single solution for light ν-mass mech.***

3 active mechanisms of the 0νββ-decay

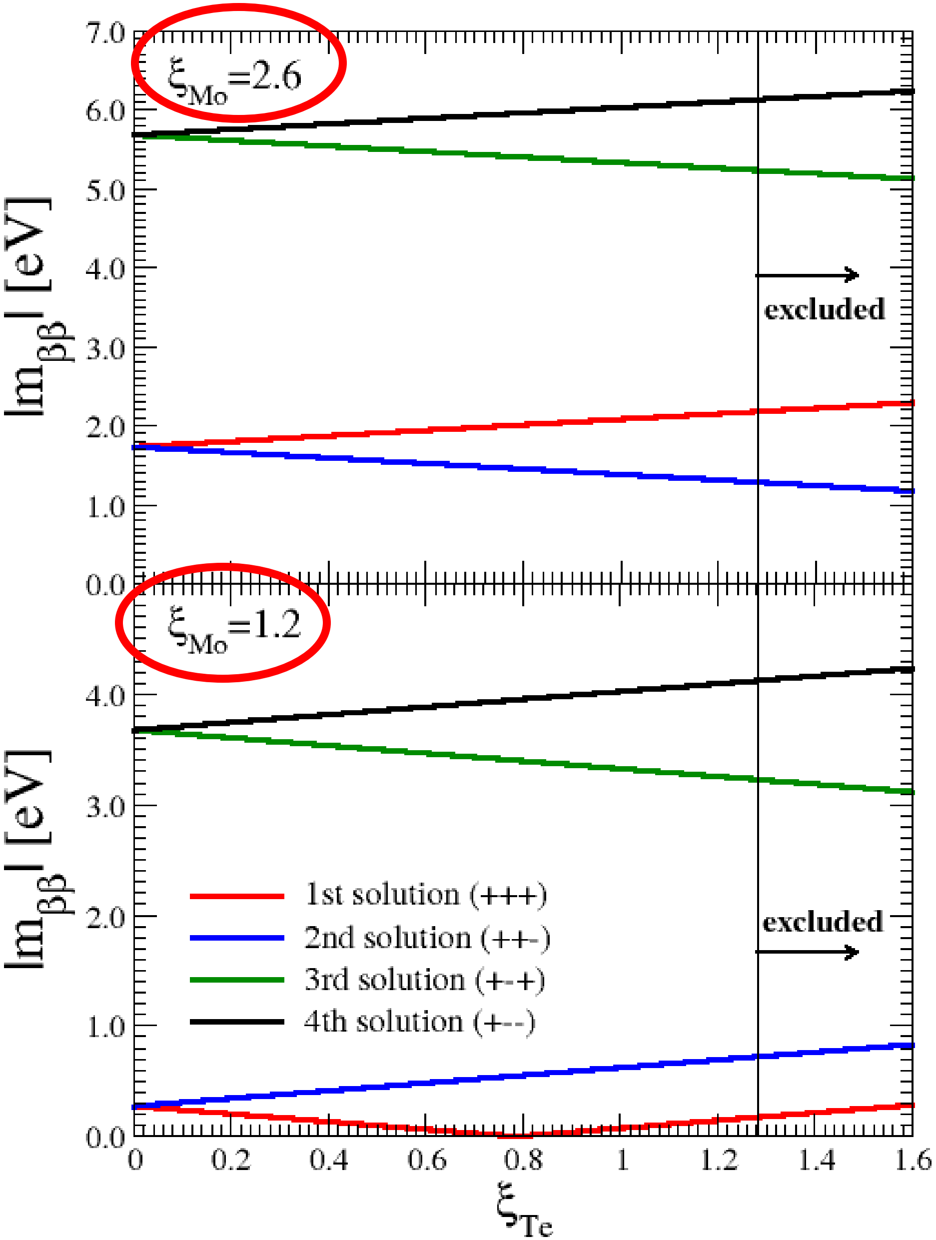
$$\frac{\pm 1}{\sqrt{T_i\, G_i}} = \frac{m_{\beta\beta}}{m_e} M_i^{\nu} + \eta_N M_i^{\eta} + \eta_{\lambda'} M_i^{\lambda'}$$

$$i = 1, 2, 3$$

assuming evidence

$$T_{1/2}^{0\nu}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} \text{ y}$$

current limits **$\xi_{Te} < 1.2$ $\xi_{Mo} < 2.6$**



Conclusions

$0\nu\beta\beta$-decay NMEs:** Significant progress achieved. Factor 2 difference (2005: factor 5). Better understanding of uncertainties. A connection to the $2\nu\beta\beta$-decay established. There is a need for supporting experiments: i) p and n removing transfer reactions (mean field); ii) Charge-changing reaction experiments (β^- and β^+ strengths); iii) $2\nu\beta\beta$-decay experiments; iv) Experiments to re-measure nuclear deformation needed

Co-existence of different mechanisms:** The non-observation of the $0\nu\beta\beta$-decay for some isotopes could be in agreement with a value of $m_{\beta\beta}$ in sub eV region. Thus, it is important to have at least two different $0\nu\beta\beta$-decay experiments for a given nucleus.