NUCLEAR STRUCTURE OF HEAVY NUCLEI IN THE DINUCLEAR SYSTEM MODEL

T. M. Shneidman, G. G. Adamian, N. V. Antonenko, R. V. Jolos

Joint Institute for Nuclear Research, Dubna, Russia

W. Scheid

Justus-Liebig-Universität, Giessen, Germany

Introduction

In the even-even isotopes of actinides and also in the heavy Ba and Ce isotopes the low–lying negative parity states are observed together with the usually presented collective positive parity states combined into rotational or quasirotational ground state bands.

Formation of the positive parity rotational or quasirotational bands is connected in general to the quadrupole collective motion.

Low–lying negative parity states connected by the strong dipole and octupole transitions with the members of the ground-state band, definitely indicates on the presence of the collective reflection-asymmetric modes.

Experimental spectrum of ^{220}Th

(W. Reviol et al., Phys. Rev. C74, 044305 (2006))

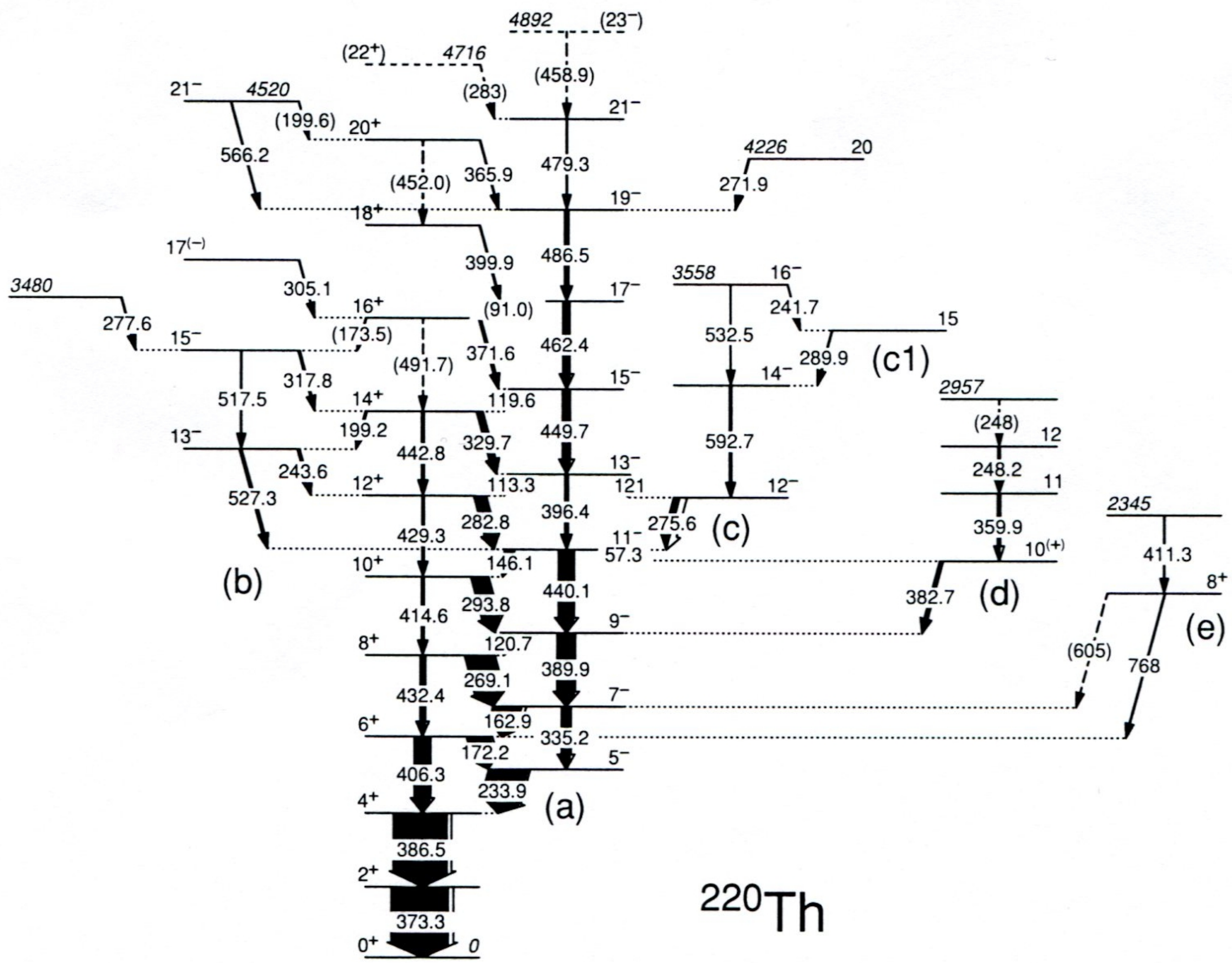


FIG. 4. Proposed level scheme for ^{220}Th. The level spins and parities are given in the figure. The highest-lying state in each sequence and the ground state are also labeled by their energy, in italics. Transitions indicated as dashed arrows, and spins and parities given in parentheses, are considered to be tentative. The energy unit is keV. The widths of the filled and open parts of the arrows are proportional to the γ-ray and internal conversion intensities, respectively.

Dinuclear system (DNS) concept

The dynamics of a reflection asymmetric collective motion can be treated as a motion in a mass-asymmetry coordinate. Such collective motion simultaneously creates deformations with even and odd-multipolarities.

$$\Psi_{p,IMK} = \sqrt{\frac{2I+1}{16\pi^2}}\left(\Phi_{n,K}(\xi)D^I_{MK} + p(-1)^{I+K}\Phi_{n,\overline{K}}(\xi)D^I_{M,-K}\right)$$

Wave function in ξ defined by the equation:

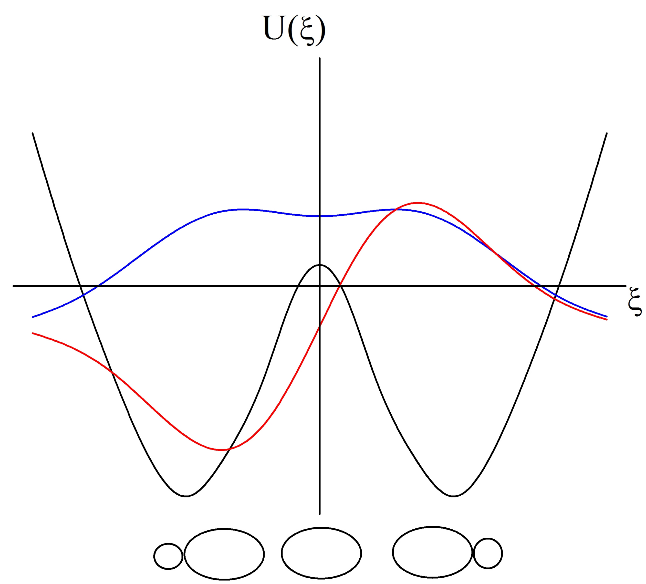
$$\left(-\frac{\hbar^2}{2B_\xi}\frac{d^2}{d\xi^2} + U(\xi) + \frac{\hbar^2}{2\Im(\xi)}I(I+1)\right)\Psi_{n,K}(\xi) = E_{n,K}\Psi_{n,K}(\xi),$$

where

$$\Im(\xi) = 0.85(\Im_1^r + \Im_2^r + m_0\frac{A_1A_2}{A}R^2)$$

Exitation spectra:

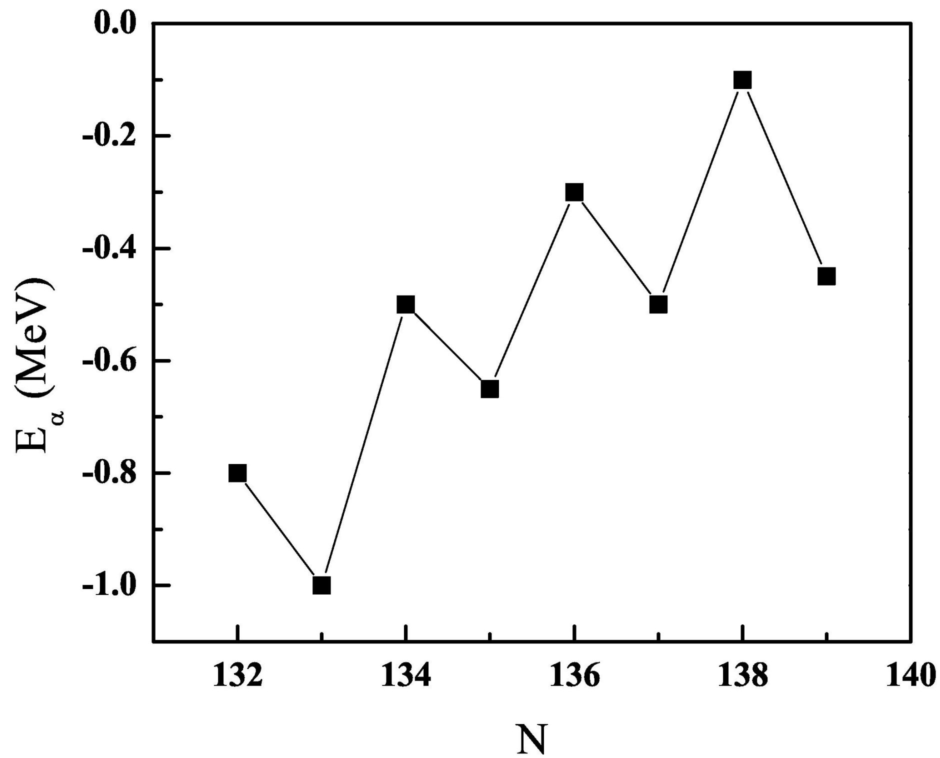
$$I^p(\text{ for } K=0) = 0^+, 1^-, 2^+ ...$$
$$I^p(\text{ for } K\neq 0) = K^\pm, (K+1)^\pm ...$$



Dinuclear system (DNS) concept

As it was shown by out calculations, only α-cluster system ${}^{A}Z \rightarrow^{(A-4)} (Z-2)+{}^{4}$He among different cluster configurations gives a significant contribution to the formation of the low-lying nuclear states.

Energies of alpha–cluster dinuclear system for different Ra isotopes



Potential Energy of the Dinuclear System

$$U(R,\xi,\beta_{2\mu}) = B_1 + B_2 - B_{12} + V(R,\xi,\beta_{2\mu})$$

where, B_1, B_2 and B_{12} are the binding energies of the fragments and the compound nucleus, respectively.

The nucleus-nucleus potential

$$V(R,\xi,\beta_{2\mu}) = V_{Coul}(R,\xi,\beta_{2\mu}) + V_{nucl}(R,\xi,\beta_{2\mu})$$

is the sum of the nuclear interaction potential $V_{nucl}(R,\xi,\beta_{2\mu})$

and of the Coulomb potential

$$V_{Coul}(R,\xi,\beta_{2\mu}) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3} R_{01}^2 \sum_{\mu} \beta_{2\mu}^* Y_{2\mu}(\theta,\phi) + \ldots$$

Double-folding potential

$$V_{nucl}(R,\xi,\beta_{2\mu}) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{R}-\mathbf{r}_2)F(\mathbf{r}_1-\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2$$

$$\rho_i(\mathbf{r}) = \frac{\rho_{00}}{1+\exp\left(\frac{s(\mathbf{r})}{a_{0i}}\right)}, \quad \rho_{00} = 0.17\ \mathrm{fm}^{-3}$$

$$F(\mathbf{r}_1-\mathbf{r}_2) = C_0\left(F_{in}\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}} + F_{ex}\left(1-\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

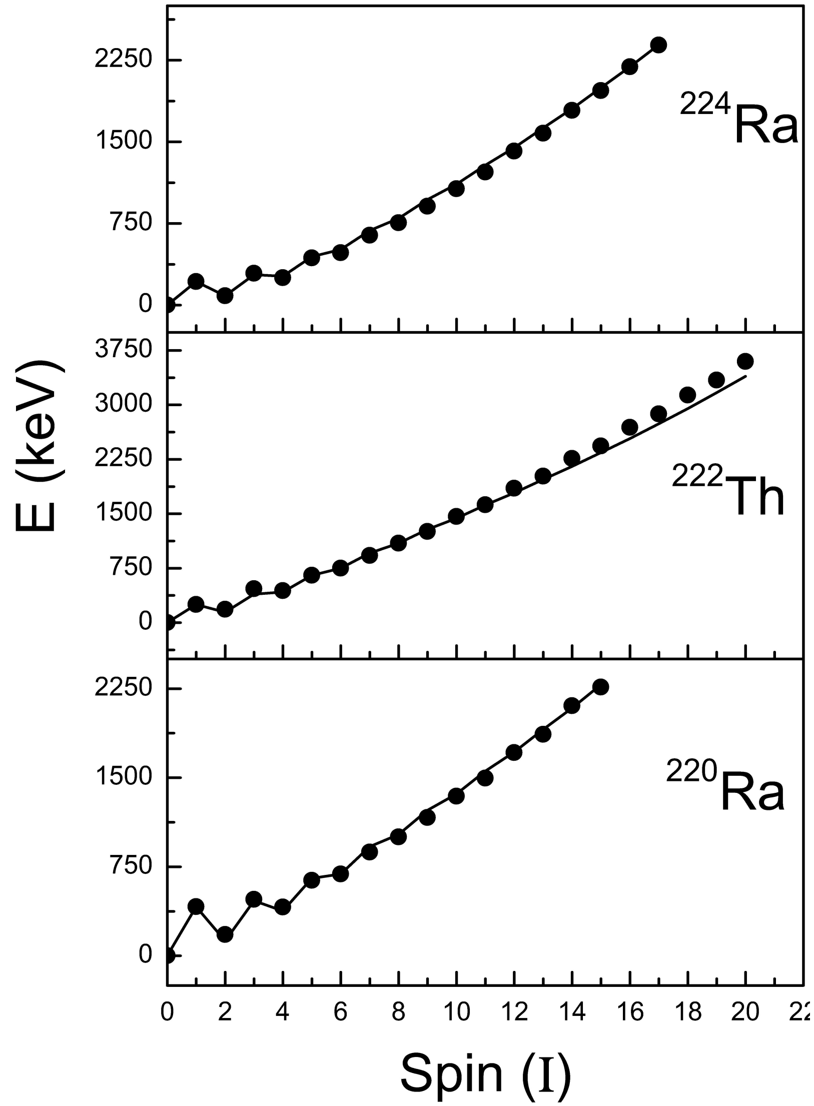
$$\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r})$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1-Z_1}{A_1}\,\frac{N_2-Z_2}{A_2}$$

$$C_0 = 300\ \mathrm{MeV\ fm}^3,\ f_{in}=0.09,\ f_{ex}=-2.59,\ f'_{in}=0.42,\ f'_{ex}=0.54$$

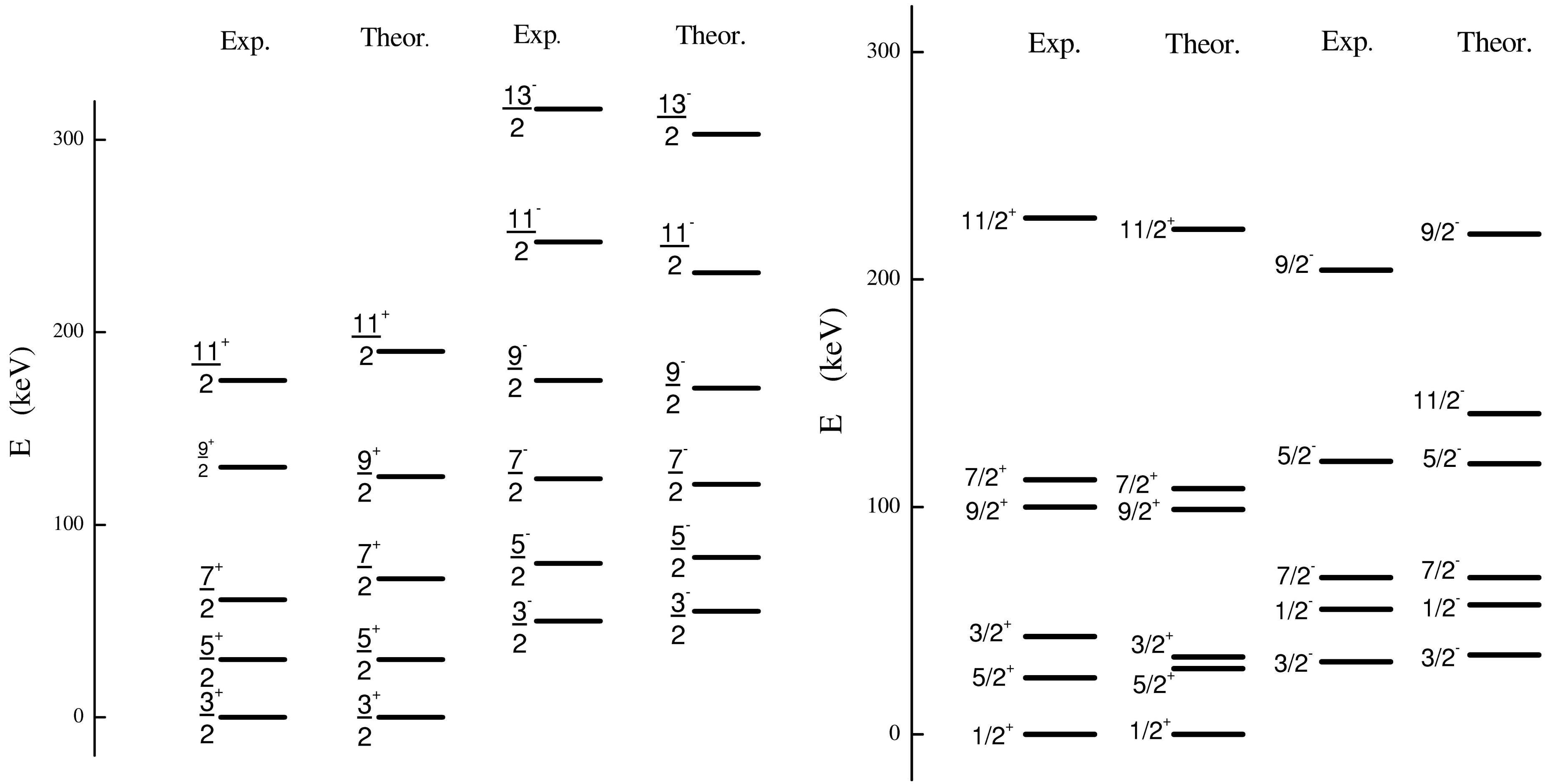
Calculations for Ra and Th isotopes

(*Phys.Rev. C 74, 034316 (2006)*)



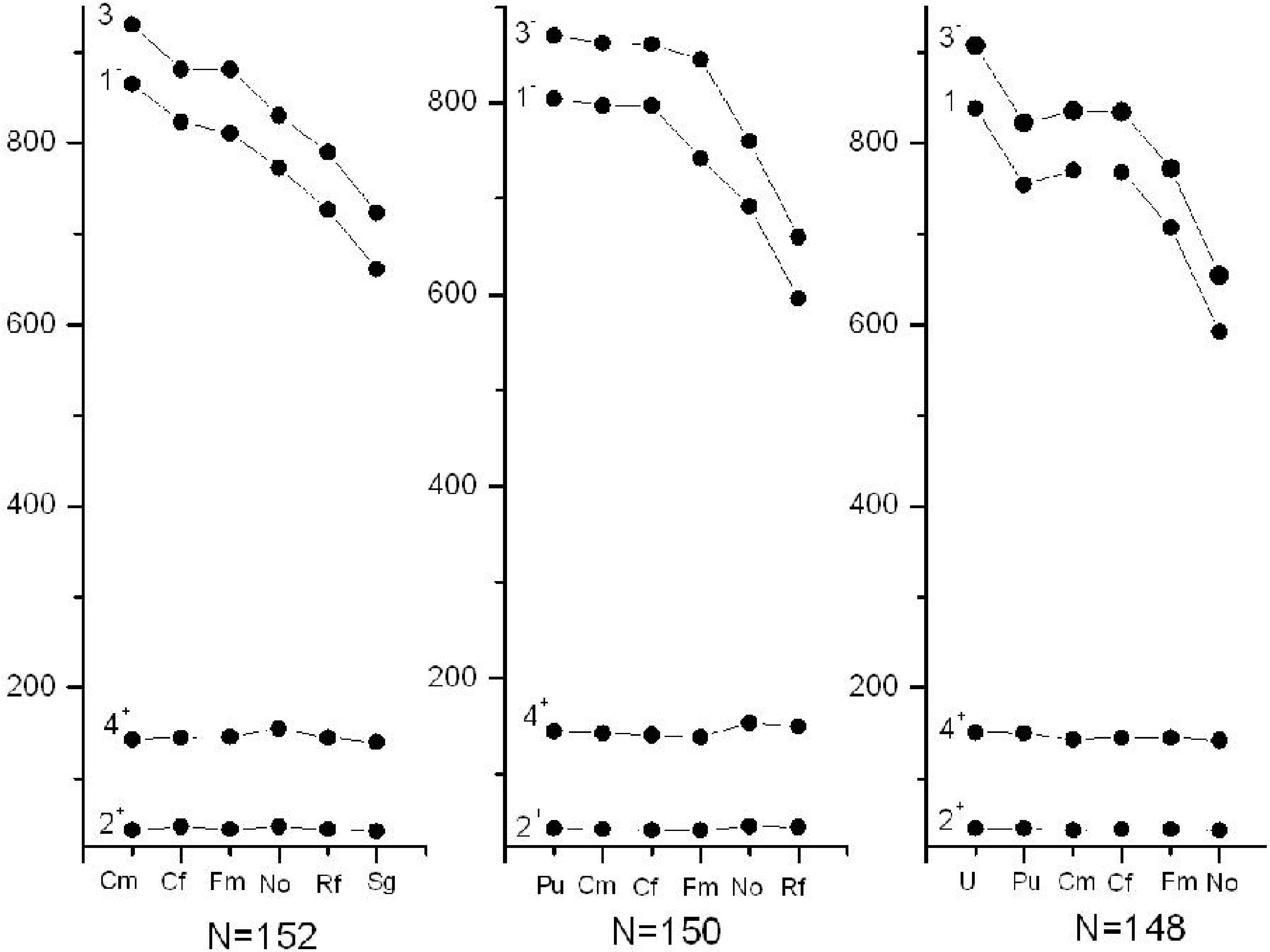
Energy spectra for ^{223}Ra and ^{225}Ra isotopes

$(Phys.Rev.\ C\ 74,\ 034316\ (2006))$



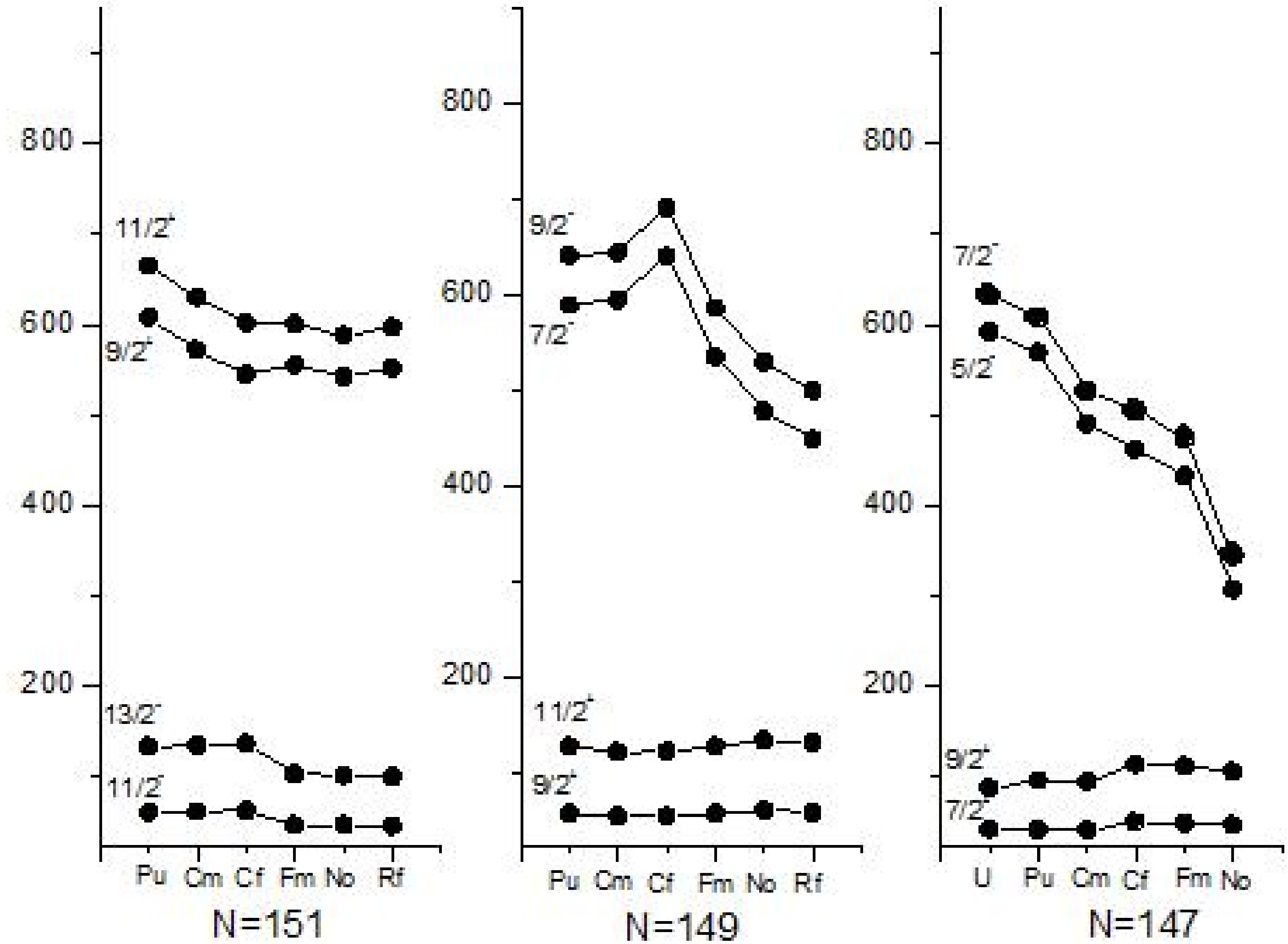
Calculations for $N = 148$, $N = 150$, $N = 152$

($Phys.Rev.\ C\ 74,\ 034316\ (2006)$)



Calculations for $N = 147$, $N = 149$, $N = 151$

$(Phys.Rev.\ C\ 74,\ 034316\ (2006))$



Dinuclear system (DNS) concept

The dinuclear system (A,Z) consists of two fragments (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$ stuck closely together by a molecular-type nucleus-nucleus potential.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$

Hamiltonian of the model (deformed heavy fragment)

If the heavy fragment of the DNS is deformed, the system tends to stay near to the pole-to-pole configuration perfoming small angular oscillations around this position.

$$\hat{H} = \hat{H}_0 + \hat{V}_{int}$$

$$\hat{H}_0 = -\frac{\hbar^2}{2B_\xi}\frac{1}{\xi}\frac{\partial}{\partial \xi}\xi\frac{\partial}{\partial \xi} + \frac{\hbar^2}{2\Im_h}\hat{l}_h + \frac{\hbar^2}{2\mu R_m^2}\hat{l}_{\mathbf{R}} + V(\xi, R_m)$$

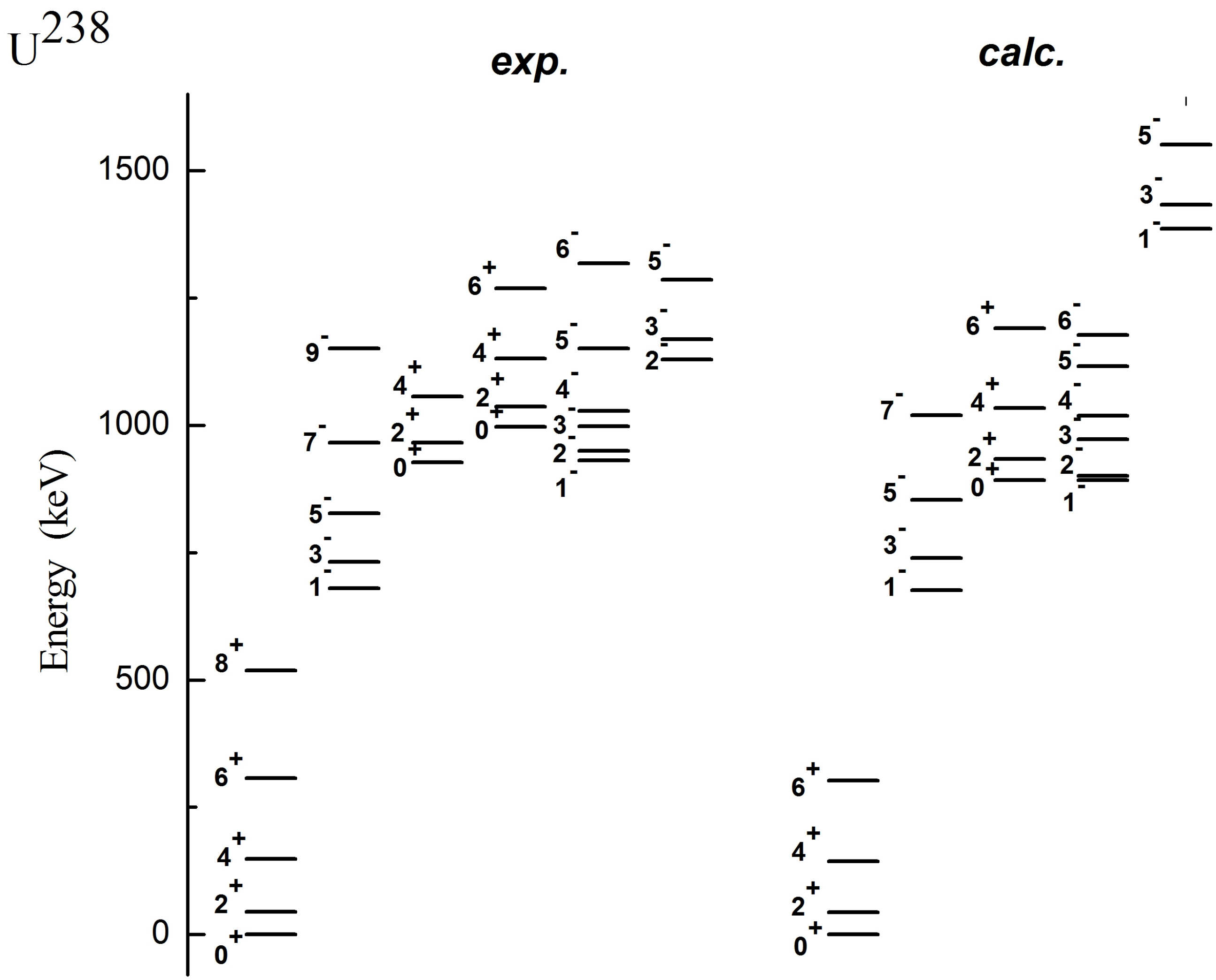
$$\hat{V}_{int} = \frac{C_0\xi}{2}\sum_\mu Y^*_{2\mu}(\Omega_h)Y_{2\mu}(\Omega_{\mathbf{R}})$$

Angular momentum operators:

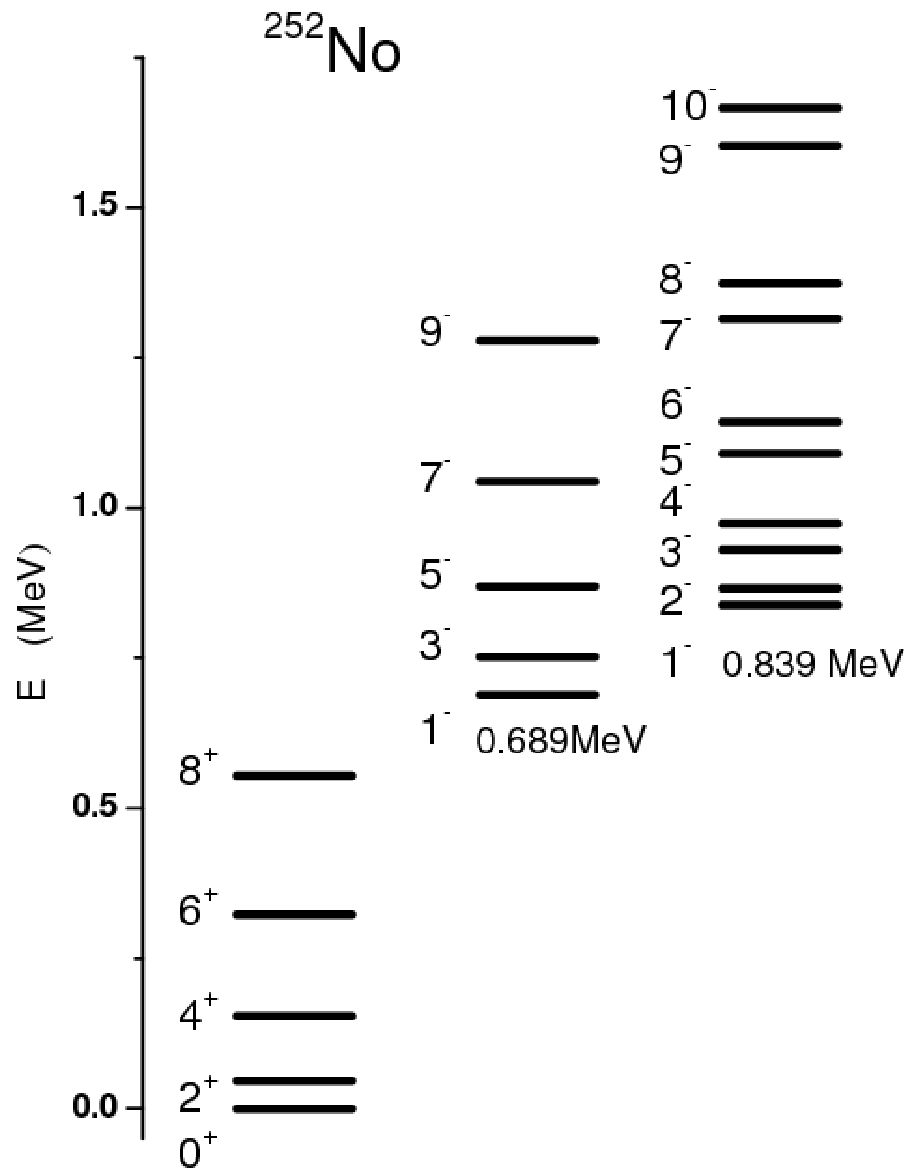
$$\hat{l}_i^2 = -\left[\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} + \frac{1}{\sin^2\theta_i}\frac{\partial^2}{\partial\phi_i^2}\right], \quad (i = h, \mathbf{R})$$

Calculated and experimental spectra of ^{238}U

(exp. data is taken from http://www.nndc.bnl.gov/nndc/ensdf/)



Calculated and experimental spectra of ^{252}No



Hamiltonian of the model (spherical heavy fragment)

For light actinide we suggest that the heavy fragment is spherical and perform harmonic quadrupole oscillations with frequency $\hbar\omega_0$.

$$\hat{H} = \hat{H}_0 + \hat{V}_{int}$$

$$\hat{H}_0 = -\frac{\hbar^2}{2B_\xi}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + \hbar\omega_0\hat{n} + \frac{\hbar^2}{2\mu R_m^2}\hat{L}^2 + V(\xi, R_m)$$

$$\hat{V}_{int} = \frac{V_0}{2}\sum_\mu \beta^*_{2\mu} Y_{2\mu}(\Omega_{\mathbf{R}}) = \frac{V_0\beta_0}{2}\left(d^+ + \tilde{d}\right) \cdot Y_{2\mu}(\Omega_{\mathbf{R}})$$

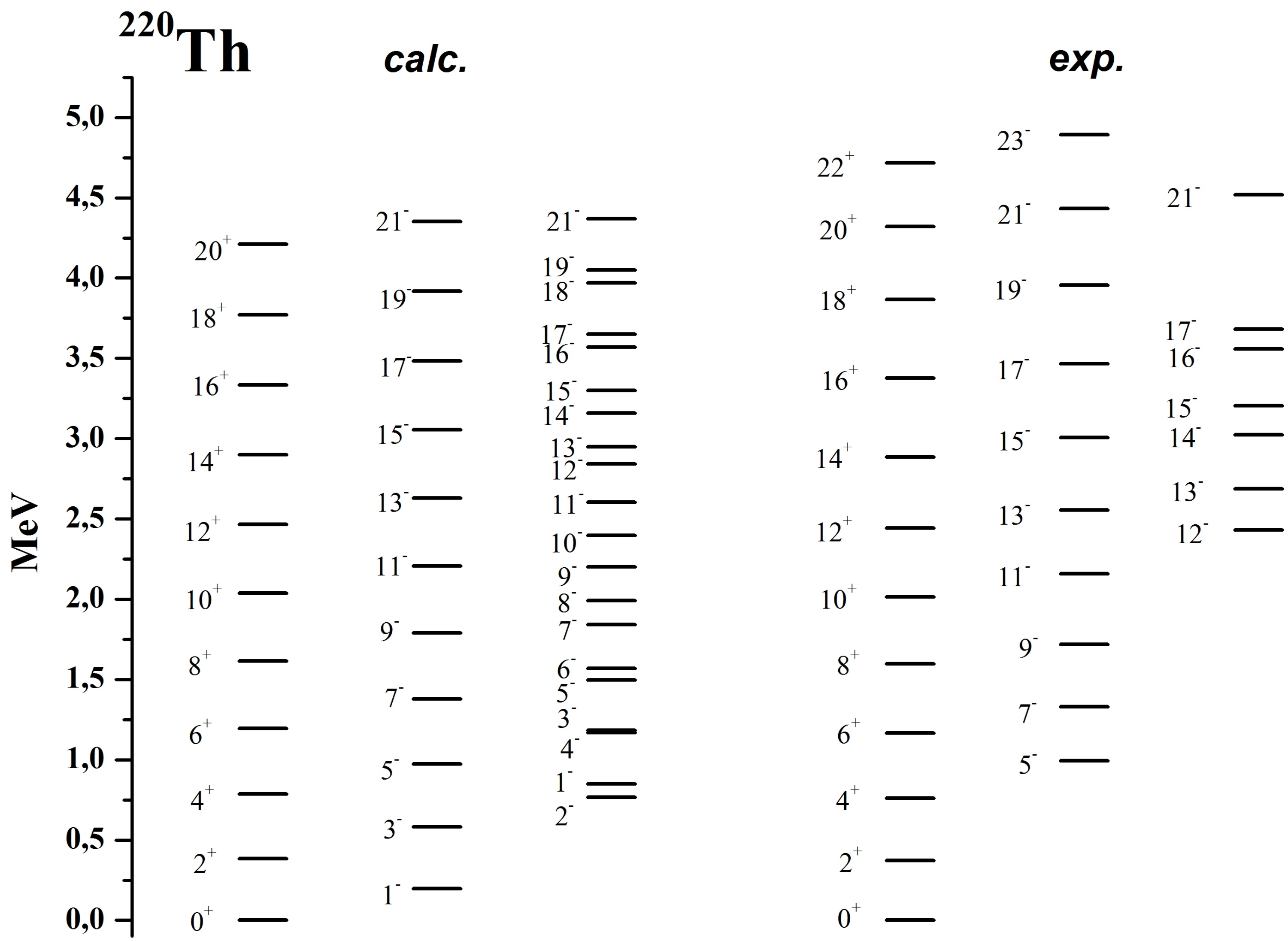
The collective quadrupole coordinates $\beta_{2\mu}$ are expressed in terms of the creation and annihilation operators of the quadrupole bosons

$$\hat{\beta}_{2\mu} = \beta_0(d^+_{2\mu} + \tilde{d}_{2\mu})$$

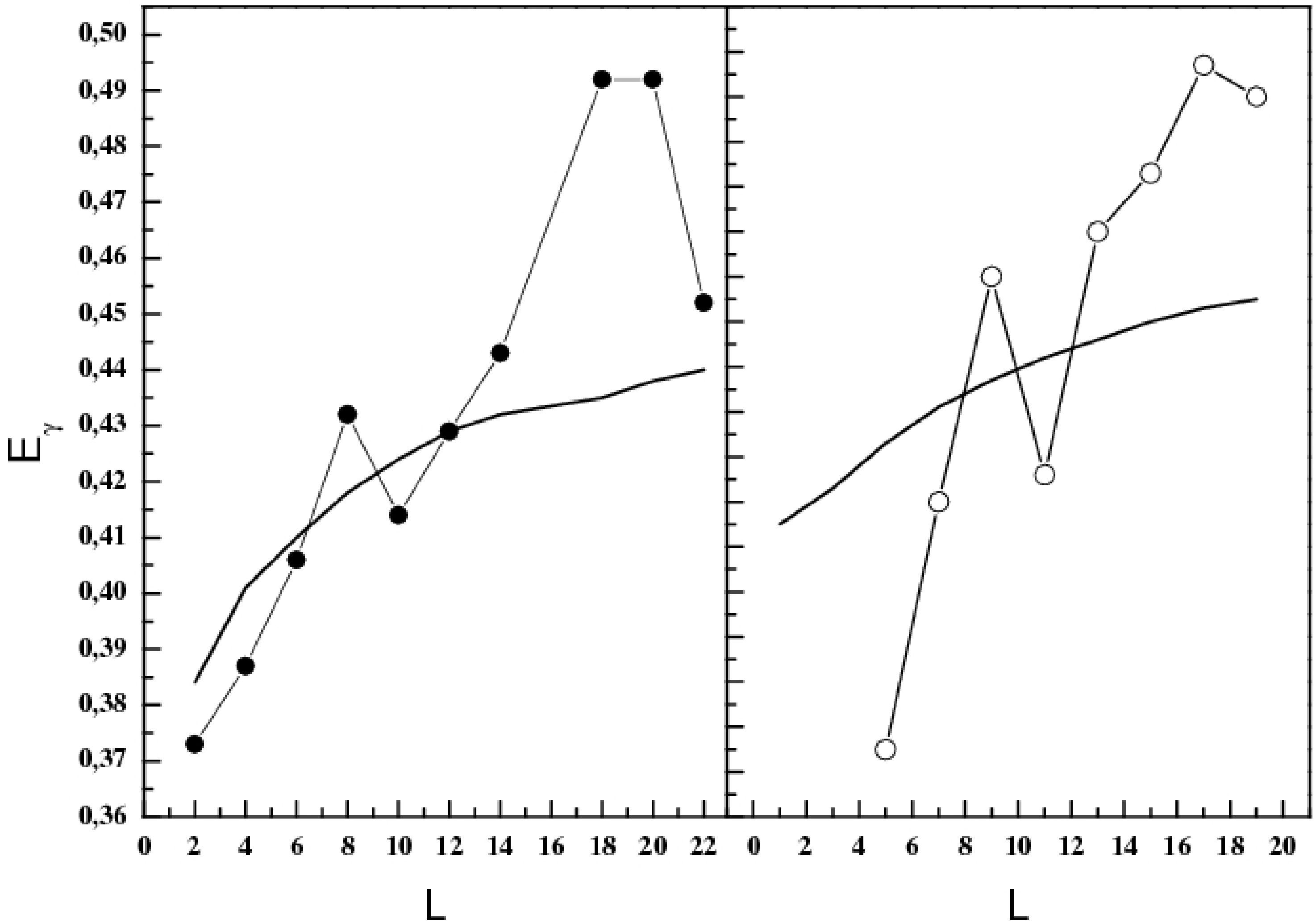
$$\beta_0 = \sqrt{\hbar/2B\omega_0}$$

Calculated and experimental spectra of ^{220}Th

(exp. data is taken from W. Reviol et al., Phys. Rev. C74, 044305 (2006))



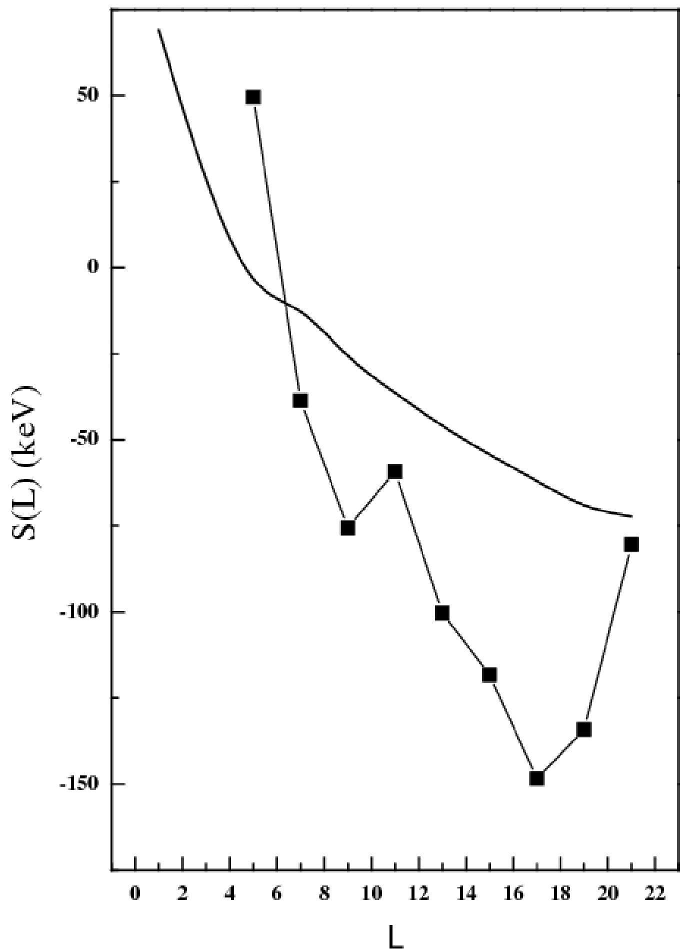
Ground state and first negative parity bands: $^{220}\mathrm{Th}$



Calculated (line) and experimental (solid circles connected by lines) energies of γ–transitions between subsequent level of the ground state band. Experimental values are taken from W. Reviol et al., Phys. Rev. C74, 044305 (2006)

Parity Splitting

$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}$$



The sign of the parity splitting is determined by the difference in energies characterizing the quadrupole vibrations of the heavy fragment and the rotation of the light fragment around the heavy one.

$$E(I^-) = \frac{1}{2}\omega(I-1) + \frac{\hbar^2}{2\mu R^2}$$

$$E(I^+) = \frac{1}{2}\omega I$$

$$S(I^-) = \frac{\hbar^2}{2\mu R^2} - \frac{1}{2}\omega\frac{2I}{(2I+1)}$$

Multipole moments and transitions

Electric multipole operators $\hat{Q}_{\lambda\mu} = \int \rho(\mathbf{r}) r^{\lambda} Y^{*}_{\lambda\mu} d\tau$

For the dinuclear system $\rho(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r}),$

$$\hat{Q}_{\lambda\mu} = \sum_{\lambda_1, \lambda_1+\lambda_2=\lambda} \sqrt{\frac{4\pi(2\lambda+1)!}{(2\lambda_1+1)!(2\lambda_2+1)!}} \left[\hat{q}^{(\lambda_1\lambda_2)}_{\lambda_1} \times Y_{\lambda_2}(\Omega)\right]_{\lambda\mu}$$

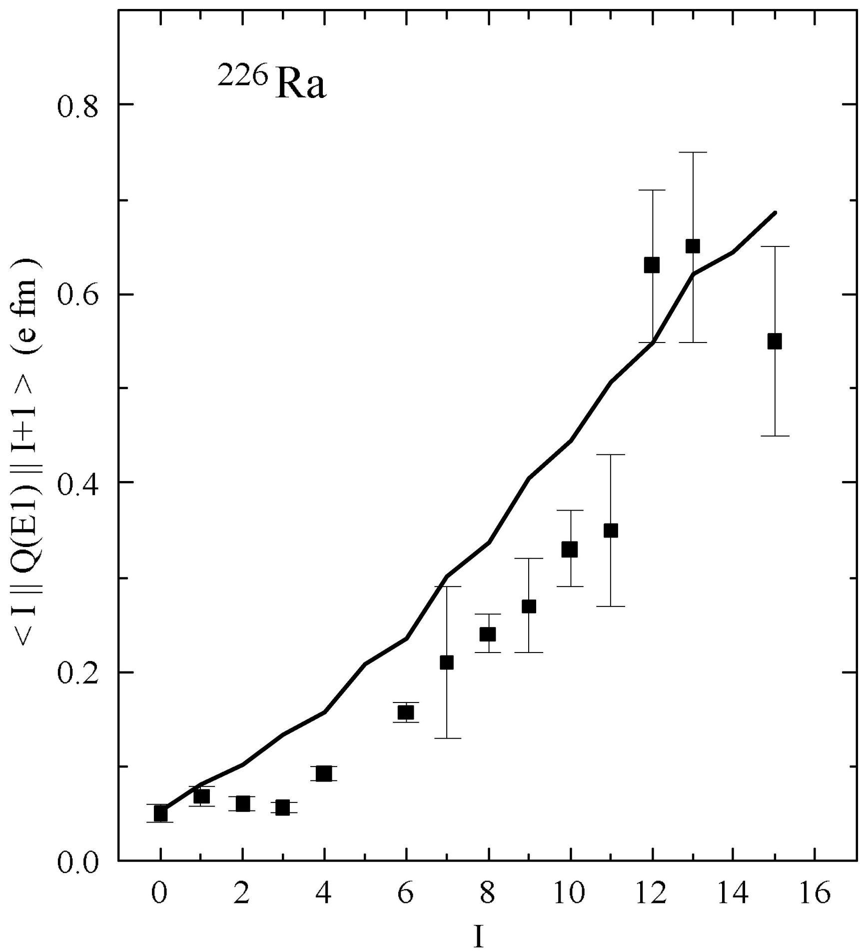
$$\hat{q}^{(\lambda_1\lambda_2)}_{\lambda_1} = \left[\left(\frac{A_1}{A}\right)^{\lambda_2} Q^{(2)}_{\lambda_1} + (-1)^{\lambda_2}\left(\frac{A_2}{A}\right)^{\lambda_2} Q^{(1)}_{\lambda_1}\right] R^{\lambda_2}$$

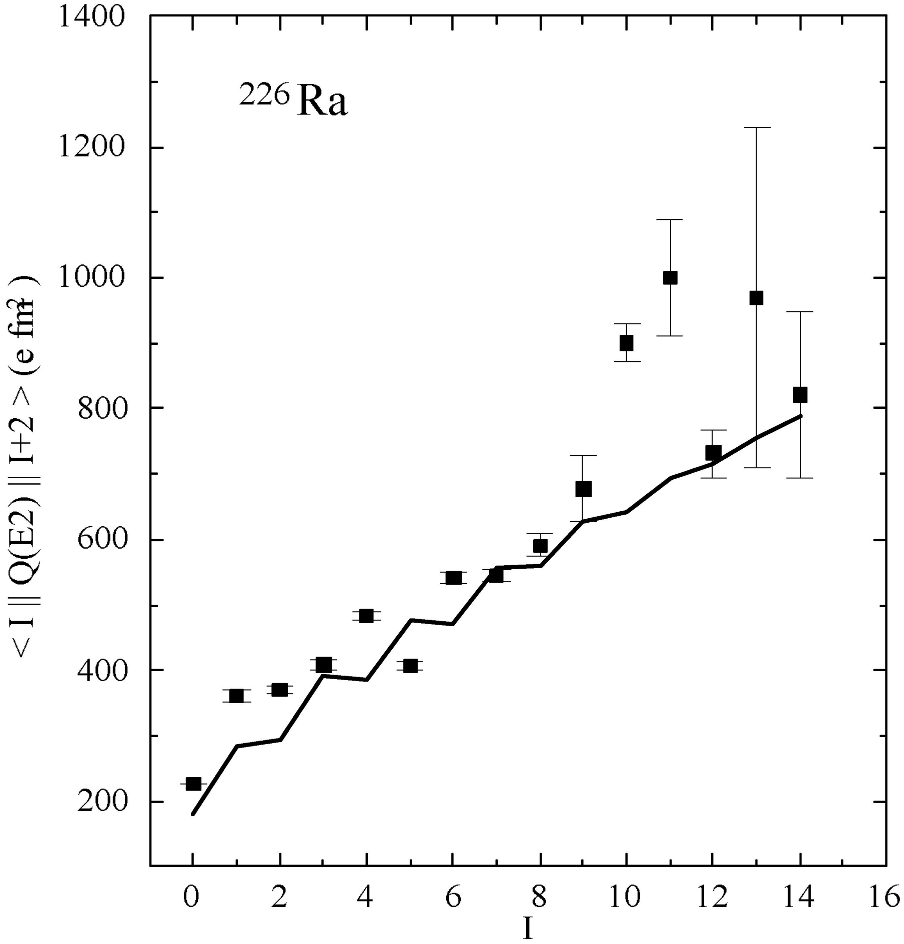
Dipole moment $Q_{1\mu} = e\frac{A_1 Z_2 - A_2 Z_1}{A} R \cdot Y_{1\mu}(\Omega)$

Quadrupole moment $Q_{2\mu} = e\frac{A_1^2 Z_2 + A_2^2 Z_1}{A^2} R^2 \cdot Y_{2\mu}(\Omega) + Q^{(2)}_{2\mu}$

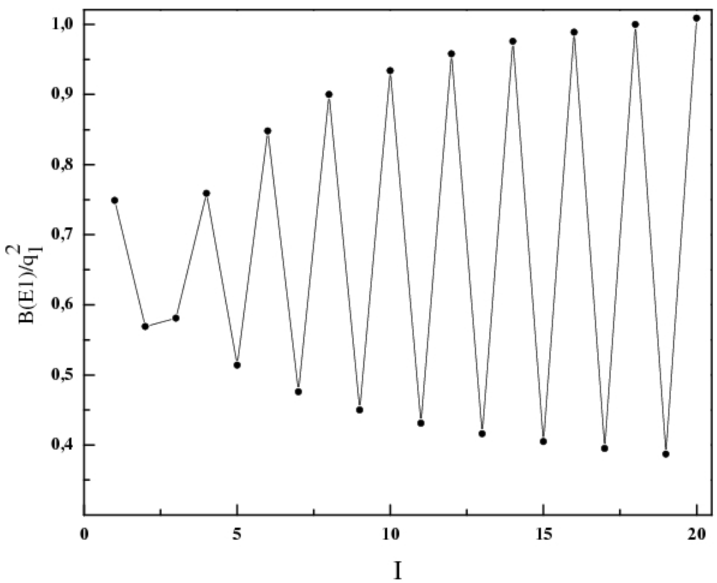
^{226}Ra: $E1$– and $E2$–transitions

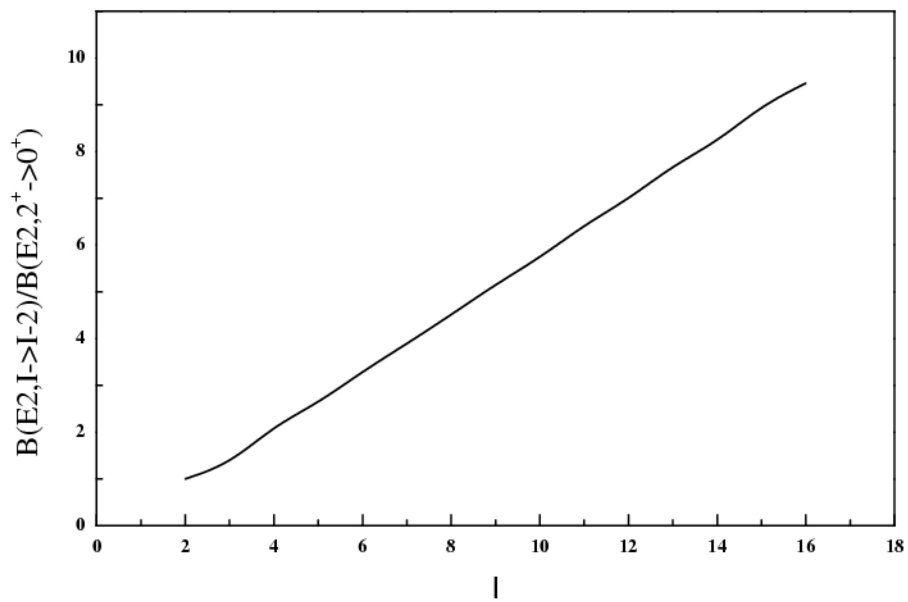
Reduced matrix elements for $E1$– and $E2$–transitions between negative parity and ground state bands as a functions of angular momentum I.

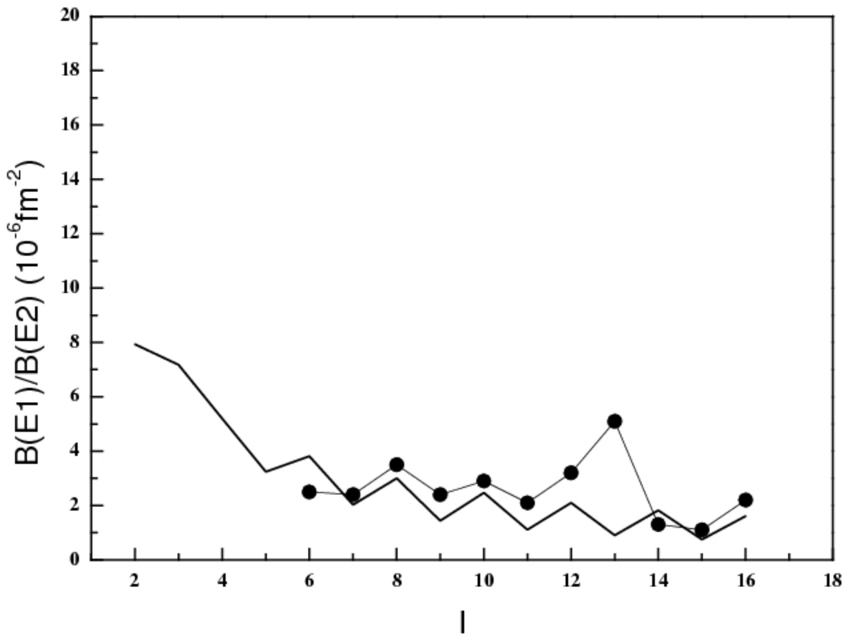




^{220}Th: $B(E1)/B(E2)$ – ratios







Calculated ratios $B(E1)/B(E2)$ for transitions from the states of negative parity lie systematically lower than the ratios for the transitions from the state of the ground state band, which is in agreement with the experimental data with the exception of data points at 13-.

Bending motion

Fixed mass asymmetry $\xi = \xi_\alpha$.

Large stiffness for the angular vibrations $C \equiv C_0 \xi_\alpha \gg 0 \Longrightarrow \sin\epsilon \approx \epsilon$

$$\hat{H} = \hat{H}_{rot} + \hat{H}_{bend} + \hat{V}_{int},$$

$$\hat{H}_{rot} = \frac{\hbar^2}{2\mu R_m^2}(\hat{L}^2 - 2\hat{L}_3')$$

$$\hat{H}_{bend} = \frac{\hbar^2}{2\Im_b}\frac{1}{\epsilon}\frac{\partial}{\partial\epsilon}\epsilon\frac{\partial}{\partial\epsilon} + \frac{\hbar^2}{2\Im_b\epsilon^2}L_3'^2 + \frac{C}{2}\epsilon^2$$

$$\hat{V}_{int} = \frac{\hbar^2}{2\mu R_m^2}\left[\left(\frac{1}{\epsilon}(L_1'L_3' + L_3'L_1'\right) + 2iL_2'\frac{1}{\sqrt{\epsilon}}\frac{\partial}{\partial\epsilon}\sqrt{\epsilon}\right]$$

(L_1', L_2', L_3') — intrinsic components of total angular momentum $\hat{L}$

In this case the rotational motion of the DNS as a whole and the angular oscillations can be approximately separated.

Bending motion

Moment of inertia $\Im_b = \frac{\Im_h \times \mu R_m^2}{\Im_h + \mu R_m^2}$
Frequency of the bending oscillations $\omega_b = \sqrt{\frac{C}{\Im_b}}$

Approximate energies (all matrix elements up to the order ϵ^2 are taken into account):

$$E_{nKp} = \hbar\omega_b(2n + K + 1) + \frac{\hbar^2}{\Im_h + \mu R_m^2}\left(I(I+1) - K^2\right)$$

Wave functions:

$$\Psi_{np,IKM} = L_{n,|K|}(\epsilon)\left(D^L_{M,K}(\Omega) + p(-1)^{I+K} D^L_{M,-K}(\Omega)\right)$$

Spectrum of bending motion for $^{238}\mathrm{U} \rightarrow ^{234}\mathrm{Th} + ^{4}\mathrm{He}$