Higher-Spin Theory and Holography

M.A.Vasiliev

MV 1504.07289

Gelfond, MV 1510.03488

Lebedev Institute, Moscow

VII round table

Dubna

27 November 2015

Plan

- **Brief introduction**

- **Holography via unfolding**

- **Field-current-field correspondence**

- $4d$ **HS theory**, $Sp(8)$ **symmetry and its breaking by interactions**

- **Invariant functionals in** AdS_4/CFT_3 **HS theory**

HS gauge theory

Higher derivatives in interactions

A.Bengtsson, I.Bengtsson, Brink (1983), Berends, Burgers, van Dam (1984)

$$S = S^2 + S^3 + \dots , \qquad S^3 = \sum_{p,q,r} (D^p \varphi)(D^q \varphi)(D^r \varphi) \rho^{p+q+r+\frac{1}{2}d-3}$$

HS Gauge Theories $(m = 0)$:

Fradkin, M.V. (1987)

$$AdS_d : \qquad [D_n, D_m] \sim \rho^{-2} = \lambda^2$$

Non-locality beyond any (=Plank) scale: Quantum Gravity?!

HS Holography

Idea of HS duality Sundborg (2001), Witten (2001), Sezgin, Sundell (2002)

AdS_4 **HS theory is dual to** $3d$ **vectorial conformal models**

Klebanov, Polyakov (2002), Petkou, Leigh (2005), Sezgin, Sundell (2005); Giombi and Yin (2009);

Maldacena, Zhiboedov (2011,2012); MV (2012); Koch, Jevicki, Jin, Rodrigues (2011-2014);

Giombi, Klebanov; Tseytlin (2013-2015); ....; Boulanger, Kessel, Skvortsov, Taronna (2015);

Bekaert, Erdmenger, Ponomarev, Sleight (2015) ...

AdS_3/CFT_2 **correspondence**

Henneaux and Rey (2010), Campoleoni, Fredenhagen, Pfenninger and Theisen (2010)

Gaberdiel and Gopakumar (2010)

Analysis of HS holography helps to uncover the origin of AdS/CFT **?!**

Despite significant progress in the construction of actions during last thirty years: A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984); Fradkin, MV (1987); ... Boulanger, Sundell (2012) ...

Construction of the generating functional for correlators was lacking

Unfolded dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \textbf{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional covariant generalization

$$\frac{\partial}{\partial t} \to d\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p}$$

$$\mathbf{dW^\Omega(x) = G^\Omega(W(x))\,, \qquad d = dx^n \partial_n}$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Φ

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \ldots \Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

$d > 1$: **Nontrivial compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

Any solution: FDA Sullivan (1968); D'Auria and Fre (1982)

The unfolded equation is invariant under the gauge transformation

$$\delta W^\Omega(x) = d\varepsilon^\Omega(x) + \varepsilon^\Phi(x) \wedge \frac{\partial G^\Omega(W(x))}{\partial W^\Phi(x)}$$

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms
- Clear group-theoretical interpretation of fields and equations in terms of modules and Chevalley-Eilenberg cohomology of a symmetry algebra h

  Background fields: flat connection of h

  Fields: h-modules

  Equations: covariant constancy conditions
- Local degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$ (as $q(t_0)$) infinite-dimensional module dual to the space of single-particle states: $C^i(x_0)$ moduli of solutions
- Independence of ambient space-time

  Geometry is encoded by $G^{\Omega}(W)$

Unfolding and holographic duality

Unfolding unifies various dualities including holographic duality

Extension of space-time without changing dynamics by letting the exterior derivative d and differential forms W live in a larger space

$$\mathsf{d} = dX^n \frac{\partial}{\partial X^n} \to \tilde{\mathsf{d}} = dX^n \frac{\partial}{\partial X^n} + d\hat{X}^{\hat{n}} \frac{\partial}{\partial \hat{X}^{\hat{n}}}, \qquad dX^n W_n \to dX^n W_n + d\hat{X}^{\hat{n}} \hat{W}_{\hat{n}},$$

$\hat{X}^{\hat{n}}$ are additional coordinates

$$\tilde{\mathsf{d}} W^\Omega(X, \hat{X}) = G^\Omega(W(X, \hat{X}))$$

Two unfolded systems in different space-times are equivalent (dual) if they have the same unfolded form. Given unfolded system generates a class of holographically dual theories in different dimensions.

Useful applications:

$sp(8)$-invariant formulation of $4d$ massless equations 2001

derivation of superfield formulations of SUSY models (Misuna, MV (2013))

HS holography 2012,2015

$3d$ conformal equations

Rank-one conformal massless equations Shaynkman, MV (2001)

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} + \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) C^3(y|x) = 0\,, \qquad \alpha, \beta = 1, 2$$

Bosons (fermions) are even (odd) functions of y^α: $C^3(-y|x) = (-1)^{p_c} C^3(y|x$

Rank-two equations: conserved currents

$$\left\{ \frac{\partial}{\partial x^{\alpha\beta}} - \frac{\partial^2}{\partial y^{(\alpha} \partial u^{\beta)}} \right\} J^3(u,\, y|x) = 0$$

Gelfond, MV (2003)

$J^3(u,\, y|x)$: **generalized stress tensor. Rank-two equation is obeyed by**

$$J^3(u,\, y\,|x) = C^3(u + y|x)\, C^3(y - u|x)$$

Primaries: $3d$ **currents of all integer and half-integer spins**

$$J^3(u, 0|x) = \sum_{2s=0}^{\infty} u^{\alpha_1} \dots u^{\alpha_{2s}} J^3_{\alpha_1 \dots \alpha_{2s}}(x)\,, \qquad \tilde{J}^3(0, y|x) = \sum_{2s=0}^{\infty} y^{\alpha_1} \dots y^{\alpha_{2s}} \tilde{J}^3_{\alpha_1 \dots \alpha_{2s}}(x$$

$$J^{3\,asym}(u, y|x) = u_\alpha y^\alpha J^{3\,asym}(x)$$

$$\Delta J^3_{\alpha_1 \dots \alpha_{2s}}(x) = \Delta \tilde{J}^3_{\alpha_1 \dots \alpha_{2s}}(x) = s + 1 \qquad \Delta J^{3\,asym}(x) = 2$$

Field-current-field correspondence

Rank-two field (current) in AdS_3 is equivalent to a rank-one field in a larger space

$$(\frac{\partial}{\partial X^{AB}}+\frac{\partial^2}{\partial y^A\partial y^B})J^3(y|X)=0\,,\qquad A,B=1,\dots,4\,,\qquad X^{AB}=X^{BA}$$

$$X^{AB}=(x^{\alpha\dot\alpha},x^{\alpha\beta},\bar{x}^{\dot\alpha\dot\beta})\,,\qquad x^{\alpha\dot\alpha}=(\mathbf{x}^{\alpha\dot\alpha},\varepsilon^{\alpha\dot\alpha}\mathbf{z})$$

Reduction to Minkowski coordinates $x^{\alpha\dot\alpha}$ gives $4d$ massless equations for all spins

$$J^3=C^4$$

$(3d,m=0)\otimes(3d,m=0)=\sum_{s=0}^{\infty}(4d,m=0)$ Flato, Fronsdal (1978)

The full system of all spins exhibits $sp(8)$ symmetry Fronsdal (1985)

Bandos, Lukierski, (1999) ; Bandos, Lukierski,D. Sorokin, (2000); MV (2001)

A rank-two field in $4d$ describes $4d$ conserved currents equivalent to a rank-one field in six dimensions

$$C^4C^4\sim J^4\sim C^6$$

Free massless fields in AdS_4

Infinite set of spins $s = 0, 1, 2 \dots$

1-form $\omega(y, \bar{y} \mid x)$, **0-form** $C(y, \bar{y} \mid x)$

$$A(y, \bar{y} \mid x) = i \sum_{n,m=0}^{\infty} \frac{1}{n!m!} y_{\alpha_1} \dots y_{\alpha_n} \bar{y}_{\dot{\beta}_1} \dots \bar{y}_{\dot{\beta}_m} A^{\alpha_1 \dots \alpha_n}{}_{,}{}^{\dot{\beta}_1 \dots \dot{\beta}_m}(x)$$

The unfolded system for free massless fields is **(1989)**

- $\star$ $$R_1(y, \overline{y} \mid x) = \eta \, \overline{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \overline{y}^{\dot{\alpha}} \partial \overline{y}^{\dot{\beta}}} \, \bar{C}^4(0, \overline{y} \mid x) + \bar{\eta} \, H^{\alpha\beta} \frac{\partial^2}{\partial y^{\alpha} \partial y^{\beta}} \, C^4(y, 0 \mid x)$$
- $\star$ $$\tilde{D}_0 C^4(y, \overline{y} \mid x) = 0$$

Zero-forms $C^4(Y|x)$ **form a Weyl module** $\sim$ **boundary current module**

Current deformation

Schematically for the flat connection $D = \mathrm{d} + w$

$$\begin{cases} D\omega^4 + L(C^4, w) = 0 \\ \tilde{D}C^4 = 0 \\ D_2 J^4 = 0 \end{cases} \Rightarrow \begin{cases} D\omega^4 + L(C^4, w) + G(w, J^4) = 0 \\ \tilde{D}C^4 + F(w, J^4) = 0 \\ D_2 J^4 = 0 \end{cases}$$

Sector of 0-forms

Gelfond, MV (2012; 2015)

J^4 can be interpreted either as a $4d$ current or as a $6d$ massless field.

$4d$ current interactions: mixed linear system of $d4$ and $d6$ fields.

Algebraically: semidirect sum of a rank-one and rank-two systems.

What is the symmetry preserved by the deformed system?!

When unmixed, both rank-one and rank-two system are $sp(8)$-invariant.

Is $sp(8)$ preserved by the deformation?

= formal consistency of the deformation with $w \in sp(8)$?

Current interactions break $sp(8)$ down to the conformal algebra $su(2,2)$

Gelfond, MV:1510.03488

AdS_4/CFT_3 holography at complex infinity

For manifest conformal invariance introduce

$$y^+_\alpha = \frac{1}{2}(y_\alpha - i\bar{y}_\alpha)\,, \qquad y^-_\alpha = \frac{1}{2}(\bar{y}_\alpha - iy_\alpha)\,, \qquad [y^-_\alpha\,, y^{+\beta}]_\star = \delta^\beta_\alpha$$

AdS_4 foliation: $x^n = (\mathbf{x}^a, \mathbf{z})$: $\mathbf{x}^a$ are coordinates of leaves $(a = 0, 1, 2,)$

Poincaré coordinate $\mathbf{z}$ is a foliation parameter. AdS infinity is at $\mathbf{z} = 0$

$$W = \frac{i}{\mathbf{z}} d\mathbf{x}^{\alpha\beta} y^-_\alpha y^-_\beta - \frac{d\mathbf{z}}{2\mathbf{z}} y^-_\alpha y^{+\alpha}$$

$$e^{\alpha\dot{\alpha}} = \frac{1}{2\mathbf{z}} dx^{\alpha\dot{\alpha}}\,, \qquad \omega^{\alpha\beta} = -\frac{i}{4\mathbf{z}} d\mathbf{x}^{\alpha\beta}\,, \qquad \bar{\omega}^{\dot{\alpha}\dot{\beta}} = \frac{i}{4\mathbf{z}} d\mathbf{x}^{\dot{\alpha}\dot{\beta}}$$

Vacuum connection can be extended to the complex plane of $\mathbf{z}$ with all components containing $d\bar{\mathbf{z}}$ being zero.

Generating functional for the boundary correlators

$$S = \frac{1}{2\pi i} \oint_{\mathbf{z}=0} \mathcal{L}(\phi)$$

An on-shell closed $(d+1)$-form $\mathcal{L}(\phi)$ for a d-dimensional boundary

$$\mathrm{d}\mathcal{L}(\phi) = 0\,, \qquad \mathcal{L} \neq \mathrm{d}M$$

Structure of the functional

The residue at $\mathbf{z}=0$ gives the boundary functional of the structure analogous to $\phi_{n_1\dots n_s}J^{n_1\dots n_s}$

$$S_{M^3}(\omega)=\int_{M^3}\mathcal{L}\,,\qquad \mathcal{L}=\frac{1}{2}\omega_{\mathbf{x}}^{\alpha_1\dots\alpha_{2(s-1)}}e_{\mathbf{x}}^{\alpha_{2s-1}}{}_{\beta}e_{\mathbf{x}}^{\alpha_{2s}\beta}(aC_{\alpha_1\dots\alpha_{2s}}(\omega)+\bar{a}\bar{C}_{\alpha_1\dots\alpha_{2s}}(\omega))$$

$C_{\alpha_1\dots\alpha_{2s}}(\omega)$ has conformal properties of currents.

$$aC_{\alpha_1\dots\alpha_{2s}}(\omega)+\bar{a}\bar{C}_{\alpha_1\dots\alpha_{2s}}(\omega)=a_-\mathcal{T}_{-\alpha_1\dots\alpha_{2s}}(\omega)+a_+\mathcal{T}_{+\alpha_1\dots\alpha_{2s}}(\omega)$$

$\mathcal{T}_-$ describes local boundary terms

$\mathcal{T}_+$ describes nontrivial correlators via the variation of S_{M_3} over the HS gauge fields $\omega_{\mathbf{x}}^{\alpha_1\dots\alpha_{2(s-1)}}$

$$\langle J(\mathbf{x}_1)J(\mathbf{x}_2)\dots\rangle=\frac{\delta^n\exp\left[-S_{M^3}(\omega,C(\omega))\right]}{\delta\omega(x_1)\delta\omega(x_2)\dots}\Big|_{\omega=0}$$

ω^{jj} has conformal dimension of the shadow field but does not describe new degrees of freedom being related to $\mathcal{T}_{\pm}^{jj}$ via unfolded equations

Computation of a_+: Didenko, Misuna, MV work in progress

Nonlinear HS equations

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = (\mathsf{d} + W) + S\,, \qquad W = dx^n W_n\,, \quad S = dz^\alpha S_\alpha + d\bar{z}^{\dot\alpha}\bar{S}_{\dot\alpha}$$

$$\mathcal{W}\star\mathcal{W} = i(dZ^A dZ_A + \eta dz^\alpha dz_\alpha B\star k\star\kappa + \bar\eta d\bar{z}^{\dot\alpha}d\bar{z}_{\dot\alpha}B\star\bar{k}\star\bar\kappa)$$

$$\mathcal{W}\star B = B\star\mathcal{W}\;, \qquad B = B(Z;Y;k,\bar{k}|x)$$

HS star-product

$$(f\star g)(Z;Y) = \frac{1}{(2\pi)^4}\int d^4U\, d^4V \exp\left[iU_A V^A\right] f(Z+U;Y+U)g(Z-V;Y+V)$$

Manifest gauge invariance

$$\delta\mathcal{W} = [\varepsilon,\mathcal{W}]_\star\,, \qquad \delta B = \varepsilon\star B - B\star\varepsilon\,, \qquad \varepsilon = \varepsilon(Z;Y;K|x)$$

Klein operator

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \kappa\star\kappa = 1$$

$$\kappa\star f(z,y) = f(-z,-y)\star\kappa$$

Invariants of the AdS_4 HS theory

The new proposal is to consider invariants that are not of the form $str(L)$ via the following extension of the HS unfolded equations

$$\mathcal{W}\star\mathcal{W}=F(\mathcal{B})+\mathcal{L}\,Id\,,\qquad \mathcal{W}\star\mathcal{B}=\mathcal{B}\star\mathcal{W}\,,\qquad \mathrm{d}\mathcal{L}=0$$

$\mathcal{W}=\mathrm{d}+W$ and $\mathcal{B}$ are differential forms of odd and even degrees, respectively (both in dx and dZ).

An appropriate choice is

$$iF(\mathcal{B})=dZ_A dZ^A+\eta\delta^2(dz)\mathcal{B}\star k\star\kappa+\bar\eta\delta^2(d\bar z)\mathcal{B}\star\bar k\star\bar\kappa+G(\mathcal{B})\delta^4(dZ)k\star\bar k\star\kappa\star\bar\kappa+\mathcal{L}I$$

$G=g+O(\mathcal{B})\,,\ g$ is the coupling constant.

$\mathcal{L}$ are x-dependent space-time differential forms of even degrees.

Density relevant to the generating functional of correlators in AdS_4/CFT_3 HS holography is a four-form $\mathcal{L}^4$

Density relevant to BH entropy is a two-form $\mathcal{L}^2$?!

Conclusions

Current interactions in $d=4$ break $sp(8)$ to conformal $su(2,2)$

Holography via unfolding

Invariant functionals via central elements of the HS algebra

Manifest holographic duality at the level of the generating functional

from the unfolded formulation of HS equations

Proposed formulation is gauge invariant, coordinate independent and

applicable to any boundaries and bulk solutions

Two-form and four-form Lagrangian densities in $4d$ HS theory:

BH charges and the boundary generating functional

AdS_3/CFT_2: Invariant functional is a two-form: boundary functional is

an integral of a one-form: holomorphicity of CFT_2

Lagrangians via contractible systems

Contractible system

$$\mathrm{d}w = \mathcal{L}\,, \qquad \mathrm{d}\mathcal{L} = 0$$

is dynamically empty: gauge transformations

$$\delta w(x) = \varepsilon(x)\,, \qquad \delta\mathcal{L}(x) = \mathrm{d}\varepsilon(x)$$

Gauge fixing $w = 0 \quad \Longrightarrow \quad \mathcal{L} = 0$

For the system

$$\mathrm{d}w + L(W) = \mathcal{L}\,, \qquad \mathrm{d}\mathcal{L} = 0$$

where $L(W)$ **is some closed function of other fields** W.

In the *canonical gauge* $w = 0$

$$\mathcal{L} = L(W)\,, \qquad \mathrm{d}L(W) = 0\,.$$

The singlet (invariant) field L **becomes a Lagrangian giving rise to an invariant action**

Vacuum geometry

$\omega = \omega^\alpha T_\alpha$: h **valued 1-form.**

$$G(\omega) = -\omega \wedge \omega \equiv -\frac{1}{2}\omega^\alpha \wedge \omega^\beta [T_\alpha, T_\beta]$$

the unfolded equation with $W = \omega$ has the zero-curvature form

$$d\omega + \omega \wedge \omega = 0\,.$$

Compatibility condition: Jacobi identity for h.

FDA: usual gauge transformation of the connection ω.

Zero-curvature equations: background geometry in a coordinate independent way.

If h is Poincare or anti-de Sitter algebra it describes Minkowski or AdS_d space-time

Linear equations in a h-invariant background are formulated in terms of fields valued in h-modules

Field equations at the boundary

Rescaling

$$C^{i\,1-i}(y,\bar{y}|\mathbf{x},\mathbf{z}) = \mathbf{z}\exp(y_\alpha\bar{y}^\alpha)T^{i\,1-i}(w,\bar{w}|\mathbf{x},\mathbf{z}) \qquad \mathbf{w}^\alpha = \mathbf{z}^{1/2}\mathbf{y}^\alpha \qquad \bar{\mathbf{w}}^\alpha = \mathbf{z}^{1/2}\bar{\mathbf{y}}^\alpha$$

$$W^{jj}(y^\pm|\mathbf{x},\mathbf{z}) = \omega^{jj}(v^-,w^+|\mathbf{x},\mathbf{z}) \qquad \mathbf{v}^\pm = \mathbf{z}^{-1/2}\mathbf{y}^\pm \quad \mathbf{w}^\pm = \mathbf{z}^{1/2}\mathbf{y}^\pm$$

In the limit $\mathbf{z}\to 0$ free HS equations take the form of current conservation equations

$$\left[\mathsf{d}_{\mathbf{x}} - i d\mathbf{x}^{\alpha\beta}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{-\beta}}\right]\mathcal{T}_{\pm}^{j\,1-j}(w^+,w^-|\mathbf{x},0) = 0$$

$$\mathcal{T}_{\pm}^{\mathbf{jj}}(\mathbf{w}^+,\mathbf{w}^-|\mathbf{x},\mathbf{0}) = \eta\mathbf{T}^{\mathbf{j}\,1-\mathbf{j}}(\mathbf{w}^+,\mathbf{w}^-\mid\mathbf{x},\mathbf{0}) \pm \bar{\eta}\mathbf{T}^{1-\mathbf{j}\,\mathbf{j}}(-i\mathbf{w}^-,i\mathbf{w}^+\mid\mathbf{x},\mathbf{0})$$

and

$$\left(\mathsf{d}_{\mathbf{x}} + 2i d\mathbf{x}^{\alpha\beta}v_\alpha^-\frac{\partial}{\partial w^{+\beta}}\right)\omega^{jj}(v^-,w^+|\mathbf{x},0) = d\mathbf{x}^{\alpha\gamma}d\mathbf{x}^{\beta\gamma}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}_{-}^{jj}(w^+,0\mid\mathbf{x},0)$$

$$D_{\mathbf{x}}\omega_{\mathbf{z}}^{jj}(v^-,w^+|\mathbf{x},0)+D_{\mathbf{z}}\omega_{\mathbf{x}}^{jj}(v^-,w^+|\mathbf{x},0) = -\frac{i}{2}d\mathbf{x}^{\alpha\beta}d\mathbf{z}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}_{+}^{jj}(w^+,0\mid\mathbf{x},0)$$

Klein operators and Supertrace

Klein operator

$$\kappa = \exp i z_\alpha y^\alpha \,, \qquad \kappa \star \kappa = 1$$

$$\kappa \star f(z,y) = f(-z,-y) \star \kappa$$

Supertrace

$$str(f(z,y)) = \frac{1}{(2\pi)^2} \int d^2u \, d^2v \exp\left[-iu_\alpha v^\beta\right] f(u,v)$$

$$str(f \star g) = str(g \star f)$$

Klein operators are well-defined with respect to the star product but have divergent supertrace

$$str(\kappa) \sim \delta^4(0)$$

In our construction invariant functionals have divergent supertrace.

HS equations have a form of de Rham cohomology in the twistor space arXiv:1502.02271

Symmetries

The system is consistent because $\mathcal{B}$ commutes with itself and Id. Gauge transformations

$$\delta\mathcal{W} = [\mathcal{W}\,, \varepsilon]_\star\,, \qquad \delta\mathcal{B} = [\mathcal{B}\,, \varepsilon]_\star\,, \qquad \varepsilon = \varepsilon(dx, x, dZ, \dots)$$

$$\delta\mathcal{B} = \{\mathcal{W}\,, \xi\}\,, \qquad \delta\mathcal{W} = \xi^A \frac{\partial F(c, \mathcal{B})}{\partial \mathcal{B}^A}\,, \qquad \xi = \xi(dx, x, dZ, \dots)$$

$$\delta\mathcal{L}(dx, x) = \mathsf{d}\chi(dx, x)\,, \qquad \delta\mathcal{W} = \chi I\,, \qquad \chi(dx, x)$$

χ- transformation implies equivalence of $\mathcal{L}$ up to exact forms allowing to choose canonical gauge $\mathcal{W}_I := \pi\mathcal{W} = 0$

π is the projection to I

$$\pi(f(Y, Z|x))) = f(0, 0|x)\,, \qquad \pi(f \star g) \neq \pi(g \star f)$$

Gauge transformation preserving the canonical gauge

$$\delta\mathcal{L} = \mathsf{d}\chi\,, \qquad \chi = -\pi\Big([\mathcal{W}\,, \varepsilon]_\star + \xi^A \frac{\partial F(c, \mathcal{B})}{\partial \mathcal{B}^A}\Big)$$

$\mathcal{L}$ is on-shell closed and gauge invariant modulo exact forms

Actions versus supertrace

Gauge invariant action

$$S = \int_{\Sigma} \mathcal{L}$$

Since $\mathcal{L}$ is closed, it should be integrated over non-contractible cycles

For AdS/CFT the singularity is at infinity

BH invariants (entropies) are associated with $(d-2)$-forms

If the HS algebra possesses a supertrace

$$\mathcal{L} = str(\mathrm{d}W + W \star W)\Big|_{dZ=0}$$

This suggests that the second term vanishes and hence $\mathcal{L}$ is exact.

Not applicable if $str(W \star W)$ is ill-defined:

$\mathcal{L}$ with well-defined $str(W \star W)$ are exact.

$\mathcal{L}$ with ill-defined $str(W \star W)$ have a chance to be nontrivial.

Boundary functionals, parity, and conformal HS theory

Parity transformation $\mathbf{z} \to -\mathbf{z}$, $\mathbf{x} \to \mathbf{x}$

$$dz^{\alpha}, z^{\alpha}, y^{\alpha}, k \quad \overset{P}{\Longleftrightarrow} \quad \bar{d}z^{\dot{\alpha}}, \bar{z}^{\dot{\alpha}}, \bar{y}^{\dot{\alpha}}, \bar{k}\,.$$

For general η HS equations are not P-invariant.

The A-model $(\eta = 1)$ and B-model $(\eta = i)$ are P-invariant

Since $\mathbf{z}^{-1} d\mathbf{z}$ is P- even, for A and B models $S = S^{loc}$ only contains boundary derivatives giving some gauge invariant boundary functional.

Actions $S^{loc}_{A,B}$ describe $3d$ conformal HS theory and differ by the parity properties of the scalar field.

Nonlocal boundary functional

Naively, $S^{nloc} = 0$ **in** A **and** B**-models.**

For general η **it is not difficult to see that**

$$\mathcal{L} \sim \omega(cos(2\varphi)R_{\mathbf{xx}} - sin(2\varphi)R_{\mathbf{zx}})\,, \qquad \eta = \exp i\varphi$$

$$R_{\mathbf{xx}} \sim \eta e_{\mathbf{x}} e_{\mathbf{x}} C + \bar{\eta} e_{\mathbf{x}} e_{\mathbf{x}} \bar{C}\,, \qquad R_{\mathbf{xz}} \sim i\eta e_{\mathbf{z}} e_{\mathbf{x}} C - i\bar{\eta} e_{\mathbf{z}} e_{\mathbf{x}} \bar{C}$$

$S^{loc} \sim cos(2\varphi)$, $S^{nloc} \sim sin(2\varphi)$. $S^{nloc} = 0$ **for** A, B **models.**

Proper definition: factors in front of $\cos(2\varphi)$ **and** $\sin(2\varphi)$

$$S^{loc}_{A,B} = S(\varphi)\Big|_{\varphi=0,\frac{\pi}{2}}\,, \qquad S^{nloc}_{A,B} = \frac{1}{2}\frac{\partial S(\varphi)}{\partial\varphi}\Big|_{\varphi=0,\frac{\pi}{2}}$$

For general η **it is impossible to separate** S^{loc} **and** S^{nloc}

$S^{loc} + S^{nloc}$ **is gauge invariant:** δS^{nloc} **can contain local terms compensating** δS^{nloc}.

Only P**-invariant** A **and** B **models allow gauge invariant local boundary functionals** $S^{loc}_{A,B}$ **= actions of the boundary conformal HS theory.**

$S^{nloc}_{A,B}$ **are gauge invariant up to local terms.**

Black holes

$4d$ GR BH is characterized by a spin-one Papapetrou field 1966.

Papapetrou two-form $\mathcal{F}$ obeys the sourceless Maxwell equations

$$\mathrm{d}_x \mathcal{F} = 0\,, \qquad \mathrm{d}_x \widetilde{\mathcal{F}} = 0\,, \qquad x \neq 0\,.$$

For Schwarzschild BH

$$\mathcal{F} = \frac{4}{r^2} dt dr\,, \qquad \widetilde{\mathcal{F}} = d\Omega$$

t and r are the time and radial coordinates. $d\Omega$ is the angular two-form.

$M\widetilde{\mathcal{F}}$ supports the BH charge. At the horizon

$$\widetilde{\mathcal{F}} = (2M)^{-2} V_H\,,$$

where V_H is the horizon volume form.

BH charge

The spin-one sector of linearized HS equations

$$\mathrm{d}\omega(x) = \left(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar y^{\dot\alpha}\partial\bar y^{\dot\beta}}C^0(Y|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}C^0(Y|x)\right)\Big|_{Y=0} + \mathcal{L}^2$$

Relation to Papapetrou field

$$\overline{H}^{\dot\alpha\dot\beta}\bar C_{\dot\alpha\dot\beta} + H^{\alpha\beta}C_{\alpha\beta} = M\mathcal{F}\,, \qquad H^{\alpha\beta} := e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}\,, \quad \overline{H}^{\dot\alpha\dot\beta} := e_{\alpha}{}^{\dot\alpha}e^{\alpha\dot\beta}$$

M **is the BH mass, zero-forms** $C_{\alpha\beta}$ **and** $\bar C_{\dot\alpha\dot\beta}$ **are (anti)self-dual components of the spin-one field strength. The Hodge dual two-form is**

$$i\left(H^{\alpha\beta}C_{\alpha\beta} - \overline{H}^{\dot\alpha\dot\beta}\bar C_{\dot\alpha\dot\beta}\right) = M\,\widetilde{\mathcal{F}}\,.$$

$C(Y|x)$ **extends the spin-two BH solution to HS fields**

For $\eta = \exp[i\varphi]$ **this gives in the canonical gauge** $\omega(x) = 0$

$$-\mathcal{L}^2 = \frac{\sin(\varphi)}{4M}V_H + M\cos(\varphi)\mathcal{F}\,.$$

The second term does not contribute since $\mathcal{F}$ **is the electric field of a point charge:** $\omega(x)$ **is the Coulomb field regular at infinity: its**

contribution to $\mathcal{L}^2$ is exact.

$\omega(x)$ for $\tilde{\mathcal{F}}$ describes a monopole solution singular at infinity due to the Dirac string: $\mathcal{L}^2$ in the canonical gauge $\omega(x) = 0$, is closed but not exact.

For the A-model with $\varphi = 0$ the proper definition is

$$Q(0) = -\frac{\partial \mathcal{L}^2(\varphi)}{\partial \varphi}\Big|_{\varphi=0}.$$

$\mathcal{L}^2$ supports BH charges.

$\mathcal{L}^2$ is closed on-shell with no Killing symmetry of a particular solution?!

No on-shell closed local $\mathcal{L}^2$ is expected in a nonlinear $4d$ field theory.

$\mathcal{L}^2$ in HS theory are in a certain sense nonlocal involving infinitely many derivatives of fields with inverse powers of Λ (flat limit is obscure).

Being independent of local variations of Σ^2, $Q = \int_{\Sigma^2} \mathcal{L}^2(\phi)$ effectively // depends on fields away from Σ^2

For asymptotically free theory at infinity $\mathcal{L}^2$ is asymptotically local, reproducing usual asymptotic charges.

HS star product versus Weyl

Formal map to the Weyl star product

$$f_W(Z;Y) = \frac{1}{(2\pi)^M}\int dSdT \exp -iS_A T^A f_{HS}(Z+S;Y+T)$$

Being equivalent for polynomials, different star products may be inequivalent beyond this class.

Weyl-Moyal star product

$$(f_W \star g_W)(Z;Y) = \frac{1}{(2\pi)^{2M}}\int dUdV \exp\left[i(-U_{1A}V_1^A + U_{2A}V_2^A)\right]$$
$$f_W(Z+U_1;Y+U_2)g_W(Z+V_1;Y+V_2)$$

The map is singular at $Z \neq 0$

$$f_W(Z;Y) = \frac{1}{(2\pi)^M}\int_0^1 d\tau(1-\tau)^{-M}\int dSdT \exp\left[-iS_A T^A + i\frac{\tau}{1-\tau}Z_A Y^A\right]$$
$$\phi\Big(\tau S + \frac{\tau}{1-\tau}Z;Y+T;\tau\Big)$$