Multi-field Born-Infeld and Supergravity

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From collaborations with P. Concha, R.D'Auria, S. Ferrara, E. Rodriguez, M. Trigiante

BI theory: a non-linear realization of (4D) electro-dynamics enjoying electric-magnetic duality symmetry:

$$\begin{aligned}\mathcal{L}_{BI} &= \frac{1}{\lambda}\left\{1-\sqrt{\left|\det\left[\eta_{\mu\nu}+\sqrt{\lambda}F_{\mu\nu}\right]\right|}\right\}\\ &= \frac{1}{\lambda}\left(1-\sqrt{1+\frac{\lambda}{2}F_{\mu\nu}F^{\mu\nu}-\frac{\lambda^2}{16}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\right)\end{aligned}$$

It admits a two-derivative realization in terms of 2 couples of Lagrange multipliers

$$\mathcal{L}' = \frac{\tilde{g}}{2\lambda}\left(\Lambda+\Sigma^2-\frac{\lambda}{2}F^2\right)+\tilde{\theta}\left(\frac{1}{4}F\tilde{F}-\frac{\Sigma}{\lambda}\right)+\frac{1}{\lambda}\left(1-\sqrt{1+\Lambda}\right)$$

Variation of $\mathcal{L}'$ w.r.to $\tilde{g}$, $\tilde{\theta}$ gives $\mathcal{L}_{BI}$!

...while variation w.r.to Λ, Σ gives a 2-derivative E-D coupled to non-dynamical scalars:

$$\tilde{\mathcal{L}}(F,\tilde{g},\tilde{\theta}) = -\frac{\tilde{g}}{4}F^2 + \frac{\tilde{\theta}}{4}F\tilde{F} - \mathcal{V}(\tilde{g},\tilde{\theta})$$

with scalar potential:

$$\mathcal{V}(\tilde{g},\tilde{\theta}) = \frac{1}{2\lambda}\left(\tilde{g} + \tilde{\theta}^2\tilde{g}^{-1} + \tilde{g}^{-1}\right) - \frac{1}{\lambda} = \frac{1}{2\lambda}\mathrm{Tr}[\mathcal{M}] - \frac{1}{\lambda}$$

where: $\mathcal{M}_{MN}[\tilde{g},\, \tilde{\theta}] = \begin{pmatrix} \tilde{g} + \tilde{\theta}\tilde{g}^{-1}\tilde{\theta} & -\tilde{\theta}\tilde{g}^{-1} \\ -\tilde{\theta}\tilde{g}^{-1} & \tilde{g}^{-1} \end{pmatrix} \in Sp(2,\mathbb{R})$.

[Rocek, Tseytlin; L.A. R. D'Auria, M. Trigiante]

We have shown $\tilde{\mathcal{L}}$ to be proportional, modulo an $Sp(2,\mathbb{R})$ rotation N (to introduce charges: $\mathrm{Tr}[\mathcal{M}]\to\mathrm{Tr}[N\cdot\mathcal{M}]$), to the bosonic lagrangian of $\mathcal{N}=2$ SuSy vector mult.:

- with a scalar potential: $\mathcal{N}=2\to\mathcal{N}=1$ SuSy breaking

in the infrared limit $\partial\phi << m\phi$, with $\phi=\theta-ig$

$\Rightarrow$ it describes the dynamics of the gauge field in the Goldstino multiplet $[1/2,1]$ of $\mathcal{N}=2\to\mathcal{N}=1$, $D=4$ theory, for $\Lambda_{\not{SuSy}}\to\infty$

[Deser-Puzalowski; Cecotti-Ferrara; Casalbuoni-et-al.;Bagger-Galperin;Rocek-Tseytlin;Komargodski-Seiberg]

We found a general rule for a 2-derivatives description "dualizing" many-vector BI, both for a generic bosonic theory & in the SuSy case (singular):

$$\mathcal{L}' = \frac{g_{IJ}}{2\lambda}\left(\Lambda^{IJ} + (\Sigma\cdot\eta\cdot\Sigma^T)^{IJ} - \frac{\lambda}{2}F^I_{\mu\nu}F^{J|\mu\nu}\right) + \theta'_{IJ}\left(\frac{1}{4}F^I\tilde{F}^J - \frac{(\Sigma\cdot\eta)^{IJ}}{\lambda}\right) + \frac{1}{\lambda}\left(C - f(\Lambda)\right),$$

with $f(\Lambda)$ such that $\mathcal{L}'|_{\bar{\Lambda};\bar{\Sigma}} = \tilde{\mathcal{L}} = -\frac{g_{IJ}}{4}F^I_{\mu\nu}F^{J|\mu\nu} + \frac{\theta_{IJ}}{4}F^I\tilde{F}^J - \mathcal{V}(g,\theta)$,

with $\mathcal{V}(g,\theta) = \frac{1}{2\lambda}\mathrm{Tr}\left(N\cdot\mathcal{M}\right) - \frac{C}{\lambda}$, where $N = \begin{pmatrix}\eta & \mathbf{0}\\ \mathbf{0} & \tilde{\eta}\end{pmatrix}$

$\Rightarrow\ f(\Lambda) = \mathrm{Tr}\left(\sqrt{(\eta+\Lambda)\cdot\tilde{\eta}}\right)$

Depending on the choice of $\eta, \tilde{\eta}$, it has different "dual" BI descriptions, both for bosonic and SuSy case, where it reproduces known results

[Ferrara-Porrati-Sagnotti; Ferrara-Sagnotti-Yeranyan]

Here I will describe a multi-vector $\mathcal{N}=2$ SuSy model admitting (in the infrared) a BI description.
I will discuss its natural interpretation as a rigid limit of $\mathcal{N}=2$ SuGra coupled to a hypermultiplet and to n_v vector multiplets, together with a SuSy-breaking scalar potential.
Two scales will play a role, $M_{P\ell}$ and $\Lambda_{\not{SuSy}}$:

- Rigid limit defined by: $\mu \equiv \frac{M_{P\ell}}{\Lambda_{\not{SuSy}}} \to \infty$
- BI limit: $\Lambda_{\not{SuSy}} \to \infty \Rightarrow \partial z^i << (\Lambda_{\not{SuSy}}) z^i$ (z^i scalars).

[L.A., D'Auria, Ferrara, Trigiante; L.A., Concha, D'Auria, Rodriguez, Trigiante]

Field content of the underlying supergravity model: $(A=1,2)$

- n_v Vector mut. $[A^I_\mu, \lambda^{iA}, z^i]$; $I,i=1,\cdots n_v$; $z^i\in\mathbb{C}$
- 1 Hypermult. $[\zeta_\alpha, q^u]$; $\alpha=1,\cdots 2; u=1,\cdots 4$
- Supergravity mult. $[g_{\mu\nu}, \psi_{\mu A}, A^0_\mu]$

In the rigid limit $\mu\to\infty$:

- Hyper + SuGra in hidden sector.
- Vector mult. in the visible sector.
  v.m. scalars masses from Fayet-Iliopoulos terms $\mathbb{P}^{xM}$, $(x=1,2,3\in SU(2))$; $M=1,\cdots 2n_v\in Sp(2n_v)$
  Scalar potential as in $\tilde{\mathcal{L}}$ "dualizing" BI:

$$\mathcal{V}_{\mathcal{N}=2}(z,\bar z)=\mathbb{P}^{xM}\mathcal{M}_{MN}(z,\bar z)\mathbb{P}^{xN}=\mathrm{Tr}(N\cdot\mathcal{M}),\ \text{for}\ N=\mathbb{P}^x\cdot\mathbb{P}^{x\,T}$$

$\Rightarrow$ It should allow $\mathcal{N}=1$-preserving vacua.

Spontaneous $\mathcal{N}=2\to\mathcal{N}=1$ in rigid SuSy has a peculiar story:

- first believed to be impossible (no-go theorems) relying on Susy Ward-identity on the fermion-shifts $\delta^A\lambda^{iB}$

$$\delta_A\,\bar\lambda^i_C\,\delta^B\lambda^{iC}=\delta^B_A\,\mathcal{V}(z,\bar z)\,,\quad (A=1,2\in U(2)_R)$$

- then shown to be possible (first by Hughes and Polchinski, in 2D), by relaxing the Ward identity to:

$$\delta_A\bar\lambda^i_C\,\delta^B\lambda^{iC}=\delta^B_A\,\mathcal{V}(\phi)+C^A{}_B$$

for a non-zero $C^A{}_B=\xi^x(\sigma^x)^A{}_B\in su(2)$ $(x=1,2,3)$.

[Ferrara-Maiani; Cecotti-Girardello-Porrati; Hughes-Polchinski; Antoniadis-Partouche-Taylor]

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Crucial role of $\xi^x = \epsilon^{xyz} m^{yI} e^z_I$ $\Leftarrow$ requires electric AND magnetic FI terms $\mathbb{P}^{xM} = \begin{pmatrix} m^{xI} \\ e^x_I \end{pmatrix}$, $M \in Sp(2n_V, \mathbb{R})$

$\Rightarrow$ With this FI, Antoniadis, Partouche, Taylor in '95 explicitly constructed the first $\mathcal{N}=2 \to \mathcal{N}=1$ rigid SuSy model.

On the other hand, the SuGra Ward identity:

$$\delta_A \bar{\lambda}^i_C \, \delta^B \lambda^{iC} + \delta_A \bar{\zeta}_\alpha \, \delta^B \zeta^\alpha - 3\delta_A \bar{\psi}_{\mu C} \, \delta^B \psi^{\mu C} = \delta^B_A \mathcal{V}(\phi)$$

allows for partial breaking in SuGra, for $\mathcal{V}|_{vac} \leq 0$, due to the negative contribution from $\psi_{\mu A}$.

[Cecotti-Girardello-Porrati; Ferrara-Girardello-Porrati; Fre-Girardello-Porrati-Trigiante]

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[Cecotti-Girardello-Porrati; Ferrara-Girardello-Porrati; Fre-Girardello-Porrati-Trigiante]

In particular it can happen that, in the vacuum:

$$\delta_A\bar{\zeta}_\alpha\,\delta^B\zeta^\alpha - 3\delta_A\bar{\psi}_{\mu C}\delta^B\psi^{\mu C} = -\xi^x(\sigma^x)_A{}^B$$

so that:

$$\delta_A\bar{\lambda}^i_C\,\delta^B\lambda^{iC} = \delta^B_A\,\mathcal{V}(\phi) + \xi^x(\sigma^x)_A{}^B$$

This requires that, in the rigid limit, gravitini and hyperini still non trivially transform into each-other, even if decoupled form the visible sector!

This describes precisely the case under investigation!

Let's discuss the $\mathcal{N} = 2$ SuGra model leading, in the rigid limit $\mu \to \infty$, to the n_v v.m. generalization of the APT model (partial SuSy breaking).
Problem already discussed, for the 1 v.m case, by Ferrara, Girardello, Porrati (FGP).
Here:

- generalization to n_v v.m.
- clarification of the origin of the FI terms as electric and magnetic charges in the SuGra model.

In the SuGra model, the charges are turned on by gauging isometries in the $\mathcal{M}_Q(q)$. We choose $\mathcal{M}_Q = SO(4,1)/SO(4)$. We will gauge 2 translational isometries t_m $(m = 1, 2)$.

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The theory contains n_v+1 electric vectors A^Λ_μ, $(\Lambda=0,I)$.
Gauging described in terms of $X_{\hat{M}}\equiv(X_\Lambda,X^\Lambda)\in Sp(2n_v+2,\mathbb{R})$ $(\hat{M}=1,\cdots 2n_v+2)$ via the embedding:

$$X_{\hat{M}}=\Theta_{\hat{M}}^{\ m}t_m\,,\quad \text{where } \Theta_{\hat{M}}^m:\mathbb{C}^{\hat{M}\hat{N}}\Theta_{\hat{M}}^m\Theta_{\hat{N}}^n=0\quad\leftrightarrow\quad \Theta_\Lambda^{\ [m}\Theta^{\Lambda|n]}=0$$

Here we take (following FGP):

- $ds^2_{\mathcal{M}_Q}=\frac{1}{2}\left(d\varphi^2+e^{2\varphi}\,dq^x dq^x\right)\,,\quad x=1,2,3=(1,m)$, isometry gen. t_m acting on q^2,q^3.
- $\Theta_{\hat{M}}^{\ m}=\left(\Theta_{\hat{M}}^{\ 1},\Theta_{\hat{M}}^{\ 2}\right)=\begin{pmatrix} e/\mu^2 & \sigma/\mu^2\\ 0 & 0\\ 0 & 0\\ m^i/\mu & 0\end{pmatrix}$: $\Theta_0^{\ [m}\Theta^{0|n]}=\Theta_I^{\ [m}\Theta^{I|n]}=0$,

This is not enough to get $\mathcal{N}=2\to\mathcal{N}=1$ (for $\mu\to\infty$)!
In this symplectic frame, it further requires a particular behavior of the special-geometry prepotential of the v.m. sector $F(X)=-\mathrm{i}(X^0)^2 f(z)$ (in special coordinates $z^i=X^I/X^0$):

$$f(z)_{FGP} \underset{\mu\to\infty}{=} \frac{1}{4}+\frac{z}{2\mu}+\frac{\phi(z)}{2\mu^2}+O\left(\frac{1}{\mu^3}\right)$$

We generalized it to $\Downarrow$

$$f(z^i) \underset{\mu\to\infty}{=} \frac{1}{4}+\frac{\eta_i z^i}{2\mu}+\frac{\phi(z^i)}{2\mu^2}+O\left(\frac{1}{\mu^3}\right)$$

so that $\Omega^{\hat{M}}=\left(1,z^i,-i(2f-z^i\partial_i f),-i\partial_i f\right)=\Omega^{\hat{M}}(z;\eta)$, and:

$$g_{i\bar{\jmath}}(z,\bar{z};\eta)=\frac{1}{\mu^2}\left\{\eta_i\eta_j-\frac{1}{2}\left(\overline{\partial_{ij}\phi}+\partial_{ij}\phi\right)\right\}=\frac{1}{\mu^2}\mathring{g}_{i\bar{\jmath}}$$

It works! We get, for $\mu \to \infty$, a rigid theory generalizing APT:

- Gravity mult. and hypermult. decouple for $\mu \to \infty$
- $\mathcal{V}^{rigid}_{\mathcal{N}=2} = \frac{e^{2\varphi}}{2} \mathcal{M}(z,\bar{z})_{MN} \mathbb{P}^{xM}\mathbb{P}^{xN}, \quad C^A{}_B = e^{2\varphi}\,\xi^x\,(\sigma^x)^A{}_B$

$$\xi^x = \frac{1}{2}\epsilon^{xyz}\,\mathbb{P}^y_{\mathcal{M}}\mathbb{P}^z_{\mathcal{N}}\mathbb{C}^{\mathcal{M}\mathcal{N}} = \epsilon^{xyz} m^{yi} e^z_i\,,$$

N.B.: here $\mathbb{P}^{xM}$ (index $M = 1,\cdots 2n_v$) non trivially related to momentum maps $\mathcal{P}^{x\hat{M}}$:
$\mathbb{P}^{x\mathcal{M}} = (m^{ix}, e^x_i)$ with $e^x_i = \eta_i e^x$ and

$$e^x = (0, e, \sigma) = (0, e^m)\,;\, m^{i\,x} = \Big(0, m^i, 0\Big) = (0, m^{im}).$$

$\Rightarrow$ Part of the charges hidden in the geometry of the moduli space!

Pattern of SuSy breaking

The rigid th. admits partial breaking for $\mathcal{V}_{\mathcal{N}=2}|_{vac} = \sqrt{\xi^x \xi^x} \neq 0$.
In this case, the effective $\mathcal{N} = 1$ potential is

$$\mathcal{V}_{\mathcal{N}=1}(z, \bar{z}) \equiv V^{(APT)}_{\mathcal{N}=2}(z, \bar{z}) - \sqrt{\xi^x \xi^x} \,,$$

and the infra-red dynamics ($\Lambda_{\cancel{SuSy}} \to \infty$) is captured by a multi-field Born-Infeld action [Ferrara, Porrati, Sagnotti].

- The squared mass-matrix of the gravitino has eigenvalues:

$$\lambda_\pm = \frac{e^{2\varphi}}{4} \left[e^2 + \left(\sigma \pm \frac{\eta_i m^i}{2} \right)^2 \right]$$

$\Rightarrow$ In general, SuSy is completely broken in the hidden sector!

Puzzles on charges

In SuGra, charges usually encoded in $\Theta_{\hat{M}}{}^{m} = (\Theta_\Lambda{}^m, \Theta^{\Lambda m})$. Here:

$$\Theta_\Lambda{}^m = (\Theta_0{}^m, \Theta_I{}^m) = \frac{1}{\mu^2}(e^m, 0), \Theta^{\Lambda m} = (\Theta^{0m}, \Theta^{Im}) = \frac{1}{\mu}(0, m^{im})$$

so that $\Theta_0{}^{[m}\Theta^{0|n]} = \Theta_I{}^{[m}\Theta^{I|n]} = 0$. $\leftarrow$ Locality condition.
On the other hand, in the rigid limit they are in $\mathbb{P}^{x\mathcal{M}} = (\mathbb{P}^{xI}, \mathbb{P}^x_I)$:

$$\mathbb{P}^{xI} = \delta^x_1 m^i, \mathbb{P}^x_I = \delta^x_m e^m \eta_i \quad \Rightarrow \epsilon^{xyz}\mathbb{P}^{yI}\mathbb{P}^z_I = \xi^x \neq 0$$

Crucial role of η_i appearing in the metric of $\mathcal{M}_{SK}$! Questions:

- Charges in the σ-mod geometry instead of in $\Theta_{\hat{M}}{}^{m}$??
- Is the rigid theory local?

Puzzles on charges

- Charges in the σ-mod geometry instead of in $\Theta_{\hat{M}}{}^{m}$??

No! in the choice of special coordinates $z^i = X^I/X^0$!
$\Rightarrow$ in the choice of the symplectic section $\Omega^M = (X^0, X^I, F_0, F_I)$.
Consider:

$$S(\eta,\mu) = \begin{pmatrix} 1 & \eta_i/\mu & 0 & 0 \\ 0 & \frac{1}{\mu}\mathbf{1_n} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\eta_i & \mu\mathbf{1_n} \end{pmatrix} \in Sp(2n_v+2,\mathbb{R}) : S\cdot\Omega = \tilde{\Omega}$$

Now $\tilde{z}^i = \tilde{X}^I/\tilde{X}^0 = \frac{z^i}{\mu+\eta_j z^j}$ and $\tilde{f}(\tilde{z}) = \frac{1}{4} + \frac{1}{2\mu^2}\tilde{\phi}(\tilde{z}) + O(\frac{1}{\mu^3})$
$\Rightarrow$ no linear term! Moreover:

$$\tilde{\Theta}_{\hat{M}}{}^{m} = \Theta_{\hat{N}}{}^{m}\cdot(S^{-1})^{\hat{N}}{}_{\hat{M}} = \frac{1}{\mu^2}\left(e^m, -\eta_i e^m, \eta_i m^{im}, m^{im}\right) = \frac{1}{\mu^2}\mathring{\Theta}_{\hat{M}}{}^{m}$$

Now $\mathring{\Theta}^{mI} = \delta_x^m \mathbb{P}^{xI} = m^{im}$, $\mathring{\Theta}_I^m = -\delta_x^m \mathbb{P}_I^x = -\eta_i e^m$

Puzzles on charges

- Is the rigid theory local?

In new frame $Sp(2n_v+2,\mathbb{R}) \underset{\mu\to\infty}{\longrightarrow} Sp(2n_v,\mathbb{R})$ manifest.
Furthermore: $\mathbb{P}^{xM} = \delta^x_m \mathbb{C}^{MN}\mathring{\Theta}_N^m$, where $\mathring{\Theta}_M{}^m = (\mathring{\Theta}_I{}^m, \mathring{\Theta}^{Im})$.

$$\mathring{\Theta}^{\Lambda[m}\mathring{\Theta}^{n]}_\Lambda = 0 \;\Rightarrow\; \mathring{\Theta}^{0[m}\mathring{\Theta}^{n]}_0 = -\mathring{\Theta}^{I[m}\mathring{\Theta}^{n]}_I = \eta_i m^{i[m}e^{n]} = \xi^1 \neq 0$$

Non locality! Due to non-trivial freezing of the graviphoton gauge-bundle in the rigid limit!
This explains why the hidden sector $(\psi_{\mu A},\zeta_\alpha)$ plays a role in the rigid theory ($C^A_B = \xi^x(\sigma^x)^A_B \neq 0$ in the SuSY Ward identity).

N.B.: non locality only affects the fermionic directions of superspace! $\Rightarrow$ the rigid theory on space-time is local!
Indeed:

$$\delta_{[1}\delta_{2]}\,A^I_\mu = -i\sqrt{2}\,\mathring{\Theta}^{I\,m}(\sigma^m)_A{}^B\,\bar{\epsilon}_{[2\,B}\gamma_\mu\,\epsilon^A_{1]}\quad(\star)$$

This is understood in supergravity: for $\Theta^{\Lambda\,m}\neq 0$, the natural symplectic frame requires to dualize $dq^m\to{}^*dq^m=dB_{(2)m}$. The gauge fields A^Λ_μ associated to them are not well defined:

$$F^\Lambda_{\mu\nu}=\partial_{[\mu}A^\Lambda_{\nu]}+\Theta^{\Lambda m}B_{m|\mu\nu}\;\Rightarrow\; dF^\Lambda=\Theta^{\Lambda m}dB_m\neq 0$$

and undergo the anti-Higgs mechanism. In the rigid limit:

$$dF^I\propto i\mathring{\Theta}^{Im}(\sigma^m)_A{}^B\bar{\psi}^A\wedge\gamma_a\psi_B\wedge V^a+\cdots\neq 0$$

Superspace counterpart of $(\star)$.

$$\mathcal{L}_{sugra} \underset{\mu\to\infty}{\longrightarrow} \mathcal{L}^{rigid}_{\mathcal{N}=2\to\mathcal{N}=1} + \mathcal{L}_{hidden}$$

$$\mathcal{L}^{rigid}_{\mathcal{N}=2\to\mathcal{N}=1} = \Lambda^2 \mathring{g}_{i\bar{\jmath}}\partial^\mu z^i \partial_\mu \bar{z}^{\bar{\jmath}} - \frac{i}{2}\mathring{g}_{i\bar{\jmath}}\left(\mathring{\bar{\lambda}}^{iA}\gamma^\mu \nabla_\mu \mathring{\lambda}^{\bar{\jmath}}_A + h.c.\right) +$$
$$+ \left(-i\mathring{\mathcal{N}}_{IJ}\mathcal{F}^{+I}_{\mu\nu}\mathcal{F}^{+J\mu\nu} + h.c.\right) + \Lambda^4\mathring{\mathcal{V}} + \Lambda\left(\mathring{\mathcal{M}}_{iAjB}\mathring{\bar{\lambda}}^{iA}\mathring{\lambda}^{jB} + \text{h.c.}\right) +$$
$$+\Lambda^{-1}\mathcal{F}^{-I}_{\mu\nu}\mathring{l}_{IJ}\left[\frac{1}{2}\nabla_i \mathring{f}^J_j \mathring{\bar{\lambda}}^{iA}\gamma^{\mu\nu}\mathring{\lambda}^{jB}\epsilon_{AB} + h.c.\right]$$

$$\mathcal{L}_{hidden} = M^2_{P\ell}\left(-\frac{R}{2} + h_{\hat{u}\hat{v}}\partial_\mu q^{\hat{u}}\partial^\mu q^{\hat{v}}\right) + 2\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}M_{P\ell}\mathcal{H}_{m|\nu\rho\sigma}A^m_{\hat{u}}\partial_\mu q^{\hat{u}} +$$
$$+\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\left(\bar{\psi}^A_\mu\gamma_\nu\rho_{A|\lambda\sigma} + h.c.\right) - \left(i\bar{\zeta}^\alpha\gamma^\mu\nabla_\mu\zeta_\alpha + h.c.\right) +$$
$$+\left(-i\mathring{\mathcal{N}}_{00}\mathcal{F}^{+0}_{\mu\nu}\mathcal{F}^{+0\mu\nu} + h.c.\right) + 6M^{mn}\mathcal{H}_{m|\mu\nu\rho}\mathcal{H}_n^{\mu\nu\rho} +$$
$$-2\mathcal{U}^{\alpha A}_{\hat{u}}\partial_\mu q^{\hat{u}}\left(\bar{\zeta}_\alpha\gamma^\nu\gamma^\mu\psi_{A\nu} + h.c.\right), \quad (q^{\hat{u}} = q^0, q^1)$$

Conclusions and outlook

- I discussed the supergravity structure hidden in partially broken rigid $\mathcal{N} = 2$ theory of n_v v.m.:
  - Part of charges hidden in moduli-space geometry. $\rightarrow$ They can be turned into the embedding tensor.
  - In the new symplectic frame the rigid theory is non-local. Non-locality only affects the fermionic directions of superspace; it is related to $\delta_\epsilon \psi_{\mu A}|_{vac}, \delta_\epsilon \zeta_\alpha|_{vac} \neq 0$ for $\mu \rightarrow \infty$
- The model has a multi-vector BI limit in the infrared. For $n_v > 1$, the SuSy case is a singular case of a class of multi-vector 2-derivative lagrangians with a dual description as multi-vector BI.