The LLog ressumation for the singular part of pion GPD

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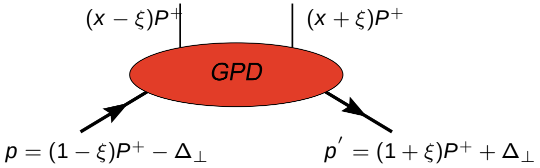
Outline

Definition of GPD



The pion GPD is defined as:

$$iT_I^{abc}H^I(x,\xi,\Delta_\perp) = \int \frac{d\lambda}{2\pi}e^{-ixP^+\lambda}\langle\pi^b(p^{'})|O^c(\lambda)|\pi^a(p)\rangle$$

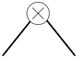
$$O^c(\lambda) = \bar{q}(-\frac{\lambda n}{2})\gamma^+ q(\frac{\lambda n}{2})$$

In the forward limit $\xi \to 0$, $\Delta_\perp \to 0$ the GPD is the parton distribution function.

$$H(x,0,0) = q(x)$$

t-dependence of GPD

The GPD is the phemenology object and it can be obtained only through models or extracted from experimental data. However the dependence of the Δ^2 and masses can be calculated explicitly in low energy effective theories, e.g. ChPT.
Introducing the effective operator $O(\lambda)$ one can build an expansion of GPD as corrections to GPD in chiral limit, which is given by the LO expression [N.Kivel&M.Polyakov,02].

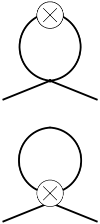


$$\mathring{H}^I(x,\xi,0) = F^I(\beta,\alpha) * \left[\delta(x-\xi\alpha-\beta) - (1-I)\xi\delta(x-\xi(\alpha+\beta))\right]$$

$F(\beta,\alpha)$ is the Double Distribution function for pion GPD in chiral limit.

Next-to-leading contribution

At the next-to-leading order one has two 1-loop diagrams [N.Kivel&M.Polyakov,02][M.Diehl,A.Manashov&A.Schafer,05]:



$$H^{I=1}(x,\xi,\Delta) = \mathring{H}^{I=1}(x,\xi,0)\left[1 - \frac{m_\pi^2 \ln m_\pi^2}{(4\pi F_\pi)^2}\right] +$$

$$\frac{a_\chi}{2}\,\frac{\theta(|x|<\xi)}{\xi}\int_{-1}^{1} d\eta R[\eta,t]\ln(R[\eta,t])\frac{d}{d\eta}\mathring{H}\left(\frac{x}{\xi\eta},\frac{1}{\eta},0\right)$$

$$R[\eta,t] = \frac{1}{(4\pi F_\pi)^2}\left(m_\pi^2 - (1-\eta^2)\frac{t}{4}\right)$$

$$a_\chi = \frac{m_\pi^2}{(4\pi F_\pi)^2} \approx 0.014$$

Singular contribution

The NLO expression for the GPD contains "singular" contribution

$$H^{I=1}_{NLO}(x,\xi,\Delta) \;\sim\; a_\chi \frac{\theta(|x|<\xi)}{\xi}$$

Singularity of that term concludes in following:

- In a regime $\xi \sim a_\chi$ this correction blows up (which is usual kinematic regime for DVCS),
- In the forward limit such contributions provides singularity in the "x"-plane:

$$H^{I=1}_{NLO}(x,0,0) \;=\; q^{I=1}_{NLO}(x) \;=\; a_\chi \ln(1/a_\chi)\delta(x)$$

The reason of appearance of such singular terms is presence in the task the second scale parameter, λ. In aim to avoid double-counting we have to resum expansion in area $\lambda m_\pi \sim 1$.

Singular terms at higher orders

One can obtain that at NNLO ChPT theory gives more singular term. Calculation up to N^3LO gives [N.Kivel&M.Polyakov,07]:

$$Q(x) = Q^{\text{reg}} - \frac{5}{3} a_\chi^2 \ln^2(1/a_\chi) \langle x \rangle \delta'(x) + \mathcal{O}(a_\chi^3)$$

$$q(x) = q^{\text{reg}}(x) - a_\chi \ln(1/a_\chi) \delta(x) - \frac{25}{108} \langle x^2 \rangle a_\chi^3 \ln^3(1/a_\chi) \delta''(x) + \mathcal{O}(a_\chi^4)$$

Investigating the structure of diagrams one can find that such singular contribution would appear in every order. At n-order term of ChPT GPD has singularity of $\theta(|x| < \xi)/\xi^n$-type (or $\delta^{(n-1)}(x)$ in forward limit).

$$q(x) = q^{\text{reg}}(x) + \sum_{n=1}^{\infty} b_n \, a_\chi^n \ln^n(1/a_\chi) \, \langle x^{n-1} \rangle \, \delta^{(n-1)}(x)$$

Calculation in the large-N approach, where $N = 3$ is number of Goldstouns, gives

$$\delta Q(x) = \sum_{n=1,3,}^{\infty} \frac{2}{N}\delta^{(n)}(x)\frac{\epsilon^{n+1}\langle x^n\rangle}{(n+1)!}\left(2+\frac{4}{n}\right) = 4\frac{\theta(|x|<\epsilon)}{N}\int_{\frac{|x|}{\epsilon}}^{1}\frac{q(\beta)}{\beta}\left(1-\frac{|x|}{\epsilon\beta}\right)d\beta$$

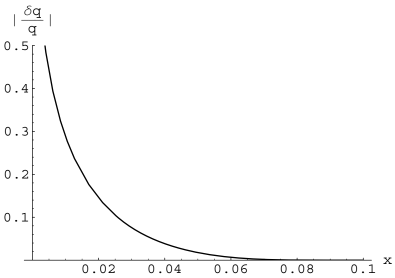
$$\delta q(x) = \sum_{n=0,2,..}^{\infty} \frac{-2}{N}\delta^{(n)}(x)\frac{\epsilon^{n+1}\langle x^n\rangle}{(n+1)!} = -2\frac{\theta(|x|<\epsilon)}{N}\int_{|x|/\epsilon}^{1}\frac{q(\beta)}{\beta}d\beta$$

$$\epsilon = \frac{N}{2}a_\chi \ln(1/a_\chi) \approx 0.09$$

This example shows that resummation solves the problem of singular terms and gives a smooth function.

The similar answer is obtained for GPD.

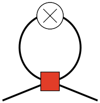
The addition of $\delta q(x)$ gives a visible contribution to PDF in chiral limit.



At $x \sim 10^{-3}$ up to 60%.

Structure of singular terms

Is it possible to calculate singular terms exactly?



- The singular terms are always proportional to the maximal power of chiral logarithm (so called Leading Log, LLog).
- Singular terms go from special class of diagrams, which have such ($\longleftarrow$) structure. Where in the red box all one-particle irreducable graphs are.
- Singular terms do not depend on mass.

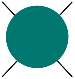
Yes, if it one can calculate the LLog coefficients of $\pi\pi$ scattering in massless ChPT.

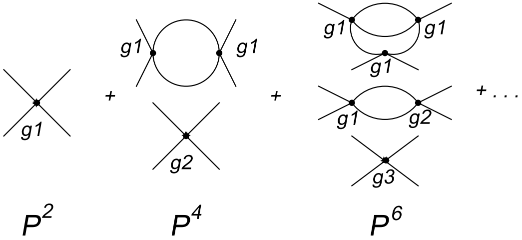
Recursive relation for LLog

The initial Lagrangian is

$$\mathcal{L}_2 \;=\; -\frac{1}{2}\pi^a\partial^2\pi^a - \frac{g_1}{8}\pi^2\partial^2\pi^2 + \mathcal{O}(\pi^6)$$

Our task is the calculation of LLog's for 4-point function:

$$G(\mu, g) =$$  $$= \sum_{n=1}^{\infty}\left(P^2\right)^n G_n(\mu, g_{i\leqslant n}) \;=$$



LLogs appear only in diagrams with g_1^n.

Recursive relation for LLog

The 4-point Green function is renorminvariant, [M.Büchler&G.Colangelo,03](for example, for ChPT).

$$\mu^2 \frac{d}{d\mu^2} G(\mu, g) \;=\; \sum_{n=1}^{\infty} \left(P^2\right)^n \mu^2 \frac{d}{d\mu^2} G_n(\mu, g_{i\leqslant n}) \;=\; 0$$

which demands the renorminvariance of G_n:

$$\mu^2 \frac{d}{d\mu^2} G_n(\mu, g_{i\leqslant n}) \;=\; \left(\mu^2 \frac{\partial}{\partial\mu^2} + \sum_i \mu^2 \frac{\partial g_i}{\partial\mu^2} \frac{\partial}{\partial g_i}\right) G_n(\mu, g_{i\leqslant n}) \;=\; 0$$

The β-function of charge g_n is given by simple poles of counterterms:

$$\mu^2 \frac{\partial g_n}{\partial\mu^2} = \beta_n(g_{i\leqslant n-1}) = \sum_{i=1}^{n-1} \beta_{i,n-i} g_i g_{n-i} + \text{Higher loop contributions}$$

$$\beta_1 = 0$$

Recursive relation for LLog

In terms of logarithms G_n has a form:

$$G_n = \sum_{k=0}^{n-1} \ln^k\left(\frac{\mu^2}{P^2}\right) R_n^k(\mu, g_{i\leqslant n-k+1})$$

This allows us to write the recursive relation:

$$R_n^k(\mu, g_{i\leqslant n-k+1}) = \frac{-1}{k}\sum_i \beta_i(g)\frac{\partial}{\partial g_i} R_n^{k-1}(\mu, g_{i\leqslant n-k+2})$$

For the LLog term one follows:

$$R_n^{n-1}(\mu, g_1) = \omega_n g_1^n = \frac{(-1)^{n-1}}{(n-1)!}\left[\sum_i \beta_i(g)\frac{\partial}{\partial g_i}\right]^{n-1} R_n^0(\mu, g_{i\leqslant n})$$

Using the property of R-operation and $\beta_1 = 0$ we obtain that

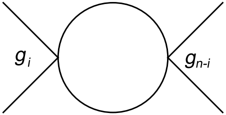
$$R_n^{n-1}(\mu, g_1) = \omega_n g_1^n = \frac{(-1)^{n-1}}{(n-1)!}\left[\sum_i \beta_i(g)\frac{\partial}{\partial g_i}\right]^{n-1} R_n^0(\mu, g_{i\leqslant n}) =$$

$$= \frac{(-1)^{n-1}}{(n-1)!}\left[\sum_i \beta_i^{(1-loop)}(g)\frac{\partial}{\partial g_i}\right]^{n-1} g_n$$

or we can rewrite

$$\omega_n = \frac{1}{n-1}\sum_{i=1}^{n-1} \beta_{i,n-i}\omega_i\omega_{n-i}$$

where $\beta_{i,n-i}$ is simple pole coefficient of the diagram:



Generation of higher vertexes and β-function

The higher vertexes are generated by initial $V_{10} = g_{10}\pi^2\partial^2\pi^2$ vertex. One can see that the next generation of Lagrangian contains two 4-pion vertexes (we introduce the subsidiary index)

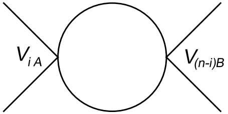
$$V_{20} = g_{20}\pi^2\left(\partial^2\right)^2\pi^2$$

$$V_{22} = g_{22}\left(\pi^a\partial_\mu\partial_\nu\pi^a\right)\left(\pi^a\partial_\mu\partial_\nu\pi^a\right)$$

Arbitrary n order of Lagrangian contains $\frac{n}{2}$ 4-pion vertexes

$$V_{nA} \;=\; g_{nA}\left(\pi^a\partial^A_\mu\pi^a\right)\left(\partial^2\right)^{n-A}\left(\pi^a\partial^A_\mu\pi^a\right)$$

We obtain the mount of β-functions, $\beta(i, A; n-i, B/C)$ from the calculation of the simple pole of diagrams and from projecting of the answer on V_{nC}



Recursive relation for LLog in ChPT

Introducing the subsidiary index do not change the general scheme discussed above. In the massless ChPT the recursive equation for the LLog coefficients takes a form

$$\omega_{nC} = \frac{1}{n-1}\sum_{i=1}^{n-1}\sum_{A,B}\beta(i,A;n-i,B/C)\omega_{iA}\omega_{n-i,B}$$

$$\omega_{1,0} = 1\,, \quad \omega_{i,C>i} = 0$$

This allows us to calculate LLog coefficients at any order numerically (has not solved yet analytically).

This equation is approved by 3-loop direct calculation, leading and next-to-leading order large-N calculation.

For example, at the 55 order of expansion LLog coefficient is $\sum_{C=0}^{55}\omega_{55,C} = 1363.1$, in contrast to the large-N approach, which gives 3.22795×10^9.

Singular part of pion GPD

Using presented method we obtain for the singular term of *n*-chiral order the following:

$$\delta Q_n(x) = \langle x^{n-1}\rangle \delta^{(n-1)}(x)(a_\chi \ln(1/a_\chi))^{n-1}\frac{1}{n!}\sum_{C=0}^{n}\left(2\omega_{nC} + N\frac{(2C)!}{C!C!}\omega_{nn}\right)$$

$$\delta q_n(x) = \langle x^{n-1}\rangle \delta^{(n-1)}(x)(a_\chi \ln(1/a_\chi))^{n-1}\frac{(-1)}{n!}\sum_{C=0}^{n}\omega_{nC}$$

There is the similar answer for the GPD's

Conclusion

- The resummation of singular terms in GPD (PDF) produces smooth function, which gives a significant effect in small-x region
- Introduced method of obtaining of LLog coefficients is valid in any massless φ^4-type non-renormalizable QFT.
- The obtained recursive equation easy transfers onto renormalizable theories.