Renormalization Group & Related Topics 2008

Renormalization-group description of nonequilibrium critical short-time relaxation processes

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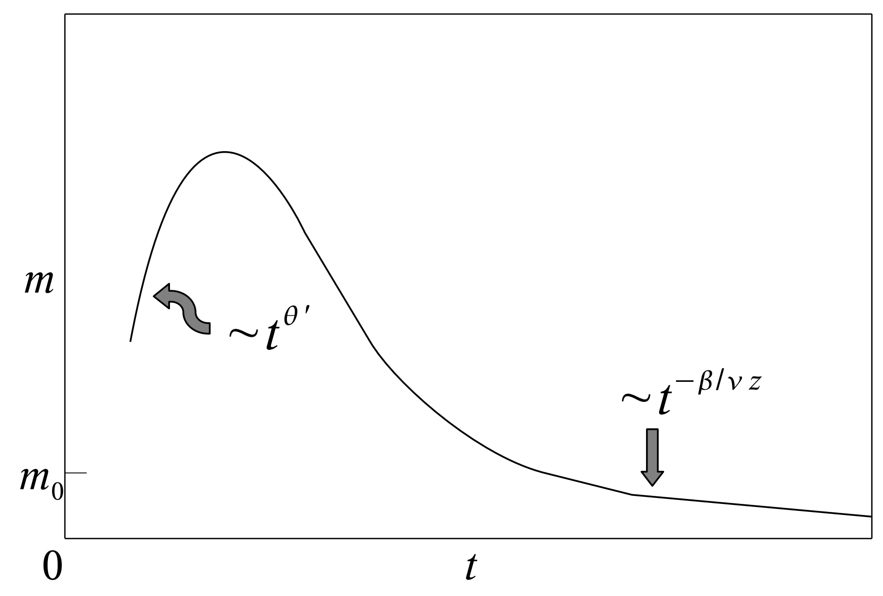
Introduction

The critical evolution of a system from the initial nonequilibrium state with a small magnetization $m_0 = m(0) \ll 1$ displays a universal scaling behavior of $m(t)$ over a short time early stage of this process, which is characterized by an anomalous increase in magnetization with time according to a power law.



Introduction

A singular part of the Gibbs potential $\Phi_{\text{sing}}(t,\tau,h,m_0)$ is characterized by a generalized homogeneity with respect to the main thermodynamic variables

$$\Phi_{\text{sing}}(t,\tau,h,m_0) = b\Phi_{\text{sing}}(b^{a_t}t, b^{a_\tau}\tau, b^{a_h}h, b^{a_m}m_0),$$

The magnetization of the system $m = -\delta\Phi/\delta h$ at the critical point is characterized by the time dependence

$$m(t,m_0) = t^{-(a_h+1)/a_t} F_m(m_0 t^{-a_m/a_t}).$$

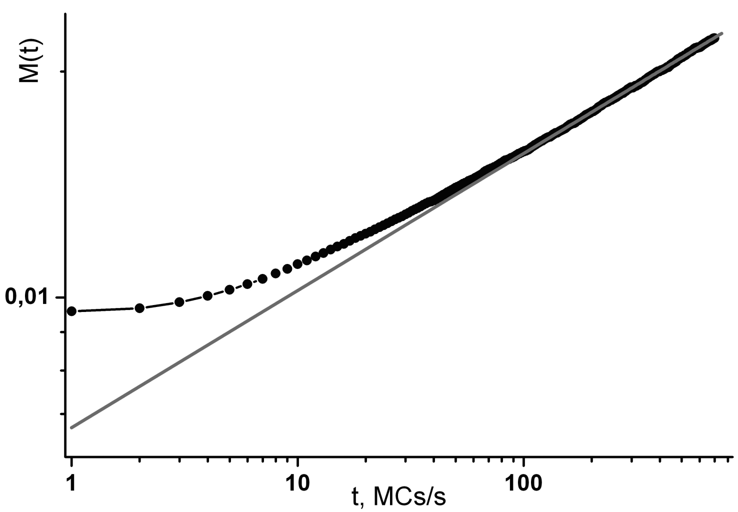
Expanding into series with respect to the small parameter $m_0\, t^{-a_m/a_t}$ lead to

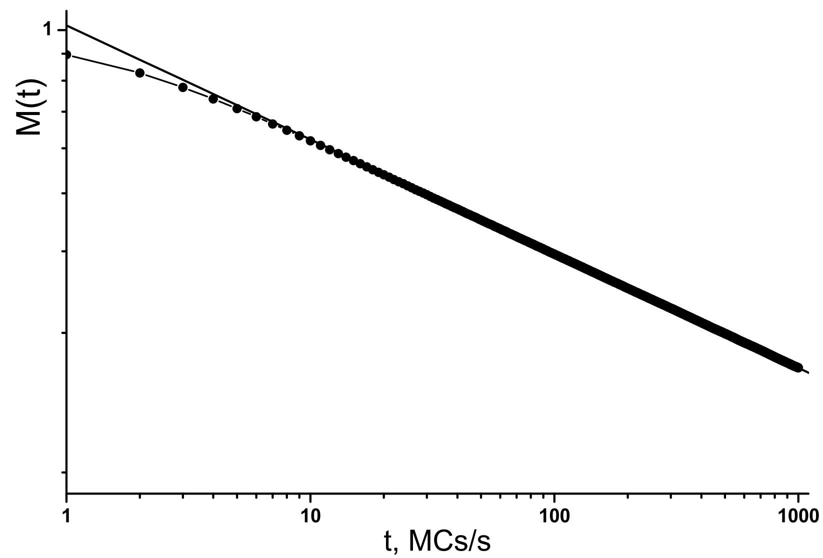
$$m(t) \sim t^{-(a_h+a_m+1)/a_t} \sim t^{\boldsymbol{\theta}}.$$

Introduction

- The evolution of the magnetization $m(t)$ in the initial time regime characterized by a new independent dynamic critical exponent θ

- For $t > t_{cr} \sim m_0^{-1/(\theta+\beta/z\nu)}$ the initial regime changes to a traditional regime of critical relaxation toward the equilibrium state, which is characterized by a time dependence of the magnetization according to the power law $m \sim t^{-\beta/\nu z}$





Introduction

ε–expansion (2-loop)

H.K.Janssen et.al., Z. Phys. B, 1989

$\theta = 0.130$, $\varepsilon \to 1$

$\theta = 0.138$, Padé-Borel summation

Monte Carlo simulations

L.Schulke, B.Zheng et.al., J.Phys.A, 1999

$\theta = 0.108(2)$

At the present work:

- Renormalization group description of the influence of nonequilibrium initial values of the order parameter on its evolution at a critical point is carried out.
- The dynamic critical exponent θ of the short time evolution of a system with an n-component order parameter is calculated within a dynamical dissipative model using the method of ε-expansion in a three-loop approximation.

Model

- Ginzburg–Landau–Wilson Hamiltonian of model

$$H_{GL}[s] = \int \mathrm{d}^{\mathrm{d}}x \left\{ \sum_{\alpha=1}^{n} \frac{1}{2!} \left[(\nabla s_\alpha(\mathbf{x}))^2 + \tau s_\alpha^2(\mathbf{x}) \right] + \frac{g}{4!} \left(\sum_{\alpha=1}^{n} s_\alpha^2(\mathbf{x}) \right)^2 \right\},$$

  where: $\boldsymbol{s}(\mathbf{x})$ - n-component order parameter field,
  $\boldsymbol{\tau}$ - reduced temperature of the phase transition,
  $\boldsymbol{g}$ - amplitude of interaction of the fluctuations.

- The distribution of an initial value of the order parameter $s(\mathbf{x}, t=0) = s_0(\mathbf{x})$

$$P[s_0] \sim \exp\left(- \int \mathrm{d}^{\mathrm{d}}x \; \frac{\tau_0}{2} \left(s_0(\mathbf{x}) - m_0(\mathbf{x}) \right)^2 \right).$$

Relaxation dynamics of the order parameter

$$\partial_t s_\alpha(x,t) = -\lambda \frac{\delta H_{GL}[s]}{\delta s_\alpha} + \zeta_\alpha(x,t),$$

where $\zeta(x,t)$ is the Gaussian random-noise source, which describes the influence of short-lived excitations with the probability functional

$$P[\zeta] \sim \exp\left[-\frac{1}{4\lambda}\int d^d x \int dt(\zeta(x,t))^2\right];$$

$$\langle\zeta_\alpha(x,t)\rangle = 0; \qquad \langle\zeta_\alpha(x,t)\zeta_\beta(x',t')\rangle = 2\lambda\,\delta_{\alpha\beta}\,\delta(x-x')\delta(t-t').$$

In relaxational dynamics described by the model A, the exponent $\boldsymbol{\theta}$ is essentially new independent dynamical exponent, which can't be expressed in terms of the static exponents.

Generating functional

- The generating functional W for the dynamic correlation functions and response functions:

$$W[h,\tilde{h}] = \ln\left\{\int \mathcal{D}(s,i\tilde{s})\exp\big(-\mathcal{L}[s,\tilde{s}] - H_0[s_0]\big)\times\right.$$
$$\left.\times\exp\left(\int \mathrm{d}^{\mathrm{d}}x \int_0^\infty \mathrm{d}t \sum_{\alpha=1}^{n}(\tilde{h}_\alpha\tilde{s}_\alpha + h_\alpha s_\alpha)\right)\right\},$$

- The action functional $\mathcal{L}$:

$$\mathcal{L}[s,\tilde{s}] = \int_0^\infty \mathrm{d}t \int \mathrm{d}^{\mathrm{d}}x \sum_{\alpha=1}^{n}\left\{\tilde{s}_\alpha\left[\dot{s}_\alpha + \lambda(\tau-\nabla^2)s_\alpha + \frac{\lambda g}{6}s_\alpha\Big(\sum_{\beta=1}^{n}s_\beta^2\Big) - \lambda\tilde{s}_\alpha\right]\right\}.$$

Correlation and response functions

An analysis of the Gaussian component of functional $\mathcal{L}$ for $g = 0$ and for the Dirichlet boundary condition ($\tau_0 = \infty$) allows the following expressions for the bare response function $G_0(p, t - t')$ and the bare correlation function $C_0^{(D)}(p, t, t')$

$$\begin{aligned} G_0(p, t - t') &= \exp\left[-\lambda(p^2 + \tau)|t - t'|\right], \\ C_0^{(D)}(p, t, t') &= C_0^{(e)}(p, t - t') + C_0^{(i)}(p, t + t'), \end{aligned}$$

where

$$\begin{aligned} C_0^{(e)}(p, t - t') &= \frac{1}{p^2 + \tau}\, e^{-\lambda(p^2+\tau)|t-t'|}, \\ C_0^{(i)}(p, t + t') &= -\frac{1}{p^2 + \tau}\, e^{-\lambda(p^2+\tau)(t+t')}. \end{aligned}$$

Renormalization

In the renormalization-group analysis of the model with allowance for the interaction of the order parameter fluctuations, singularities appearing in the dynamic correlation functions and response functions in the limit as $\tau \to 0$ were eliminated using the procedure of dimensional regularization and the scheme of minimum substraction followed by reparametrization of the Hamiltonian parameters and by multiplicative field renormalization in the generating functional W:

$$
\begin{aligned}
s &\to Z_s^{1/2} s, & \tilde{s} &\to Z_{\tilde{s}}^{1/2} \tilde{s}, \\
\lambda &\to (Z_s/Z_{\tilde{s}})^{1/2} \lambda, & \tau &\to Z_s^{-1} Z_\tau \mu^2 \tau, \\
g &\to Z_g Z_s^{-2} \mu^\varepsilon g, & \tilde{s}_0 &\to (Z_{\tilde{s}} Z_0)^{1/2} \tilde{s}_0,
\end{aligned}
$$

where $\varepsilon = 4 - d$ and μ is a dimensional parameter.

Response function

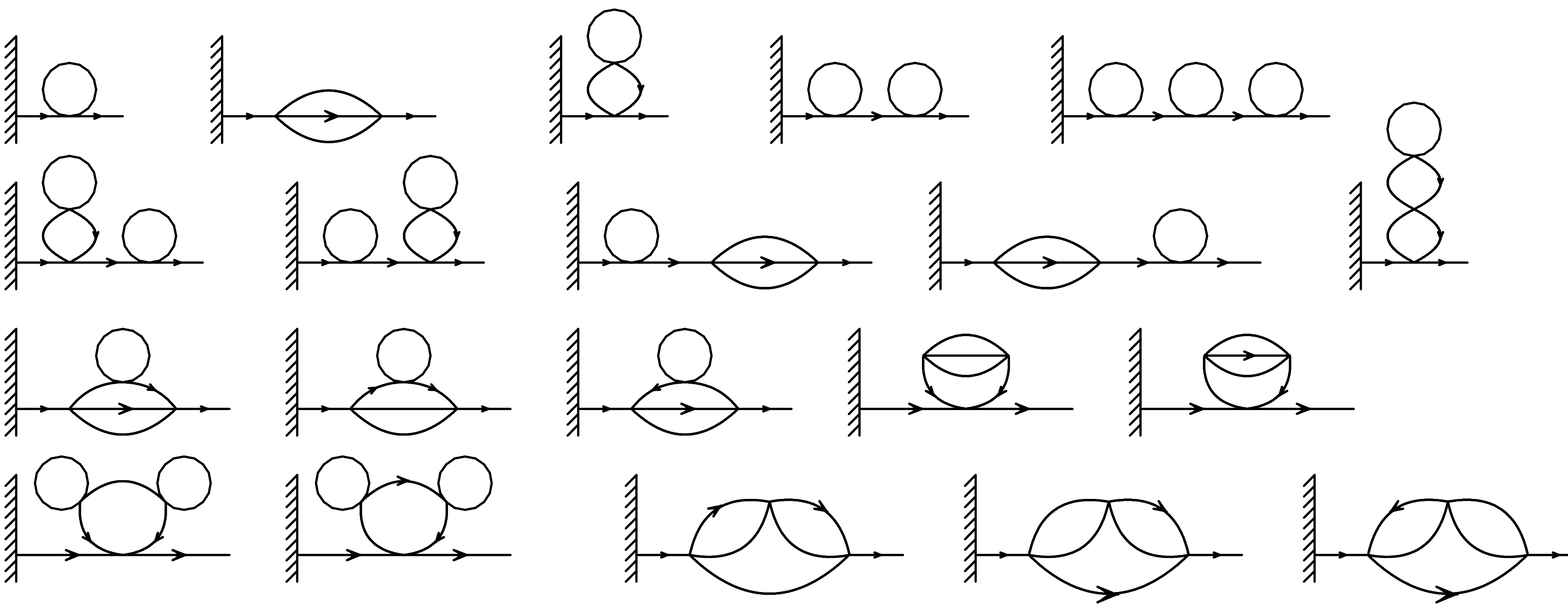
Introduction of the initial conditions into the theory makes necessary to renormalize the response function $\langle s(p,t)\tilde{s}_0(-p,0)\rangle$, which determines the influence of the initial state of the system on its relaxation dynamics.

$$G^{(i)}_{1,1}(p,t) = \langle s(p,t)\tilde{s}_0(-p,0)\rangle = \int\limits_0^t \mathrm{d}t' \, \bar{G}_{1,1}(p,t,t') \, \Gamma^{(i)}_{1,0}(p,t')_{[\tilde{s}_0]}.$$

- $\bar{G}_{1,1}(p,t,t')$ is determined by the equilibrium component of the correlator $C_0^{(e)}$

Diagrams

- The one-particle vertex function $\Gamma^{(i)}_{1,0}(p,t)_{[\tilde{s}_0]}$ with a single field insertion $\tilde{s}_0$ in the three-loop approximation is described by the diagrams:



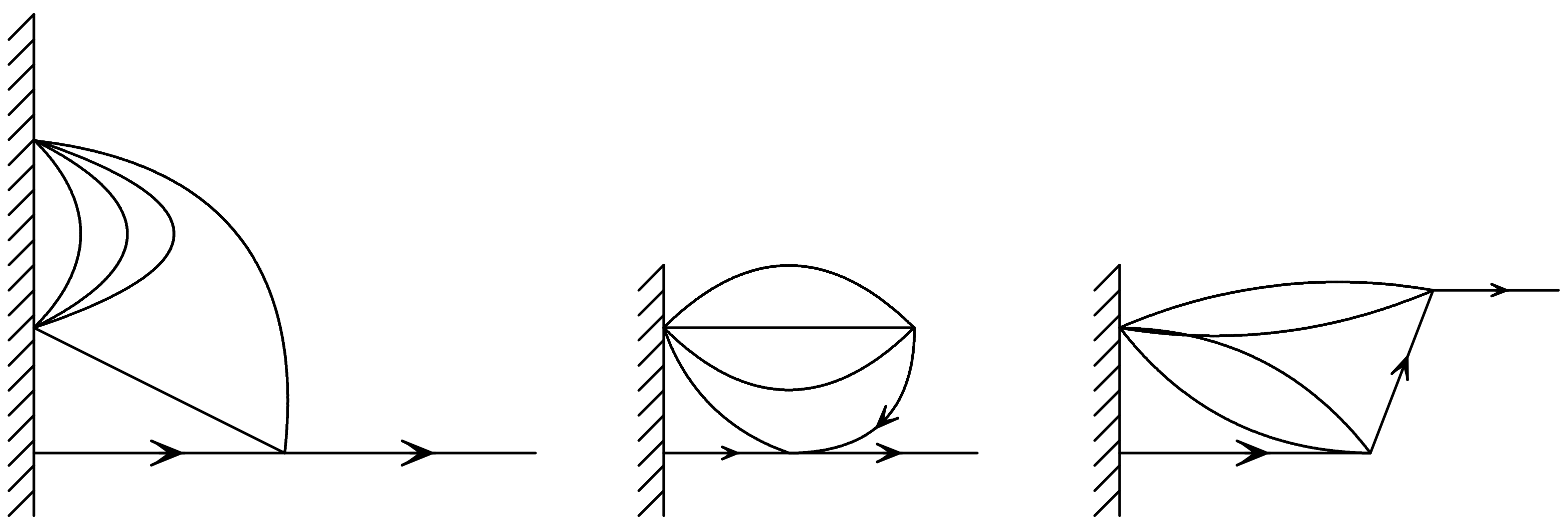
Fluctuation corrections

The additional vertex function $\Gamma^{(eq)}_{1,0}$, which is localized on the surface t = 0, appears due to averaging over the initial fields

$$G^{(eq)}_{1,1}(p, t - t') = \int_{t'}^{t} dt'' \bar{G}_{1,1}(p, t, t'') \Gamma^{(eq)}_{1,0}(p, t'')_{[\tilde{s}(t')]}.$$

Fluctuation corrections

Fluctuation corrections to dynamical response function caused by the initial nonequilibrium states appear only in the third order of theory



Renormalization-group procedure

The invariance with respect to the renormalization-group transformations of the generalized connected Green's function $G^{\tilde{M}}_{N,\tilde{N}} \equiv \langle [s]^N [\tilde{s}]^{\tilde{N}} [\tilde{s}_0]^{\tilde{M}} \rangle$ can be expressed in terms of the renormalization-group Callan–Symanzik differential equation:

$$\left\{ \mu\partial_\mu + \zeta\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta\partial_g + \frac{N}{2}\gamma + \frac{\tilde{N}}{2}\tilde{\gamma} + \frac{\tilde{M}}{2}(\tilde{\gamma}+\gamma_0) + \zeta\tau_0^{-1}\partial_{\tau_0^{-1}} \right\} G^{\tilde{M}}_{N,\tilde{N}} = 0.$$

Renormalization-group procedure

For a short time regime of nonequilibrium critical relaxation, the only essentially new quantity is the renormalization-group function γ_0. In the three-loop approximation it is expressed as follows:

$$\gamma_0 = -\frac{n+2}{6} g \left(1 + \left(\ln 2 - \frac{1}{2} \right) g - 0.0988989 \, (n + 3.13882) \, g^2 \right) + O(g^4).$$

Results

$$\boldsymbol{z} = 2+\frac{\varepsilon^2}{2}\left(6\ln\frac{4}{3}-1\right)\frac{n+2}{(n+8)^2}\left[1+\varepsilon\left(\frac{6(3n+14)}{(n+8)^2}-0.4384812\right)\right],$$

$$\boldsymbol{\theta} = \frac{(n+2)}{4(n+8)}\,\varepsilon\left(1+\frac{6\varepsilon}{(n+8)^2}\left(n+3+(n+8)\ln\frac{3}{2}\right)-\right.$$

$$\left.-\frac{7.2985}{(n+8)^4}\,\varepsilon^2\left(n^3+17.3118n^2+153.2670n+383.5519\right)\right)+O(\varepsilon^4).$$

Results

The calculated values of the critical exponent θ for Ising, XY and Heisenberg models and comparison it's with Monte Carlo results

Method	Exponent θ value: Ising	XY	Heisenberg
2-loop approximation			
$\varepsilon = 1$ substitution	0.130	0.154	0.173
Padé-Borel summation	**0.138**	**0.170**	**0.197**
3-loop approximation			
$\varepsilon = 1$ substitution	0.0791	0.0983	0.115
Padé-Borel summation	**0.1078(22)**	**0.1289(23)**	**0.1455(25)**
MC results	**0.108(2)**	**0.144(10)**	
	B.Zheng et.al., 1999	V.V.Prudnikov et.al., 2007	

Prudnikov V.V., Prudnikov P.V. et al., JETP, 2008

Conclusions

- The field theory description of the nonequilibrium critical relaxation of a system within the dynamical model A was presented.
- It was shown that only beginning with a three-loop approximation an additional vertex function $\Gamma_{1,0}^{(eq)}$ appears localized on the surface of initial states ($t = 0$), which provides fluctuation corrections to the dynamic response function due to the influence of nonequilibrium initial state.
- Using three-loop approximation it is possible to obtain the values of the exponent θ describing the short time evolution in close agreement with Monte Carlo results.