ERBL and DGLAP evolution for transversity distributions:

two-loop calculations in covariant gauge

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Plan

Main objects - transversely polarized distributions

- Parton distribution function (PDF)
- Distribution amplitude (DA)

DGLAP and ERBL evolution equations

- 1-loop kernels for transversely polarized case
- 2-loop kernels for transversely polarized case
- Analyses of the kernel structure

Solutions of evolution equations

- NLO ERBL solution for ρ^T DA φ_ρ
- Estimate of the size of NLO evolution effects

Conclusions

Parton distribution function in inclusive reactions

According to factorization theorems [Efremov&Radyushkin 1978] the hadronic tensor $H_{\mu\nu}$ can be represented as:

$$H_{\mu\nu} \sim \sum_i C(x; \mu_F^2, Q^2) * f_i(x, \mu_F^2) + O(\frac{1}{Q^2})$$

Usually $\mu_F^2 = Q^2$.

$C(x, \mu^2) = A\delta(1-x) + \sum_{n\geq 1} a_s^n(\mu^2) C_n(x, \mu^2)$ (where $a_s = \alpha_s/(4\pi)$) — coefficient function — calculable in pQCD.

$$f_i(x, Q^2) \ \sim \ \int dz e^{ix(zp)} < P|\bar{q}(z)\hat{\Gamma} U(z,0) q(0)|P >_{z^2=0}$$

$$U(z,0) \ = \ P\exp(ig \int_0^z A_\mu(\tau) d\tau^\mu)$$

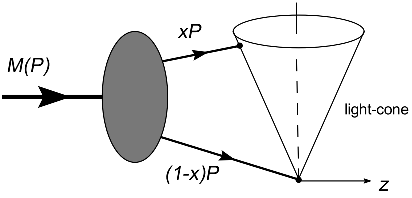
— parton distribution function (PDF),
cannot be obtained within pQCD.

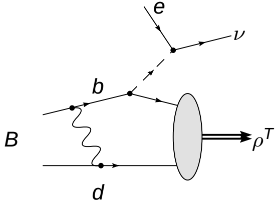
But evolution can be calculated in pQCD.

Distribution amplitudes in exclusive reactions

DA introduced in similar way:

$$\varphi_M(x, Q^2) \sim \int dz e^{ix(zp)} < 0|\bar{q}(z)\hat{\Gamma} U(z,0) q(0)|M(P) >_{z^2=0}$$





$B \to \rho\, e\, \nu$ decay

$$A_i^{B\to\rho}(t) =$$

$$C(t;x) \otimes \varphi_\rho(x;\mu^2) + O(\frac{1}{M_B^2})$$

Main object – transversely polarized distributions

For transversely polarized distributions:

$$\hat{\Gamma} = \sigma^{\mu\nu}$$

- Forward distribution $h_1(x;\mu^2)$ (for transversely polarized hadron) is

  chirally-odd $\Rightarrow$ absent in fully inclusive DIS

  But it appears in hadron-hadron collisions (Drell–Yan process)

$$h_1(x) = f_{\uparrow}(x) - f_{\downarrow}(x)$$

$$h_1(x;\mu^2) \sim p_\mu \varepsilon_\nu \int dz e^{ix(pz)} < h(p,s)|\bar{q}(0)\sigma^{\mu\nu} q(z)|h(p,s) > \Big|_{z^2=0}$$

- Distribution amplitude for transversely polarized ρ^T-meson :

$$\varphi_\rho^T(x;\mu^2) \sim p_\mu \varepsilon_\nu \int dz e^{ix(pz)} < 0|\bar{d}(0)\sigma^{\mu\nu} u(z)|\rho^T(p) > \Big|_{z^2=0}$$

DGLAP and ERBL evolution equations

DGLAP and ERBL evolution equation

Distribution functions $f_i(x,\mu^2)$ evolves according to DGLAP [72-77] equation:

$$\mu^2 \frac{df(x;\mu^2)}{d\mu^2} = \int_0^1 \delta(x-yz)\, P(z)\, f(y;\mu^2)\, dz\, dy \equiv (P * f)(x;\mu^2);$$

$$\gamma(N) = \int_0^1 P(z)\, z^N\, dz\ .$$

Amplitudes $\varphi(x,\mu^2)$ evolves according to ERBL [79-80] equation:

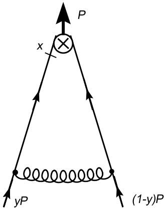
$$\mu^2 \frac{d\varphi(x;\mu^2)}{d\mu^2} = \int_0^1 V(x,y)\, \varphi(y;\mu^2)\, dy \equiv (V \otimes \varphi)(x;\mu^2)$$

Our goals are the kernels:

$$P^T(x) = a_s\, P_0^T(x) + a_s^2\, P_1^T(x) + \dots\ ,$$

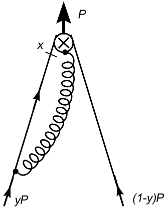
$$V^T(x,y) = a_s\, V_0^T(x,y) + a_s^2\, V_1^T(x,y) + \dots$$

1-loop ERBL kernel for transversely polarized DA's



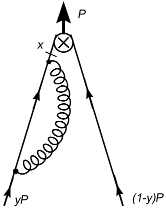
$V_0(x,y) = -C_F \boldsymbol{\xi} \delta(y-x)$

$P_0(z) = -C_F \boldsymbol{\xi} \delta(1-z)$



$V_0(x,y) = 2C_F \cdot \left(\mathcal{C}\theta(y>x)\frac{x}{y}\frac{1}{y-x}\right)_+$

$P_0 = C_f \left(\frac{4z}{1-z}\right)_+$



$V(x,y) = -C_F(1-\boldsymbol{\xi})\delta(y-x)$

$P_0(z) = -C_F(1-\boldsymbol{\xi})\delta(1-z)$

Here $\mathcal{C} = 1 + \{x \to \bar{x}, y \to \bar{y}\}$

$$V_0^T(x,y) = a_s C_F \left[2\left(\mathcal{C}\theta(y>x)\frac{x}{y}\frac{1}{y-x}\right)_+ - \delta(y-x) \right]$$ [Brodsky&Lepage, 80]

$$P_0^T(x) = a_s C_F \left[\left(\frac{4x}{1-x}\right)_+ - \delta(1-x) \right]$$ [Artru&Mekfi, 90]]

Solution of ERBL equation: LO [Efremov&Radyushkin, 1980]

$$y\bar{y}V_0(x,y) \;=\; x\bar{x}V_0(y,x)$$

Gegenbauer polynomials $C_n^{\nu}(x)$ are eigenfunctions of one-loop kernel:

$$\left(V_0 \otimes \psi_n\right)(x) \;=\; \gamma_n \psi_n(x) \;, \qquad \psi_n(x) \;= 6x\bar{x}\; C_n^{3/2}(2x-1)$$

Expanding distribution amplitudes in Gegenbauer series

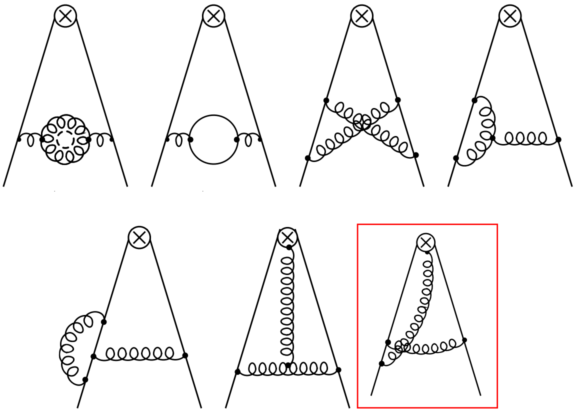
$$\varphi(x,\mu_0^2) \;=\; \sum_{n=0}^{\infty} a_n(\mu_0^2)\psi_n(x) \;,$$

we easily obtain a solution of one-loop ERBL equation

$$\varphi(x,\mu^2) \;=\; \sum_{n=0}^{\infty} \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right)^{-\gamma_n/b_0} a_n(\mu_0^2)\psi_n(x)$$

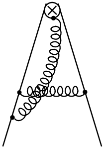
NLO ERBL/DGLAP kernels

$$V, P = a_s \partial_{a_s} \, K_1 \, R'(G), \; R' \text{ – incomplete } R\text{-operation,}$$
diagrams G are:



$+\,\mathbf{11}$ more simple diagrams

NLO ERBL kernel: an example

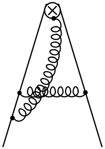


Here $F^T = \frac{x}{y}\frac{1}{y-x}$

$$V(x,y) = 2C_f(C_f - \frac{C_a}{2})\Big\{\mathcal{C}\theta(y>x)\Big[\frac{2}{y}\ln\left(1-\frac{x}{y}\right) - \frac{2}{y\bar{y}}\ln\frac{x}{y}\ln\left(1-\frac{x}{y}\right) + \frac{2}{y\bar{y}}\ln x\ln\bar{x} + F^T\left(4\mathrm{Li}_2\left(1-\frac{x}{y}\right) + \ln^2\frac{x}{y}\right)\Big] + G^T(x,y)\Big\}$$

$$G^T(x,y) = -4\mathcal{C}\Big\{\theta(y>x)\left(\bar{F}^T\ln\bar{x}\ln y - F^T\left[\mathrm{Li}_2(x)+\mathrm{Li}_2(\bar{y})\right] + \frac{\pi^2}{6}F^T\right) + \theta(\bar{y}<x)\Big((F^T-\bar{F}^T)\left[\mathrm{Li}_2\left(1-\frac{x}{y}\right)+\frac{1}{2}\ln^2 x\right] + F^T\left[\mathrm{Li}_2(\bar{y})-\ln x\ln y\right] + \bar{F}^T\mathrm{Li}_2(\bar{x})\Big)\Big\}$$

NLO DGLAP kernel: an example



Here $p_0 = \dfrac{4x}{1-x}$

$$P_{qq}(x) = C_F\left(C_F - \frac{C_A}{2}\right)\left[p_0\left(4\mathrm{Li}_2(1-x) - \ln^2 x\right) + 8\ln\bar{x}\right]$$

$$P_{q\bar{q}}(x) = 2\,C_F\left(C_F - \frac{C_A}{2}\right)\,p_0\left(\mathrm{Li}_2\left(\frac{|x|}{1+|x|}\right) - \mathrm{Li}_2\left(\frac{1}{1+|x|}\right) + \frac{1}{2}\ln^2|x| - \ln|x|\ln(1+|x|)\right)$$

Structure of kernel

$$\text{2-loop diagram} \quad \boldsymbol{G} = \boldsymbol{W}(\text{remnant}) * \boldsymbol{E}(\text{subgraph})$$

$$P\ (\ V\ ) = 2\boldsymbol{K_1}\ \boldsymbol{R}'(\ \boldsymbol{W} * \boldsymbol{E}) = \boxed{\boldsymbol{W} * \boldsymbol{E} - \boldsymbol{W} * \boxed{\boldsymbol{E}}}$$

we introduce the generalized 1-loop kernel:

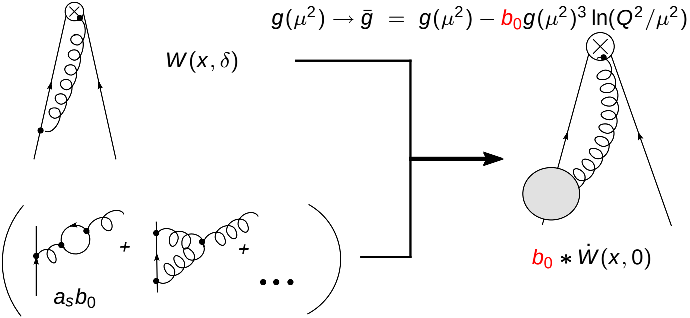
$$W(x;\delta) \ = \ 4\frac{x^{1+\delta}}{1-x} \quad \left(\ \frac{1}{y} w\left(\frac{x}{y};\delta\right)\ \right)$$

It is easy to see that

- singular part $\boxed{\boldsymbol{W}} = \dfrac{W(x;0)}{\delta} \ = \ \dfrac{p_0(x)}{\delta}$
- finite part $\boldsymbol{W} - \boxed{\boldsymbol{W}} = \dot{W}(x;\delta)_{\delta=0} \ = \ \dot{p}_0(x) \ = \ 4\dfrac{x\ln(x)}{1-x}$

Structure of kernel

A: Renormalization of charge g



Diagrams, which contain corrections to quark-gluon subgraph $E_{qg}(\delta)$, have in dimensional regularization the following form:

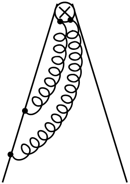
$$E_{qg}(\delta) \;=\; \frac{b_0}{\delta} + \dot{E}(0) + O(\delta)$$

$$2\boldsymbol{KR'}\Big(G \;=\; W(x;\delta) * E_{qg}(\delta)\Big) \;=$$

$$-\frac{1}{\delta^2} b_0 W(x;0) + \frac{1}{\delta} b_0 \dot{W}(x;0) + \frac{1}{\delta} W(x;0) * \dot{E}_{qg}(0) + O(\delta^0)$$

Structure of kernel

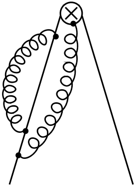
B: Renormalization of 1-loop composite operator



These diagrams contain corrections to one-loop kernel:

$$2\boldsymbol{KR'}\Big(G = W(x;\delta) * W(x;\delta)\Big) =$$

$$-\frac{1}{\delta^2} W(x;0) * W(x;0) + \frac{1}{\delta} W(x;0) * \dot{W}(x;0) + ..$$



Our results: NLO DGLAP kernel

$$P_1^T(x) = C_F\left\{\dot{p}_0 * \left(b_0\mathbb{1} - P_0\right) + p_0(x)\left[C_A\left(\frac{67}{9} - \frac{\pi^2}{3}\right) - \frac{20}{9}N_f T_r\right]\right\}_+$$
$$+C_F\left(C_F - \frac{1}{2}C_A\right)\left[4\bar{x} - 2\left(p_0(x)\ln^2(x)\right)_+\right]$$
$$+\delta(1-x)C_F\left[C_F\frac{43}{2} - C_A\frac{365}{18} + N_f T_r\frac{26}{9}\right.$$
$$\left. +(C_F - \frac{C_A}{2})8\left(\zeta(3) - \frac{\pi^2}{3}\right)\right]$$

This Feynman-gauge result coincides with the "light-cone" gauge calculation of Vogelsang '98

Our results: NLO ERBL kernel

$$V_1^T(x,y) = C_F\Big\{ \dot{V}_0 \otimes \left(b_0 \mathbb{1} - V_0\right) + [\boldsymbol{g_+ \,, \otimes V_0}]$$

$$+2\mathcal{C}\theta(y>x)F^T\left[C_A\left(\frac{67}{9}-\frac{\pi^2}{3}\right)-\frac{20}{9}N_f\;T_r\right]\Big\}_+$$

$$+\,C_F\left(C_F-\frac{C_A}{2}\right)\cdot 2\left\{\mathcal{C}\left[2\theta(y>x)\frac{x}{y}+2\theta(\bar{y}<x)\frac{\bar{x}}{y}\right]+2G^T(x,y)\right\}$$

$$+\,\delta(y-x)C_F\left[C_F\frac{27}{2}-C_A\frac{221}{18}+N_fT_r\frac{26}{9}\right]$$

This Feynman-gauge result coincides with the "prediction" from [Belitsky&Müller&Freund '00]

$$g(x,y) \;=\; -2\mathcal{C}\frac{\theta(y>x)}{y-x}\ln\left(1-\frac{x}{y}\right)$$,[D. Müller '94]

First two terms (red and blue) broke the conformal symmetry of V_1^T.

Check V with P

Kernel $P(x)$ can be obtained from $V(x,y)$ through transformation [D. Müller et al., 94]:

$$P(z) = \lim_{\tau\to 0} \frac{1}{|\tau|} V\left(\frac{z}{\tau}, \frac{1}{\tau}\right)$$

$$\begin{aligned}
V_0(x,y) &\to P_0(x) \\
\dot{V}_0(x,y) &\to \dot{P}_0(x) \\
\\
\dot{V}_0 \otimes \left(b_0 - V_{0+}\right) &\to \dot{p}_0 * \left(b_0 - P_0\right) \\
\mathcal{C}\left[2\theta(y > x)\frac{x}{y}\right] &\to 4(1-x) \\
(\text{standard } \theta) G(x,y) &\to -p_0(x) \ln^2(x) \\
(\text{nonstandard } \theta) G(x,y) &\to P^T_{1q\bar{q}} \\
\\
[g_+ , \otimes V_0] &\to 0
\end{aligned}$$

Solution of ERBL/DGLAP equations

Solution of ERBL equation in NLO [Mikhailov&Radyushkin, 1986]

At two-loop level, the solution of ERBL equation is not diagonal in $\{\psi_n(x)\}$-basis. The approximate solution for $\psi_n, \psi_n(x) \to \Phi_n(x,\mu^2)$ reads

$$\Phi_n(x,\mu^2) = \exp\left(\int_{a_s(\mu_0^2)}^{a_s(\mu^2)} \frac{\gamma_n(a)}{\beta(a)} da\right) \cdot \left[\psi_n(x) + a_s \sum_{m>n} \frac{d_{mn}}{N_m} \psi_m(x)\right],$$

$$\gamma_n(a_s) = a_s \gamma_n^{(0)} + a_s^2 \gamma_n^{(1)},$$
$$\beta(a_s) = -a_s^2 b_0 - a_s^3 b_1,$$

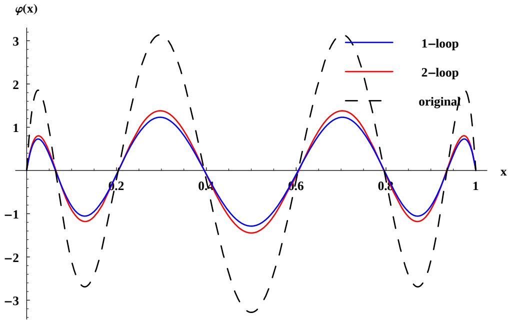
$$d_{mn} = \frac{Z_{mn}}{\gamma_n^{(0)} - \gamma_m^{(0)} - b_0}\left[1 - \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right)^{\frac{\gamma_n^{(0)} - \gamma_m^{(0)} - b_0}{b_0}}\right],$$

$$Z_{nm} = C_n^{3/2} \otimes V \otimes \psi_m, \quad N_n = C_n^{3/2} \otimes \psi_m = \frac{(n+1)(n+2)}{4(2n+3)} \delta_{mn}$$

γ_n is the diagonal term of matrix Z_{mn}: $\gamma_n = \dfrac{Z_{nn}}{N_n}$

Solution of ERBL equation in NLO (continued)

As an example we show the evolution of the 6-th harmonic term $6x\bar{x}C_6^{3/2}(1-2x)$ from 1 GeV^2 to 25 GeV^2:



Effect of NLO is less than 1% for the important quantity $\langle\frac{\psi_6(x)}{x}\rangle$.

For other harmonics

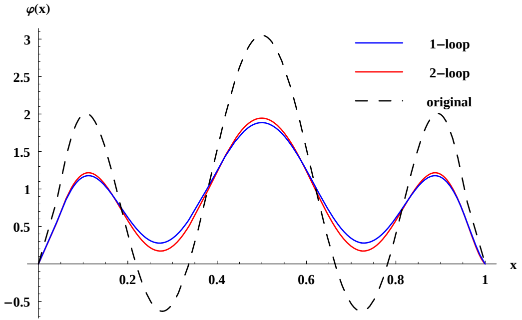
$\langle\frac{\psi_0(x)}{x}\rangle = 4\%$, $\langle\frac{\psi_2(x)}{x}\rangle = 1.4\%$, $\langle\frac{\psi_4(x)}{x}\rangle < 1\%$.

Solution of ERBL equation in NLO (continued)

The ρ-meson model DA of [Bakulev&Mikhailov, 00] at 1 GeV^2 has the form:

$$\varphi_\rho^T(x, \mu_0^2 = 1 \text{ GeV}^2) \;=\; \psi_0(x) + 0.29\psi_2(x) + 0.41\psi_4(x) - 0.32\psi_6(x)$$

At 25 GeV^2 we have:



Normalized difference between 1- and 2-loop evolution consists of about 4% for $\langle \frac{\psi_\rho^T(x)}{x} \rangle$ (32% at x =0.3).

Conclusions

- Obtain DGLAP and ERBL evolution kernels in LO and NLO for transversely polarized distributions.
- NLO ERBL kernel for transversely polarized meson DA is the new result.
- Solution of NLO ERBL equation for transversely polarized DA's has been obtained. All the element of approximate solution can be obtained in analytical form.
  NLO evolution effect for transversely polarized meson DA found to be rather large.