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Small x behavior of parton distributions. Analytical and "frozen" coupling constants

OUTLINE

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1. Introduction

A. The overlape between (x, Q^2) range of parton densities contributed for LHC processes and parton densities (PD) fitted at HERA and fixed target experiments, is not completly same (see, for example, fig. from (R.S.Thorne et al, 2005)).
So, direct application of modern sets of parton distributions may be not so correct.
B. The larger uncertanties for many processes at LHC came from restricted knowledge of parton distributions.
So, The knowledge of (small x behavior = high-energy asymptotics of) parton densities (the quark one $f_q(x, Q^2)$ and the gluon one $f_g(x, Q^2)$) is very important for many processes.

C. The deep-inelastis scattering (DIS) process is the basic one to extract PD, because the DIS structure functions (SF) $F_k(x,Q^2)$ $(k=2,3,L)$ relate with PD

$$F_k(x,Q^2) \;=\; \sum_{i=q,g} C_{k,i}(x) \otimes f_i(x,Q^2), \tag{1}$$

where the simbol $\otimes$ marks the Mellin convolution

$$f_1(x) \otimes f_2(x) \;\equiv\; \int_x^1 \frac{dy}{y} f_1(x/y) f_2(y) \tag{2}$$

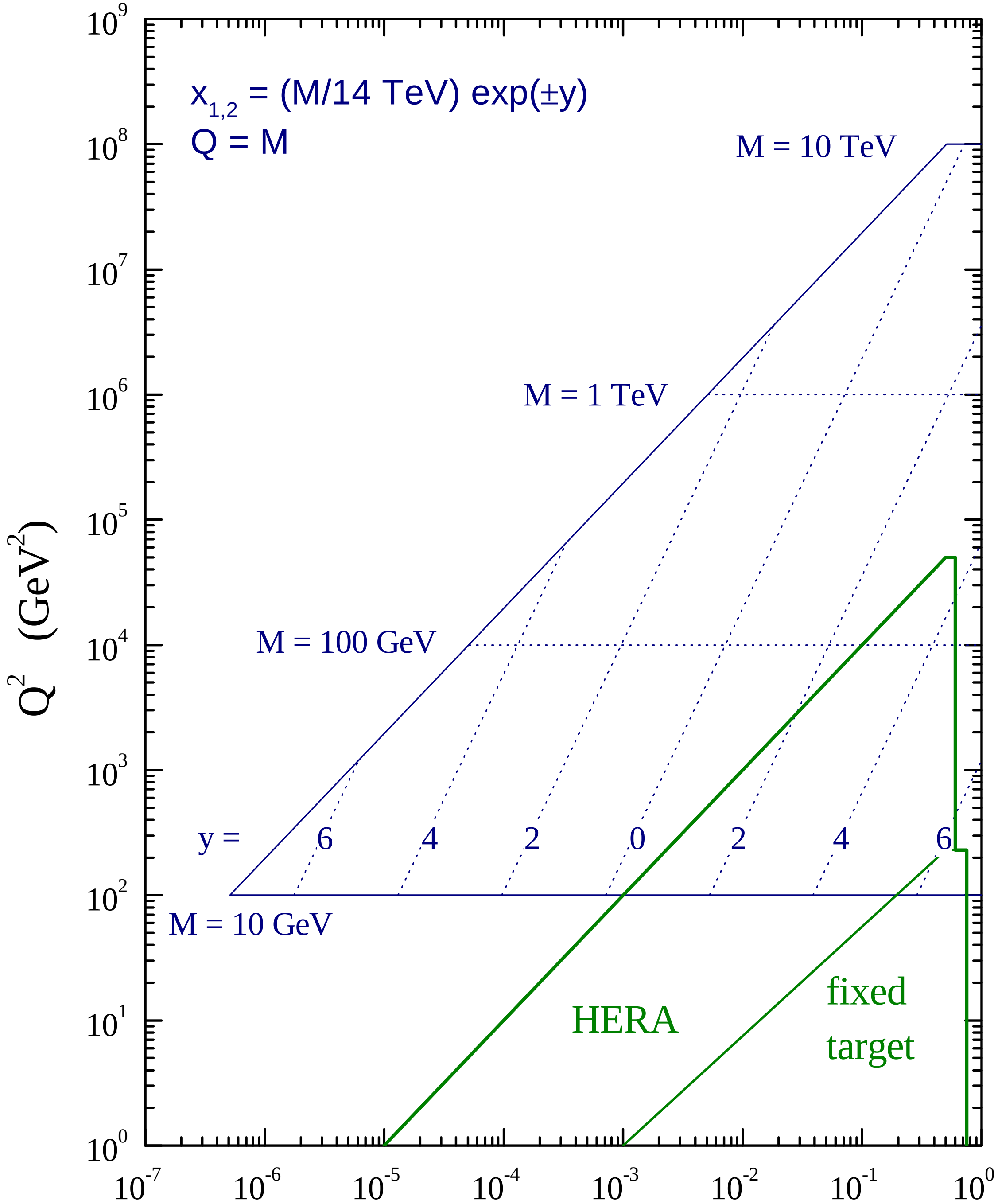
The best measured SF $F_2(x, Q^2)$ and $F_3(x, Q^2)$ relate directly with the quarks density at the leading order (LO) of perturbation theory (PT)

$$F_{2,3}(x, Q^2) = f_q(x, Q^2) + O(\alpha_s) \tag{3}$$

and the SF $F_L(x, Q^2)$ depends mostly on gluon denstity at low x

$$\begin{aligned} F_L(x, Q^2) &= \alpha_s(Q^2)\,[B^{(0)}_{k,q}(x) \otimes f_q(x, Q^2) \\ &+ B^{(0)}_{k,g}(x) \otimes f_g(x, Q^2)] + O(\alpha_s^2) \end{aligned} \tag{4}$$

LHC parton kinematics



2. Introduction to DIS

A. Deep-inelastic scattering cross-section:

$$\sigma \sim L^{\mu\nu} F^{\mu\nu}$$

Hadron part $F^{\mu\nu}$ ($Q^2 = -q^2 > 0$, $x = Q^2/[2(pq)]$):

$$\begin{aligned} F^{\mu\nu} &= (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) F_1(x, Q^2) \\ &- (p^\mu - \frac{(pq)}{q^2} q^\mu)(p^\nu - \frac{(pq)}{q^2} q^\nu) \frac{2x}{q^2} F_2(x, Q^2) + ..., \end{aligned}$$

where $F_k(x, Q^2)$ ($k = 1, 2, 3, L$) - are DIS SF and q and p are photon and hadron (parton) momentums.

B. Wilson operator expansion: Mellin moments $M_k(j,Q^2)$ of DIS SF $F_k(x,Q^2)$ can be represented as sum

$$M_k(j,Q^2) = \sum_{a=NS,SI,g} \underbrace{C_k^a(j,Q^2/\mu^2)}_{\text{Coeff. function}} A_a(j,\mu^2),$$

where $A_a(j,\mu^2) =< N|\mathcal{O}^a_{\mu_1,\ldots,\mu_j}|N >$ are matrix elements of the Wilson operators $\mathcal{O}^a_{\mu_1,\ldots,\mu_j}$.

C. The matrix elements $A_a(j,\mu^2)$ are Mellin moments of the unpolarized and polarized PD $f_a(j,\mu^2)$ and $\tilde{f}_a(j,\mu^2)$.

DGLAP [= Renormgroup] equations:

$$\begin{aligned}\frac{d}{d\ln Q^2} f_a(x,Q^2) &= \int_x^1 \frac{dy}{y} \sum_b W_{b\to a}(x/y)\, f_b(y,Q^2)\,,\\ \frac{d}{d\ln Q^2} \tilde{f}_a(x,Q^2) &= \int_x^1 \frac{dy}{y} \sum_b \tilde{W}_{b\to a}(x/y)\, \tilde{f}_b(y,Q^2)\,. \end{aligned} \qquad (5)$$

The anomalous dimensions (AD) $\gamma_{ab}(j)$ of the twist-2 Wilson operators $\mathcal{O}^a_{\mu_1,\ldots,\mu_j}$ (hereafter $a_s = \alpha_s/(4\pi)$)

$$\begin{aligned}\gamma_{ab}(j) &= \int_0^1 dx\; x^{j-1} W_{b\to a}(x) &= \sum_{m=0}^{\infty} \gamma_{ab}^{(m)}(j) a_s^m,\\ \tilde{\gamma}_{ab}(j) &= \int_0^1 dx\; x^{j-1} \tilde{W}_{b\to a}(x) &= \sum_{m=0}^{\infty} \tilde{\gamma}_{ab}^{(m)}(j) a_s^m.\end{aligned}$$

All parton densities are multiplies by x, t.e.

structure function = combination of parton densities.

3. Method

(C.Lopez and F.J.Yndurain, 1980,1981), (A.V.K., 1994)

Here I present briefly the method, which leads to the possibility to replace the Mellin convolution of two functions

$$f_1(x) \otimes f_2(x) \equiv \int_x^1 \frac{dy}{y} f_1(x/y) f_2(y) \tag{6}$$

by a simple products at small x.

A. So, if $f_1(x) = B_k(x, Q^2)$ is perturbatively calculated Wilson kernel and $f_2(x) = x f_a(x, Q^2) \sim x^{-\delta}$ at $x \to 0$, then

$$f_1(x) \otimes f_2(x) \;\approx\; M_k(1+\delta, Q^2)\, f_2(x) \qquad (7)$$

where $M_k(1+\delta, Q^2)$ is the analytical continuation to non-integer arguments of the Mellin moment $M_k(n, Q^2)$ of $B_k(x, Q^2)$:

$$M_k(n, Q^2) \;=\; \int_0^1 x^{n-2} B_k(x, Q^2) \qquad (8)$$

The equation (7) is correct if the moment $M_k(n, Q^2)$ has no singularity at $n \to 1$.

B. The general case

$(M(n)$ **contains the singularity at** $n \to 1)$:

the form of subasymptotics of $f_2(x)$ starts to be important.

Let PD have the different forms:

- Regge-like form $xf_R(x) = x^{-\delta}\tilde{f}(x)$,
- Logarithmic-like form $xf_L(x) = x^{-\delta}ln(1/x)\tilde{f}(x)$,
- Bessel-like form $xf_I(x) = x^{-\delta}I_k(2\sqrt{\hat{d}ln(1/x)})\tilde{f}(x)$,

where $\tilde{f}(x)$ and its derivative $\tilde{f}'(x) \equiv d\tilde{f}(x)/dx$ are smooth at $x = 0$ and both are equal to zero at $x = 1$:

$$\tilde{f}(1) \; = \; \tilde{f}'(1) \; = \; 0$$

Then $(i = R, L, I)$

$$f_1(x) \otimes f_2(x) \;\approx\; \tilde{M}_k(1+\delta_i, Q^2)\, f_2(x),$$

where $\tilde{M}_{1+\delta_i} = M_{1+\delta}$ with $1/\delta \to 1/\tilde{\delta}_i$.

Regge-like behavior:

$$1/\tilde{\delta}_R \;=\; 1/\delta \left[1 - x^{\delta} \frac{\Gamma(1-\delta)\Gamma(\nu)}{\Gamma(1+\nu-\delta)}\right],$$

where $x f_R(x) \sim (1-x)^{\nu}$ at $x \to 1$.

The seconfd term comes from low part of convolution integral

$$f_1(x) \otimes f_2(x) \;\equiv\; \int_x^1 \frac{dy}{y} f_1(x/y) f_2(y) \qquad (9)$$

So,

$$\frac{1}{\tilde{\delta}_R} = \frac{1}{\delta} \quad \text{if} \quad x^{\delta} << 1$$

and

$$\frac{1}{\tilde{\delta}_R} = \ln\frac{1}{x} - \left[\Psi(1+\nu) - \Psi(1)\right] \quad \text{if} \quad \delta = 0$$

Analogously, for nonRegge behavior at $\delta \to 0$

$$\frac{1}{\tilde{\delta}_L} = \frac{1}{2}\ln\frac{1}{x} + O(1/ln(1/x)),$$

$$\frac{1}{\tilde{\delta}_I} = \sqrt{\frac{ln(1/x)}{\hat{d}}}\,\frac{I_{k+1}(2\sqrt{\hat{d}ln(1/x)})}{I_k(2\sqrt{\hat{d}ln(1/x)})}$$

4. Double-logarithmic approach

(A.V.K. and G.Parente, 1998),
(A.Yu.Illarionov, A.V.K. and G.Parente, 2004)

1 Leading order without quarks (a pedagogical example)

At the momentum space, the solution of the DGLAP equation in this case has the form

$$M_g(n,Q^2) \;=\; M_g(n,Q_0^2)e^{-d_{gg}(n)s},$$

where $M_g(n,Q^2)$ are the moments of the gluon distribution,

$$s = ln\left(\frac{\alpha(Q_0^2)}{\alpha(Q^2)}\right), \quad \alpha(Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \quad \text{and} \quad d_{gg} = \frac{\gamma_{gg}^{(0)}(n)}{2\beta_0}$$

The terms $\gamma_{gg}^{(0)}(n)$ and β_0 are respectively the LO coefficients of the gluon-gluon AD and the QCD β-function.

For any perturbatively calculable variable $Q(n)$, it is very convenient to separate the singular part when $n \to 1$ (denoted by "$\widehat{Q}$") and the regular part (marked as "$\overline{Q}$"):

$$Q(n) \; = \; \frac{\widehat{Q}}{n-1} + \overline{Q}(n)$$

Then, the above equation can be represented by the form

$$M_g(n,Q^2) \; = \; M_g(n,Q_0^2) e^{-\hat{d}_{gg} s_{LO}/(n-1)} e^{-\overline{d}_{gg}(n) s_{LO}},$$

with $\hat{\gamma}_{gg} = -8C_A$ and $C_A = N$ for $SU(N)$ group.

Finally, if one takes the flat boundary conditions

$$x f_a(x,Q_0^2) \; = \; A_a, \quad \to \quad M_a(n,Q_0^2) \; = \; \frac{A_a}{n-1} \qquad (10)$$

(A.D.Rujula, S.L.Glashow, H.D.Politzer, S.B.Treiman, F.Wilczek and A.Zee, 1974)

Then, expanding the second exponential in the above equation

$$M_g^{cdl}(n,Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\left(-\hat{d}_{gg} s_{LO}\right)^k}{(n-1)^{k+1}}$$

and using the Mellin transformation for $(ln(1/x))^k$:

$$\int_0^1 dx x^{n-2} (ln(1/x))^k \;=\; \frac{k!}{(n-1)^{k+1}}$$

we immediately obtain the well known double-logarithmic behavior

$$f_g^{cdl}(x,Q^2) \;=\; A_g \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(-\hat{d}_{gg} s_{LO}\right)^k (ln(1/x))^k \;=\; A_g I_0(\sigma_{LO}),$$

where $I_0(\sigma_{LO})$ is the modified Bessel function with argument $\sigma_{LO} = 2\sqrt{\hat{d}_{gg} s_{LO} ln(x)}$. (R.D.Ball and S.Forte, 1994),

1.2 The more general case

For a regular kernel $\tilde{K}(x)$, having Mellin moment **(nonsingular at $n \to 1$)**

$$K(n) \; = \; \int_0^1 dx x^{n-2} \tilde{K}(x)$$

and the PD $f_a(x)$ in the form $I_\nu(\sqrt{\hat{d} ln(1/x)})$ we have the following equation

$$\tilde{K}(x) \otimes f_a(x) \; = \; K(1) f_a(x) + O(\sqrt{\frac{\hat{d}}{ln(1/x)}})$$

So, one can find the general solution for the LO gluon density without the influence of quarks

$$f_g(x,Q^2) \ = \ A_g I_0(\sigma_{LO}) e^{-\overline{d}_{gg}(1) s_{LO}} \ + \ O(\rho_{LO}),$$

where (R.D.Ball and S.Forte, 1994)

$$\rho_{LO} = \sqrt{\frac{\hat{d}_{gg} s_{LO}}{ln(x)}} = \frac{\sigma_{LO}}{2ln(1/x)}\,, \quad \overline{\gamma}_{gg}^{(0)}(1) = 22 + \frac{4}{3} f$$

and

$$\overline{d}_{gg}(1) = 1 + \frac{4f}{3\beta_0}$$

with f as the number of active quarks.

At the momentum space, the solution of the DGLAP equation at LO has the form (*after diagonalization*)

$$M_a(n,Q^2) \;=\; M_a^{+}(n,Q^2) + M_a^{-}(n,Q^2) \quad \text{and}$$
$$M_a^{\pm}(n,Q^2) \;=\; M_a^{\pm}(n,Q_0^2)e^{-d_{\pm}(n)s} = M_a^{\pm}e^{-\hat{d}_{\pm}s/(n-1)}e^{-\overline{d}_{\pm}(n)s},$$

where

$$M_a^{\pm}(n,Q^2) \;=\; \varepsilon_{ab}^{\pm}(n)M_b(n,Q^2), \qquad d_{ab} = \frac{\gamma_{ab}^{(0)}(n)}{2\beta_0},$$
$$d_{\pm}(n) \;=\; \frac{1}{2}[(d_{gg}(n)+d_{qq}(n))$$
$$\pm\; (d_{gg}(n)-d_{qq}(n))\sqrt{1+\frac{4d_{qg}(n)d_{gq}(n)}{(d_{gg}(n)-d_{qq}(n))^2}}]$$
$$\varepsilon_{qq}^{\pm}(n) \;=\; \varepsilon_{gg}^{\mp}(n) = \frac{1}{2}(1+\frac{d_{qq}(n)-d_{gg}(n)}{d_{\pm}(n)-d_{\mp}(n)}),$$

$$\varepsilon^{\pm}_{ab}(n) = \frac{d_{ab}(n)}{d_{\pm}(n) - d_{\mp}(n)} (a \neq b)$$

As the singular (when $n \to 1$) part of the $+$ component of the anomalous dimension is !!! $\hat{d}_+ = \hat{d}_{gg} = -4C_A/\beta_0$!!! while the $-$ component does not exist: !!! $(\hat{d}_- = 0)$!!! , we consider below both cases separately.

2.1 The "+" component

The analysis of the "$+$" component is practically identical to the case studied before. The only difference lies in the appearance of new terms $\varepsilon^{+}_{ab}(n)$!!! . If they are expanded in the vicinity of $n = 1$ in the form $\varepsilon^{+}_{ab}(n) = \overline{\varepsilon}^{+}_{ab} + (n-1)\tilde{\varepsilon}^{+}_{ab}$, !!! then for the terms $\overline{\varepsilon}^{+}_{ab}$ multiplying $M_b(n, Q^2)$, we have the same results as in previous section:

$$\overline{\varepsilon}^{+}_{ab} M_b(n, Q^2) \ \stackrel{\mathcal{M}^{-1}}{\longrightarrow} \ \overline{\varepsilon}^{+}_{ab} A_b I_0(\sigma_{LO}) e^{-\overline{d}_{+}(1) s_{LO}} \ + \ O(\rho_{LO}),$$

where the symbol $\stackrel{\mathcal{M}^{-1}}{\longrightarrow}$ denotes the inverse Mellin transformation. The values of σ and ρ coincide with those defined in the previous section because $\hat{d}_{+} = \hat{d}_{gg}$.

The terms $\tilde{\varepsilon}^{+}_{ab}$ that come with the additional factor $(n-1)$ in front, lead to the following results

$$(n-1)\tilde{\varepsilon}^{+}_{ab}\frac{A_b}{(n-1)}e^{-\hat{d}_+ s_{LO}/(n-1)} = \tilde{\varepsilon}^{+}_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{\left(-\hat{d}_+ s_{LO}\right)^k}{(n-1)^k}$$

$$\xrightarrow{\mathcal{M}^{-1}} \tilde{\varepsilon}^{+}_{ab}A_b \sum_{k=0}^{\infty}\frac{1}{k!}\frac{1}{(k-1)!}\left(-\hat{d}_+ s_{LO}\right)^k (ln(1/z))^{k-1}$$

$$= \tilde{\varepsilon}^{+}_{ab}A_b\rho_{LO}I_1(\sigma_{LO}),$$

i.e. the additional factor $(n-1)$ in momentum space leads to replacing the Bessel function $I_0(\sigma_{LO})$ by $\rho_{LO}I_1(\sigma_{LO})$ in x-space.

Thus, we obtain that the term $\varepsilon^{+}_{ab}(n)M_b(n,Q^2)$ leads to the following contribution in x space !!! :

$$(\overline{\varepsilon}^{+}_{ab}I_0(\sigma_{LO}) + \tilde{\varepsilon}^{+}_{ab}\rho_{LO}I_1(\sigma_{LO}))A_b e^{-\overline{d}_+(1)s_{LO}} + O(\rho_{LO})$$

Because the Bessel function $I_\nu(\sigma)$ has the ν-independent asymptotic behavior !!! $e^\sigma/\sqrt{\sigma}$ at $\sigma \to \infty$ (i.e. $x \to 0$), the second term is $O(\rho)$ and must be kept only !!! when $\overline{\varepsilon}^{+}_{ab} = 0$. This is the case for the quark distribution at the LO approximation.

Using the concrete AD values, one has

$$f^+_g(x,Q^2) \;=\; (A_g + \frac{4}{9}A_q) I_0(\sigma_{LO}) e^{-\overline{d}_+(1)s_{LO}} \;+\; O(\rho_{LO}) \quad \text{and}$$

$$f^+_q(x,Q^2) \;=\; \frac{f}{9}(A_g + \frac{4}{9}A_q)\rho_{LO} I_1(\sigma_{LO}) e^{-\overline{d}_+(1)s_{LO}} \;+\; O(\rho_{LO})$$

where $\overline{d}_+(1) = 1 + 20f/(27\beta_0)$.

2.2 the "–" component

In this case the anomalous dimension is regular !!! and one has

$$\varepsilon^{-}_{ab}(n) A_b e^{-d_-(n)s} \xrightarrow{\mathcal{M}^{-1}} \overline{\varepsilon}^{-}_{ab}(1) A_b e^{-d_-(1)s_{LO}} + O(x)$$

Using the concrete AD values !!! , we have

$$f_g^-(x, Q^2) = -\frac{4}{9} A_q e^{-d_-(1)s_{LO}} + O(x) \text{ and}$$
$$f_q^-(x, Q^2) = A_q e^{-d_-(1)s_{LO}} + O(x),$$

where $d_-(1) = 16f/(27\beta_0)$.

Finally we present the full small x asymptotic results for PD and F_2 structure function at LO of perturbation theory:

$$f_a(x,Q^2) = f_a^+(x,Q^2) + f_a^-(x,Q^2) \text{ and}$$
$$F_2(x,Q^2) = e \cdot f_q(z,Q^2)$$

where f_q^+,f_g^+, f_q^- and f_g^- were already given before and $e = \Sigma_1^f e_i^2/f$ is the average charge square of the f active quarks.

Extension to NLO is trivial and can be found in (A.V.K. and G.Parente, 1998)

So, we resume the steps we have followed to reach the small x approximate solution of DGLAP shown above:

- Use the n-space exact solution.
- Expand the perturbatively calculated parts (AD and coefficient functions) in the vicinity of the point $n = 1$.
- The singular part with the form

$$A_a (n-1)^k e^{-\hat{d} s_{LO}/(n-1)}$$

leads to Bessel functions in the x-space in the form

$$A_a \left(\frac{\hat{d} s_{LO}}{ln x}\right)^{(k+1)/2} I_{k+1}(2\sqrt{\hat{d} s_{LO} ln x})$$

- The regular part $B(n)\exp(-\overline{d}(n)s_{LO})$ leads to the additional coefficient

$$B(1)exp(-\overline{d}(1)s_{LO}) + O(\sqrt{\hat{d}s_{LO}/lnx})$$

behind of the Bessel function !!! in the x-space. Because the accuracy is $O(\sqrt{\hat{d}s_{LO}/lnx})$, it is necessary to use only the first nonzero term !!! , i.e. all terms $(n-1)^k$ in front of $\exp(-\hat{d}/(n-1))$, with the exception of one with the smaller k value, can be neglected.

- If the singular part at $n \to 1$ is absent, i.e. $\hat{d} = 0$, the result in the x-space is determined by $B(1)exp(-\overline{d}(1)s_{LO})$ with accuracy $O(x)$.

3. Fits of HERA data

At low x, the structure function $F_2(x, Q^2)$ is related to parton densities as (A.V.K. and G.Parente, 1998)

at LO

$$F_2(x, Q^2) = \frac{5}{18} f_q(x, Q^2)$$

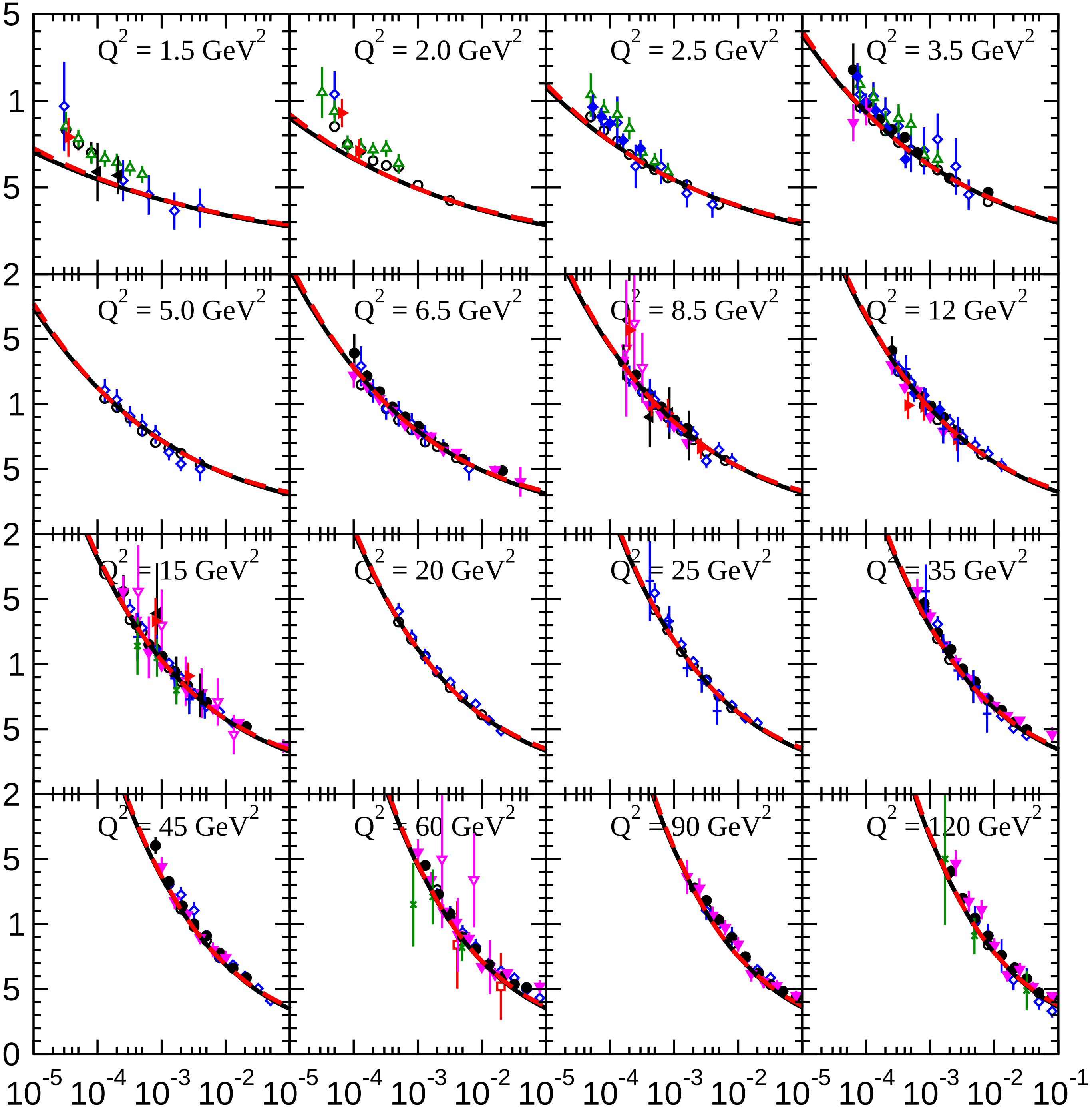
at NLO

$$F_2(x, Q^2) = \frac{5}{18} \left[f_q(x, Q^2) + \frac{2f}{3} a_s(Q^2) f_g(x, Q^2) \right].$$

Fits of HERA experimental data of the structure function $F_2(x, Q^2)$ (A.Yu.Illarionov, A.V.K. and G.Parente, 2004)

!!! Only two parameters: A_q and A_g

Λ_{QCD} **cannon be extract in small** x **Physics.**



Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 12 GeV^2
Q^2 = 15 GeV^2
Q^2 = 20 GeV^2
Q^2 = 25 GeV^2
Q^2 = 35 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 90 GeV^2
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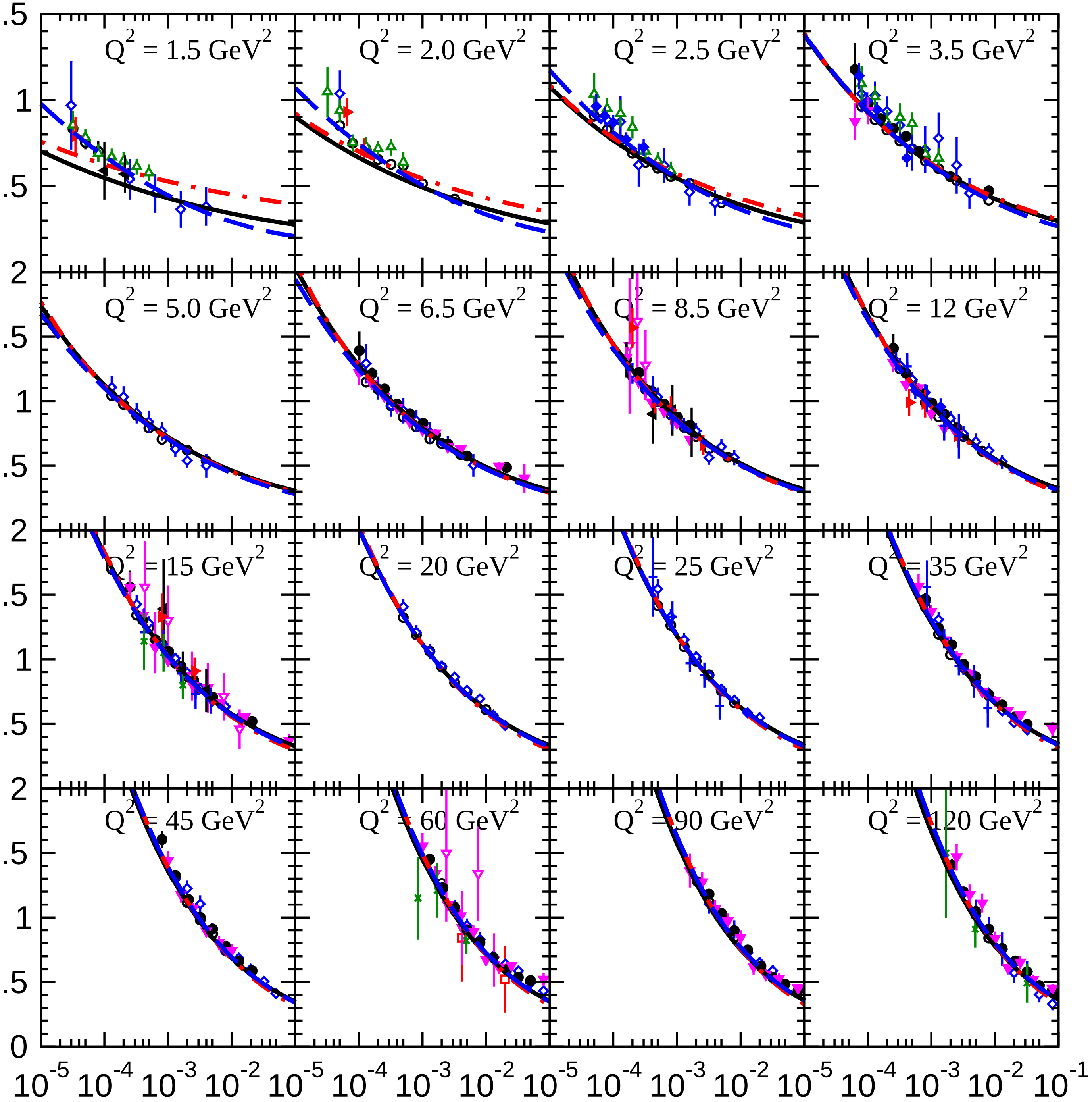

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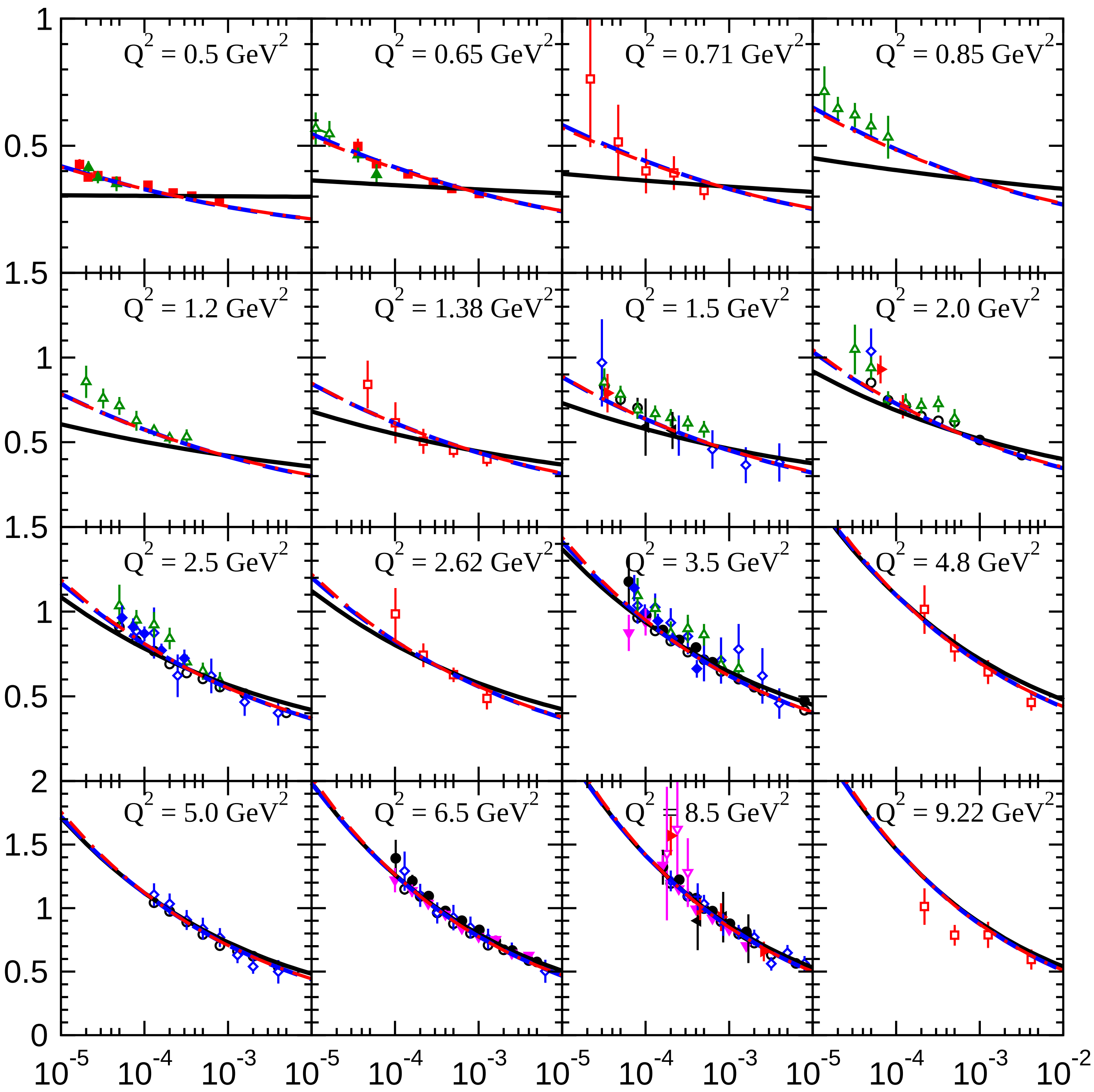


Q^2 = 0.5 GeV^2
Q^2 = 0.65 GeV^2
Q^2 = 0.71 GeV^2
Q^2 = 0.85 GeV^2
Q^2 = 1.2 GeV^2
Q^2 = 1.38 GeV^2
Q^2 = 1.5 GeV^2
Q^2 = 2.0 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 2.62 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 4.8 GeV^2
Q^2 = 5.0 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 8.5 GeV^2
Q^2 = 9.22 GeV^2

The double-logarithmic behaviour can mimic a power law shape over a limited region of x, Q^2.

$$f_a(x,Q^2) \sim x^{-\lambda_a^{eff}(x,Q^2)} \text{ and } F_2(x,Q^2) \sim x^{-\lambda_{F2}^{eff}(x,Q^2)}$$

The quark and gluon effective slopes $\lambda_a^{eff} = -\frac{d}{d\ln x}\ln f_a(x,Q^2)$ are reduced by the NLO terms that leads to the decreasing of the gluon distribution at small x. For the quark case it is not the case, because the normalization factor A_q^+ of the "$+$" component produces an additional contribution undampening as $\sim (lnx)^{-1}$.

The gluon effective slope λ_g^{eff} is larger !!! than the quark slope λ_q^{eff}, which is in excellent agreement with other studies. Indeed

$$\lambda_g^{eff}(x,Q^2) = \frac{f_g^+(x,Q^2)}{f_g(x,Q^2)} \cdot \rho \cdot \frac{I_1(\sigma)}{I_0(\sigma)}$$

$$\lambda_q^{eff}(x,Q^2) = \frac{f_q^+(x,Q^2)}{f_q(x,Q^2)}$$

$$\cdot \ \rho \cdot \frac{I_2(\sigma)(1-\overline{d}_{+-}^q(1)\alpha(Q^2)) + 20\alpha(Q^2)I_1(\sigma)/\rho}{I_1(\sigma)(1-\overline{d}_{+-}^q(1)\alpha(Q^2)) + 20\alpha(Q^2)I_0(\sigma)/\rho}$$

$$\lambda_{F2}^{eff}(x,Q^2) = \frac{\lambda_q^{eff} \cdot f_q^+ + (2f)/3\alpha(Q^2) \cdot \lambda_g^{eff}(\cdot f_g^+}{f_q(x,Q^2) + (2f)/3\alpha(Q^2) \cdot f_g(x,Q^2)}$$

The effective slopes λ_a^{eff} and λ_{F2}^{eff} depend on the magnitudes A_a of the initial PD and also on the chosen input values of Q_0^2 and Λ.

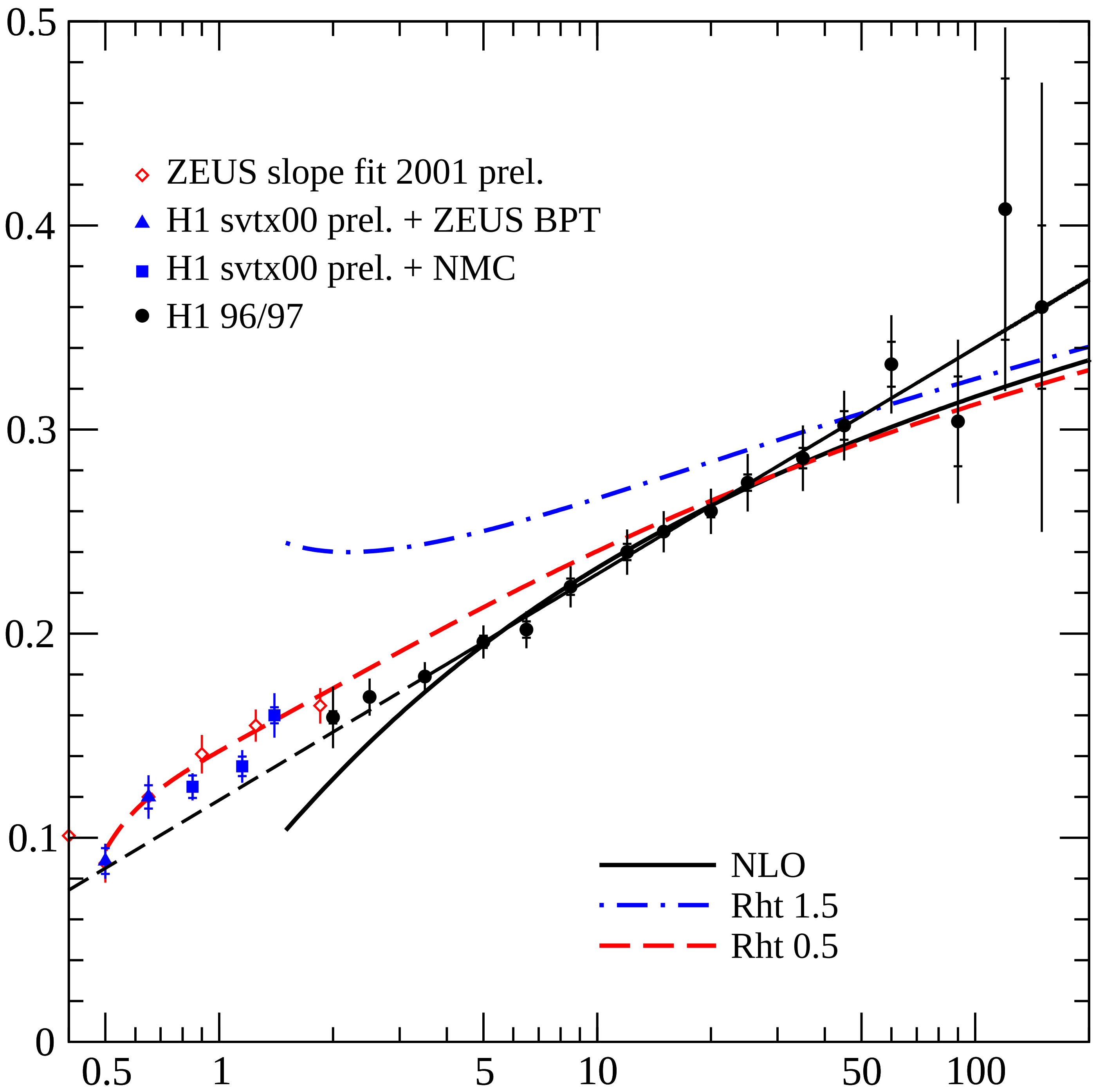
At quite large values of Q^2, where the "$-$" component is not relevant, the dependence on the magnitudes of the initial PD disappear, having in this case for the asymptotic values:

$$
\begin{aligned}
\lambda_g^{eff,as}(x,Q^2) &= \rho\frac{I_1(\sigma)}{I_0(\sigma)} \approx \rho - \frac{1}{4\ln(1/x)} \\
\lambda_q^{eff,as}(x,Q^2) &= \rho\cdot\frac{I_2(\sigma)(1-\overline{d}^q_{+-}(1)\alpha(Q^2)) + 20\alpha(Q^2)I_1(\sigma)/\rho}{I_1(\sigma)(1-\overline{d}^q_{+-}(1)\alpha(Q^2)) + 20\alpha(Q^2)I_0(\sigma)/\rho} \\
&\approx (\rho - \frac{3}{4\ln(1/x)})(1-\frac{10\alpha(Q^2)}{(\hat{d}_+ s + \hat{D}_+ p)}) \\
\lambda_{F2}^{eff,as}(x,Q^2) &= \lambda_q^{eff,as}\frac{1+6\alpha(Q^2)/\lambda_q^{eff,as}}{1+6\alpha(Q^2)/\lambda_g^{eff,as}} + \ O(\alpha^2(Q^2)) \\
&\approx \lambda_q^{eff,as}(x,Q^2) + \frac{3\alpha(Q^2)}{\ln(1/x)},
\end{aligned}
$$

where simbol $\approx$ marks approximations obtained by expansions of modified Bessel functions $I_n(\sigma)$. These aprroximations should be correct only at very large σ values (i.e. at very large Q^2 and/or very small x).

Both slopes λ_a^{eff} decrease with decreasing x !!! . A x dependence of the slope should not appear for a PD with a Regge type asymptotic ($x^{-\lambda}$) and precise measurement of the slope λ_a^{eff} may lead to the possibility to verify the type of small x PD asymptotics !!! .

Coefficients of HT terms strongly depend on set of the experimental data


0.5
0.4
0.3
0.2
0.1
0
0.5
1
5
10
50
100
ZEUS slope fit 2001 prel.
H1 svtx00 prel. + ZEUS BPT
H1 svtx00 prel. + NMC
H1 96/97
NLO
Rht 1.5
Rht 0.5

5. Analytical and "frozen" coupling constabts

Two modifications of the coupling constant

A. More phenomenological.

(G.Curci, M.Greco and Y.Sristava, 1979), (M.Greco, G. Penso and Y.Sristava, 1980), (N.N.Nikolaev and B.M.Zakharov, 1991,1992), (B.Badelek,J.Kwiecinski and A.Stasto, 1997), (A.M.Badalian and Yu.A.Simonov, 1997)

We introduce freezing of the coupling constant by changing its argument $Q^2 \longrightarrow Q^2 + M_\rho^2$, where M_ρ is sually the ρ-meson mass. Thus, in the formulae of the previous Sections we should do the following replacement

$$a_s(Q^2) \rightarrow a_{fr}(Q^2) \equiv a_s(Q^2 + M_\rho^2) \qquad (11)$$

B. Theoretical approach.

Incorporates the Shirkov-Solovtsov idea (D.V.Shirkov and L.I.Solovtsov, 1997), about analyticity of the coupling constant that leads to the additional its power dependence.

(K.A.Milton, A.V. Nesterenko, O.Solovtsova, G. Svetic, C. Valenzuela, I. Schmidt, O. Teryaev, N. Stefanis, A. Bakulev, S. Mikhailov, ...)

Then, in the formulae of the previous Section the coupling constant $a_s(Q^2)$ should be replaced as follows

$$a_{an}^{LO}(Q^2) = a_s(Q^2) - \frac{1}{\beta_0}\frac{\Lambda_{LO}^2}{Q^2 - \Lambda_{LO}^2} \tag{12}$$

at the LO approximation and

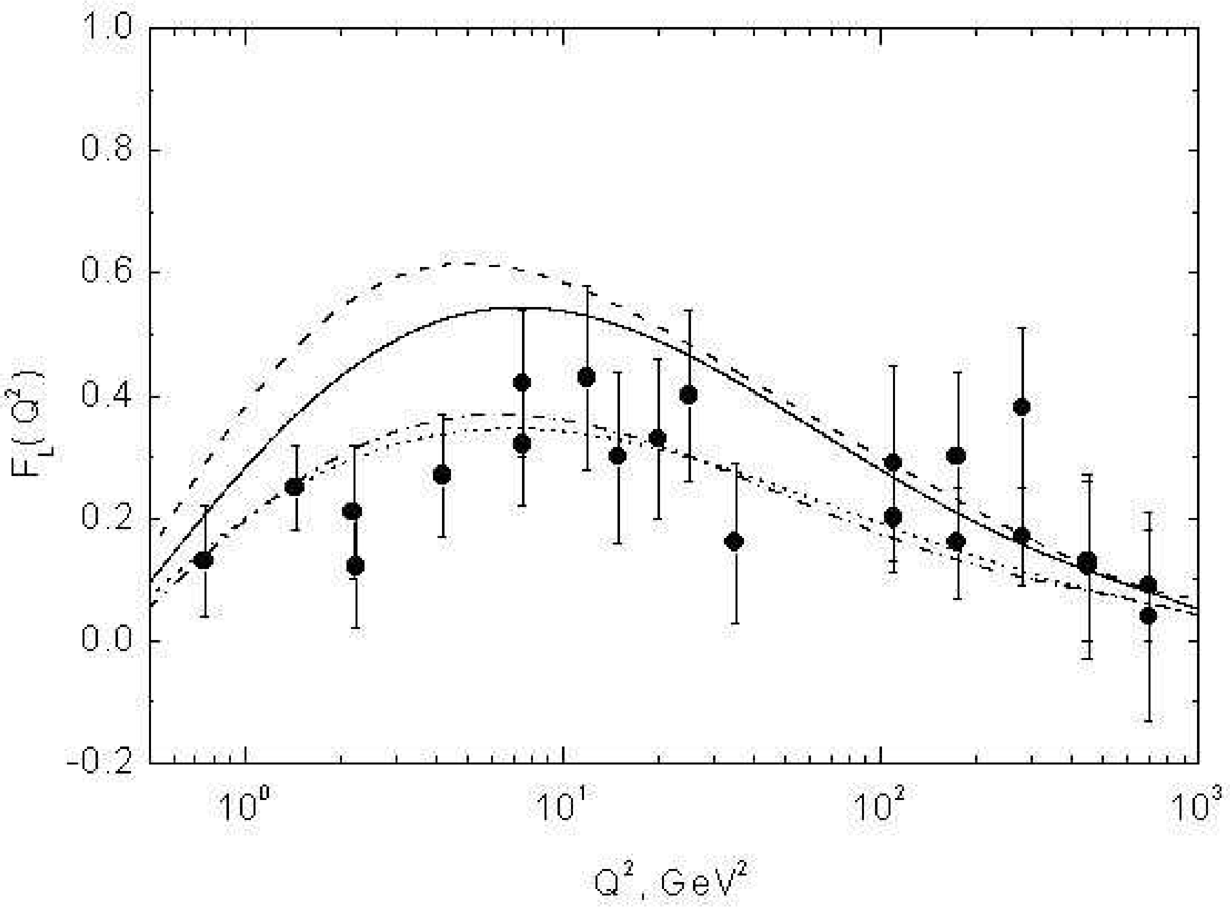
$$a_{an}(Q^2) = a_s(Q^2) - \frac{1}{2\beta_0}\frac{\Lambda^2}{Q^2 - \Lambda^2} - \frac{1}{\beta_0}\sum_{k=1}^{\infty}\left(\frac{\Lambda^2}{Q^2}\right)^k C_k[f] \tag{13}$$

at the NLO approximation, where the expansion coeficients $C_k[f]$,

$$C_k[f] = \int_0^\infty \frac{\exp\left[-k(\beta_1/\beta_0^2)(1+t)\right]}{(1+t+\ln t)^2 + \pi^2}\, dt\,, \tag{14}$$

depend on the number of flavors f. They are numerically small and decrease rapidly.

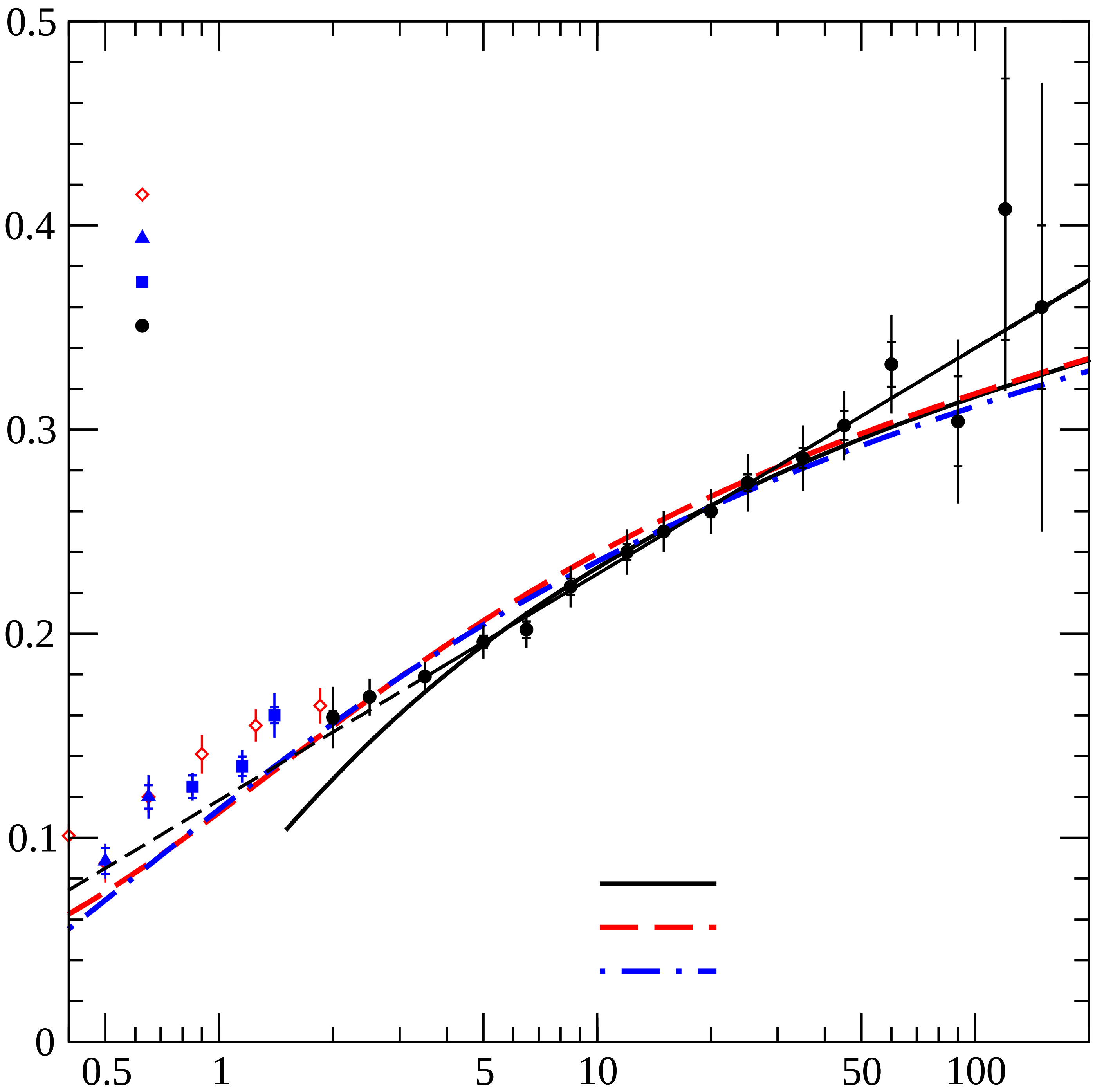
!!! One example of application of the analytical and “frozen” coupling constants: (A.V.Kotikov, A.V.Lipatov and N.P.Zotov, 2004)



- Usage of the analytical and “frozen” coupling constants leads to improvement with data
- Really, no difference between results based on the analytical and “frozen” coupling constants.

Similar observation was shown in Prof. Faustov talk

The preliminary results for slope of the SF F_2


0.5
0.4
0.3
0.2
0.1
0
0.5
1
5
10
50
100

Conclusion

- I have demonstrated the low x asymptotics of parton densities and SF F_2.
- Low x asymptotics of F_2 are in good agreement with data from HERA at $Q^2 \geq 2.5$ GeV^2.
- [**preliminary**] Usage of the analytical and "frozen" coupling constants leads to improvement with data from HERA at $Q^2 \leq 2.5$ GeV^2.

  The results based on the analytical and "frozen" coupling constants are very similar.

Next steps:

- To finish above preliminary studies.
- To analyse of some LHC processes using the analytical and "frozen" coupling constants.