Renormalization Group and Related Topics
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Ultraviolet divergences, vacuum energy and the Standard Model physics

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A.Yu. Kamenshchik, A. Tronconi, G.P. Vacca and G. Venturi, *Vacuum energy and spectral function sum rules*, Phys. Rev. D75: 083514,2007

G.L. Alberghi, A.Yu. Kamenshchik, A. Tronconi, G.P. Vacca and G. Venturi, *Vacuum Energy and Standard Model Physics*, arXiv:0804.4782 [hep-th]

Vacuum energy in quantum field theory

Every boson field mode has the vacuum energy

$$E_{\mathrm{boson}} = \frac{\hbar\omega}{2}.$$

Every fermion field mode has the vacuum energy

$$E_{\mathrm{fermion}} = -\frac{\hbar\omega}{2}.$$

The normal ordering prescription eleminates the vacuum energy, but its influence should be taken into account in the presence of gravity.

Suggestion of W. Pauli: cancellations between bosons and fermions

In his lectures "Selected Topics in Field Quantization" (1950-51) W. Pauli asked
"whether these zero-point energies [from Bosons and Fermions] can compensate each other."

Now we know that the exact cancellation of zero-point energy is realized in the theories with an exact supersymmetry.
However, the Nature does not reveal an exact supersymemtry.

Idea of Ya.B. Zeldovich: vacuum energy is responsible for the cosmological constant (1967-68)

The vacuum energy density and the pressure of a boson field:

$$\varepsilon = \frac{1}{2}\frac{1}{(2\pi\hbar)^3}\int_0^\infty c\sqrt{p^2+\mu^2}\,4\pi p^2 dp = KI(\mu),$$

$$P = K\cdot\frac{1}{3}\int_0^\infty \frac{p^2}{\sqrt{p^2+\mu^2}}p^2 dp = KF(\mu),$$

$$\mu = mc.$$

Having many fields, one writes

$$\varepsilon = \int_0^\infty f(\mu) I(\mu) d\mu,$$
$$P = \int_0^\infty f(\mu) F(\mu) d\mu.$$

Here $f(\mu)$ can be regarded as a spectral regularizing function of the Pauli-Villars type.
The conditions of the cancellation of the quartic, quadratic and logarithmic divergences are:

$$\int_0^\infty f(\mu) d\mu = 0,$$
$$\int_0^\infty f(\mu)\mu^2 d\mu = 0,$$
$$\int_0^\infty f(\mu)\mu^4 d\mu = 0.$$

Then the finite parts are:

$$\varepsilon = \frac{1}{8}\int_0^\infty f(\mu)\mu^4 \ln \mu d\mu,$$
$$P = -\frac{1}{8}\int_0^\infty f(\mu)\mu^4 \ln \mu d\mu,$$
$$P = -\varepsilon.$$

"The field theory with relativistically invariant regularization does not require a zero vacuum energy, but it leads naturally to the situation characterized by a cosmological constant." (Zeldovich).

Our approach

1. The particle content of a theory is such that UV divergences do not appear insofar boson and fermions should compensate each other as Pauli suggested (but only for divergent parts).
2. The study of the finite value of the effective cosmological constant provided the UV divergences are cancelled.
3. The study of a minimal extension of the Standard Model compatible with an observable small value of the cosmological constant.

The conditions of the cancellation of the ultraviolet divergences (the Minkowski background)

$$N_{\text{boson degrees of freedom}} = N_{\text{fermion degrees of freedom}},$$

$$\sum m_s^2 + 3\sum m_V^2 = 2\sum m_F^2,$$

$$\sum m_s^4 + 3\sum m_V^4 = 2\sum m_F^4.$$

De Sitter spacetime

For the case of the curved de Sitter spacetime, equations giving conditions of cancellation of quadratic and logarithmic ultraviolet divergences are more involved. Some examples of these conditions for simple particle physics models are presented in our paper.

We would like to study the final part of the vacuum energy, provided that the constraints, imposed by requirement of its ultraviolet finiteness are satisfied.

$$\varepsilon = \frac{1}{8}\left(\sum m_s^4 \ln m_s + 3\sum m_V^4 \ln m_V - 2\sum m_F^4 \ln m_F\right).$$

It should be comparable with the present value of the cosmological constant. This value is very small in comparison with masses of some constituents of the Standard Model of elementary particles.

$$\sum m_s^4 \ln m_s + 3\sum m_V^4 \ln m_V - 2\sum m_F^4 \ln m_F = 0.$$

Minimal extension of the Standard Model

The number of fermion degrees of freedom - 96.
The number of boson degrees of freedom -27.
We need 69 boson degrees of freedom, part of which belongs to Higgs bosons.
We would like to have some minimal extension of the Standard Model, which leaves intact the fermionic degrees of freedom adding hypothetical bosons.

Main Result

Such an extension does not exist. We show that introducing new bosonic fields and providing the cancellation of the ultraviolet divergences in the vacuum energy density, one finds that the finite part of the effective cosmological constant is huge and cannot be lowered below some rather big number (of the order of the mass of the top quark elevated to the fourth power) . That can be interpreted as a necessity of introducing of new heavy fermions.

Standard Model and vacuum energy balance

The particles with greatest masses:

1. top quark - $m_t \approx 170 Gev$
2. $W^{\pm}$ bosons - $m_W \approx 0.47 m_t$
3. Z^0 boson - $m_Z \approx 0.53 m_t$

Compare with
bottom quark - $m_b \approx 4.5 Gev$
τ-lepton - $m_\tau \approx 2 Gev$.

Contributions of heavy particles and hypothetical massive bosons

$$R^2 \equiv 12m_t^4 - 6m_W^4 - 3m_Z^4 \approx 11.5,$$

$$h \equiv 12m_t^2 - 6m_W^2 - 3m_Z^4 \approx 9.83,$$

$$L \equiv 12m_t^4 \ln m_t^2 - 6m_W^4 \ln m_W^2 - 3m_Z^4 \ln m_Z^2 \approx 0.743.$$

The squared masses of massive boson fields -
$x_1, x_2, \ldots, x_n, \quad x_i > 0$.
The constraints:

$$\sum_{i=1}^{n} x_i^2 = R^2,$$

$$\sum_{i=1}^{n} x_i = h.$$

Question

Can the function

$$\phi(x_1, x_2, \cdots, x_n) \equiv \sum_{i=1}^{n} x_i^2 \ln x_i$$

be equal to L on the constraint surface ?

Constraint surface

It is $(n-2)$ - dimensional sphere - an intersection of $(n-1)$ - dimensional sphere and $(n-1)$ - dimensional plane.

It exists if

$$n > \frac{h^2}{R^2} \approx 8.4.$$

The minimal number of massive boson degrees of freedom is

$$n_0 \equiv \left[\frac{h^2}{R^2} + 1\right] = 9.$$

Calculation of the minimal value of $\phi(x_i)$
Result

$$\phi(x_i)_{min} = (n_0 - 1)x_0^2 \ln x_0 + y_0^2 \ln y_0,$$

where

$$x_0 = \frac{h}{n_0} + \sqrt{\frac{R^2}{n_0(n_0 - 1)} - \frac{h^2}{n_0^2(n_0 - 1)}},$$

$$y_0 = \frac{h}{n_0} - (n_0 - 1)\sqrt{\frac{R^2}{n_0(n_0 - 1)} - \frac{h^2}{n_0^2(n_0 - 1)}}.$$

Calculation of the minimal value of $\phi(x_i)$ Technique

An auxiliary function

$$F(x_1, \cdots, x_n) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \ln x_i^2 - \lambda(\sum_{i=1}^{n} x_i^2 - R^2) - \mu(\sum_{i=1}^{n} x_i - h),$$

where λ and μ are the Lagrange multipliers.

We look for stationary points of the function F.
Differentiation with respect to the Lagrange multipliers gives the constraints.

The differentiation with respect to x_i

gives the system of equations:

$$x_i^2 \ln x_i - x_i - 2\lambda x_i - \mu = 0, \quad i = 1, \cdots, n.$$

Consider a solution
$\bar{x}_1, \cdots, \bar{x}_n, \bar{\lambda}, \bar{\mu}$.
Let us choose
$\bar{x}_1 \neq \bar{x}_2$.
Then

$$\bar{\lambda} = 1 + \frac{\bar{x}_1 \ln \bar{x}_1^2 - \bar{x}_2 \ln \bar{x}_2^2}{\bar{x}_1 - \bar{x}_2},$$

$$\bar{\mu} = \frac{\bar{x}_1 \bar{x}_2 (\ln \bar{x}_2^2 - \ln \bar{x}_1^2)}{\bar{x}_1 - \bar{x}_2}.$$

If $x_3 = x_4 = \cdots = x_k = x_1$ while $x_{k+1} = x_{k+2} = \cdots = x_n$, where $3 \leq k \leq n$ the system will be satisfied.

Thus, if one subset of $\bar{x}_i$ has one value and its complement has another value such set of values of x_i provides us with an extremum point of the function F, or in other words with the conditional extremum point of the function ϕ.

If all $\bar{x}_i$ are divided into these two classes, the possible values of $\bar{x}_1$ and $\bar{x}_2$ can be found exactly by solving the constraints.

$$x = x(k,n) = \frac{h}{n} + \sqrt{\frac{R^2(n-k)}{nk} - \frac{h^2(n-k)}{n^2k}},$$

where $1 \le k \le n-1$

$$y = y(k,n) = \frac{h}{n} - \frac{k}{n-k}\sqrt{\frac{R^2(n-k)}{nk} - \frac{h^2(n-k)}{n^2k}}.$$

The value of y is positive if

$$k < n_0 \le n.$$

The analysis of the stationarity equations shows that stationary points whose coordinates have three different values can exist. However, if such points exist, at least one these three values is negative and they are of no interest.

The minimum of the function ϕ on the constraint surface can be achieved only in the staionary points with the coordinates described above or on the boundary of the region of the positivity, where at least one of the coordinates is equal to zero. In this last case the problem is reduced to the problem with the lower dimensionality n.

The lowest dimensionality case $n = n_0$

All the values of x_i are positive. The extremum points are achieved in the points with $x(k,n), y(k,n)$ given above. Consider the function

$$\phi_1(k,n) = kx^2(k,n)\ln x(k,n) + (n-k)y(k,n)^2 \ln y(k,n).$$

We have proven that

$$\frac{d\phi_1(k,n)}{dk} < 0$$

and

$$\frac{d\phi_1(k,n)}{dn} > 0.$$

Minimum value of the function $\phi(k, n)$

is achieved at

$$n = n_0, \quad k = n_0 - 1$$

It is equal to

$$\phi_{1\ min} = (n_0 - 1)x^2(n_0 - 1, n_0)\ln x(n_0 - 1, n_0)$$
$$+y^2(n_0 - 1, n_0)\ln y(n_0 - 1, n)0).$$

For our values of n_0, R^2, h

$$\phi_{1\ min} \approx 1.95$$

which is greater than

$$L \approx 0.743.$$

The solution of the equation

$$\phi = L$$

on the constraint surface does not exist. Moreover the difference $\phi_{min} - L$ giving the minimal value of the effective cosmological constant is huge - of order of m_t^4.

It is necessary to introduce some new fermions.

Presence of new fermions

modifies the quantities

$$\tilde{R}^2 = R^2 + \sum n_f m_f^4,$$
$$\tilde{h} = h + \sum n_f m_f^4,$$
$$\tilde{L} = L + s \sum n_f m_f^4 \ln m_f^2.$$

The condition for the existence of a solution of the equation

$$\phi = \tilde{L}$$

is

$$\phi_{1\ min}(\tilde{n}_0, \tilde{R}^2, \tilde{h}) < \tilde{L} < \phi_{1\ max}(\tilde{n}_0, \tilde{R}^2, \tilde{h}).$$

Numerical results

For a Majorana spinor ($n_f = 2$)

$$m_f > 1.52 m_t$$

For a Dirac spinor ($n_f = 4$)

$$m_f > 1.46 m_t$$

For a colored quark ($n_f = 12$)

$$0.354 m_t < m_f < 0.359 m_t$$

or

$$0.4 m_t < m_f < 0.66 m_t$$

or

$$0.689 m_t < m_f < 0.691 m_t$$

or

$$m_f > 1.4 m_t$$

The lightest additional boson mass m_B (Higgs ?)

For a Majorana spinor with the mass $m_f^2 = 2.5 m_t^2$

$$111\ Gev < m_B < 139\ Gev$$

For a Majorana spinor with the mass $m_f^2 = 3 m_t^2$

$$115\ Gev < m_B < 172\ Gev$$

For a Majorana spinor with the mass $m_f^2 = 3.5 m_t^2$

$$112\ Gev < m_B < 178\ Gev$$

For a Majorana spinor with the mass $m_f^2 = 4 m_t^2$

$$86\ Gev < m_B < 177\ Gev$$

Dedication

- Lectures of D.V. Shirkov and the problem of ultraviolet divergences in quantum field theory
- How to get finite numbers after elimination of the ultraviolet divergences ?
- How to connect these finite numbers and observable/measurable quantities ?