TOWARDS COHERENT RG DESCRIPTION OF FRUSTRATED MAGNETS

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Plan

1. An object: stacked triangular antiferromagnets. Fixed points (FP) picture:

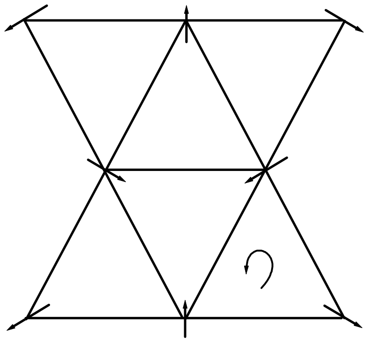
- perturbative RG: no stable FP
- non-perturbative RG: no stable FP
- but: non-perturbative stable FP found within perturbative RG.

2. A method: fixed-d RG approach.

3. Convergence of numerical results and behaviour of FPs with change of d.

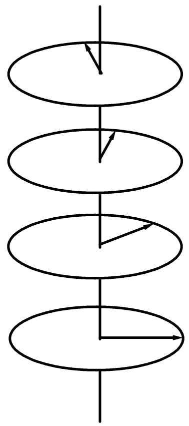
Non-collinear ordering

STA



$$\mathcal{H} = \frac{1}{2}\sum_{\langle\Delta\rangle} J\,\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} - \frac{1}{2}\sum_{\langle|\rangle} J'\,\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'},$$

Helimagnets



$$\mathcal{H} = -\frac{1}{2}\sum_{\langle \text{n.n.}\rangle} J\,\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} + \frac{1}{2}\sum_{\langle|\text{n.n.n.}\rangle} J'\,\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'}$$

Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \int \mathrm{d}^d R \left\{ \frac{1}{2}\left[\mu_0^2(\phi_1^2+\phi_2^2) + (\nabla\phi_1)^2 + (\nabla\phi_2)^2\right] + \frac{u_0}{4!}\left[\phi_1^2+\phi_2^2\right]^2 + \frac{v_0}{4!}\left[(\phi_1\cdot\phi_2)^2 - \phi_1^2\phi_2^2\right] \right\}.$$

Experiments

- there are two groups of incompatible exponents:

  $N = 2$: i. $CsMnBr_3$, $CsNiCl_3$, $CsMnI_3$, Tb: $\beta \sim 0.237(4)$

  ii. Ho, Dy: $\beta \sim 0.389(7)$

  $N = 3$: i. **A**, **B**, VCl_2, VBr_2: $\beta \sim 0.230(8)$

  ii. $CsNiCl_3$, $CsMnI_3$, **C**: $\beta \sim 0.287(8)$

- $\eta < 0$ for group i

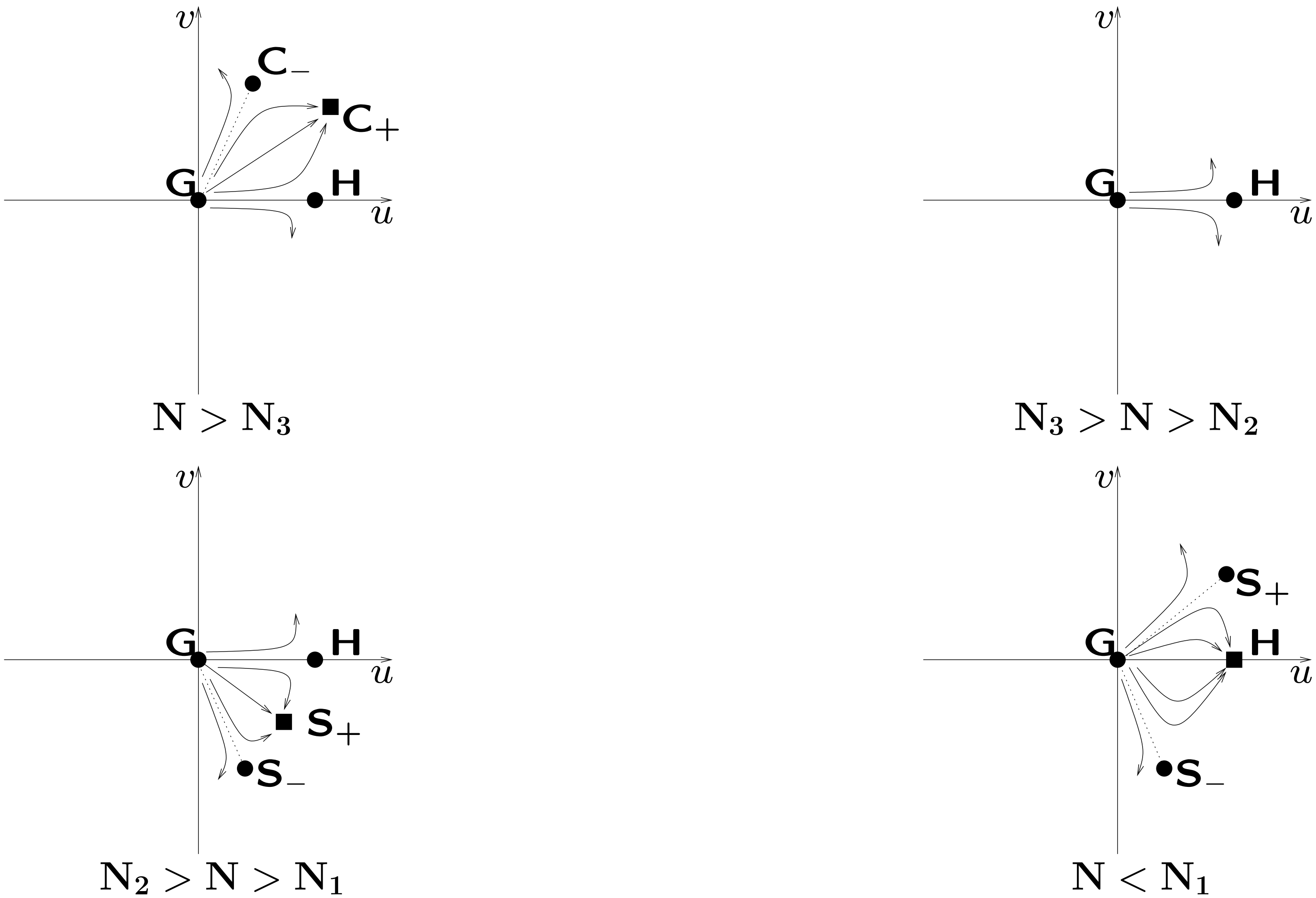
- scaling relations are violated

MC simulations

$N = 2$
- for STA exponents are compatible with group i
- $\eta < 0$ for STA
- 1^{st} order transition for STAR–GLW

$N = 3$
- for STA β is compatible with group ii
- $\eta < 0$
- β differs for different systems
- 1^{st} order transition for STAR–GLW

Theory: RG analysis



FPs and RG flows of the STA model. Unstable FPs are shown by discs, stable FPs are shown by squares. Three marginal dimensions N_1, N_2, N_3 govern the FP picture. Kawamura'88, ...

Marginal dimensions

E.g. pseudo-ε-expansion for N_3, $d = 3$ (Yu.H. et al.'04):

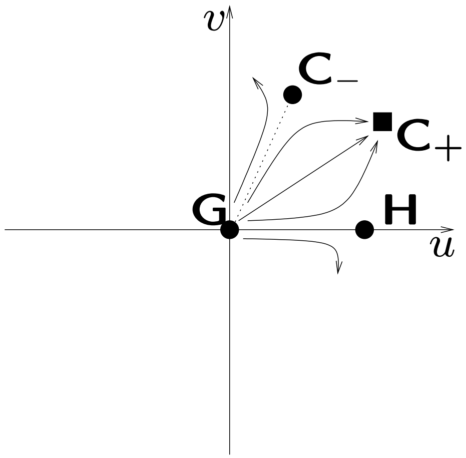
$$N_3 = 21.798 - 15.621\,\tau + 0.262\,\tau^2 - 0.151\,\tau^3 - 0.039\,\tau^4 - 0.030\,\tau^5,$$

Padé analysis:

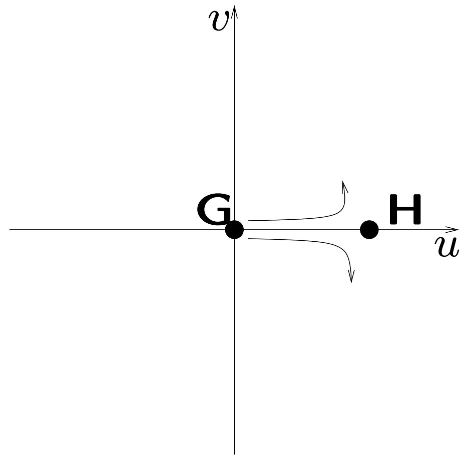
$$N_3 = \begin{bmatrix} 21.798 & 6.177 & 6.439 & 6.288 & 6.249 & 6.220 \\ 12.698 & 6.435 & 6.344 & 6.236 & \frac{6.126}{1.318} & \\ 9.827 & 6.290 & 6.230 & \frac{6.182}{1.751} & & \\ 8.463 & 6.247 & \frac{6.155}{1.453} & & & \\ 7.695 & 6.217 & & & & \\ 7.220 & & & & & \end{bmatrix}$$

$$N_3 = 6.23(21), N_2 = 1.99(4), N_1 = 1.43(2)$$

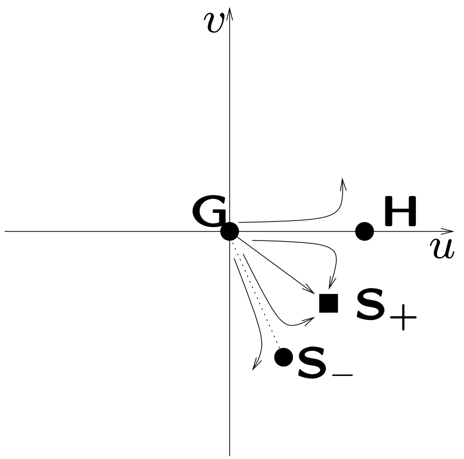
Recall FP picture:



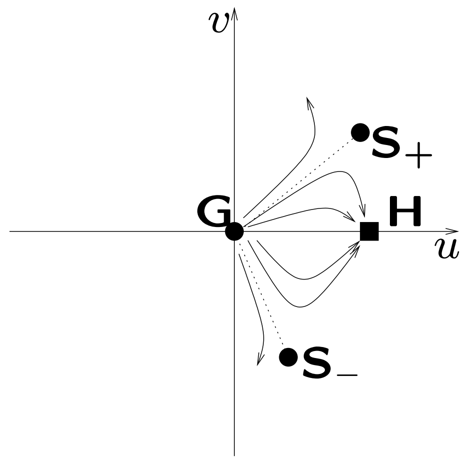
$\mathbf{N} > \mathbf{6.23(21)}$



$\mathbf{6.23(21)} > \mathbf{N} > \mathbf{1.99(4)}$



$\mathbf{1.99(4)} > \mathbf{N} > \mathbf{1.43(2)}$



$\mathbf{N} < \mathbf{1.43(2)}$

Therefore: no stable accesible FPs for $N = 2$, $N = 3$ at $d = 3$

FP picture for $N = 2$, $N = 3$, current situation

- Perturbative RG: no stable accessible FP found (Antonenko et al.'95, Yu.H. et al.'04, Calabrese et al.'04)

- Non-perturbative RG: no stable accessible FP found (Tissier et al.'00, Delamotte et al.'04)

- BUT: non-perturbative FP found within perturbative RG approach (Calabrese et al.'04, Pelissetto et al.'01)

How to judge whether a FP is not an artifact of the calculation procedure?

Perturbative field-theoretical RG: expansions and fixed-dimension approaches

$$\mathcal{H} = -\frac{J}{2}\sum_{\langle \mathbf{R},\mathbf{R}'\rangle} \vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} \quad \longleftrightarrow \quad \mathcal{H}_{\text{eff}} = \int \mathrm{d}^d R \left\{\frac{1}{2}\Big((\nabla\phi)^2 + \mu_0^2\phi^2\Big) + \frac{u_0}{4!}\phi^4\right\},$$

RG flow equation:

$$\frac{\mathrm{d}u}{\mathrm{d}\ln\ell} = \beta(u).$$

Fixed point:

$$\beta(u^*) = 0.$$

E.g. β-function in $\overline{\text{MS}}$ scheme (known in 5 loops, Kleinert et al.'1991):

$$\beta(u) = -u(\varepsilon - u + 3u^2(3N+14)/(N+8) + \cdots).$$

ε-expansion (Wilson, Fisher'72):

$$u^* = \varepsilon + 3\varepsilon^2(3N + 14)/(N + 8) + \cdots.$$

Fixed d (fixed ε) approach (Parisi'80; Schloms, Dohm'87):

(a) (b)

β-function of the 3d $N = 1$ model in successive perturbation theory orders ranging from 1 to 5 as shown by the labels in the figures. **(a)**: naïve evaluation, **(b)**: resummation taking into account asymptotic properties of the series.

Analysis of the RG functions for two couplings

- fixed d approach:

$$\beta_u(u,v,)|_{d=3} = 0,$$
$$\beta_v(u,v)|_{d=3} = 0$$

- resummation:

$$\beta_u^{\text{res}}(u,v) = 0,$$
$$\beta_v^{\text{res}}(u,v) = 0$$

- peculiarities: resummation with respect to u at fixed v

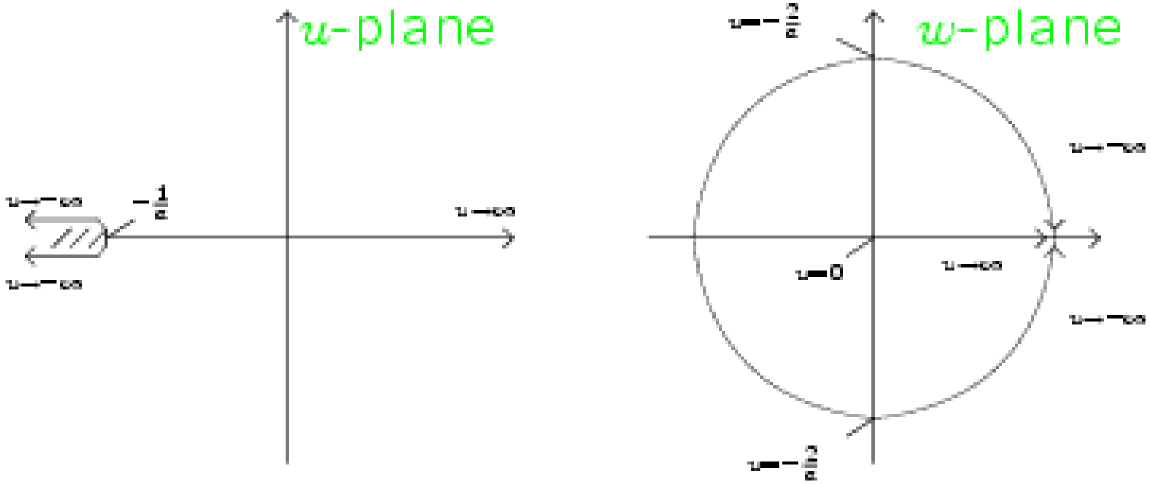
$$f(u,z) = \sum_n f_n(z)\ u^n, \qquad z = v/u.$$

Borel resummation based on conformal mapping

RG function: $f(u) = \sum_n f_n \; u^n,$ $\qquad f_{n\to\infty} \sim (-a)^n \, n! \, n^b$

Its Borel-Leroy transform: $B(u) = \sum_k \frac{f_n}{\Gamma(k+1+b)} u^k,$

$f(u) = \int_0^\infty e^{-t} B(ut) t^b \mathrm{d}t$



$$u = \frac{4}{a} \frac{w}{(1-w)^2} \qquad\qquad w = \frac{(1+au)^{1/2}-1}{(1+au)^{1/2}+1}$$

Resummed expression for f:

$$f_R(u,z) = \sum_n d_n(\alpha, a(z), b; z) \int_0^\infty dt \; \frac{e^{-t}\, t^b \, [\omega(ut;z)]^n}{[1-\omega(ut;z)]^\alpha}$$

with

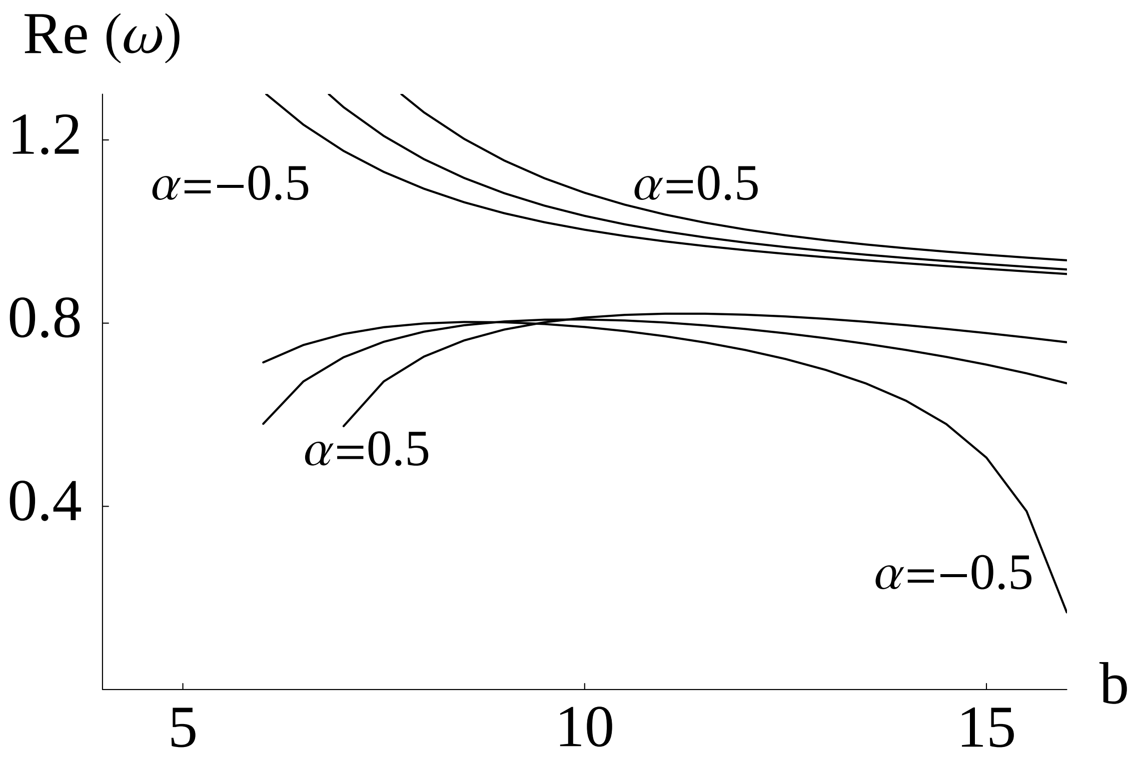
$$\omega(u;z) = (\sqrt{1+a(z)\,u} - 1)/(\sqrt{1+a(z)\,u} + 1)$$

Parameters $a(z)$, b, and α are determined by:

- $f_{n\to\infty} \sim (-a(z))^n \, n! \, n^b$

- $f(u \to \infty, z) \sim u^{\alpha/2}$.

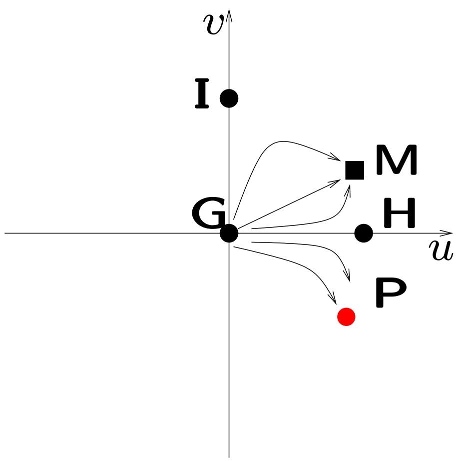
Convergence of the numerical results (STA)



The (real part of the) critical exponent ω as a function of b at five (upper curves) and four (lower curves) loops for $\alpha = -0.5, 0$ and 0.5 for the frustrated model ($N = 3$).

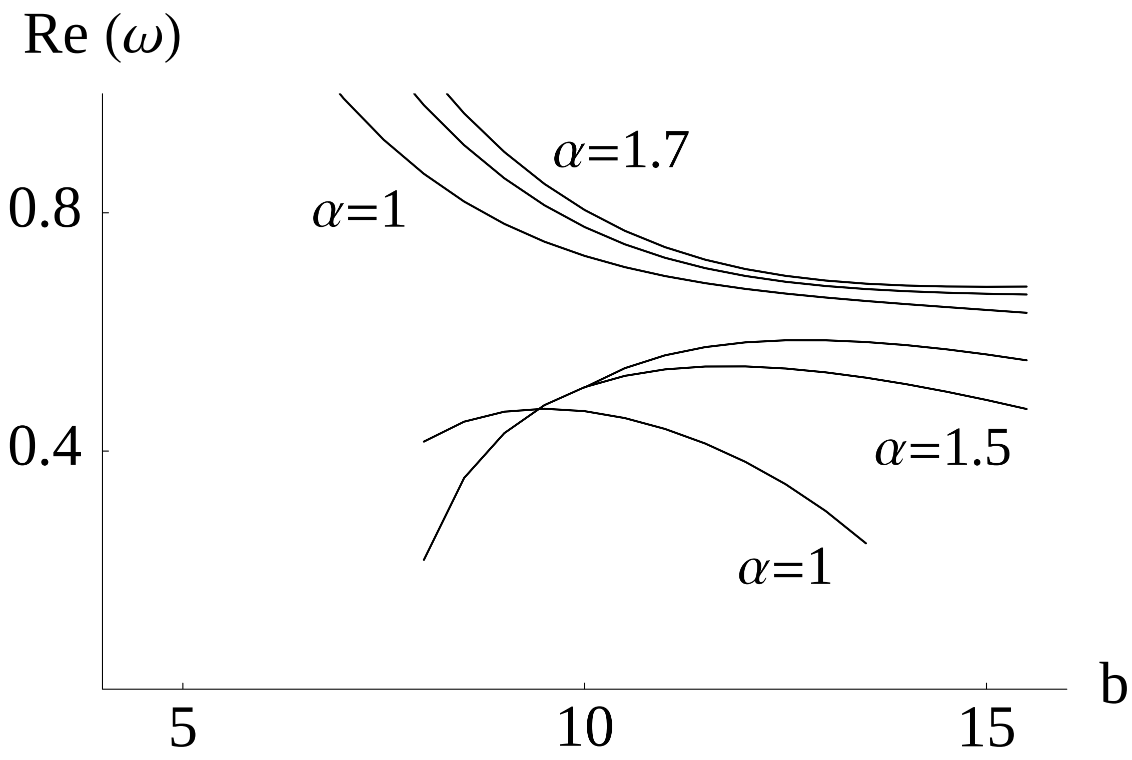
Similar analysis for the cubic model

$$\mathcal{H}_{\rm eff} = \int \mathrm{d}^d R \left\{ \frac{1}{2} \left((\nabla\phi)^2 + \mu_0^2 \phi^2 \right) + \frac{u_0}{4!}\phi^4 + \frac{v_0}{4!} \sum_{i=1}^{m} (\phi^i)^4 \right\},$$



Stable FP **P** exists for any $N \leq 7.5$ and lies in the region of Borel summability $u + v > 0$

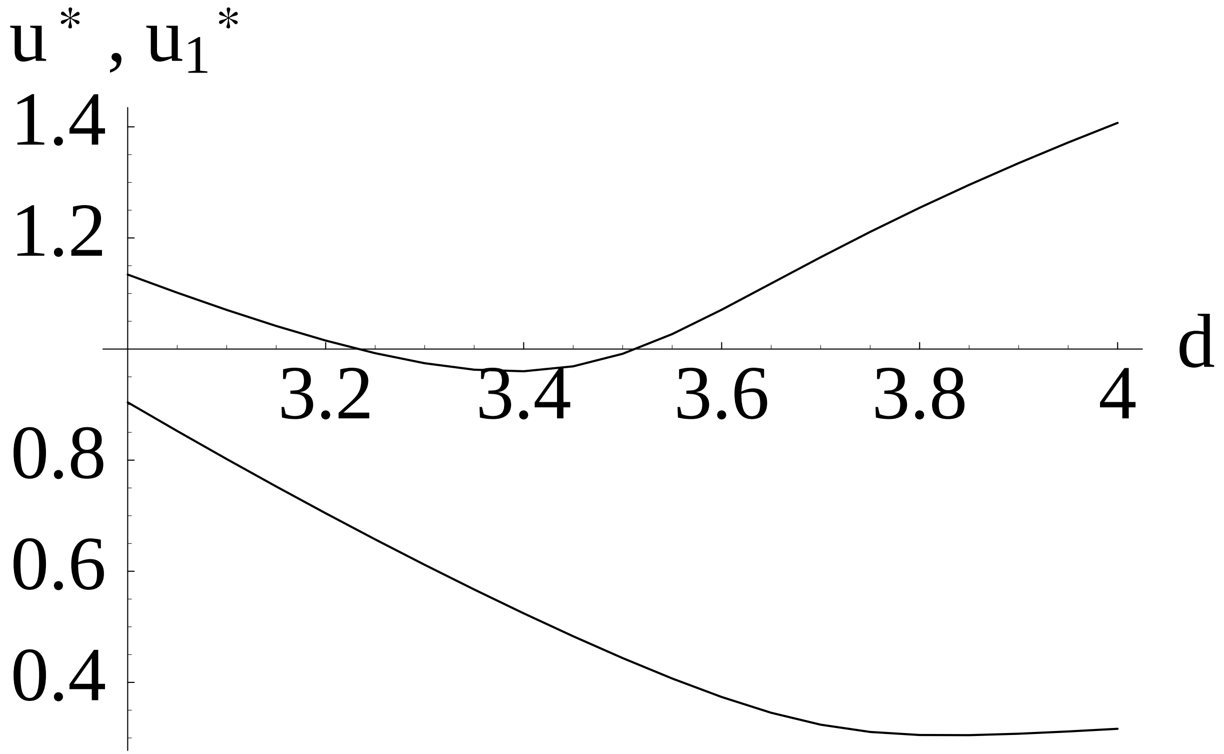
Convergence of the numerical results (cubic)



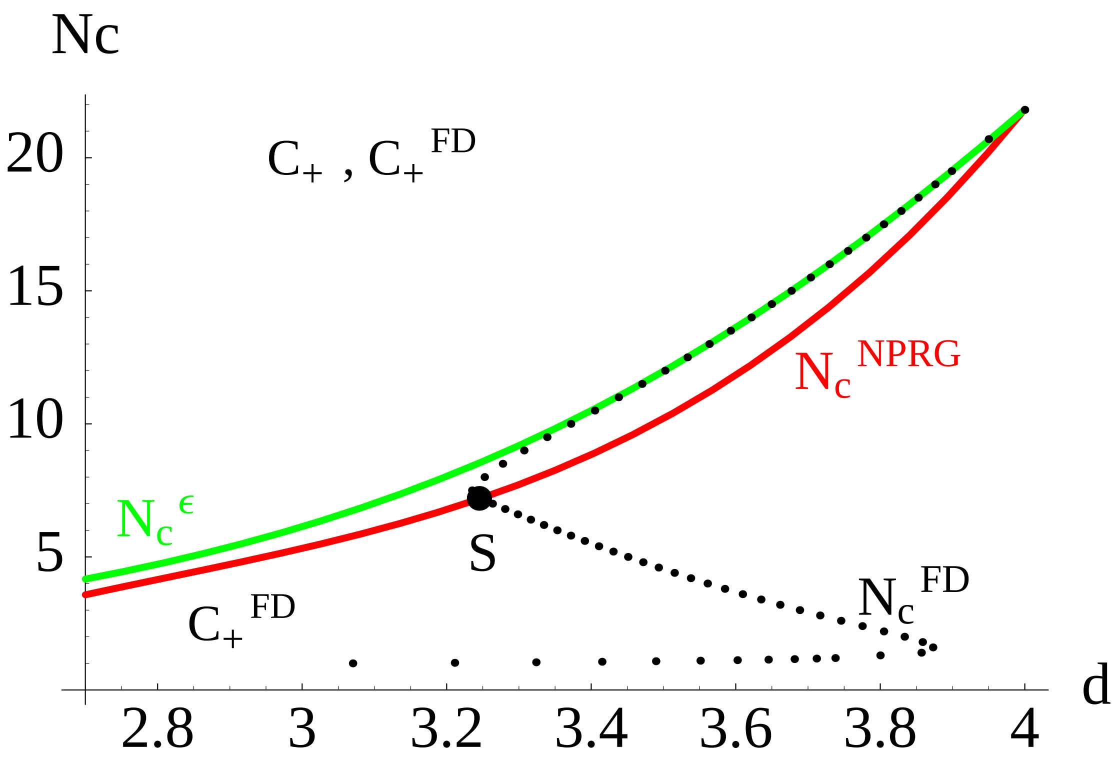
The (real part of the) critical exponent ω as a function of b at five (upper curves) and four (lower curves) loops for $\alpha = 1, 1.5$ and 1.7 for the cubic model (N=2).

Therefore, *the convergence of the perturbative numerical results is not enough to judge whether a FP is a genuine one or an artifact of the analysis.*

Behaviour of FPs with change of d



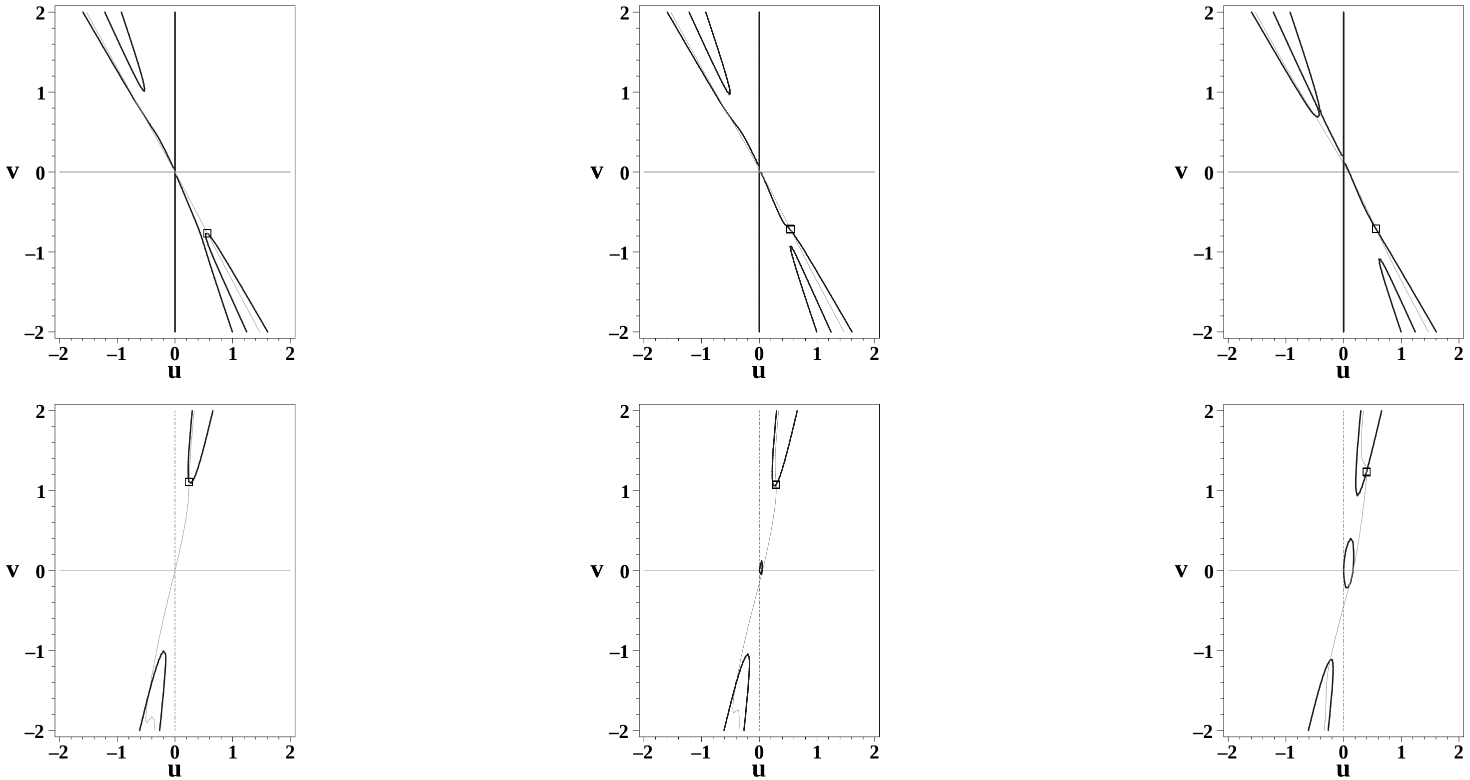
The u^* coordinate of the FP P ($N = 2$, upper curve) and the $v \equiv u_1^*$ coordinate of the FP C_+^{FD} ($N = 3$, lower curve) as functions of d.



Curves $N_3(d) \equiv N_c(d)$ obtained within the ϵ-expansion (N_c^{ϵ}), the $\overline{\text{MS}}$ scheme without ϵ-expansion (N_c^{FD}) and the NPRG approach (N_c^{NPRG}). The resummation parameters for the $\overline{\text{MS}}$ curve are $a = 1/2$, $b = 10$ and $\alpha = 1$. The part of the curve N_c^{FD} below S corresponds to a regime of non-Borel-summability.

At $d = 3.99...$

$\varepsilon = 0$ $\varepsilon = 0.1$ $\varepsilon = 0.3$



CUBIC

FRUST.

Lines of zeros of the β-functions for the cubic and frustrated model in 5 loops. No resummation is applied for small ε. Crossing of the lines corresponds to a fixed point. One of the "spurious"fixed points is shown by a square.

If a FP survives as a non-Gaussian FP at upper critical dimension, it is a signal that it is a spurious one

How to judge whether a FP is not an artifact of the calculation procedure?

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