Multicriticality and dynamic scaling

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Outline

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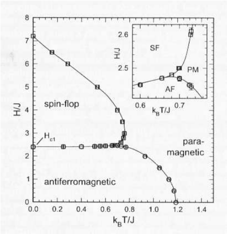
Antiferromagnet in a magnetic field

$$H_{AF} = J\sum_{ij}\left[A(S_i^x S_j^x + S_i^y S_j^y) + S_i^z S_j^z\right] - H^z \sum_i S_i^z$$

$$H_{singl-ion} = D\sum_i (S_i^z)^2$$

$d = 3$ $n_\perp = 2$ $n_| = 1$



A=0.8 D/J=0

Monte Carlo simulations: Bannasch, Selke, cond-mat 0807.1019v1; Holtschneider, Selke, Leidl, Phys. Rev. B **72** 064443 (2005)

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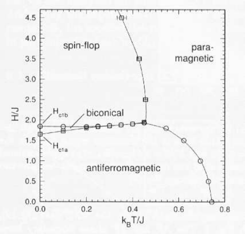
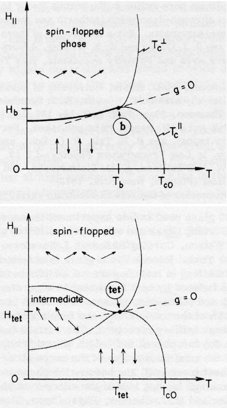
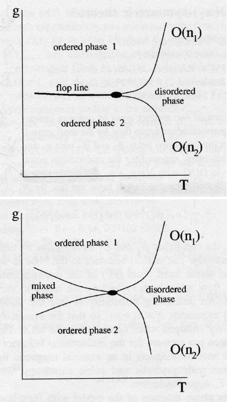


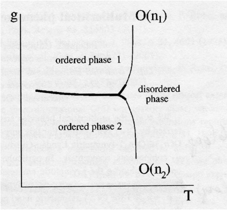
A=0.8 D/J=0.2

Monte Carlo simulations: Bannasch, Selke, cond-mat 0807.1019v1; Holtschneider, Selke, Leidl, Phys. Rev. B **72** 064443 (2005)

Phase diagrams with a bi-, tetracritical or triple point







Domany, Mukamel, Fisher,

Phys. Rev. B **15** 5432 (1977)

Lyuksyutov, Pokrovskii, Khmelnitskii, Sov. Phys. JETP **42** 923 (1975); Kosterlitz, Nelson, Fisher, Phys. Rev. B **13** 412 (1976): 1loop

Prudnikov^2, Fedorenko, JETP Lett. **68**, 950 (1998): 2 loop massive RG scheme, Borel summed ; Calabrese, Pelisetto, Vicari, Phys. Rev. B **67** 054505 (2003): 5 loop ε-expansion

Field theoretic functional

$O(n_{\|}) \oplus O(n_{\perp})$ symmetry

$$\mathcal{H}_{Bi}(x) = \int d^d x \Bigg\{ \frac{1}{2}\mathring{r}_{\perp}\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0} + \frac{1}{2}\sum_{\alpha=1}^{n_{\perp}} \nabla_{\alpha}\vec{\phi}_{\perp 0}\cdot\nabla_{\alpha}\vec{\phi}_{\perp 0}$$

$$+\frac{1}{2}\mathring{r}_{\|}\vec{\phi}_{\| 0}\cdot\vec{\phi}_{\| 0} + \frac{1}{2}\sum_{i=1}^{n_{\|}} \nabla_{i}\vec{\phi}_{\| 0}\cdot\nabla_{i}\vec{\phi}_{\| 0}$$

$$+\frac{\mathring{u}_{\perp}}{4!}\left(\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0}\right)^2 + \frac{\mathring{u}_{\|}}{4!}\left(\vec{\phi}_{\| 0}\cdot\vec{\phi}_{\| 0}\right)^2 + \frac{2\mathring{u}_{\times}}{4!}\left(\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0}\right)\left(\vec{\phi}_{\| 0}\cdot\vec{\phi}_{\| 0}\right)\Bigg\}$$

F., Holovatch, Moser, to appear in Phys. Rev. E (2008); arXiv:0808.0314: 2loop minimal subtraction RG scheme, Borel summed

Flow equations for the static couplings

$$\beta_{u_\perp} = -\varepsilon u_\perp + \frac{(n_\perp + 8)}{6} u_\perp^2 + \frac{n_\parallel}{6} u_\times^2 - \frac{(3n_\perp + 14)}{12} u_\perp^3 - \frac{5n_\parallel}{36} u_\perp u_\times^2 - \frac{n_\parallel}{9} u_\times^3$$

$$\beta_{u_\times} = -\varepsilon u_\times + \frac{(n_\perp + 2)}{6} u_\perp u_\times + \frac{(n_\parallel + 2)}{6} u_\times u_\parallel + \frac{2}{3} u_\times^2 - \frac{(n_\perp + n_\parallel + 16)}{72} u_\times^3$$
$$- \frac{(n_\perp + 2)}{6} u_\times^2 u_\perp - \frac{(n_\parallel + 2)}{6} u_\times^2 u_\parallel - \frac{5(n_\perp + 2)}{72} u_\perp^2 u_\times - \frac{5(n_\parallel + 2)}{72} u_\times u_\parallel^2;$$

$$\beta_{u_\parallel} = -\varepsilon u_\parallel + \frac{(n_\parallel + 8)}{6} u_\parallel^2 + \frac{n_\perp}{6} u_\times^2 - \frac{(3n_\parallel + 14)}{12} u_\parallel^3 - \frac{5n_\perp}{36} u_\parallel u_\times^2 - \frac{n_\perp}{9} u_\times^3 .$$

No real fixed points at $\varepsilon = 1$ → Borel summation → β_a^{Borel}

$$l \frac{du_a}{dl} = \beta_{u_a}^{Borel}(\{u\})$$

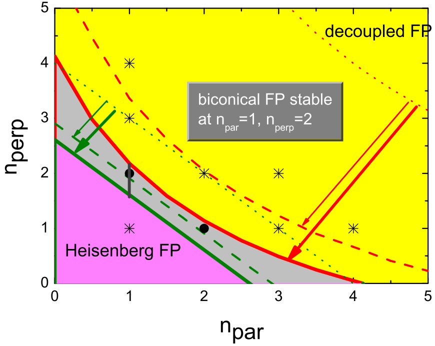
Fixed points and stability exponents

FP	$u^\star_\perp$	$u^\star_\times$	$u^\star_\parallel$
$\mathcal{G}$	0	0	0
$\mathcal{H}(n_\perp)$	$u^{\mathcal{H}(n_\perp)}$	0	0
$\mathcal{H}(n_\parallel)$	0	0	$u^{\mathcal{H}(n_\parallel)}$
$\mathcal{D}$	$u^{\mathcal{H}(n_\perp)}$	0	$u^{\mathcal{H}(n_\parallel)}$
$\mathcal{H}(n_\perp + n_\parallel)$	$u^{\mathcal{H}(n_\perp+n_\parallel)}$	$u^{\mathcal{H}(n_\perp+n_\parallel)}$	$u^{\mathcal{H}(n_\perp+n_\parallel)}$
$\mathcal{B}$	$u^{\mathcal{B}}_\perp$	$u^{\mathcal{B}}_\times$	$u^{\mathcal{B}}_\parallel$
$\mathcal{U}_1$	$u^{\mathcal{U}_1}_\perp$	$u^{\mathcal{U}_1}_\times$	$u^{\mathcal{U}_1}_\parallel$
$\mathcal{U}_2$	$u^{\mathcal{U}_2}_\perp$	$u^{\mathcal{U}_2}_\times$	$u^{\mathcal{U}_2}_\parallel$

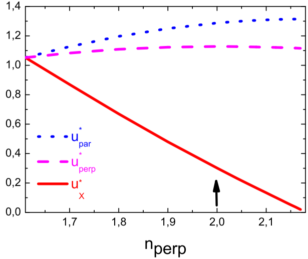
Regions of stable fixed points

Static 'phase diagram'

Arrows: shift of stability borderlines in resummed two loop order



Biconical fixed point values



dotted: 1loop KNF
dashed: 2loop massive P^2F
solid: 2loop minimal subtraction FMH

Fixed points and stability exponents for $n_\parallel = 1$ and $n_\perp = 2$

Small transient exponent due to nearby stability border line

FP	$u_\perp^\star$	$u_\times^\star$	$u_\parallel^\star$	ω_1	ω_2	ω_3
$\mathcal{G}$	0	0	0	-1	-1	-1
$\mathcal{H}(2)$	1.141	0	0	0.581	-0.461	-1
$\mathcal{H}(1)$	0	0	1.315	-1	-0.552	0.565
$\mathcal{D}$	1.141	0	1.315	0.581	-0.014	0.566
$\mathcal{H}(3)$	1.002	1.002	1.002	0.597	0.407	-0.036
$\mathcal{B}$	1.128	0.301	1.287	0.583	0.554	0.01

Table: Fixed points and stability exponents of the $O(1) \oplus O(2)$ model obtained by the Padé-Borel resummation within two loops. Biconical FP $\mathcal{B}$ is stable.

Exponents: Definitions $i = \perp, \parallel$

Relations to the field theoretic ζ-functions

$$\eta_i = -\zeta^\star_{\phi_i}$$

$$\gamma_i = \frac{2 + \zeta^\star_{\phi_i}}{2 - \zeta^\star_+}$$

only one exponent for the correlation lengths!

$$\nu^{-1} = 2 - \zeta^\star_+ \equiv \nu_+^{-1}$$

$$\gamma_i = \nu(2 - \eta_i)$$

$$\phi = \frac{2 - \zeta^\star_-}{2 - \zeta^\star_+}$$

$$\nu_-^{-1} \equiv 2 - \zeta^\star_-$$

$$\phi = \frac{\nu_+}{\nu_-}$$

only one exponent for the specific heat!

$$\alpha = \frac{\epsilon - 2\zeta^\star_+}{2 - \zeta^\star_+} = 2 - \frac{d}{2 - \zeta^\star_+} = 2 - d\nu$$

Scaling laws fulfilled for FP H and B

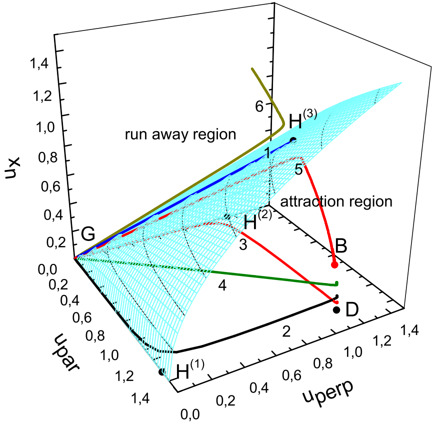
Exponents: Results

FP	$\eta_\perp$	$\eta_\parallel$	$\gamma_\perp$	$\gamma_\parallel$	ν_+	ν_-	ϕ	α
$\mathcal{B}$	0.037	0.037	1.366	1.366	0.696	0.692	1.144[1]	-0.088
$\mathcal{H}(3)$	0.040	0.040	1.411	1.411	0.720	0.564	1.275[1]	-0.160
$\mathcal{B}$	0	0	1.222	1.222	0.611	0.503	1.176	*0.167*
$\mathcal{H}(3)$	0	0	1.227	1.227	0.611	0.505	1.136	*0.167*
$\mathcal{B}$	0.037(5)	0.037(5)	*1.37(7)*	*1.37(7)*	0.70(3)	*0.56(3)*	1.25(1)	*-0.10(9)*
$\mathcal{H}(3)$	0.0375(45)	0.0375(45)	1.382(9)	1.382(9)	0.7045(55)	0.559(17)	*1.259(23)*	*-0.114(17)*

Critical exponents of the $O(1) \oplus O(2)$ model obtained by resummation of the two-loop RG series at fixed $d = 3$ in different FPs (first two rows of the table). Our data is compared with the results of first order ε-expansion, and resummed fifth order ε-expansion. Numbers, shown in italic were obtained via familiar scaling relations.

[1] Pole in the Padé approximant is present.

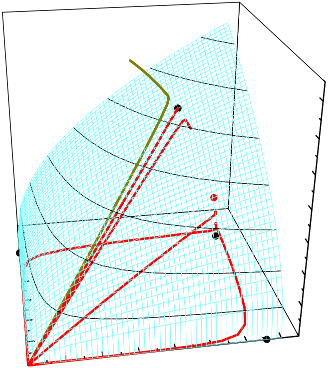
Attraction regions



Mean field condition for tetracritcality

$$\Delta = u_{\|} u_{\perp} - u_{\times}^2 > 0$$

Attraction regions



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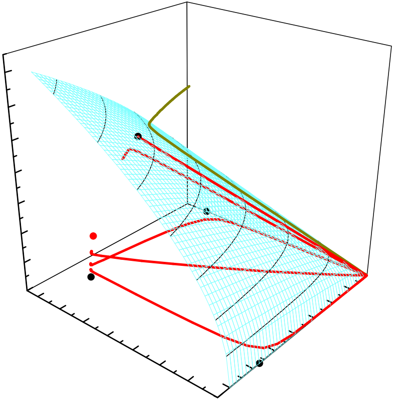

Mean field condition for tetracritcality

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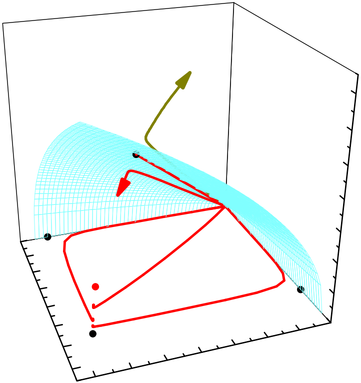
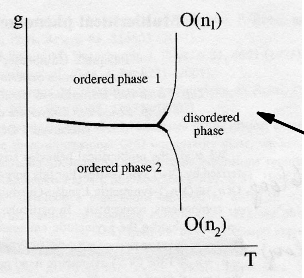
Attraction regions

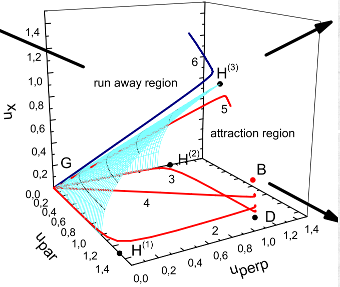


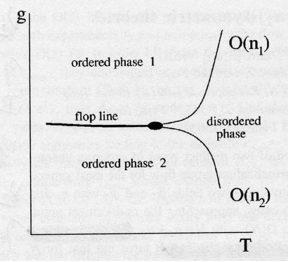
Mean field condition for tetracritcality

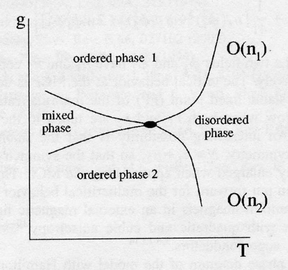
$$\Delta = u_{\|} u_{\perp} - u_{\times}^2 > 0$$

Consequences for the phase diagram









$u_{\parallel}(0)$, $u_{\perp}(0)$, $u_{\times}(0)$
are functions of
$H_{\parallel}$, T and exchange coupling,
exchange, cubic anisotropy etc.

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static couplings	model A	model B
dynamic couplings		model J
cons. density (CD)	yes	
two time scales; time scale ratio w		
static couplings	model C	model D
dynamic couplings	model E,G	model H
both couplings	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	model B
dynamic c.		model J
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	model C	model D
dynamic c.	model E,G	model H
both c.	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		model J
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	model C	model D
dynamic c.	model E,G	model H
both c.	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		isotropic FM
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	model C	model D
dynamic c.	model E,G	model H
both c.	model F	

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Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		isotropic FM
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	AFM in mag. field	model D
dynamic c.	model E,G	model H
both c.	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		isotropic FM
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	AFM in mag. field	model D
dynamic c.	planar FM, isotropic AFM	model H
both c.	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		isotropic FM
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	AFM in mag. field	model D
dynamic c.	planar FM, isotropic AFM	gas-liquid
both c.	model F	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Dynamic Classification

Dynamic models		
conservation of OP	no	yes
static c.	uniaxial AFM	uniaxial FM
dynamic c.		isotropic FM
cons. density (CD)	yes	
two time scales; time scale ratio w		
static c.	AFM in mag. field	model D
dynamic c.	planar FM, isotropic AFM	gas-liquid
both c.	superfluid He^4,AFM in mag. field	

Hohenberg, Halperin, Rev. Mod. Phys. **49** 435 (1977); F., Moser, J. Phys. A: Math. Gen. **39** R207 (2006)

Strong and weak dynamic scaling

STRONG SCALING

Fixed point value of the time scale ratio $w^\star \neq 0$, finite
One dynamical critical exponent $z_{OP} = z_{CD}$;
one characteristic frequency ω_c

WEAK SCALING

Fixed point value of the time scale ratio $w^\star = 0, \infty$
Two dynamical critical exponents z_{OP} different from z_{CD};
two characteristic frequencies

Prominent examples for weak dynamic scaling: dynamics at superfluid transition and the magnetic transition of a planar antiferromagnet (models F and E for the case $n = 2$ in $d = 3$)

Dynamical critical exponents

STRONG SCALING

$$z = 2 + \zeta_{\Gamma_{OP}}$$

A finite FP value of the (static or dynamic) coupling leads to an exact expression for z
Examples: Model C: $z = 2 + \alpha/\nu$
Model E,G: $z = 3/2$ Model J: $z = (5 - \eta)/2$

WEAK SCALING

$$z_{OP} = 2 + \zeta_{\Gamma_{OP}} \qquad z_{CD} = 2 + \zeta_{\Gamma_{CD}}$$

A finite FP value of the (static or dynamic) coupling leads to an exact expression for $z_{OP} + z_{CD}$
Examples: Model E,G: $z_{OP} + z_{CD} = 3$
Model H: $z_{OP0} + z_{CD} = 5$

Flow equation

$$\ell \frac{dw}{d\ell} = \beta_w(couplings, w)$$

$$\beta_w(couplings, w) = w\Big(\zeta_{\Gamma_{OP}}(couplings, w) - \zeta_{\Gamma_{CD}}(couplings, w)\Big)$$

Matching condition for flow paramter ℓ

$$\ell^{2z_{vanHove}} = \left(\frac{\xi_0}{\xi}\right)^{2z_{vanHove}} + \left(\frac{2\omega\xi_0^{z_{vanHove}}}{\Gamma_{OP}(\ell)}\right)^2$$

Dynamic model I

perpendicular components of the alternating magnetization $\vec{\phi}_{\perp 0}$

$$\frac{\partial \vec{\phi}_{\perp 0}}{\partial t} = -\overset{o}{\Gamma}_{\perp} \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + g_{\perp} \vec{\phi}_{\perp 0} \times \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + g_m \vec{\phi}_{\perp 0} \times \vec{e}_z \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + \vec{\theta}_{\phi_{\perp}}$$

z-component of the alternating magnetization $\phi_{\parallel 0}$

$$\frac{\partial \phi_{\parallel 0}}{\partial t} = -\overset{o}{\Gamma}_{\parallel} \frac{\delta \mathcal{H}_{Bi}}{\delta \phi_{\parallel 0}} + \theta_{\phi_{\parallel}}$$

z-component of the magnetization m_0

$$\frac{\partial m_0}{\partial t} = \overset{o}{\lambda} \nabla^2 \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + g_m \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + \theta_m$$

V. Dohm, H.-K. Janssen, Phys. Rev. Lett. **39** 946 (1977) 1loop

Dynamic model II

$$\mathcal{H}_{Bi}(x)=\int d^dx\Bigg\{\frac{1}{2}\mathring{r}_\perp\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0}+\frac{1}{2}\sum_{\alpha=1}^{n_\perp}\nabla_\alpha\vec{\phi}_{\perp 0}\cdot\nabla_\alpha\vec{\phi}_{\perp 0}$$

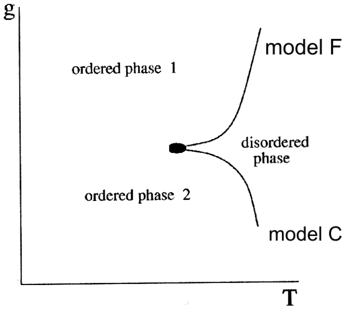
$$+\frac{1}{2}\mathring{r}_\|\vec{\phi}_{\|0}\cdot\vec{\phi}_{\|0}+\frac{1}{2}\sum_{i=1}^{n_\|}\nabla_i\vec{\phi}_{\|0}\cdot\nabla_i\vec{\phi}_{\|0}$$

$$+\frac{\mathring{u}_\perp}{4!}\left(\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0}\right)^2+\frac{\mathring{u}_\|}{4!}\left(\vec{\phi}_{\|0}\cdot\vec{\phi}_{\|0}\right)^2+\frac{2\mathring{u}_\times}{4!}\left(\vec{\phi}_{\perp 0}\cdot\vec{\phi}_{\perp 0}\right)\left(\vec{\phi}_{\|0}\cdot\vec{\phi}_{\|0}\right)$$

$$+\frac{1}{2}\overset{o}{\gamma}_{m\perp}\ m_0\vec{\phi}^2_{\perp 0}+\frac{1}{2}\overset{o}{\gamma}_{m\|}\ m_0\phi^2_{\|0}-\overset{o}{h}_m\ m_0\Bigg\}$$

Dynamic universality classes

Isotropic antiferromagnet in a magnetic field
OP and CD measurable !;
neutron scattering



Model F: superfluid transition; OP not measuable

$$\frac{\partial \vec{\phi}_{\perp 0}}{\partial t} = -\overset{\circ}{\Gamma}_{\perp} \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + g_{\perp} \vec{\phi}_{\perp 0} \times \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + g_m \vec{\phi}_{\perp 0} \times \vec{e}_z \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + \vec{\theta}_{\phi_{\perp}}$$

$$\frac{\partial m_0}{\partial t} = \overset{\circ}{\lambda} \nabla^2 \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + g_m \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + \theta_m$$

ζ-functions: F., Moser, Phys. Rev. Lett. **89**, 125301 (2002); Erratum **93**, 229902 (2004)

model C

$$\frac{\partial \phi_{\| 0}}{\partial t} = -\overset{\circ}{\Gamma}_{\|} \frac{\delta \mathcal{H}_{Bi}}{\delta \phi_{\| 0}} + \theta_{\phi_{\|}}$$

$$\frac{\partial m_0}{\partial t} = \overset{\circ}{\lambda} \nabla^2 \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + +\theta_m$$

ζ-functions: F., Moser, Phys. Rev. Lett. **91**, 030601 (2003)

Model A

$$\frac{\partial \vec{\phi}_{\perp 0}}{\partial t} = -\mathring{\Gamma}_{\perp} \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + \vec{\theta}_{\phi_{\perp}} \qquad \frac{\partial \vec{\phi}_{\| 0}}{\partial t} = -\mathring{\Gamma}_{\|} \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\| 0}} + \vec{\theta}_{\phi_{\|}}$$

$$\mathcal{H}_{Bi}(x) = \int d^d x \Bigg\{ \frac{1}{2} \mathring{r}_{\perp} \vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} + \frac{1}{2} \sum_{\alpha=1}^{n_{\perp}} \nabla_{\alpha} \vec{\phi}_{\perp 0} \cdot \nabla_{\alpha} \vec{\phi}_{\perp 0}$$
$$+ \frac{1}{2} \mathring{r}_{\|} \vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0} + \frac{1}{2} \sum_{i=1}^{n_{\|}} \nabla_{i} \vec{\phi}_{\| 0} \cdot \nabla_{i} \vec{\phi}_{\| 0}$$
$$+ \frac{\mathring{u}_{\perp}}{4!} \left(\vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} \right)^2 + \frac{\mathring{u}_{\|}}{4!} \left(\vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0} \right)^2 + \frac{2 \mathring{u}_{\times}}{4!} \left(\vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} \right) \left(\vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0} \right) \Bigg\}$$

Time scale ratio

$$v = \frac{\Gamma_{\parallel}}{\Gamma_{\perp}}$$

$v^{\star} \neq 0, \infty$ finite $\rightarrow$ strong dynamic scaling $\rightarrow$ one time scale

$v^{\star} = 0, \infty$ $\rightarrow$ weak dynamic scaling $\rightarrow$ two time scales

What is new?

Dohm, Janssen, Phys. Rev. Lett. **39** 946 (1977) used the static one loop order result. The Heisenberg FP was stable, the stability border lines far away!

Now the Borel summed two loop order result is used. The biconical FP is stable and the stability border line to the decoupeld FP is nearby!

Dynamical exponents

$$\zeta_{\Gamma_\perp} = \frac{n_\perp+2}{36}\, u_\perp^2 \left(3\ln\frac{4}{3}-\frac{1}{2}\right) + \frac{n_\parallel}{36}\, u_\times^2 \left[\frac{2}{v}\ln\frac{2(1+v)}{2+v}+\ln\frac{(1+v)^2}{v(2+v)}-\frac{1}{2}\right]$$

$$\zeta_{\Gamma_\parallel} = \frac{n_\parallel+2}{36}\, u_\parallel^2 \left(3\ln\frac{4}{3}-\frac{1}{2}\right) + \frac{n_\perp}{36}\, u_\times^2 \left[2v\ln\frac{2(1+v)}{1+2v}+\ln\frac{(1+v)^2}{1+2v}-\frac{1}{2}\right]$$

$$z_\parallel = 2+\zeta_\parallel^\star \qquad z_\perp = 2+\zeta_\perp^\star$$

$$z_\parallel^{\mathcal{B}} = z_\perp^{\mathcal{B}} = z^{\mathcal{B}} = 2.052$$

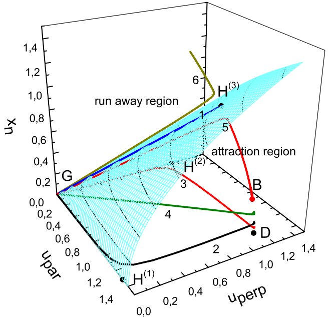
Transient exponent

$$\omega_v = \left(\frac{\partial\beta_v}{\partial v}\right)_{u_\times^\star, v^\star} = u_\times^{\star 2}\frac{v^\star}{18}\left(\frac{n_\parallel}{v^\star}\ln\frac{2(1+v^\star)}{2+v^\star} + n_\perp\ln\frac{2(1+v^\star)}{1+2v^\star}\right)$$

$$\omega_v^{\mathcal{B}} = 0.004$$

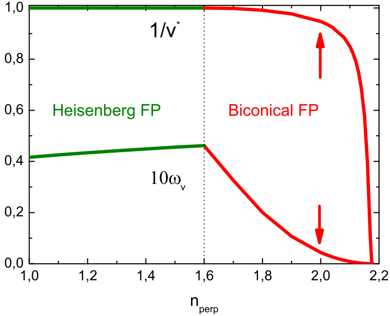
Attraction regions

Static couplings

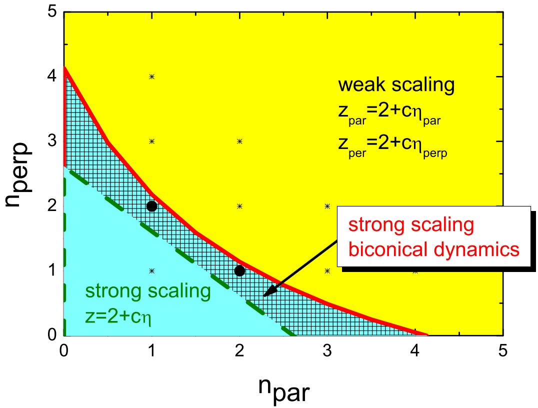


Timescale ratio v

Biconical fixed point $v^{\mathcal{B}} = 1.055$



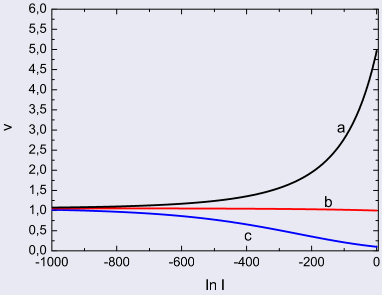
Dynamical 'phase diagram'



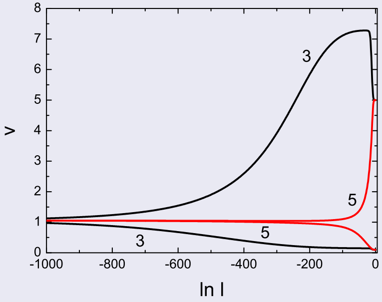
F., Holovatch, Moser, submitted to Phys. Rev. E (2008)

Flow of the time scale ratio v

At the biconical static FP



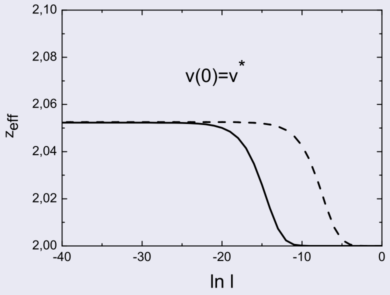
Starting in the static background



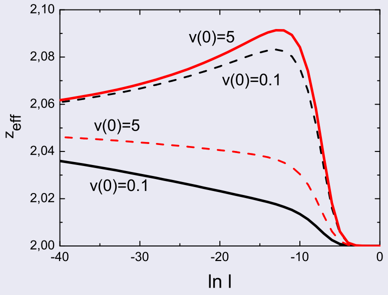
Effective dynamic exponents

$z_{||}$ solid $z_{\perp}$ dashed

At the biconical dynamical FP (3)



Starting in the background (5)



Relaxational dynamics including conservation of magnetization

$$\frac{\partial \vec{\phi}_{\perp 0}}{\partial t} = -\mathring{\Gamma}_{\perp} \frac{\delta \mathcal{H}_{Bi}}{\delta \vec{\phi}_{\perp 0}} + \vec{\theta}_{\phi_{\perp}} \qquad \frac{\partial \phi_{\| 0}}{\partial t} = -\mathring{\Gamma}_{\|} \frac{\delta \mathcal{H}_{Bi}}{\delta \phi_{\| 0}} + \theta_{\phi_{\|}}$$

$$\frac{\partial m_0}{\partial t} = \mathring{\lambda}_m \nabla^2 \frac{\delta \mathcal{H}_{Bi}}{\delta m_0} + \theta_m$$

$$\mathcal{H}_{Bi}(x) = \int d^d x \Bigg\{ \frac{1}{2} \mathring{r}_{\perp} \vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} + \frac{1}{2} \sum_{\alpha=1}^{n_{\perp}} \nabla_{\alpha} \vec{\phi}_{\perp 0} \cdot \nabla_{\alpha} \vec{\phi}_{\perp 0} + \frac{1}{2} \mathring{r}_{\|} \vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0}$$

$$+ \frac{1}{2} \sum_{i=1}^{n_{\|}} \nabla_i \vec{\phi}_{\| 0} \cdot \nabla_i \vec{\phi}_{\| 0} + \frac{\mathring{u}_{\perp}}{4!} \left(\vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} \right)^2 + \frac{\mathring{u}_{\|}}{4!} \left(\vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0} \right)^2$$

$$+ \frac{2\mathring{u}_{\times}}{4!} \left(\vec{\phi}_{\perp 0} \cdot \vec{\phi}_{\perp 0} \right) \left(\vec{\phi}_{\| 0} \cdot \vec{\phi}_{\| 0} \right) + \frac{1}{2} \mathring{\gamma}_{m\perp} m_0 \vec{\phi}^2_{\perp 0} + \frac{1}{2} \mathring{\gamma}_{m\|} m_0 \phi^2_{\| 0} - \mathring{h}_m \, m_0 \Bigg\}$$

Time scale ratios

$$v = \frac{\Gamma_{\parallel}}{\Gamma_{\perp}} \qquad w_{\parallel} = \frac{\Gamma_{\parallel}}{\lambda_m} \qquad w_{\perp} = \frac{\Gamma_{\perp}}{\lambda_m} \qquad \frac{w_{\parallel}}{w_{\perp}} = v$$

$v^{\star} \neq 0, \infty$ finite $\rightarrow$ strong dynamic scaling $\rightarrow$ one time scale

$v^{\star} = 0, \infty$ $\rightarrow$ weak dynamic scaling $\rightarrow$ two time scales

Dynamical ζ-functions: Fixed points and exponents

$$\zeta_{\Gamma_\perp}^{(C)}(\{u\},\{\gamma\},\{w\})) = \zeta_{\Gamma_\perp}^{(A)}(u_\perp, u_\times, v) + [\zeta_{\Gamma_\perp}^{(C_\perp)}(u_\perp, \gamma_\perp, w_\perp) - \zeta_{\Gamma_\perp}^{(A_\perp)}(u_\perp)]$$

$$-\frac{n_\parallel}{4}\frac{w_\perp\gamma_\perp\gamma_\parallel}{1+w_\perp}\left[\frac{2}{3}u_\times + \frac{w_\perp\gamma_\perp\gamma_\parallel}{1+w_\perp}\right]\left(1+\ln\frac{2v}{1+v} - \left(1+\frac{2}{v}\right)\ln\frac{2(1+v)}{2+v}\right)$$

$$\zeta_{\Gamma_\parallel}^{(C)}(\{u\},\{\gamma\},\{w\})) = \zeta_{\Gamma_\parallel}^{(A)}(u_\parallel, u_\times, v) + [\zeta_{\Gamma_\parallel}^{(C_\parallel)}(u_\parallel, \gamma_\parallel, w_\parallel) - \zeta_{\Gamma_\parallel}^{(A_\parallel)}(u_\parallel)]$$

$$-\frac{n_\perp}{4}\frac{w_\parallel\gamma_\parallel\gamma_\perp}{1+w_\parallel}\left[\frac{2}{3}u_\times + \frac{w_\parallel\gamma_\parallel\gamma_\perp}{1+w_\parallel}\right]\left(1+\ln\frac{2}{1+v} - (1+2v)\ln\frac{2(1+v)}{1+2v}\right)$$

$$\zeta_{\lambda_m} = 2\zeta_m$$

$$z_\parallel = 2 + \zeta_\parallel^\star \qquad z_\perp = 2 + \zeta_\perp^\star \qquad z_m = 2 + \zeta_{\lambda_m}^\star$$

F., Holovatch, Moser unpublished (2008)

Flow equation for asymmetric couplings $\gamma_{\|}$ and $\gamma_{\perp}$

$$\vec{\gamma} \equiv \begin{pmatrix} \gamma_{\perp} \\ \gamma_{\|} \end{pmatrix} \qquad \text{condition} \qquad \gamma_{\perp} = f(\gamma_{\|}, \{u\})$$

coupling only to 'magnetic' conserved density not to energy density

$$\ell \frac{d\vec{\gamma}}{d\ell} = \left(\left[-\frac{\varepsilon}{2} + \frac{1}{2} \vec{\gamma}^{T} \cdot \mathbf{B}_{\phi^2}(\{u\}) \cdot \vec{\gamma} \right] \mathbf{1} + \zeta^{T}_{\phi^2}(\{u\}) \right) \cdot \vec{\gamma}$$

In two loop order:

$$\mathbf{B}_{\phi^2}(\{u\}) = \begin{pmatrix} \frac{n_{\perp}}{2} & 0 \\ 0 & \frac{n_{\|}}{2} \end{pmatrix} + \mathcal{O}(\{u^2\})$$

Dohm, *Report of the Kernforschungsanlage Jülich Nr. 1578* (1979)

There: $\mathcal{H}$eisenberg - O(n) symmetric fixed point

Here: $\mathcal{B}$iconical - $O(n_{\|}) \oplus O(n_{\perp})$ symmetric fixed point!

Scaling exponent for m: $2\frac{\phi}{\nu} - 3 = 0.287$

Fixed points for asymmetric couplings $\gamma_\parallel$ and $\gamma_\perp$

Stable FP: coupling only to 'magnetic' density $\gamma_+^\star = 0$ ($\zeta_m^\star = 2\frac{\phi}{\nu} - 3$)

$$\gamma_\parallel^{\star 2} = \frac{2\frac{\phi}{\nu} - 3}{\frac{n_\parallel}{2} + \frac{n_\perp}{2}\left(\frac{[\zeta_{\phi^2}^\star]_{21}}{\zeta_-^\star - [\zeta_{\phi^2}^\star]_{11}}\right)^2} \qquad \gamma_\perp^{\star 2} = \left(\frac{[\zeta_{\phi^2}^\star]_{21}}{\zeta_-^\star - [\zeta_{\phi^2}^\star]_{11}}\right)^2 \gamma_\parallel^{\star 2}$$

$\mathcal{H}$eisenberg - O(n) symmetric fixed point: $\frac{[\zeta_{\phi^2}^\star]_{21}}{\zeta_-^\star - [\zeta_{\phi^2}^\star]_{11}} = -\frac{n_\parallel}{n_\perp}$

$\mathcal{B}$iconical - $O(n_\parallel) \oplus O(n_\perp)$ symmetric fixed point: $\frac{[\zeta_{\phi^2}^\star]_{21}}{\zeta_-^\star - [\zeta_{\phi^2}^\star]_{11}} = f(\{u^\star\})$

Dynamic scaling of the correlation function

Shape functions $\mathcal{F}_i$ Kawasaki functions f_i

$$C_i(\xi, k, \omega, w) = \frac{C_i^{(st)}(\xi, k)}{\omega_i(\xi, k)} \mathcal{F}_i\left(x, y, w(\ell[\xi, k, \omega])\right) \qquad i = OP,\ CD$$

$$x = k\xi \ , \qquad y_i = \frac{\omega}{\omega_i(\xi, k)} \ , \qquad \omega_i = A_i k^{z_i} f_i(x)$$

$w(\ell)$ solution of flow equations or $\sim w^\star$

ℓ from matching condition ot *ell* $\rightarrow \infty$

$$C_{OP}^{(st)}(\xi, k) = k^{-2+\eta} g_{OP}(x) \ , \qquad C_{CD}^{(st)}(\xi, k) = g_{CD}(x)$$

Dynamic vertex functions i = OP, CD 1loop order

$$\mathring{C}_i(\xi,k,\omega) = -\frac{\mathring{\Gamma}_{\tilde{i}\tilde{i}}(\xi,k,\omega)}{|\mathring{\Gamma}_{i\tilde{i}}(\xi,k,\omega)|^2}.$$

$$\mathring{\Gamma}_{i\tilde{i}}(\xi,k,\omega) = -i\omega\mathring{\Omega}_{i\tilde{i}}(\xi,k,\omega) + \mathring{\Gamma}_i^{st}(\xi,k)\mathring{\Gamma}^{(d)}_{i\tilde{i}}(\xi,k,\omega)$$

$$\mathring{\Gamma}_{\tilde{i}\tilde{i}}(\xi,k,\omega) = -2\Re\left[\mathring{\Gamma}^{(d)}_{i\tilde{i}}(\xi,k,\omega)\mathring{\Omega}_{i\tilde{i}}(\xi,k,\omega)\right]$$

$$\mathring{\Omega}_{i\tilde{i}} = 1 + \mathring{\gamma}\mathring{W}_{i\tilde{i}} \qquad \mathring{\Gamma}^{(d)}_{i\tilde{i}} = 2\mathring{\Gamma}_i k^{a_i} + \mathring{g}\mathring{G}_{i\tilde{i}}$$

model A,C   model E,G

both in model F

Model C

$$\frac{\partial \vec{\phi}_0}{\partial t} = -\mathring{\Gamma} \frac{\delta H}{\delta \vec{\phi}_0} + \vec{\theta}_\phi \qquad \frac{\partial m_0}{\partial t} = \mathring{\lambda} \nabla^2 \frac{\delta H}{\delta m_0} + \theta_m \, .$$

$$H = \int d^d x \left\{ \frac{1}{2} \mathring{\tau} \, \vec{\phi}_0^2 + \frac{1}{2} \sum_{i=1}^{n} (\nabla \phi_{i0})^2 + \frac{\mathring{\tilde{u}}}{4!} \vec{\phi}_0^4 + \frac{1}{2} a_m m_0^2 + \frac{1}{2} \mathring{\gamma} m_0 \vec{\phi}_0^2 - \mathring{h}_m \, m_0 \right\}$$

Time scale ratio $w = \frac{\Gamma}{\lambda}$; borderline to weak scaling at $d = 3$ is n_c

$n = 1, \ d = 3$	$\gamma^{\star 2}$	$w^\star$	$z = 2 + \alpha/\nu$	ω_w	n_c
1 loop, ϵ-expan.	2/3	1	2.33	1/6	2
2 loop, ϵ-expan.	0.2	0.56	2.01	0.375	2
2 loop, at $d = 3$	0.35	0.49	2.18	0.045	1.3

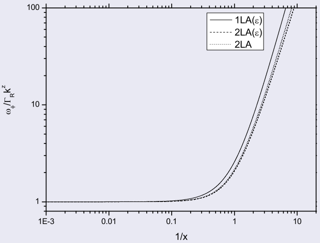
Strong dynamic scaling at $d = 3$ and $n = 1$ $\gamma^\star \neq 0$, finite
Model A dynamics at $d = 3$ and $n = 2$ $\gamma^\star = 0$

Characteristic frequencies of model C

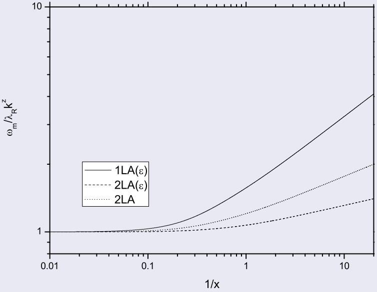
$$\omega_i^{-1}(k,\xi) = C_i(\xi, k, \omega = 0)/2C_i^{st}(k,\xi).$$

$$f_{OP}(x) = (1+\frac{1}{x^2})^{z_{OP}}\left(1-\frac{\zeta_\Gamma(w)}{2}\left(1+a_{OP}(x,w)\right)\right). \qquad f_{CD}(x) = \left(1+\frac{1}{x^2}\right)^{z_{CD}(w)-2}\left(1-\frac{\zeta_\lambda(w)}{2}\left(1-a_{CD}(x,w)\right)\right)$$

OP



CD

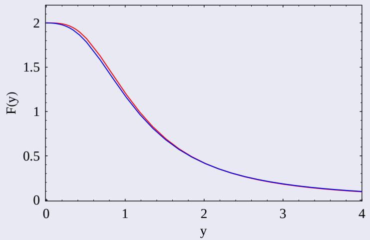


OP Shape functions

$$\mathcal{F}_{OP} = \frac{2}{y^2+1}\left(1 + \frac{\zeta_\Gamma(w)}{2(y^2+1)}\Big\{ + (y^2-1)A(x,y,w) - 2yB(x,y,w)\Big\}\right)$$

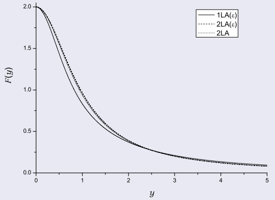
Same one loop expressions A and B for model C and E

Model E: at T_c



Janssen, Z. Phys. B 26, 127 (1977)

model C: at T_c



Dudka, F., Moser, unpublished (2008)

Amplitude ratio R

$$R = \lim_{\xi\to\infty}\lim_{k\to 0}\left[\frac{\omega_{CD}(k,\xi)}{\omega_{OP}(k,\xi)(k\xi)^2}\right] = \frac{1}{w^\star}\left[1+\frac{\gamma^{\star 2}}{2}\left(\frac{n}{2}+\frac{w^{\star 2}}{1+w^\star}\ln\frac{w^\star}{1+w^\star}\right)\right]$$

$n=1,\ d=3$	$\gamma^{\star 2}$	$w^\star$	$z=2+\alpha/\nu$	ω_w	n_c	R
1 loop, ϵ-expan.	2/3	1	2.33	1/6	2	1.05
2 loop, ϵ-expan.	0.2	0.56	2.01	0.375	2	1.79
2 loop, at $d=3$	0.35	0.49	2.18	0.045	1.3	2.04

Dudka, F., Moser unpublished (2008)

Fixed point values $i^\star$, dynamical exponent z, dynamical transient exponent ω_w, and amplitude ratio R.

For $n > n_c = 1.3$ the strong scaling fixed point is unstable.