Exclusive QED radiative corrections in NLO renormalization group

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Outline

- RG approach for large logs in QED
- NLO QED version of the factorization theorem
- Ansatz for the master formula describing exclusive observables in $\mathcal{O}\left(\alpha^2 L^1\right)$
- Particular second order contributions:
  - 2-loop Soft + Virtual
  - 2 Hard photons
  - 1 Hard photon $\otimes$ 1-loop Soft + Virtual
  - Pairs
- Outlook

RG for Large Logs in Radiative Corrections

RG is a very powerful approach based on the principle of scale invariance

In high energy physics RG helps to study the dependence of observable results on the energy scale

Evolution equations for Large Logs of the energy scale were first derived in early '70 for scalar QED. And immediately extended for the QCD case, known now as DGLAP. First application of the method for spinor QED in the LLA was made by Kuraev and Fadin only in '85. Why?

- methods of equivalent electrons (γ) did the job
- $\alpha_{QED} \ll \alpha_{QCD}$
- difference in the degree of inclusiveness

N.B. Energy scale and light mass enter into the evolution equations as initial data

QED Factorization Theorem (I)

The QCD factorization theorem can be adopted for the QED NLO case *e.g.* for Bhabha scattering:

$$\mathrm{d}\sigma = \int_{\bar{z}_1}^{1} \mathrm{d}z_1 \int_{\bar{z}_2}^{1} \mathrm{d}z_2 \sum_{a,b,c,d=e^{\pm},\gamma} \mathcal{D}^{\mathrm{str}}_{ae}(z_1) \mathcal{D}^{\mathrm{str}}_{be}(z_2) \Big(\mathrm{d}\sigma^{(0)}(z_1,z_2)$$

$$+ \quad \mathrm{d}\bar{\sigma}^{(1)}(z_1,z_2) + \mathcal{O}\left(\alpha^2 L^0\right) \Big) \int_{\bar{y}_1}^{1} \frac{\mathrm{d}y_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{\mathrm{d}y_2}{Y_2} \mathcal{D}^{\mathrm{frg}}_{ec}\Big(\frac{y_1}{Y_1}\Big) \mathcal{D}^{\mathrm{frg}}_{ed}\Big(\frac{y_2}{Y_2}\Big),$$

where $\sigma^{(0)}(a+b \to c+d)$ is the Born-level partonic cross section, $\bar{\sigma}^{(1)}$ is the $\overline{\mathrm{MS}}$ subtracted $\mathcal{O}\left(\alpha\right)$ contribution,

$$\mathcal{D}^{\mathrm{str,frg}}_{e^-e^-}(z) = \delta(1-z) + \frac{\alpha}{2\pi} d^{(1)}(z,\mu_0,m_e) + \frac{\alpha}{2\pi} L P^{(0)}(z)$$

$$+ \left(\frac{\alpha}{2\pi}\right)^2 \Bigg(\frac{1}{2} L^2 P^{(0)} \otimes P^{(0)}(z) + L P^{(0)} \otimes d^{(1)}(z,\mu_0,m_e)$$

$$+ \quad L P^{(1,\gamma)\mathrm{str,frg}}_{e^-e^-}(z) + L P^{(1,\mathrm{pair})\mathrm{str,frg}}_{e^-e^-}(z) \Bigg) + \mathcal{O}\left(\alpha^2 L^0, \alpha^3\right)$$

QED Factorization Theorem (II)

Applications of RG approach to compute higher order QED LLA corrections is a standard issue for e^+e^- colliders, DIS and other

Exponentiated and explicit formulae for electron structure functions in LLA are known up to $\mathcal{O}\left(\alpha^5\right)$ [Przybycien 1992; A.A. 1999].

Direct application of the NLO formulae is possible only for exclusive in photon emission angle processes:
1st: Berends *et al.* 1987 (ISR in e^+e^- annihilation)
2nd: A.A. & K.Melnikov 15 year later
(FSR in muon decay)

QED Master Formula Ansatz

Using slicing in the photon energy, we cast the corrected cross section in the form

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \mathrm{d}\sigma^{(1)}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(1)}_{\mathrm{H}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} + \mathrm{d}\sigma^{(3)LO} + \ldots$$

For many observables we need to know the complete kinematics including hard photon angles, which are integrated over in the QCD-like formula.
Let us decompose the $\mathcal{O}\left(\alpha^2 L^{2,1}\right)$ hard radiation contribution

$$\mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} = \mathrm{d}\sigma^{(2)}_{\mathrm{HH(coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{HH(s-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(n-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(coll)}}$$

where slicing in the photon emission angle is applied:

- **“coll” means collinear photon(s) with $\vartheta_\gamma < \theta_0 \ll 1$,**
- **“n-coll” means non-collinear photon with $\vartheta_\gamma > \theta_0$,**
- **“HH(s-coll)” means semi-collinear kinematics, *i.e.* one collinear photon and one non-collinear**

Particular NLO contributions (1)

The combined effect of virtual corrections and soft photon emission ones within the $\mathcal{O}\left(\alpha^2 L^1\right)$ can be obtained by convolution of the structure functions with the kernel cross section according to the general factorization theorem. Here one requires only one non-trivial convolution

$$\frac{\alpha}{2\pi} L \int\limits_{1-\Delta}^{1} \mathrm{d}z \int\limits_{0}^{1} \frac{\mathrm{d}x}{x} P^{(0)}\left(\frac{z}{x}\right) \mathrm{d}\bar{\sigma}^{(1)}(x)$$

This integral can be found for any relevant process as demonstrated in [A.A., E. Scherbakova, ZhETF Pis'ma 2006] for the large-angle Bhabha case by getting $\mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}}$ in agreement with the complete $\mathcal{O}\left(\alpha^2\right)$ calculation.

QED Collinear Radiation Factors in NLO (1a)

Recent results (crucial for ILC):
2-loop virtual QED RC to Bhabha scattering

- **NLO QED in $\mathcal{O}(\alpha^2 L^1)$: [1] E. Glover *et al.* 2001; A.A. & E. Scherbakova 2006**
- **Massless case in $\mathcal{O}(\alpha^2)$: [2] Z. Bern *et al.* 2001**
- **Complete $\mathcal{O}(\alpha^2)$ omitting only $\mathcal{O}(\alpha^2 m_e^2/s)$: [3] A. Penin 2005**

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- Matching:
  [1] + [2] = [3] by T. Becher & K. Melnikov 2007

Particular NLO contributions (2)

Emission of two hard photons, HH, can be considered in three regions:

1. non-collinear: $\theta_{1,2} > \vartheta_0$
   suited for Monte Carlo simulation
2. semi-collinear: $\theta_1 > \vartheta_0$ and $\theta_2 < \vartheta_0$
   in $\mathcal{O}\left(\alpha^2 L\right)$ has factorized form $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \otimes R_{\mathrm{H}}^{\mathrm{ISR,FSR}}(z)$
3. collinear: $\theta_{1,2} < \vartheta_0$ is described by the HH radiation factor convoluted with the Born

Emission of one hard photon in $\mathcal{O}\left(\alpha^2 L\right)$ can be sliced into two domains:

1. non-collinear: $\theta_\gamma > \vartheta_0$ as a product of two factors
   $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \times \delta_{\mathrm{Soft+Virt}}^{\mathrm{LO}}$
2. collinear: $\theta_\gamma < \vartheta_0$ is described by the collinear NLO H radiation factor (see below)

QED Collinear Radiation Factors in NLO (1)

A. A., E. Scherbakova, **Phys. Lett. B 660 (2008) 37**

$$\mathrm{d}\sigma[a(p_1)+b(p_2)\to c(q_1)+d(q_2)+\gamma(k\sim(1-z)p_1)]$$
$$=\mathrm{d}\hat{\sigma}[a(zp_1)+b(p_2)\to c(q_1)+d(q_2)]\otimes R_{\mathrm{H}}^{\mathrm{ISR}}(z)$$

Emission of collinear photons in FSR and ISR with conditions

$$\vartheta_\gamma<\vartheta_0,\qquad \frac{m}{E}\ll\vartheta_0\ll 1,\qquad l_0=\ln\frac{\vartheta_0^2}{4},\qquad \frac{E_\gamma}{E}>\Delta\ll 1$$

In $\mathcal{O}(\alpha)$ the result is well known:

$$R_{\mathrm{H}}^{\mathrm{ISR}}(z)=\frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{4E^2}{m^2}-1+l_0\right)+1-z+\mathcal{O}\left(\frac{m^2}{E^2}\right)+\mathcal{O}\left(\vartheta_0^2\right)\right]$$

QED Collinear Radiation Factors in NLO (2)

Emission of two collinear photons (HH) in the same direction is described by a one-fold integral of results from [A.A. et al., Nucl. Phys. B 483 (1997) 83]:

$$R_{\mathrm{HH}}^{\mathrm{ISR}}(z) = \left(\frac{\alpha}{2\pi}\right)^2 L \Bigg\{ (L + 2l_0) \left(\frac{1+z^2}{1-z} (2\ln(1-z) - 2\ln\Delta - \ln z) \right.$$
$$\left. + \frac{1+z}{2}\ln z - 1 + z \right) + \frac{1+z^2}{1-z}\left(\ln^2 z + 2\ln z - 4\ln(1-z) + 4\ln\Delta \right)$$
$$+ (1-z)\left(2\ln(1-z) - 2\ln\Delta - \ln z + 3 \right) + \frac{1+z}{2}\ln^2 z \Bigg\}$$

FSR factor is restored with help of the Gribov-Lipatov relation generalized for the collinear emission case:

$$R_{\mathrm{HH}}^{\mathrm{FSR}}(z) = -z R_{\mathrm{HH}}^{\mathrm{ISR}}\left(\frac{1}{z}\right)\Bigg|_{\ln\Delta \to \ln\Delta - \ln z;\ l_0 \to l_0 + 2\ln z}$$

QED Collinear Radiation Factors in NLO (3)

Emission of one collinear hard photon accompanied by one-loop soft and virtual correction (H(S+V)) is received using the NLO QED splitting functions

$$R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z) \otimes \mathrm{d}\hat{\sigma}(z) = \delta^{(1)}_{(\mathrm{S+V})} R^{\mathrm{ISR}}_{\mathrm{H}}(z) \otimes \mathrm{d}\sigma^{(0)}(z)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 L \Bigg[2\frac{1+z^2}{1-z}\left(\mathrm{Li}_2\,(1-z) - \ln(1-z)\ln z\right)$$
$$-(1+z)\ln^2 z + (1-z)\ln z + z \Bigg] \otimes \mathrm{d}\sigma^{(0)}(z),$$

$$\delta^{(1)}_{(\mathrm{S+V})} = \frac{\mathrm{d}\sigma^{(1)}_{\mathrm{Soft}} + \mathrm{d}\sigma^{(1)}_{\mathrm{Virt}}}{\mathrm{d}\sigma^{(0)}}\,,$$

where $\sigma^{(0)}(z)$ is the boosted Born cross section, and $\delta^{(1)}_{(\mathrm{S+V})}$ is the relative $\mathcal{O}\left(\alpha\right)$ Soft + Virtual radiative correction with $E^{\mathrm{Soft}}_{\gamma} < \Delta E$. The corresponding FSR factor is received again using the Gribov-Lipatov relation.

Pair Corrections

Leptonic and hadronic pair corrections are important for a number of precision observables. Exclusive treatment here is of ultimate importance. Monte Carlo has to be used for real or for hard pairs, then soft and virtual ones can be treated analytically (semi-analytically for the hadronic case).

Singlet and non-singlet NLO pair contributions in $\mathcal{O}\left(\alpha^2 L\right)$ to inclusive observables can be described within the QCD-like factorization approach.

But if we have a MC for hard pairs, we can extract analytically the soft+virtual part, so that

$$\mathrm{d}\sigma^{(2)}_{\mathrm{pair}} = \mathrm{d}\sigma^{(2)MC}_{\mathrm{H\ \ pair}} + \mathrm{d}\sigma^{(0)} \times \delta^{(2)}_{\mathrm{S+V\ \ pair}}$$

Approximations

- In $\mathcal{O}\left(\alpha^1\right)$ radiation factor terms of the order $\mathcal{O}\left(m^2/E^2\right)$ and $\mathcal{O}\left(\theta_0^2\right)$ can be restored if required
- The collinear cone can be transformed into any other form
- $\mathcal{O}\left(\alpha^2 L^0\right)$ terms in particular kinematical domains are process-dependent and can be added if known
- Negatively weighted events within this approach are possible but not numerous

Outlook

- The ansatz for the treatment of $\mathcal{O}\left(\alpha^2 L^1\right)$ QED radiative corrections to exclusive observables is described
- The ansatz is suited for analytical calculations and MC simulations
- Many processes can be treated in this way
- $\mathcal{O}\left(\alpha^2 L^0\right)$ contributions can be put into the same structure

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- Many processes can be treated in this way
- $\mathcal{O}\left(\alpha^2 L^0\right)$ contributions can be put into the same structure
- Renormalization group approach is for sure one of the main tools in HEP