Axial anomaly and BABAR data

Y.N. Klopot[1] , A.G. Oganesian[2], O.V. Teryaev[1]

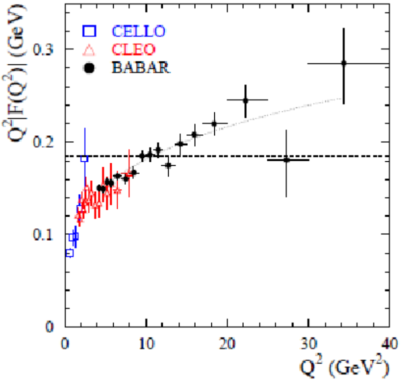
[1]Bogoliubov Laboratory of Theoretical Physics, JINR,
Dubna, Russia

[2]Institute of Theoretical and Experimental Physics,
Moscow, Russia

International Workshop "Bogoliubov readings"
JINR
Dubna

Outline

- The last data of BaBar collaboration [Phys. Rev. D 80, 052002 (2009)] show unexpectedly large values of photon-pion transition form factor at large Q^2 (violation of QCD factorization!)



- [Dolgov, Zakharov, Nucl. Phys. B27,1971]
- Dispersive representation of anomaly: exact sun rules (ASR).
  Opportunity to find high precision results and relations.
  Pion width [Ioffe, Oganesian, Phys.Lett. B647,2007] for real photons case with very high accuracy.
- !! ASR is valid for virtual photons also [Horejsi, Teryaev, Z.Phys.C65, 1995]
  It will be shown that ASR is actually saturated only by infinite number of resonances- anomaly reveals itself as a collective effect of meson spectrum.
- Allows: ... and transition formfactor pion->2γ
  Exactness af ASR -significant
- The data CELLO, CLEO,(consistent to LCSR with usual DA)
- but BaBar collaboration [Phys. Rev. D 80, 052002 (2009)] show unexpectedly large values of photon-pion transition form factor at large Q^2 (violation of QCD factorization!)
  It is impossible to explain BaBar data on $F_{\pi\gamma}(Q^2)$ at large Q^2 by use of usual form of pion distribution [Khodjamirian 0909.2154, Mikhailov, Stefanis, Nucl. Phys. B821 2009]

- [Dolgov, Zakharov, Nucl. Phys. B27,1971]
- Dispersive representation of anomaly: exact sun rules (ASR). Opportunity to find high precision results and relations. Pion width [Ioffe, Oganesian, Phys.Lett. B647,2007] for real photons case with very high accuracy.
- !! ASR is valid for virtual photons also [Horejsi, Teryaev, Z.Phys.C65, 1995] It will be shown that ASR is actually saturated only by infinite number of resonances- anomaly reveals itself as a collective effect of meson spectrum.
- Allows: ... and transition formfactor pion->2γ Exactness af ASR -significant
- The data CELLO, CLEO,(consistent to LCSR with usual DA)
- but BaBar collaboration [Phys. Rev. D 80, 052002 (2009)] show unexpectedly large values of photon-pion transition form factor at large Q^2 (violation of QCD factorization!) It is impossible to explain BaBar data on $F_{\pi\gamma}(Q^2)$ at large Q^2 by use of usual form of pion distribution [Khodjamirian 0909.2154, Mikhailov, Stefanis, Nucl. Phys. B821 2009]

- There were suggested several models explaining this contradiction [Radyushkin Phys.Rev.D80:094009(2009),Dorokhov1003.4693,Mikhailov&Pimikov&Stef 1006.2936]
  [Mikhailov, Pimikov, Stefanis 1006.2936] give some arguments against this enhancement.
- Let us find out what can be learned about the meson transition form factors behaviour at large Q^2 in QCD beyond the factorization from the anomaly sum rule.
- Our (non-perturbative) QCD method does not imply QCD factorization and is valid even if QCD factorization is broken. By use of the axial anomaly (in the dispersive approach) we can get the exact relations between possible corrections to lower states and continuum
- We will show that even small continuum corrections provides a possibility of relatively large corrections to the lower states.

- There were suggested several models explaining this contradiction [Radyushkin Phys.Rev.D80:094009(2009),Dorokhov1003.4693,Mikhailov&Pimikov&Stef 1006.2936]
  [Mikhailov, Pimikov, Stefanis 1006.2936] give some arguments against this enhancement.
- Let us find out what can be learned about the meson transition form factors behaviour at large Q^2 in QCD beyond the factorization from the anomaly sum rule.
- Our (non-perturbative) QCD method does not imply QCD factorization and is valid even if QCD factorization is broken. By use of the axial anomaly (in the dispersive approach) we can get the exact relations between possible corrections to lower states and continuum
- We will show that even small continuum corrections provides a possibility of relatively large corrections to the lower states.

- There were suggested several models explaining this contradiction [Radyushkin Phys.Rev.D80:094009(2009),Dorokhov1003.4693,Mikhailov&Pimikov&Stef 1006.2936]
  [Mikhailov, Pimikov, Stefanis 1006.2936] give some arguments against this enhancement.
- Let us find out what can be learned about the meson transition form factors behaviour at large Q^2 in QCD beyond the factorization from the anomaly sum rule.
- Our (non-perturbative) QCD method does not imply QCD factorization and is valid even if QCD factorization is broken. By use of the axial anomaly (in the dispersive approach) we can get the exact relations between possible corrections to lower states and continuum
- We will show that even small continuum corrections provides a possibility of relatively large corrections to the lower states.

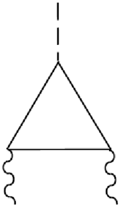
Anomaly sum rule

The VVA amplitude

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0|T\{J^5_\alpha(0)J_\mu(x)J_\nu(y)\}|0\rangle, \qquad (1)$$

where $J^5_\alpha = (\bar{u}\gamma_5\gamma_\alpha u - \bar{d}\gamma_5\gamma_\alpha d)$

$$\langle 0|J^5_\alpha(0)|\pi^0(p)\rangle = i\sqrt{2}p_\alpha f_\pi, \qquad (2)$$



The VVA triangle graph amplitude can be presented as a tensor decomposition

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\,\varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\,\varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+ F_5\,k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\,q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned} \quad (3)$$

We consider the following case:

$$k^2 = 0, Q^2 = -q^2 \quad (4)$$

Dispersive approach to axial anomaly leads to [Hořejší1985,Hořejší&Teryaev1995]:

$$\int_{4m^2}^{\infty} A_3(s; Q^2, m^2)ds = \frac{1}{2\pi} \tag{5}$$

$$A_3 \equiv Im(F_3 - F_6)/2$$

(in the paper $F_3 = -F_6$ is obtained from direct calculation perturbative calculation)

- Holds for any Q^2 and any m^2.
- Does not have α_s corrections.
- Does not have nonperturbative corrections ('t Hooft's principle).

Transition form factors of mesons

The form factor $\pi^0 \to \gamma^* \gamma$ is defined from the matrix element:

$$\int d^4 x e^{ikx} \langle \pi^0(q)| T\{J_\mu(x) J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{\pi\gamma\gamma}, \quad (6)$$

The VVA amplitude

$$T_{\alpha\mu\nu}(k,q) = \int d^4 x d^4 y e^{(ikx+iqy)} \langle 0| T\{J^5_\alpha(0) J_\mu(x) J_\nu(y)\}|0\rangle, \quad (7)$$

where $J^5_\alpha = (\bar{u}\gamma_5\gamma_\alpha u - \bar{d}\gamma_5\gamma_\alpha d)$

$$\langle 0|J^5_\alpha(0)|\pi^0(p)\rangle = i\sqrt{2} p_\alpha f_\pi, \quad (8)$$

Three-point correlation function $T_{\alpha\mu\nu}(k,q)$ has pion and higher states contributions:

$$T_{\alpha\mu\nu}(k,q) = \frac{i\sqrt{2}f_\pi}{p^2 - m_\pi^2} p_\alpha k^\rho q^\sigma \epsilon_{\mu\nu\rho\sigma} F_{\pi\gamma\gamma} + (higher\ states). \quad (9)$$

Using the kinematical identities

$$\delta_{\alpha\beta}\epsilon_{\sigma\mu\nu\tau} - \delta_{\alpha\sigma}\epsilon_{\beta\mu\nu\tau} + \delta_{\alpha\mu}\epsilon_{\beta\sigma\nu\tau} - \delta_{\alpha\nu}\epsilon_{\beta\sigma\mu\nu} + \delta_{\alpha\tau}\epsilon_{\beta\sigma\mu\nu} = 0\ , \quad (10)$$

we can single out the pion contribution to $\frac{1}{2}(F_3 - F_6)$ (imaginary part is taken w.r.t. p^2):

$$\frac{1}{2} Im(F_3 - F_6) = \sqrt{2} f_\pi \pi F_{\pi\gamma}(Q^2)\delta(s - m_\pi^2)\ , \quad (11)$$

- $\underline{Q^2 = 0:}$ pion contribution saturates ASR:

$$F_{\pi\gamma\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \tag{12}$$

- $\underline{Q^2 \neq 0:}$ Factorization approach to pQCD [Lepage&Brodsky1980]:

$$F_{\pi\gamma\gamma}(Q^2) = \frac{\sqrt{2}f_\pi}{3Q^2}\int_0^1 dx \frac{\varphi_\pi\left(x, Q^2\right)}{x} + \mathcal{O}(1/Q^4), \tag{13}$$

Asymptote at large Q^2: [Efremov&Radyushkin1980]

$$\varphi_\pi^{\mathrm{asymp}}(x) = 6x(1-x)$$

$$F_{\pi\gamma\gamma}^{\mathrm{asymp}}(Q^2) = \frac{\sqrt{2}f_\pi}{Q^2} + \mathcal{O}(1/Q^4). \tag{14}$$

Transition form factors of mesons

- At $Q^2 \neq 0$ anomaly sum rule (37) cannot be saturated by pion contribution due to $1/Q^2$ behavior, we need to consider higher states.
- The higher mass pseudoscalar states have the same behavior and suppressed by the factor m_π^2/m_{res}^2 as follows from the PCAC (since $\partial_\mu J^3_\mu$ should vanish in the chiral limit).
- The contribution of longitudinally polarized a_1 is given by the similar equation to (14) (at large Q^2) [Ball,Braun, Phys. Rev.D54, 1996b]and transversally polarized a_1 contribution decrease at least not slower. Actually, the same is true for all the higher mesons.
- for the case $Q^2 \neq 0$ anomaly relation (37) cannot be explained in terms of any finite number of mesons due to the fact that all transition form factors are decreasing functions.

Transition form factors of mesons

- At $Q^2 \neq 0$ only *infinite* number of higher states can saturate anomaly sum rule. $\Rrightarrow$
  Axial anomaly is a genuine collective effect of meson spectrum! in contrast with the case of two real photons $Q^2 = 0$, where the anomaly sum rule is saturated by pion contribution only.
- this conclusion does not depend on any choice of meson distribution amplitudes (even flat).
- A way to account higher resonances- use quark-hadron duality

Transition form factors of mesons

- At $Q^2 \neq 0$ only *infinite* number of higher states can saturate anomaly sum rule. $\Rrightarrow$
Axial anomaly is a genuine collective effect of meson spectrum! in contrast with the case of two real photons $Q^2 = 0$, where the anomaly sum rule is saturated by pion contribution only.
- this conclusion does not depend on any choice of meson distribution amplitudes (even flat).
- A way to account higher resonances- use quark-hadron duality

π+continuum

$$\int_{4m^2}^{\infty} A_3(s;Q^2)ds = \frac{1}{2\pi}$$

π+continuum.

$$A_3\left(s,Q^2\right) = \sqrt{2}\pi f_\pi \delta(s-m_\pi^2) F_{\pi\gamma\gamma}\left(Q^2,0\right) + A_3^{QCD}\theta(s-s_0), \qquad (15)$$

Continuum contribution from one-loop PT calculation:

$$A_3^{QCD}(s,Q^2,0) = \frac{Q^2}{(s+Q^2)^2}. \qquad (16)$$

Anomaly sum rule:

$$\frac{1}{2\pi} = \sqrt{2}\pi f_\pi F_{\pi\gamma\gamma}(Q^2) + \frac{1}{2\pi}\int_{s_0}^{\infty} ds \frac{Q^2}{(s+Q^2)^2}, \qquad (17)$$

$$F_{\pi\gamma\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_0}{s_0+Q^2}, \qquad (18)$$

[Brodsky&Lepage1981] ($s_0 = 4\pi^2 f_\pi^2$):

$$F^{\mathrm{BL}}_{\pi\gamma\gamma}(Q^2,0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{1}{1+Q^2/(4\pi^2 f_\pi^2)}. \qquad (19)$$

π+continuum

$$\int_{4m^2}^{\infty} A_3(s;Q^2)ds = \frac{1}{2\pi}$$

π+continuum.

$$A_3\left(s,Q^2\right) = \sqrt{2}\pi f_\pi \delta(s-m_\pi^2) F_{\pi\gamma\gamma}\left(Q^2,0\right) + A_3^{QCD}\theta(s-s_0), \qquad (15)$$

Continuum contribution from one-loop PT calculation:

$$A_3^{QCD}(s,Q^2,0) = \frac{Q^2}{(s+Q^2)^2}. \qquad (16)$$

Anomaly sum rule:

$$\frac{1}{2\pi} = \sqrt{2}\pi f_\pi F_{\pi\gamma\gamma}(Q^2) + \frac{1}{2\pi}\int_{s_0}^{\infty} ds \frac{Q^2}{(s+Q^2)^2}, \qquad (17)$$

$$F_{\pi\gamma\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_0}{s_0+Q^2}, \qquad (18)$$

[Brodsky&Lepage1981] ($s_0 = 4\pi^2 f_\pi^2$):

$$F^{\mathrm{BL}}_{\pi\gamma\gamma}(Q^2,0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{1}{1+Q^2/\left(4\pi^2 f_\pi^2\right)}. \qquad (19)$$

$\pi + a_1 +$continuum

$$\frac{1}{2\pi} = \sqrt{2}\pi f_\pi F_{\pi\gamma\gamma}(Q^2) + I_{a_1} + \frac{1}{2\pi}\int_{s_1}^{\infty} ds \frac{Q^2}{(s+Q^2)^2} \qquad (20)$$

Estimation for a_1 contribution to sum rule:

$$I_{a_1} = \frac{1}{2\pi} Q^2 \frac{s_1 - s_0}{(s_1+Q^2)(s_0+Q^2)} \qquad (21)$$

(correct asymptotes at small and large Q^2)

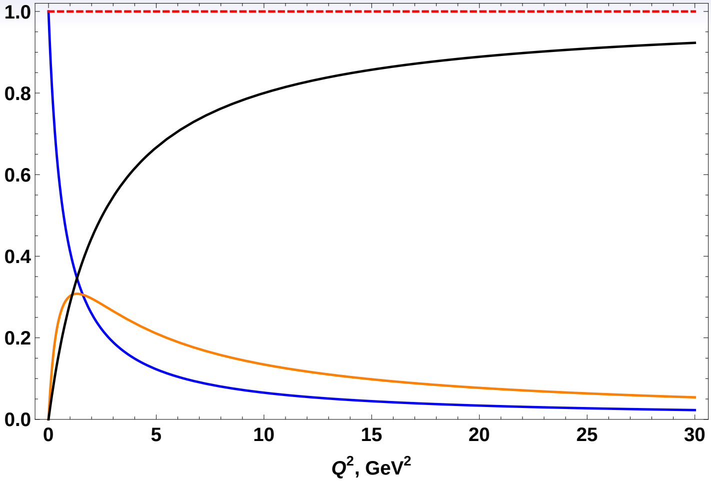


Figure: Relative contributions π^0 (blue curve), a_1 (orange curve) mesons and continuum (black curve) to ASR (intervals of duality are $0.7, 1.8$ and continuum threshold is 2.5 GeV^2)

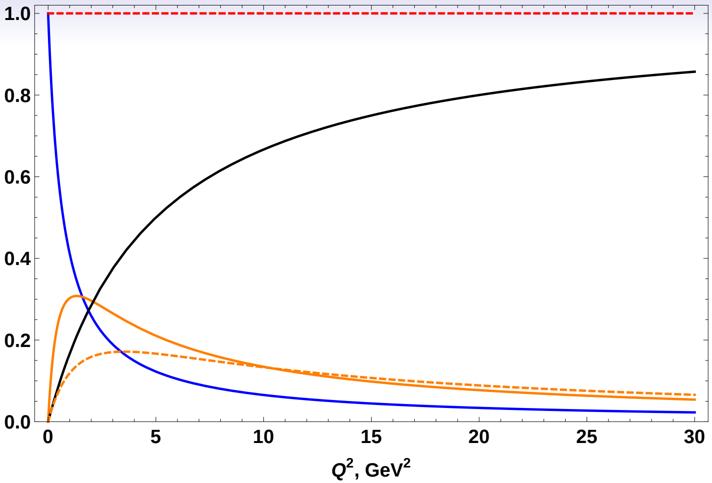


Figure: Relative contributions of π^0 (blue curve), $a_1(1260)$ (orange curve) and $a_1(1640)$(orange dashed curve) mesons, intervals of duality are 0.7 GeV^2, 1.8 GeV^2 and 2.5 GeV^2 respectively, and continuum (black curve), continuum threshold is $s_2 = 5.0\ GeV^2$

Corrections interplay and experimental data

- The anomaly sum rule

$$\frac{1}{2\pi} = \int_0^\infty A_3(s;Q^2)ds = I_\pi + I_{a_1} + I_{cont}$$

is the exact relation (does not have any corrections).

- The continuum contribution

$$I_{cont} = \int_{s_0}^\infty A_3(s;Q^2)ds$$

may have perturbative as well as power corrections.

- In order to preserve the sum rule there must be compensating corrections to the lower states.

Corrections interplay and experimental data

- Model "π+continuum".

  The contributions of pion and continuum read:

$$I_\pi = \sqrt{2} f_\pi F_{\pi\gamma\gamma*}(Q^2) = \frac{1}{2\pi} \frac{s_0}{s_0 + Q^2},$$

$$I_{cont} = \frac{1}{2\pi} \frac{Q^2}{s_0 + Q^2}.$$

  If the corrections to pion and continuum are δI_π and δI_{cont}:

$$I_\pi = I_\pi^0 + \delta I_\pi,$$

$$I_{cont} = I_{cont}^0 + \delta I_{cont}$$

  then the ratio of relative corrections to continuum and pion is

$$\left| \frac{\delta I_{cont} / I_{cont}^0}{\delta I_\pi / I_\pi^0} \right| = \frac{s_0}{Q^2}.$$

Corrections interplay and experimental data

- Model "π+continuum".

  The contributions of pion and continuum read:

$$I_\pi = \sqrt{2} f_\pi F_{\pi\gamma\gamma*}(Q^2) = \frac{1}{2\pi}\frac{s_0}{s_0 + Q^2},$$

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0 + Q^2}.$$

  If the corrections to pion and continuum are δI_π and δI_{cont}:

$$I_\pi = I_\pi^0 + \delta I_\pi,$$

$$I_{cont} = I_{cont}^0 + \delta I_{cont}$$

  then the ratio of relative corrections to continuum and pion is

$$\left|\frac{\delta I_{cont}/I_{cont}^0}{\delta I_\pi/I_\pi^0}\right| = \frac{s_0}{Q^2}.$$

Corrections interplay and experimental data

- For instance, for $Q^2 = 20\ GeV^2$, $s_0 = 0.7\ GeV^2$ we get

$$\left| \frac{\delta I_{cont}/I^0_{cont}}{\delta I_\pi / I^0_\pi} \right| \simeq 0.03 \,. \qquad (22)$$

- Continuum correction can be suppressed (relatively to main term) both parametrically and numerically, but lead to pion formfactor corrections, which are not suppressed (neither parametrically nor numerically)!!!

Corrections interplay and experimental data

- To illustrate our conclusion, we assume the correction to continuum at large Q^2 is $\delta I_{cont} = -cs_0 \frac{\ln(Q^2/s_0)+b}{Q^2}$. This correction preserves asymptote of continuum contribution at large Q^2. Contributions of pion and continuum to ASR then have the following explicit form:

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}\,, \tag{23}$$

$$I_{\pi} = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}\,. \tag{24}$$

- Relying on the BaBar data we can estimate the relative corrections to continuum. For $s_0 = 0.7\ GeV^2$ one found:

$$b = -2.74, \quad c = 0.045. \tag{25}$$

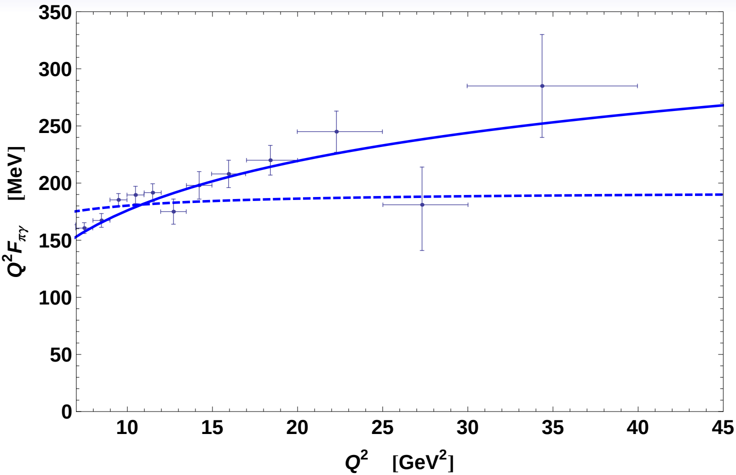

350
300
250
200
150
100
50
0
10
15
20
25
30
35
40
45
$Q^2F_{\pi\gamma}$ [MeV]
Q^2 [GeV^2]

Figure:

.

Summary

- When both photons are real ($Q^2 = 0$) the ASR saturates by pion contribution only. However, when one of the photons is virtual ($Q^2 \neq 0$) we immediately get different situation: ASR can be saturated only with a full meson spectrum (any finite number of mesons cannot saturate the ASR). The axial anomaly is a collective effect of meson spectrum.
- The exactness of the ASR reveals the relation between corrections to continuum and to lower mass states contributions. Relatively small corrections to continuum preserving it's asymptotes may change the pion form factor asymptote at large Q^2.
- The last experimental data on pion transition form factor $F_{\pi\gamma\gamma^*}$ at large Q^2 allows us to get the estimation for the possible continuum corrections.

Thank you for your attention!

Backup

Bose symmetry implies:

$$\begin{aligned} F_1(k,p) &= -F_2(p,k), \\ F_3(k,p) &= -F_6(p,k), \\ F_4(k,p) &= -F_5(p,k). \end{aligned} \tag{26}$$

One can show also that

$$F_6(k,p) = -F_3(k,p) \tag{27}$$

vector Ward identities

$$k^\mu T_{\alpha\mu\nu} = 0, \qquad p^\nu T_{\alpha\mu\nu} = 0 \tag{28}$$

In terms of formfactors, the identities (28) read

$$\begin{aligned} F_1 &= k.p\, F_3 + p^2\, F_4 \\ F_2 &= k^2\, F_5 + k.p\, F_6 \end{aligned} \tag{29}$$

Anomalous axial-vector Ward identity for the amplitude (3) is [Adler'69]:

$$q^\alpha T_{\alpha\mu\nu}(k,p) = 2m T_{\mu\nu}(k,p) + \frac{1}{2\pi^2}\varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma \tag{30}$$

Backup

$$T_{\mu\nu}(k,p) = G\ \varepsilon_{\mu\nu\rho\sigma} k^\rho p^\sigma \qquad (31)$$

where G is the relevant form factor. In terms of form factors, eq.(30) reads

$$F_2 - F_1 = 2mG + \frac{1}{2\pi^2} \qquad (32)$$

For the form factors F_3, F_4 and G one may write unsubtracted dispersion relations

$$F_j(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t-q^2} dt, \qquad j = 3,4 \qquad (33)$$

$$G(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{B(t)}{t-q^2} dt$$

Backup

From (27) and (29) it is easy to see that for the considered kinematical configuration one has

$$F_2 - F_1 = (p^2 - q^2)F_3 - p^2 F_4 \tag{34}$$

Using now (33) and taking into account that the imaginary parts of the relevant formfactors satisfy non-anomalous Ward identities, in particular

$$(p^2 - t)A_3(t) - p^2 A_4(t) = 2mB(t) \tag{35}$$

one gets finally

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \tag{36}$$

Comparing eq.(36) with (32) one may thus observe that the occurrence of the axial anomaly is equivalent to a "sum rule"

$$\int_{4m^2}^{\infty} A_3(t; p^2, m^2)dt = \frac{1}{2\pi} \tag{37}$$

(which must hold for an arbitrary m and for any p^2 in the considered region).