OPE, heavy – quark mass, and decay constants of heavy mesons from QCD sum rules

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A new extraction of the decay constants of D, D_s, B, and B_s mesons from the two-point function of heavy-light pseudoscalar currents is presented. The main emphasis of this talk is laid on the uncertainties in these quantites, both related to the OPE for the relevant correlators and to the extraction procedures of the method of sum rules.

Based on W.Lucha, D.Melikhov, S.Simula "Decay constants of heavy pseudoscalar mesons from QCD sum rules" arXiv:1008.2698

A QCD sum-rule calculation of hadron parameters involves two steps:

I. Calculating the operator product expansion (OPE) series for a relevant correlator

For heavy-light currents, one observes a very strong dependence of the OPE for the correlator (and, consequently, of the extracted decay constant) on the heavy-quark mass used, i.e., on-shell (pole), or running $\overline{\text{MS}}$ mass.

We make use of the three-loop OPE for the correlator by Chetyrkin et al, reorganized in terms of $\overline{\text{MS}}$ mass, in which case OPE exhibits a reasonable convergence.

II. Extracting the parameters of the ground state by a numerical procedure

 NEW :

(a) Make use of the new more accurate duality relation based on Borel-parameter-dependent threshold.

Allows a more accurate extraction of the decay constants and provides realistic estimates of the intrinsic (systematic) errors — those related to the limited accuracy of sum-rule extraction procedures.

(b) Study the sensitivity of the extracted value of f_P to the OPE parameters (quark masses, condensates,...). The corresponding error is referred to as OPE uncertainty, or statistical error.

1. Basic object:

$$\Pi(p^2) = i \int dx e^{ipx} \langle 0|T\left(j_5(x)j_5^\dagger(0)\right)|0\rangle, \qquad j_5(x) = (m_Q + m)\bar{q}i\gamma_5 Q(x)$$

2. Calculate OPE series and its Borel transform ($p^2 \to \tau$):

Green functions in Minkowski space $\to$ evolution operator in Euclidean space, τ related to Euclidean time

$$\Pi(\tau) = \int\limits_{(m_Q+m_u)^2}^{\infty} e^{-s\tau}\rho_{\text{pert}}(s, \alpha, m_Q, \mu)\, ds + \Pi_{\text{power}}(\tau, m_Q, \mu),$$

- $\rho_{\text{pert}}(s, \mu) = \rho^{(0)}(s) + \frac{\alpha_s(\mu)}{\pi}\rho^{(1)}(s) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \rho^{(2)}(s) + \cdots$
- $\Pi_{\text{power}}(\tau, \mu)$ **– power expansion in τ in terms of condensates.**

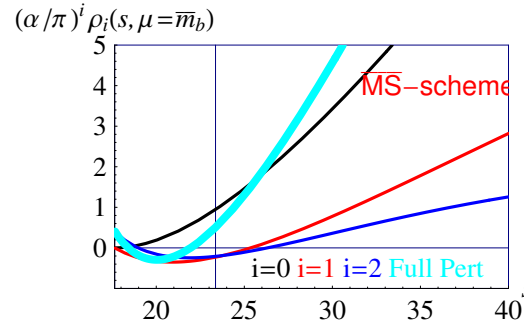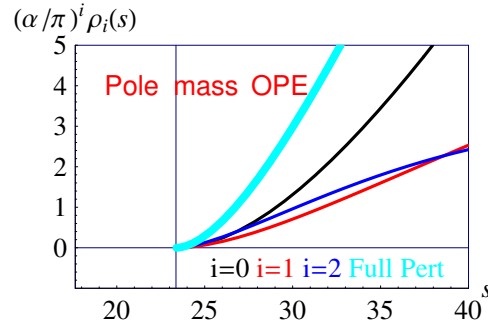### Quark – hadron duality assumption :

$$f_Q^2 M_Q^4 e^{-M_Q^2\tau} = \int\limits_{(m_Q+m_u)^2}^{s_{\text{eff}}} e^{-s\tau}\rho_{\text{pert}}(s, \alpha, m_Q, \mu)\, ds + \Pi_{\text{power}}(\tau, m_Q, \mu) \equiv \Pi_{\text{dual}}(\tau, \mu, s_{\text{eff}})$$

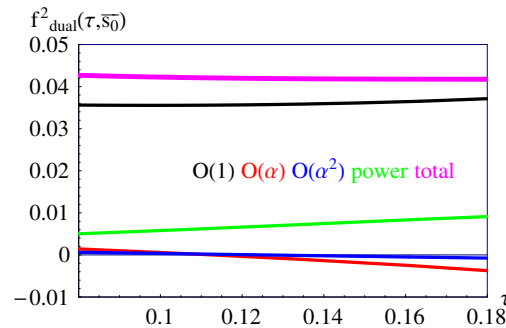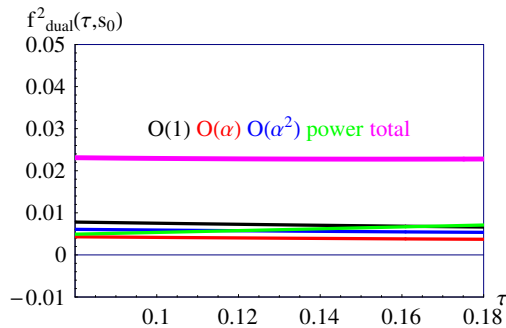# OPE : heavy – quark pole mass or running mass ?

Spectral densities

$$\rho(m_b, \alpha_s, s) \rightarrow \Pi(m_b, \alpha_s, \tau) \rightarrow \Pi(m_b(\overline{m}_b, \alpha_s), \alpha_s, \tau) \rightarrow \Pi(\overline{m}_b, \alpha_s, \tau) \rightarrow \rho(\overline{m}_b, \alpha_s, s)$$

To α_s^2-accuracy, $m_{b,pole} = 4.83$ GeV $\leftrightarrow \overline{m}_b(\overline{m}_b) = 4.20$ GeV

- **In pole mass scheme poor convergence of perturbative expansion**

- **In $\overline{\text{MS}}$ scheme the perturbative spectral density has negative region**

Extracted decay constant

- **Decay constant in pole mass shows NO hierarchy of perturbative contributions**

- **Decay constant in $\overline{\text{MS}}$-scheme shows such hierarchy. Numerically, f_P using pole mass $\ll f_P$ using $\overline{\text{MS}}$ mass.**

	OPE	mQ, GeV		fB, MeV
Aliev [1983]	$O(\alpha)$	pole : 4.8		130 (± 20 %)
Narison [2001]	$O(\alpha^2)$	pole : 4.7	$\overline{MS}$: 4.05	$203 \pm 23_{OPE}$
Jamin [2001]	$O(\alpha^2)$	pole : 4.83	$\overline{MS}$: 4.21 ± 0.05	$215 \pm 19_{OPE}$
Our results	$O(\alpha^2)$		$\overline{MS}$: 4.25 ± 0.025	$193 \pm 13_{OPE} \pm 4_{syst}$

Quark – hadron duality assumption :

$$f_Q^2 M_Q^4 e^{-M_Q^2 \tau} = \int_{(m_Q+m_u)^2}^{s_{\text{eff}}} e^{-s\tau} \rho_{\text{pert}}(s, \alpha, m_Q, \mu)\, ds + \Pi_{\text{power}}(\tau, m_Q, \mu) \equiv \Pi_{\text{dual}}(\tau, \mu, s_{\text{eff}})$$

In order the l.h.s. and the r.h.s. have the same τ-behavior

s_{eff} is a function of τ (and μ) : $s_{\text{eff}}(\tau, \mu)$

The "dual" mass: $\quad M_{\text{dual}}^2(\tau) = -\frac{d}{d\tau} \log \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)).$

If quark-hadron duality is implemented "perfectly", then M_{dual} should be equal to M_Q;
The deviation of M_{dual} from the actual meson mass M_Q measures the contamination of the dual
correlator by excited states. Better reproduction of $M_Q \rightarrow$ more accurate extraction of f_Q.

Taking into account τ-dependence of s_{eff}
improves the accuracy of the duality approximation.

Obviously, in order to predict f_Q, we need to fix s_{eff}. How to fix s_{eff}?

● *For a given trial function $s_{\text{eff}}(\tau)$ there exists a variational solution which minimizes the deviation of the dual mass from the actual meson mass in the τ-"window".*

# Our new algorithm for extracting ground – state parameters when M_Q is known

(i) Consider a set of Polynomial τ-dependent Ansaetze for s_{eff}: $s_{\text{eff}}^{(n)}(\tau) = \sum\limits_{j=0}^{n} s_j^{(n)}(\tau)^j.$

(ii) Minimize the squared difference between the "dual" mass M_{dual}^2 and the known value M_Q^2 in the τ-window. This gives us the parameters of the effective continuum threshold.

(iii) Making use of the obtained thresholds, calculate the decay constant.

(iv) Take the band of values provided by the results corresponding to <u>linear</u>, <u>quadratic</u>, and <u>cubic</u> effective thresholds as the characteristic of the intrinsic uncertainty of the extraction procedure.

Illustration: D-meson
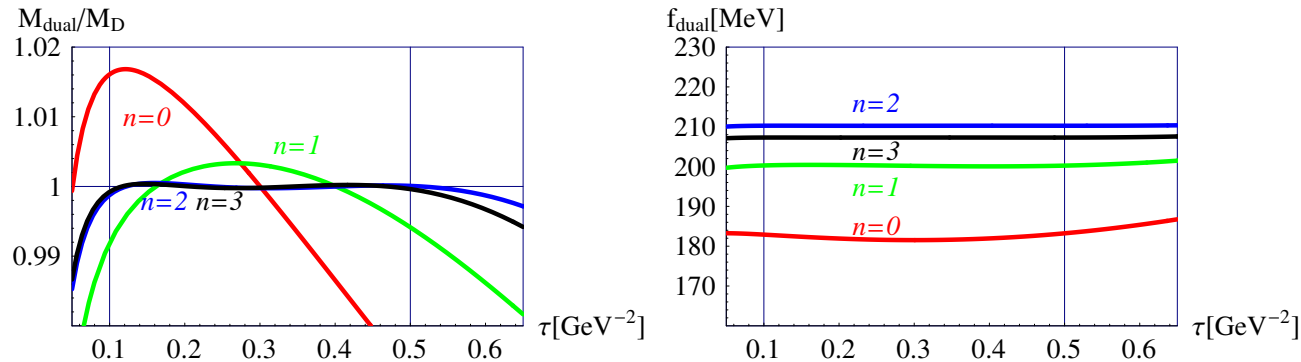
Extraction of f_D

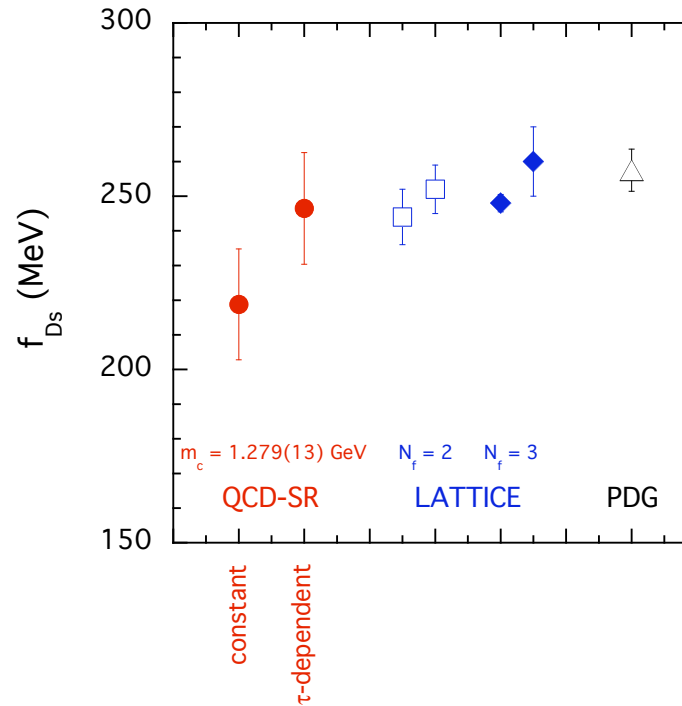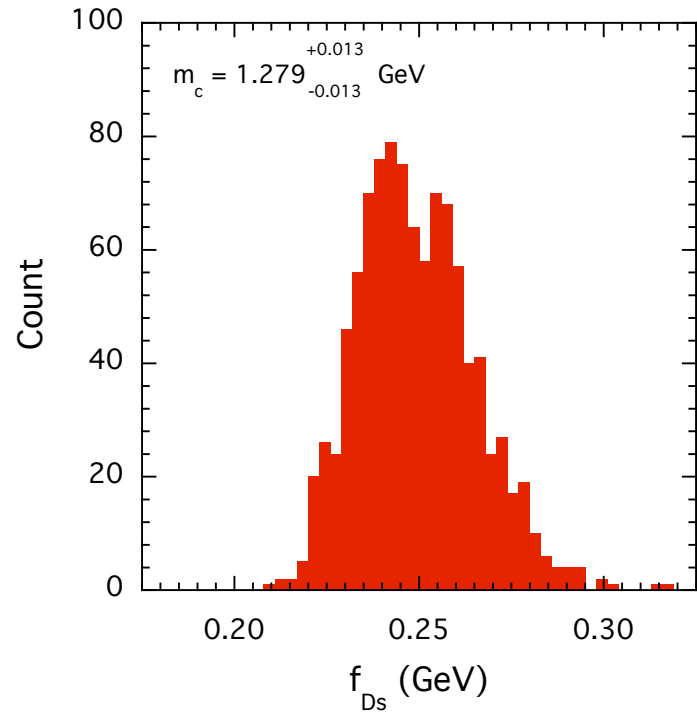$m_c(m_c) = 1.279 \pm 0.013\,\text{GeV}, \mu = 1 - 3\,\text{GeV.}$
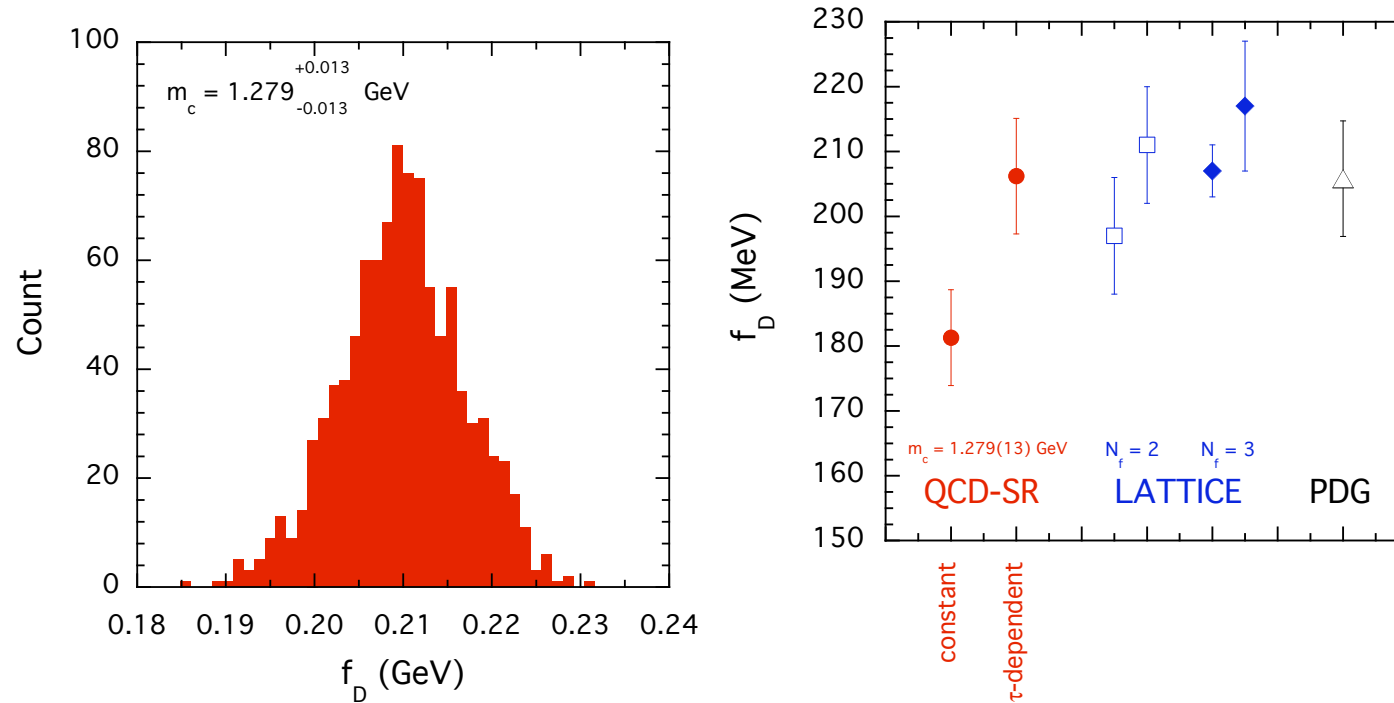
$$\mathbf{f_D = 206.2 \pm 7.3_{OPE} \pm 5.1_{syst}\ MeV}$$

`fD (const) = 181.3 ± 7.4_OPE MeV`

The effect of τ-dependent threshold is visible!

Extraction of f_{Ds}

$m_c(m_c) = 1.279 \pm 0.013\,\text{GeV}, \mu = 1 - 3\,\text{GeV}.$

$\mathbf{f_{D_s} = 246.5 \pm 15.7_{OPE} \pm 5_{syst}\ MeV}$

$\texttt{f}_{\texttt{Ds}} \texttt{(const)} = \texttt{218.8} \pm \mathbf{16.1}_{\texttt{OPE}}\ \texttt{MeV}$

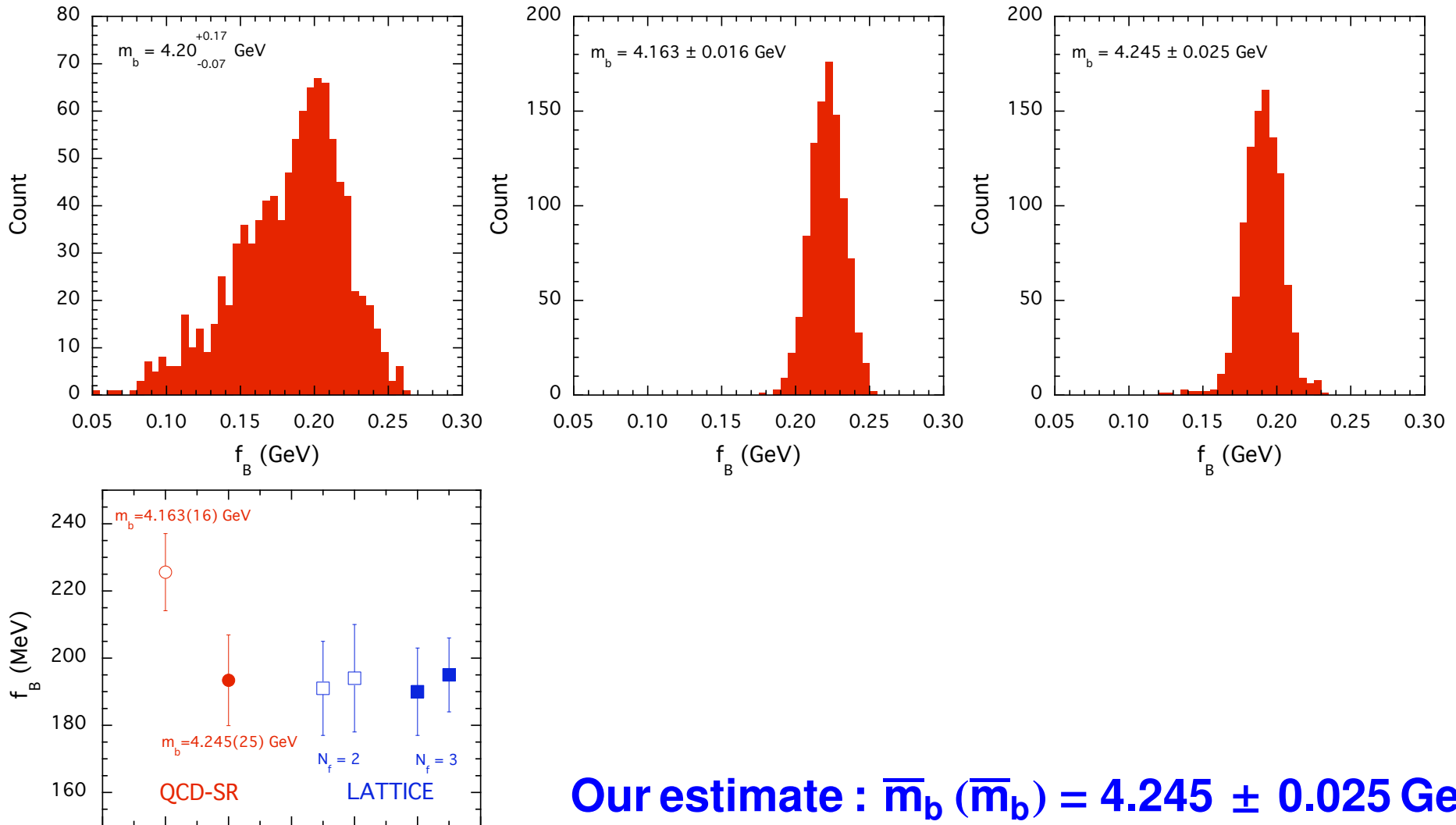## Extraction of f_B: a very strong sensitivity to $m_b(m_b)$
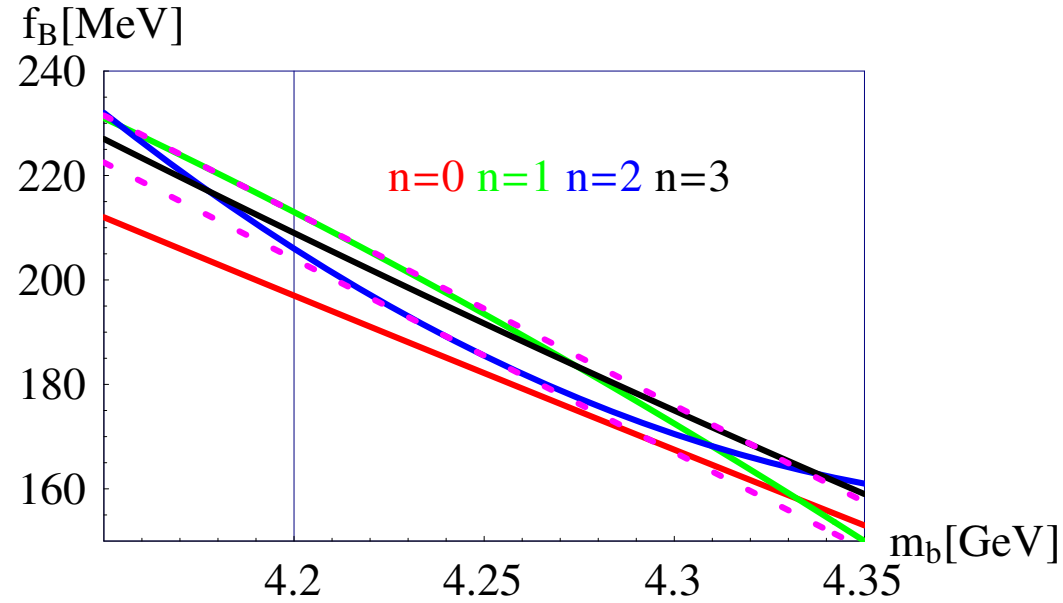
τ-dependent effective threshold:

$$f_B^{\text{dual}}(m_b, \langle \bar{q}q \rangle, \mu = m_b) = \left[206.5 \pm 4 - 37 \left(\frac{m_b - \textbf{4.245 GeV}}{\textbf{0.1 GeV}} \right) + 4 \left(\frac{\langle \bar{q}q \rangle^{1/3} - \textbf{0.267 GeV}}{\textbf{0.01 GeV}} \right) \right] MeV,$$

$\pm$ 10 MeV on $m_b \to \mp$ 37 MeV on f_B!

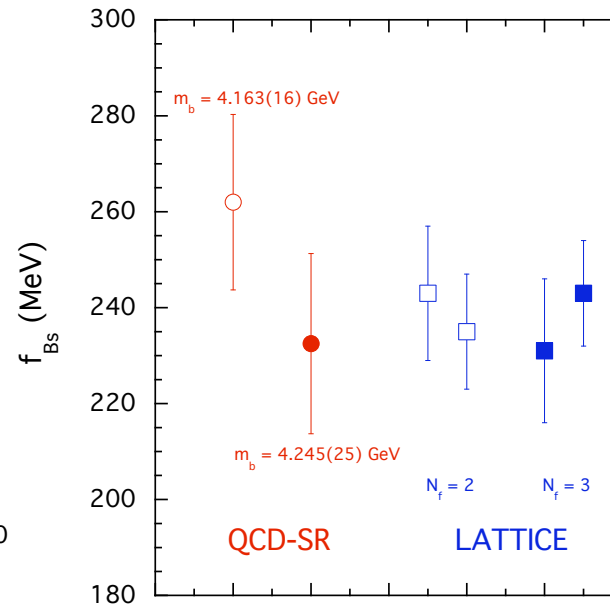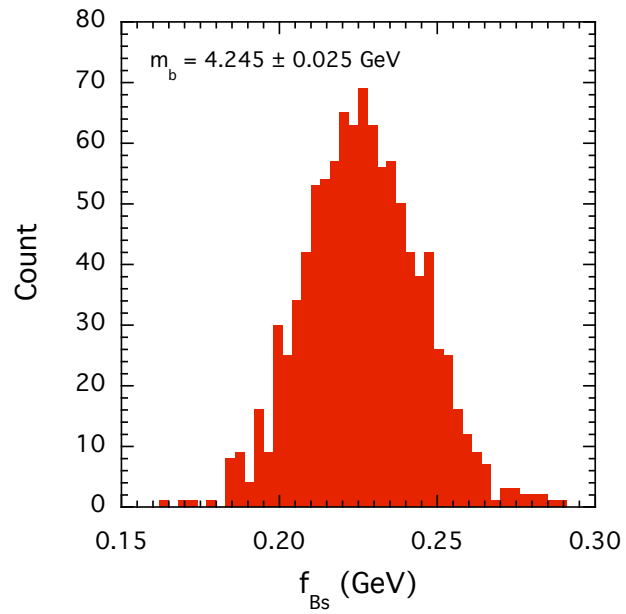**The prediction for f_B is not feasible without a very precise knowledge of m_b:**

Our estimate : $\overline{m}_b\,(\overline{m}_b) = 4.245 \pm 0.025$ GeV

$f_B = 193.4 \pm 12.3_{OPE} \pm 4.3_{syst}$ MeV

`fB (const) = 184 ± 13`$_{OPE}$ `MeV`

Extraction of f_{Bs}

$$f_{Bs} = 232.5 \pm 18.6_{OPE} \pm 2.4_{syst} \text{ MeV}$$

$$f_{Bs} \text{ (const)} = 218 \pm 18_{OPE} \text{ MeV}$$

Conclusions

The effective continuum threshold s_{eff} is an important ingredient of the method which determines to a large extent the numerical values of the extracted hadron parameters. Finding a criterion for fixing s_{eff} poses a problem in the method of sum rules.

• τ-dependence of s_{eff} emerges naturally when trying to make quark-hadron duality more accurate. For those cases where the ground-state mass M_Q is known, we proposed a new algorithm for fixing s_{eff}. We have tested that our algorithm leads to the extraction of more accurate values of bound-state parameters than the standard algorithms used in the context of sum rules before.

• τ-dependent s_{eff} is a useful concept as it allows one to probe realistic intrinsic uncertainties of the extracted parameters of the bound states.

• We obtained predictions for the decay constants of heavy mesons f_Q which along with the "statistical" errors related to the uncertainties in the QCD parameters, <u>for the first time</u> include realistic <u>"systematic" errors</u> related to the uncertainty of the extraction procedure of the method of QCD sum rules.

• Matching our SR estimate to the average of the recent lattice results for f_B allowed us to obtain a rather accurate estimate

$$\overline{m}_b(\overline{m}_b) = 4.245 \pm 0.025 \,\text{GeV}.$$

Interestingly, this range does not overlap with a very accurate range reported by Chetyrkin et al. Why?

Im $\Pi(s)$

f_B

theoretical
physical

s_{eff}

s

m_b^2

M_B^2

$s_{cont} = (M_{B^*} + m_\pi)^2$