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Two-component liquid model of quark-gluon plasma

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The model. A reminder

In hydrodynamic approximation:

$$
\begin{aligned}
j^\mu &= n u^\mu + f^2 \partial^\mu \phi \\
T^{\mu\nu} &= (\epsilon + P) u^\mu u^\nu + P \eta^{\mu\nu} + f^2 \partial^\mu \phi \partial^\nu \phi \\
u^\mu \partial_\mu \phi &= \mu
\end{aligned} \tag{1}
$$

u^μ - 4 velocity of an element of the liquid,
representing normal component
ϕ - scalar field, representing superfluid component

Two independent notions,

$$
\rho_{tot} = \rho_n + \rho_s \tag{2}
$$

Outline

1. Why liquid at all?
2. Why two components?
3. Where is the scalar filed in QCD?
4. Crucial checks?

A. Equation of state

$$\epsilon(T) = \left[\epsilon(T)_{ideal\ gas}\right](1 - \delta) \quad (3)$$
$$\delta \approx 0.1 - 0.15$$

That is, close to the ideal gas

Known since long (from the lattice)

Knowledge resulted in "Lost Decades"(E. Shuryak)

Viscosity η

Viscosity is similar to friction

$$F_x = \eta \cdot (Area) \cdot \frac{\partial v_x}{\partial y}$$

Enters also energy-momentum tensor $T^{\mu\nu}$

Fits to the RHIC data

$$\left(\frac{\eta}{s}\right)_{plasma} = (0.03 - 0.3) \approx \frac{1}{4\pi}$$

Closest to the ideal liquid among all known substances

For the ideal gas, to the contrary,

$$\left(\frac{\eta}{s}\right)_{ideal\ gas} \longrightarrow \infty$$

Low η/s is confirmed on the lattice

Quantum effects

From kinetics:

$$\eta \sim \epsilon \cdot \tau_{free}$$

For the entropy:

$$s \sim n \cdot k_B$$

From the uncertainty relation

$$\frac{\epsilon}{n}\tau_{free} \geq \hbar$$

Uncertainty relation in hidrodynamic (Kovtun, Son, Starinetz, (2005))

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Violated "experimentally'?

Two components ?

Is it possible to reconcile EoS as for gas and viscosity as for liquid?
What is special about the viscosity?
Let there be two components. Intuitively

$$\frac{1}{\eta_{tot}} = \frac{c_1}{\eta_1} + \frac{c_2}{\eta_2} \qquad (*)$$

where $c_{1,2}$are weight factors

$$c_1 + c_2 = 1$$

Status of (*):
– true in case of solutions,
– true in superfluidity
(for capillary motion)

Two components

Plasma $=$ gas $+$ superfluid
explains readily:

1. $c_1 > c_2$ - Equation of State close to gas
2. $\eta_2 \approx 0$ responsible for low viscosity
3. $\eta_2 \approx 0$ explains large quantum effects
4. note that even in the limit $T \to \infty$ superfluidity could survive:

$$\lim_{T \to \infty} c_2 \to \frac{1}{(\ln T)^3}$$

Conclusions to part I

1. Quantum relativistic liquid seems to have been discovered
2. Two-component model allows to at least memorize the basic properties of the quark-gluon plasma

In no way does this imply that we do have two components.
Turn to dynamics

Constraints on dynamics

Crucial point: existence of effective scalar fields.
One needs:

1. a complex field,
$$\phi^* \neq \phi$$
2. field condensed,
$$< \phi > \neq 0$$
3. 3d поле (not 4d)
$$< \phi(\mathbf{r}) > \neq 0 \, , \; \partial_t \phi = \mu$$
4. $< \phi > \neq 0$ should not violate known quantum numbers

Constraints look too strong: a no-go theorem?

Thermal scalar

Deconfinement phase transition in strings language

It is known since long that if $T \to T_c$ from below the partition function reduces to that of a single scalar

$$m_\beta^2 \;=\; \frac{(\beta-\beta_H)\beta_H}{(2\pi^2)(\alpha')^2}$$

where $\beta_H = 1/T_c$.

$$F \;=\; -\beta \ln Z \;=\; -\beta \int_0^\infty \frac{dL}{L} \frac{\exp(-m_\beta^2 L l_s)}{(L l_s)^{d/2}}$$

Comments on the thermal scalar

1. we have a scalar field in the polymer representation
2. at $\beta = \beta_H$ the mass becomes tachyonic
3. It is an effective field since the scale is fixed as l_s, while for an elementry scalar $l_s \to a$, where a is the lattice spacing

The thermal scalar vs dynamical constraints

1. a complex field, $\phi \neq \phi'$, since the sum is over closed paths
2. the U(1) quantum number is of topological nature: wrapping in opposite directions around the compact time coordinate
3. the field is 3d field
4. Thus all the constraints are satisfied, below T_c

What happens at $T > T_c$?

If phase transition is of second order (might be relevant to SU(2)), there exists a kind of analytic continuation to the tachyonic region, $T > T_c$.
There exists then a condensate, there is light degree of freedom but only close to $T = T_c$. At higher temperatures everything is massive. Not good for light scalar

For larger N_c the phase transition is of first order. Looks even worse for a light scalar
Turn to holography + lattices to clarify what happens at $T > T_c$

Conclusions to part II

1. Example of thermal scalar demonstrates that stringy approach does bring in scalar fields.
2. Surprisingly, with properties close to what is needed. The issue of quantum numbers is resolved
3. However, this is true below T_c.
   Above T_c no sign for massless scalars

Extra dimensions

In infrared, QCD probably is to be completed as string theory in extra dimensions.

Geometry below T_c:

1. spatial 3d
2. Time direction compact, $1/T$
3. z-direction, loosely, $z \sim 1/Q$, with a horizon, $z < z_h$
4. One more compact direction chirality related: instantons wrap around it

Phase transition as change of dimensions

New result from holography: deconfinement is

a $4d \quad \to 3d$ transition

for non-perturbative physics, where

non-perturbative $\equiv$ low-dimensional defects

the 3d picture found at $T < T_c$ gets frozen at $T = T_c$

Change of geometry at $T = T_c$

Phase transition in dual language is
Hawking-Page transition:

Below T_c: radius of $\theta =$ direction $R_\tau(z) = const$
radius of chirality direction $R_\theta(z_H) = 0$
and wrapping around θ-direction does not cost classical action (instantons)

Above T_c: radius of θ direction $R_\theta(z) = const$
radius of τ direction $R_\tau(z_H) = 0$
and wrapping around time does not cost classical action

Confirmation/discovery on the lattice

Implications of the change of geometry:

If below T_c there is a percolating surface in 4d
Then at $T > T_c$ the surface looks in the time direction:

$$(2d \text{ in } 4d) \rightarrow (1d \text{ in } 3d) + (\text{static in time})$$

Furthermore

Percolating 1d defect in 3d ≡ condensed 3d scalar field

Static correlator of momentum densities

consider

$$G^{i,k}(\mathbf{q}) = \int d^4x \exp(i\mathbf{q}\cdot\mathbf{x})\langle T^{i0}(x), T^{j0}(0)\rangle$$

in geberal

$$G^{i,k}(\mathbf{q}) = G^T(\mathbf{q}^2)\left(\delta^{ij} - \frac{\mathbf{q}_i\mathbf{q}_j}{\mathbf{q}^2}\right) + G^L(\mathbf{q}^2)\frac{\mathbf{q}_i\mathbf{q}_j}{\mathbf{q}^2}$$

Low-energy theorems:

$$G^L(0) = -(sT + \mu\rho_{tot}) \qquad G^T(0) = (-sT + \mu\rho_n)$$

or

$$\lim_{\mathbf{q}\to 0} G^{ik}(\mathbf{q}) = \frac{\mathbf{q}_i\mathbf{q}_j}{\mathbf{q}^2}\rho_s\mu$$

Superfluidity as non-analyticity at $\mathbf{q} \to 0$.
Well-suited for the lattice (zero Matsubara frequency)

From confinement to superfluidity?

1. Confinement as evidence of dual formulations: strings correctly describe the lattice defects
2. At $T = T_c$ 4d surfaces, or vortices become condensed 3d fields might provide superfluidity
3. Emerging two-component model does well phenomenologically
4. Weak point: there might be a few scalar fields and their interaction might eliminate superfluidity
5. Superfluid component as a source of non-analyticity at $\mathbf{q} \to 0$